Research Article

Impulsive State Feedback Control of the Rhizosphere Microbial Degradation in the Wetland Plant

Zhong Zhao, Liuyong Pang, Zhanping Zhao, and Chengguang Luo

Department of Mathematics, Huanghuai University, Zhumadian, Henan 463000, China

Correspondence should be addressed to Zhong Zhao; zhaozhong8899@163.com

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The rhizosphere microbe plays an important role in removing the pollutant generated from industrial and agricultural production. To investigate the dynamics of the microbial degradation, a nonlinear mathematical model of the rhizosphere microbial degradation is proposed based on impulsive state feedback control. The sufficient conditions for existence of the positive order-1 or order-2 periodic solution are obtained by using the geometrical theory of the semicontinuous dynamical system. We show the impulsive control system tends to an order-1 periodic solution or order-2 periodic solution if the control measures are achieved during the process of the microbial degradation. Furthermore, mathematical results are justified by some numerical simulations.

1. Introduction

The constructed wetland is regarded as the most efficient and cost-effective system that involves microbes consuming pollutant generated from industrial and agricultural production. However, degradation ability of the microbe in the constructed wetland is affected by many factors such as temperature, pH, dissolved oxygen, and substrate concentration [1]. Studies have shown that excessive pollutant strongly affects the dynamic behavior of microbe, which can lead to the incidence of the sustained oscillation and decrease degradation ability of the microbe under certain conditions [2]. For the purpose of improving degradation ability of the microbe and decreasing the inhibition effect of the substrate, it is necessary to control the pollutant concentration lower than a certain level.

The removal of the sewage from a water body by the biodegradation has been investigated by many researchers [3–15]. In [14], pollutant removal process of a constructed wetland is simulated by the following model:

$$\begin{aligned}\frac{dT}{dt} &= f(S)T - \sigma T, \\ \frac{dW}{dt} &= Q - (\gamma + \alpha)W,\end{aligned}$$

$$\frac{dS}{dt} = \gamma W - \frac{f(S)T}{Y} - \phi S, \quad (1)$$

where $f(S)$ denotes the mangrove growth rate which is chosen to follow Monod kinetics ($f(S) = \beta S/(k + S)$, $\beta > 0$, $k > 0$). $T(t)$ is the mangrove biomass concentration, $W(t)$ denotes the total nitrogen concentration in wastewater, and $S(t)$ shows the total nitrogen concentration in soil solution. The meaning of other parameters is also given in [14]. Scott et al. [15] propose a mathematical model for dispersal of bacterial inoculants colonizing the water rhizosphere to describe bacterial growth and movement in the rhizosphere.

Recently, there are many studies on the geometric theory of semicontinuous dynamical system which are applied into the chemostat model [16–20]. Considering the product inhibition during the microorganism culture, Guo and Chen [16] develop a mathematical model concerning a chemostat with impulsive state feedback control to investigate the periodicity of bioprocess and an order-1 periodic solution is obtained by the existence criteria of periodic solution. The mathematical model of a continuous bioprocess with impulsive state feedback control is formulated [18] and the analysis of the bioprocess stability is presented. As far as I know, little information is available concerning the dynamical research on the microbial degradation of the rhizosphere.

The main purpose of this paper is to construct a mathematical model of the microbial degradation with impulsive state feedback control and understand the dynamics of the microbial degradation in the constructed wetland.

The paper is organized as follows: a mathematical model of the microbial degradation with impulsive state feedback control is proposed in Section 2. In Section 3, the qualitative analysis of system without impulsive control is given. Furthermore, the existence of order-1 and order-2 periodic solutions is investigated in Section 4. Finally, we give some numerical simulations and a brief discussion.

2. Model Description and Preliminaries

In the constructed wetland, it is showed that the rhizosphere microbe makes an important contribution to the degradation of pollutant [1, 21]. Based on references [1, 16–21], we give some assumptions to investigate the dynamics of the microbial degradation of the rhizosphere.

