Cooperative Passenger Inflow Control in Urban Mass Transit Network with Constraint on Capacity of Station

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Received 24 July 2015; Revised 6 October 2015; Accepted 12 October 2015

1. Introduction

More and more passengers travel by urban mass transit (UMT) because UMT is rapid, punctual, and green. With the increasing number of passengers, the capacity of urban mass transit network (UMTN) cannot satisfy the trip demands of passengers. There are lots of passengers who queue and wait because of insufficient capacity of trains [1]. Dangers are brought while there are too many waiting passengers in stations. In the cities in China, such as, Beijing, Guangzhou and Shanghai, the UMT corporations usually control passenger inflow in stations where there are too many waiting passengers to assure safety [2, 3].

The approaches of passenger inflow control include controlling passenger inflow in a special station [4], in two stations [5], in stations on a special line [6], or in stations on different lines [7]. In the studies above, the model of controlling passenger inflow in a special station, two stations, and stations on a special line has been described accurately and solved by some methods. Controlling passenger inflow of stations in different lines is described in a qualitative way and there is no model and no effective method.

In UMTN, many trips involve at least one transfer [8]. Transferring passenger takes a great proportion of waiting passengers in transfer stations. The number of waiting passengers cannot be controlled under safety limitation only by controlling inflow of this transfer station. It is necessary to limit the inflow of original station which constitutes the transferring passengers and waiting passengers in this transfer station. So the cooperative inflow control involving stations in the whole network is needed to reduce the pressure of transfer station and improve the safety and efficiency of passengers’ trips. This will be described in detail in Section 2.1.

Thus we build model to describe cooperative passenger inflow control in the whole network (CPICN). This model and the solving method will provide theoretic support for planning control measure in real operation.

The rest of this paper is organized as follows. Section 2 analyzes the necessity and evaluation criteria of CPICN.
Section 3 builds the model, including assumption, objective, and constraints. Section 4 solves the model by PSO algorithm. Section 5 carries out numerical experiments to prove the feasibility and effectiveness of this model and solving method. Section 6 summarizes the contributions in this paper and puts forward the further research.

2. Analyzing CPICN

2.1. The Necessity of CPICN. A simple network is constructed as shown in Figure 1 to illustrate the composition of waiting passengers in transfer station. Figure 1(b) describes some detail in Figure 1(a). In the simple network there are 5 stations which are \( s_0, s_1, \ldots, s_4 \). \( s_0 \) is a transfer station. We note the waiting area (such as platform) in \( s_0 \) on line 1 and line 2 as \( s_{01} \) and \( s_{02} \), respectively. Denote \( PL_{ij} \) as the safety limitation of \( s_{ij} \) \(( j \in \{1, 2\}) \) and \( W_{ij} \) as the number of waiting passengers in \( s_{ij} \). When \( W_{ij} > PL_{ij} \), \( s_{ij} \) is not safe and it is necessary to control number of waiting passengers to satisfy \( W_{ij} \leq PL_{ij} \) to assure the safety of \( s_{ij} \).

Suppose train tr departs from \( s_3 \), goes to \( s_4 \), and passes by \( s_{02} \). The passengers who wait for tr at \( s_{02} \) consist of passengers who depart from \( s_1 \) or \( s_2 \) and go to \( s_4 \) and passengers who enter \( s_0 \) and depart from \( s_{02} \). Here denote \( W_{ij} \) \(( i \neq 0 \) as the passengers who enter \( s_i \) \(( i \neq 0 \) algight from train at \( s_{ij} \) and wait tr in \( s_{ij} \). When the number of transferring passengers satisfies \( \sum_{i=3}^{4} W_{i-0} > PL_{i-0} \), even though there are no passengers who enter \( s_0 \) and no passengers who get on tr at \( s_3 \), the safety of \( s_0 \) cannot be guaranteed because of too many waiting passengers. Under this condition, it is necessary to limit the number of inflow at \( s_i \) \(( i \neq 0 \) to assure \( W_{ij} \leq PL_{ij} \).

According to above illustration, it is necessary to use method of CPICN to assure safe of transfer station in real network when there are too much transferring passengers.

2.2. How to Evaluate Solution of CPICN. Passenger inflow control can change the number of passengers waiting outside or inside of stations. The waiting time of passengers can be changed along with it. The waiting time of a passenger may consist of several parts shown in Figure 2. These waiting parts can be classified into three types: \( W_1, W_2, \) and \( W_3 \). \( W_1 \) represents the waiting time outside of station caused by inflow control, \( W_2 \) represents the waiting time used to wait for the first coming train, and \( W_3 \) represents the waiting time used to wait for the next coming train which can be boarded because of insufficient capacity of the first coming train. \( W_2 \) is inevitable for a trip. \( W_1 \) and \( W_3 \) are extra delay for passengers’ trip, which can be changed and controlled by inflow control. Here we name the sum of \( W_1 \) and \( W_3 \) as time of delay \( (TD) \).