- (a) Suppose the pollutant and rhizosphere microbe are uniformly distributed inside the rhizosphere. $S(t)$ denotes the pollutant concentration discharged from the household and industrial sources. $x(t)$ is the concentration of the rhizosphere microbe.
- (b) It is assumed the pollutant is discharged into the plant rhizosphere from outside with a constant Q .
- (c) α is the rate of decrease due to biochemical and other factors, which is proportional to the pollutant concentration $S(t)$.
- (d) The growth of the rhizosphere microbe is assumed to follow the Monod equation involving the pollutant concentration $S(t)$ as well as the microbial concentration $x(t)$ (i.e., $\mu Sx/\delta(K+S)$), where μ is the maximum specific growth rate and the constant δ is yield term and K is a half-saturation constant.
- (e) The mortality of the rhizosphere microbe is denoted as D . Considering the nutrient recycling, we suppose the fraction of the death microbe is converted into the substrate and r ($0 < r \leq 1$) is the fraction of the conversion.
- (f) Let h show a predetermined threshold, which is a minimum allowed value harmless to human health. When pollutant concentration is higher than the critical threshold h (which can be measured in advance), control measures are taken to reduce the pollutant concentration below the critical threshold; that is, we will inoculate the microbe into the plant rhizosphere so as to improve microbial ability of degradation and τ denotes the amount of releasing the microbe into the plant rhizosphere. At the same time, we will control the emissions of pollutant. E ($0 < E < 1$) is the effect of controlling the pollutant sources.

According to the above assumption, we formulate the following model:

$$\begin{aligned} \frac{dS}{dt} &= Q - \alpha S - \frac{\mu Sx}{\delta(K+S)} + rDx, \\ \frac{dx}{dt} &= \frac{\mu Sx}{(K+S)} - Dx, \\ S &< h, \\ \Delta S &= -ES, \\ \Delta x &= \tau, \\ S &= h, \end{aligned} \quad (2)$$

where $\Delta S = S(t^+) - s(t)$ and $\Delta x = x(t^+) - x(t)$.

Next, we will discuss the existence of periodic solution of (2) by using the geometry theory of semicontinuous dynamic system. For convenience, we give some definitions and lemmas.

Definition 1 (see [22]). Let M denote the impulsive set and N be the phase set. Suppose $g : N \rightarrow N$ is a mapping. For any point $P \in N$, there exists $t_1 > 0$ such that $F(P) = f(P, t_1) = P_1 \in M$, $P_1^+ = \varphi(P_1) \in N$, and then $g(P) = l(P_1^+) - l(P)$ is called the successor function of point P and the point P_1^+ is called the successor point of P .

To investigate the dynamics of system (2), we construct the following Poincaré map. Choose two sections P_0 and P_1 as the Poincaré sections, where $P_0 = \{(S, x) \mid S = (1 - E)h, x \geq \tau\}$ and $P_1 = \{(S, x) \mid S = h, x \geq 0\}$. Assume the point $B_k(h, x_k)$ is on the Poincaré section P_1 , then $B_k^+((1 - E)h, x_{k+1})$ lies on the section P_0 due to the impulsive effect, and the trajectory starting from the initial value B_k^+ intersects the section P_1 at the point $B_{k+1} = (h, x_{k+1})$, where x_{k+1} is determined by x_k and the parameter τ . Therefore, we get the Poincaré map as follows:

$$x_{k+1} = F(\tau, x_k). \quad (3)$$

3. Qualitative Analysis for System (2)

Before discussing the periodic solution of system (2), we should consider the qualitative property of (2) without the impulsive effect.

Lemma 2. Suppose $\omega(t) = (S(t), x(t))$ is a solution of (2) subject to $\omega(0^+) \geq 0$, then $\omega(t) \geq 0$ for all $t \geq 0$, and furthermore $\omega(t) > 0$, $t \geq 0$ if $\omega(0^+) > 0$.

Theorem 3. The system (2) is ultimately bounded.

Proof. Define a function $V(t) = S(t) + x(t)/\delta$. We calculate the upper right derivative of $V(t)$ along a solution of the first and second equations of system (2) and get the following differential equation:

$$\frac{dV}{dt} = Q - \alpha S + \left(r - \frac{1}{\delta}\right) Dx \leq Q - \beta V(t), \quad (4)$$

where $\beta = \min\{\alpha, (1/\delta - r)D\delta\}$. We have

$$S(t) + \frac{x(t)}{\delta} \leq \lim_{t \rightarrow \infty} V(t) = \frac{Q}{\beta}. \quad (5)$$

According to the positivity of $S(t)$ and $x(t)$, we obtain $S(t) \leq Q/\beta$ and $x(t) \leq Q/\beta$. The proof is completed. $\square$

Next, we explore the asymptotical behavior of the system (2) without impulsive effect.

$$\begin{aligned} \frac{dS}{dt} &= Q - \alpha S - \frac{\mu S x}{\delta_1 (K + S)} + r D x, \\ \frac{dx}{dt} &= \frac{\mu S x}{(K + S)} - D x. \end{aligned} \quad (6)$$

An equilibrium point of system (6) satisfies the system

$$\begin{aligned} Q - \alpha S - \frac{\mu S x}{\delta_1 (K + S)} + r D x &= 0, \\ \frac{\mu S x}{(K + S)} - D x &= 0. \end{aligned} \quad (7)$$

It can be seen that system (7) has a microbe-free equilibrium of the form $E_0 = (Q/\alpha, 0)$. We start by analyzing the behavior of the system (7) near E_0 . The characteristic equation of the linearization of (6) near E_0 is

$$\det \begin{pmatrix} -\alpha - \lambda & rD \\ 0 & \frac{\mu Q}{k\alpha + Q} - D - \lambda \end{pmatrix} = 0. \quad (8)$$

Two eigenvalues are $\lambda_1 = -\alpha$ and $\lambda_2 = \mu Q/(k\alpha + Q) - D$, respectively. We obtain the microbe-free equilibrium of system (6) is unique and locally asymptotically stable if the condition $\mu Q/(k\alpha + Q) < D$ holds.

Define $R_1 = \mu Q/D(k\alpha + Q)$.

Theorem 4. *The microbe-free equilibrium E_0 is asymptotically stable if $R_1 < 1$. E_0 is unstable if $R_1 > 1$.*

From (7), it follows that when the trivial equilibrium E_0 of system (6) is asymptotically stable then positive equilibrium does not exist. When $R_1 > 1$, system (6) has a unique positive equilibrium $E_1(S^*, x^*)$, where $S^* = Dk/(\mu - D)$, $x^* = \delta(Q - \alpha S^*)(k + S^*)/((\mu - rD\delta)S^* - rD\delta k) = \delta(Q\mu - QD - \alpha Dk)/\mu Dk(\mu - D)(1 - r\delta)$, $\mu > D$. We can easily obtain $S^* > 0$ and $x^* > 0$ for $R_1 > 1$. Next, we analyze the stability of the positive equilibrium $E^*(S^*, x^*)$. The characteristic equation at $E^*(S^*, P^*)$ is

$$\det \begin{pmatrix} \lambda + \alpha + \frac{\mu k x^*}{\delta (k + S^*)^2} & -rD + \frac{\mu S^*}{\delta (k + S^*)} \\ -\frac{\mu k x^*}{(k + S^*)^2} & \lambda + D - \frac{\mu S^*}{k + S^*} \end{pmatrix} = 0. \quad (9)$$

We know $\lambda^2 + (\alpha + \mu k x^*/\delta(k + S^*)^2)\lambda + (\mu k x^*/(k + S^*)^2)(\mu S^*/\delta(k + S^*) - rD) = 0$. After a few computations, we have $\lambda_1 + \lambda_2 < 0$, $\lambda_1 \lambda_2 > 0$ for $R_1 > 1$. Therefore, we have.

Theorem 5. *If $R_1 > 1$ holds, the positive equilibrium of system (6) is locally asymptotically stable.*

Now, let us discuss the global stability of system (2). Firstly, we will give the following lemma.

Lemma 6. *Suppose $\Gamma(t) = (S(t), x(t))$ is a periodic orbit of the system (2) with the period T . The bounded region R consists of all the points in phase plane Γ . Denote*

$$\Delta = \int_0^T \left[\frac{\partial f_1}{\partial S}(S(t), x(t)) + \frac{\partial f_2}{\partial x}(S(t), x(t)) \right] dt, \quad (10)$$

where $dS/dt \triangleq f_1(S, x)$ and $dx/dt \triangleq f_2(S, x)$. Then we obtain

$$\begin{aligned} \Delta &= - \int_0^T \left[\alpha + \frac{\mu k x(t)}{\delta_1 (k + S(t))^2} \right] dt \\ &\quad + \int_0^T \left[\frac{\mu S(t)}{k + S(t)} - D \right] dt \leq 0. \end{aligned} \quad (11)$$

Proof. From system (2), we obtain $\int_0^T [(\mu S(t)/(k + S(t)) - D)dt = \int_0^T d(\ln x(t))$. Obviously, we have $\int_0^T (1/x)(dx/dt) = 0$ because of the periodicity of $x(t)$. Therefore, we obtain $\Delta = - \int_0^T [\alpha + \mu k x(t)/\delta_1 (k + S(t))^2] dt \leq 0$. $\square$

Theorem 7. *The positive equilibrium $E^*(S^*, x^*)$ is globally asymptotically stable if $R_1 > 1$, where R_1 is defined above.*

Proof. From Theorem 5, we know that $E^*(S^*, x^*)$ is locally asymptotically stable. According to Lemma 6, we obtain the periodic solution is stable if it exists around $E^*(S^*, x^*)$. However, it is impossible according to Poincaré-Bendixson Theorem, which implies that $E^*(S^*, x^*)$ is globally asymptotically stable. $\square$

4. Existence of the Periodic Solution

When $h < kD/(\mu - D)$, the trajectory from the initial value $(S(0), x(0))$ with $S(0) < h$ will intersect the Poincaré section P_1 infinite times; while $h \geq kD/(\mu - D)$, the trajectory from the initial value $(S(0), x(0))$ with $S(0) < h$ will tend to the equilibrium (S^*, x^*) after the impulsive effect of the finite times. In order to investigate the impact of the impulsive effect on the system (2), we have two cases: (a) $h < kD/(\mu - D)$ and (b) $h \geq kD/(\mu - D)$.

4.1. Existence of the Order-1 Periodic Solution for $h < kD/(\mu - D)$. The impulsive set $S = h$ intersects the isoclinical line $dS/dt = 0$ at the point $D(h, x_D)$, where x_D is satisfied $x_D = \delta(Q - \alpha h)(k + h)/((\mu - rD\delta)h - rD\delta k)$. The phase set $S = (1 - E)h$ intersects the isoclinical line $dS/dt = 0$ at the point $H((1 - E)h, x_H)$, where x_H is satisfied $x_H = \delta(Q - \alpha(1 - E)h)(k + (1 - E)h)/((\mu - rD\delta)(1 - E)h - rD\delta k)$.

Choose $A_1((1 - E)h, \varepsilon)$, where ε is small enough and $\varepsilon < \tau$. According to the qualitative analysis, the trajectory from the point A_1 reaches the point $B_1(h, \varepsilon_1)$ on the Poincaré section P_1 , next jumps onto the point $A_2((1 - E)h, \varepsilon_1 + \tau)$, and then

FIGURE 1: (a) and (b) The existence of order-1 periodic solution for $h < kD/(\mu - D)$. (c) The existence of order-1 or order-2 periodic solution for $h \geq kD/(\mu - D)$.

reaches the point $B_2(h, \varepsilon_2)$ again. Obviously, the point A_2 is above the point A_1 due to impulsive effect $\Delta x = \tau$ and $\tau + \varepsilon_1 > \varepsilon$. Therefore, The point A_2 is the successor point of the point A_1 , and in view of the property of the successor function, we have

$$f(x_{A_2}) - f(x_{A_1}) > 0. \quad (12)$$

Suppose the trajectory from the point $H((1-E)h, x_H)$ interacts the Poincaré section P_1 at the point $G(h, l)$ and next jumps onto the Poincaré section P_0 at the point $H_1((1-E)h, \gamma_1)$. There are two possible cases between the point H and H_1 . One is that the point H_1 is below the point H , and the other case is that point H_1 is above the point H .

For the point H_1 being below the point H (see Figure 1(a)), the point H_1 is the successor point of the point H , we have

$$f(x_{H_1}) - f(x_H) < 0. \quad (13)$$

Hence, there must be a point $F((1-E)h, x_F)$ such that $f(x_F) = 0$; that is, system (2) has an order-1 periodic solution.

When the point H_1 is above the point H , the trajectory from the point H_1 interacts the Poincaré section at the point $G_1(h, l_1)$ and next jumps onto the point $H_2((1-E)h, \gamma_2)$. Obviously, the point H_2 is below the point H_1 according to the impulsive effect and qualitative property of system (2) (see Figure 1(b)). The point H_2 is the successor point of the point H_1 ; we obtain

$$f(x_{H_2}) - f(x_{H_1}) < 0. \quad (14)$$

From (12) and (14), we obtain system (2) exists an order-1 periodic solution.

4.2. Existence of the Periodic Solution for $h \geq kD/(\mu - D)$. Suppose the Poincaré section $S = h$ intersects the isocline line $dS/dt = 0$ at the point $D(h, x_D)$ and the Poincaré section $S = (1-E)h$ intersects the isocline line $dS/dt = 0$ at the point $H((1-E)h, x_H)$. There must exist a trajectory Γ passing through the point D and tangent to the Poincaré section $S = h$ at the point $D(h, x_D)$ with $x_D = \delta(Q - \alpha h)(k + h)/((\mu - rD\delta)h - rD\delta k)$ (see Figure 1(c)). Furthermore, the

trajectory Γ crosses the Poincaré section $S = (1-E)h$ at the point $E((1-E)h, \bar{x}_1)$ and the point $F((1-E)h, \bar{x}_2)$ with $\bar{x}_1 < \delta(Q - \alpha h)(k + h)/((\mu - rD\delta)h - rD\delta k) < \bar{x}_2$. For $x \in (\bar{x}_1, \bar{x}_2)$, the trajectory passing through the point $((1-E)h, x)$ will not interact with the Poincaré section $S = h$ and will ultimately tend to the positive equilibrium $E^*(S^*, x^*)$. For any trajectory of (2) will pass through the point $((1-E)h, x)$, $x \in (\bar{x}_1, \bar{x}_2)$ after some impulsive effects on the Poincaré section P_1 , they ultimately tend to the positive equilibrium $E^*(S^*, x^*)$ and there is not any periodic solution. For $\tau > \bar{x}_2$, a trajectory of system (2) will intersect the Poincaré section $S = (1-E)h$ infinite times due to the impulsive effect $\Delta S = -ES$, $\Delta x = \tau$. When the condition $\tau > \bar{x}_2$ is satisfied, we have $0 < B_i(h, x_i) < B_j(h, x_j) < \delta(Q - \alpha h)(k + h)/((\mu - rD\delta)h - rD\delta k)$ for any two points $B_i(h, x_i)$ and $B_j(h, x_j)$. The corresponding impulsive points $B_i^+(h, x_i + \tau)$ and $B_j^+(h, x_j + \tau)$ are above the point F . According to the property of the vector field, we obtain that $0 < x_{j+1} < x_{i+1} < \delta(Q - \alpha h)(k + h)/((\mu - rD\delta)h - rD\delta k)$; that is,

$$0 < x_{j+1} < x_{i+1} < \frac{\delta(Q - \alpha h)(k + h)}{(\mu - rD\delta)h - rD\delta k}, \quad (15)$$

$$\text{for } 0 < x_i < x_j < \frac{\delta(Q - \alpha h)(k + h)}{(\mu - rD\delta)h - rD\delta k}, \quad \tau > \bar{x}_2.$$

For $x_0 \in (0, \delta(Q - \alpha h)(k + h)/((\mu - rD\delta)h - rD\delta k))$ and $\tau > \bar{x}_2$, we consider the Poincaré map $x_i = F(\tau, x_{i-1})$, $i = 1, 2, \dots, n$. If $x_1 = x_0$, then system (2) has an order-1 periodic solution. If $x_0 \neq x_1$, $x_0 = x_2$, we obtain that system (2) has an order-2 periodic solution.

For convenience, we investigate the more general thing in the following, that is, $x_0 \neq x_1 \neq x_2 \neq \dots \neq x_n$ ($n \geq 3$).

- (a) For $x_0 < x_1$, it follows from $x_2 < x_1$, which leads to the relation of x_0, x_1, x_2 to be one of the following cases according to the qualitative property of system (2).

FIGURE 2: For $h < kD/(\mu - D)$, the existence of order-1 periodic solution with the parameters $\mu = 0.4$, $Q = 6$, $k = 4$, $r = 0.01$, $D = 0.03$, $E = 0.1$, $\tau = 4$, $h = 0.2$, $\alpha = 0.2$, and $\delta = 0.01$.

(1_a) $x_2 < x_0 < x_1$, in the case, it follows from (15) that $x_2 < x_1 < x_3$. Repeating the above process, the following circumstances are obtained:

$$0 < \dots < x_{2n} < \dots < x_2 < x_0 < Z_1 < \dots < x_{2n+1} < \dots < \frac{\delta(Q - \alpha h)(k + h)}{(mu - rD\delta)h - rD\delta k}. \quad (16)$$

(2_a) When $x_0 < x_2 < x_1$, similarly, we have

$$x_0 < x_2 < \dots < x_{2n} < \dots < x_{2n+1} < \dots < x_3 < x_1 < \frac{\delta(Q - \alpha h)(k + h)}{(mu - rD\delta)h - rD\delta k}. \quad (17)$$

(b) When $x_0 > x_1$, again from (15), we have $x_1 < x_2$. The result leads to the relation of x_0, x_1 , and x_2 to be one of the following cases:

(1_b)

$$0 < \dots < x_{2n+1} < \dots < x_1 < x_0 < x_2 < \dots < x_{2n} < \dots < \frac{\delta(Q - \alpha h)(k + h)}{(mu - rD\delta)h - rD\delta k}. \quad (18)$$

(2_b) If $x_1 < x_2 < x_0$, similarly, we also have

$$0 < x_1 < \dots < x_{2n+1} < \dots < x_{2n} < \dots < x_2 < x_0 < \frac{\delta(Q - \alpha h)(k + h)}{(mu - rD\delta)h - rD\delta k}. \quad (19)$$

Furthermore, in case (1_a) of (a), we obtain $\lim_{n \rightarrow \infty} x_{2n} = \xi_1$ and $\lim_{n \rightarrow \infty} x_{2n+1} = \xi_2$, where $0 < \xi_1 < \xi_2 < \delta(Q - \alpha h)(k + h)/((mu - rD\delta)h - rD\delta k)$. Hence we obtain $\xi_1 = F(\tau, \xi_2)$ and

FIGURE 3: For $h \geq kD/(\mu - D)$, the existence of order-1 periodic solution with the parameters $\mu = 0.4$, $Q = 5$, $r = 0.02$, $D = 0.03$, $E = 0.071$, $\tau = 7.6$, $\alpha = 0.2$, $\delta = 0.02$, and $h = 0.6233$.

$\xi_2 = F(\tau, \xi_1)$; therefore, system (2) has an order-2 periodic solution, which is orbitally asymptotically stable. Similarly, for case (2_a) of (a) and (2_b) of (b), system (2) has an orbitally asymptotically stable order-2 periodic solution. In case (1_b) of (b), system (2) also has an order-2 periodic solution, which is orbitally asymptotically stable.

5. Numerical Simulations and Discussion

In this paper, we have formulated the mathematical model of microbial degradation with the impulsive state feedback control and obtained the existence of order-1 periodic solution and order-2 periodic solution by using the geometry theory of semicontinuous dynamic system. System (2) without the impulsive effect has two equilibria, $E_0(Q/\alpha, 0)$ and $E^*(S^*, x^*)$; we show the microbe-free equilibrium $E_0(Q/\alpha, 0)$ is asymptotically stable if $R_1 < 1$. The positive equilibrium $E^*(S^*, x^*)$ is globally asymptotically stable if $R_1 > 1$, which

means that pollutant concentration will reach unacceptable level if no measurement is adopted. In order to reduce the pollutant concentration to the predetermined threshold, which is less harmful to human health, the model of the microbial degradation is proposed. By using the geometric theory of semicontinuous dynamical system, we obtain system (2) exists an order-1 periodic solution for $h < kD/(\mu - D)$, which implies the control measurement is effective even if the initial concentration of pollutant is very high and the example is given as follows:

$$\frac{dS}{dt} = 6 - \alpha S - \frac{4Sx}{0.01(4 + S)} + 0.0003x,$$

$$\frac{dx}{dt} = \frac{4Sx}{(4 + S)} - 0.03x,$$

$$S < 0.2,$$

$$\begin{aligned}\Delta S &= -0.1S, \\ \Delta x &= 4, \\ S &= 0.2.\end{aligned}\tag{20}$$

System (20) can be simulated in Figure 2. We see the trajectory starting through the initial value (0.6, 0.3) is attracted by the order-1 periodic solution (see Figure 2). When $h \geq kD/(\mu - D)$, system (2) exists an order-2 periodic solution starting from the initial value (0.13, 0.36) with the parameters $\mu = 0.4$, $Q = 5$, $r = 0.02$, $D = 0.03$, $E = 0.071$, $\tau = 7.6$, $\alpha = 0.2$, $\delta = 0.02$, and $h = 0.6233$ (see Figure 3).

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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