Denote \( TDOC_p \) as the time used by passenger \( p \) to wait outside of station because of inflow control, which corresponds to \( W_1 \). Denote \( TDIO_p \) as the extra time used to wait for next train that can be boarded, which corresponds to \( W_3 \). TD for a special passenger \( p \) can be described as follows:

\[
TD_p = TDOC_p + TDIO_p.
\] (1)

Inflow control leads to less TD for some passengers while more TD for some other passengers. So we use ATD to evaluate the solution which represents average TD. It is computed according to (2), where \( PA \) is the set of passengers.
in whole network and PAC is the number of passengers in PA. For the fixed transport capacity of network, if ATD is less, more passengers are served and the control solution is better

\[ \text{ATD} = \sum_{p \in \text{PA}} \frac{\text{TD}_p}{\text{PAC}}. \]  

(2)

Moreover, it exists that some passengers have longer TD than others. If TD of passenger \( p \) is much longer than TD of passenger \( p' \in \text{PA} - \{ p \} \), it is unfair for passenger \( p \). So we define MTD to represent the longest TD, which can be described as (3). The solution is better while MTD is less

\[ \text{MTD} = \max \left( \text{TD}_p \right), \quad p \in \text{PA}. \]  

(3)

Thus, the general efficiency and individual interest are both considered. The solution of CPICN is needed to satisfy minimum ATD and MTD.

3. Building Model of CPICN

3.1. Assumptions of Model. To simplify the problem and highlight the key conceptions, some assumptions are given as follows.

(1) The time-dependent O-D trip demand is known and discretized into time segments. It is a uniform random passenger arrival distribution in a time segment \( \Delta t \).

(2) The control solution is time varying for every station.

(3) Passengers queue and board trains according to first-in-first-out discipline.

(4) All passengers who enter transit network will reach their planned destination and will not give up the trip at midway.

(5) All transit trains have fixed capacity and operate precisely according to specified timetables.

(6) Capacity of platform is used as the capacity of station supplied for waiting passengers.

(7) In a transfer station, different lines use different platforms.

3.2. Model of CPICN. According to Section 2.2, the objective of CPICN should minimize ATD and MTD, which is described as (4a). The constraints of model encompass capacity of station, capacity of train, O-D demand, and flow conservation, which are described as (4b)–(4e)

\[
\begin{align*}
\text{min} \quad & \text{ATD} \land \text{MTD} \\
\text{S.T.} \quad & W^j_i \leq P^j_i \\
& t_{p_{tr}} \leq t_{c_{tr}} \\
& c^{ts}_i = w^{ts}_i \\
& t_{p_{tr}} - a^{ts}_{l_{tr}} + a^{ts}_{l_{tr}} = t_{p_{tr}}^{i+1}.
\end{align*}
\]  

(4a)  

(4b)  

(4c)  

(4d)  

(4e)

Equation (4c) represents that the number of passengers aboard train \( tr \) should not exceed the capacity of \( tr \), where \( t_{p_{tr}} \) is denoted as number of passengers aboard \( tr \), as capacity of \( tr \).

In (4d), \( c^{ts}_i \) is the maximum number of passengers allowed to enter station \( s_i \) under control in time segment \( ts \), and \( w^{ts}_i \) is the number of passengers who want to enter station \( s_i \) in this time segment. \( w^{ts}_i \) can be expressed as (5), where \( a^{ts}_i \) is the number of arrival passengers who arrive at \( s_i \) in time segment \( ts \), and \( ts - 1 \) is the time segment before \( ts \)

\[
\begin{align*}
& w^{ts}_i = a^{ts}_i + w^{ts-1}_i - c^{ts-1}_i, \quad ts > 1, \\
& w^1_i = a^1_i.
\end{align*}
\]  

(5)

Equation (4e) corresponds to assumption (4a), (4b), (4c), (4d), and (4e), where \( t_{p_{tr}} \) is the number of passengers on \( tr \) when \( tr \) arrives at station \( s_i \). \( a^{ts}_{l_{tr}} \) and \( ab_i \) represent the number of passengers alighting from \( tr \) and boarding \( tr \) at \( s_i \), respectively. \( t_{p_{tr}}^{i+1} \) represents the number of passengers on \( tr \) when \( tr \) arrives at next station \( s_{i+1} \).

4. Solution Procedure

4.1. Method Used to Solve Model. According to assumption (2), the solution of model can be expressed as in (6), where \( n \) is the number of stations and \( m \) is the number of time segments (the solution of model):

\[
C = \begin{bmatrix}
  c^1_1 & \cdots & c^m_1 \\
  \vdots & \ddots & \vdots \\
  c^n_1 & \cdots & c^n_m
\end{bmatrix} 
\]  

(6)

The maximum numbers of passengers allowed to enter station in different time segments of different stations influence each other for the constraints of model. Because of the complexity, particle swarm optimization (PSO) for constrained optimization problem is chosen to solve the model.

Particle swarm optimization (PSO) originally was developed by Eberhart and Kennedy in 1995 [9], which belongs to a class of methods known as evolutionary computation. It hypothesizes that there are \( m \) particles in the \( D \) dimensions’ space. Fitness function used to evaluate particles in space. For the particle \( i \) of iterative \( t + 1 \) generation, the position and velocity of particles are adjusted according to the position and velocity in the previous \( t \) generation, as follows:

\[
\begin{align*}
& v_{id} \left( t + 1 \right) = v_{id} \left( t \right) + c_1 r_1 \times \left( p_{sid} \left( t \right) - x_{id} \left( t \right) \right) \\
& + c_2 r_2 \times \left( p_{gdi} \left( t \right) - x_{id} \left( t \right) \right), \\
& x_{id} \left( t + 1 \right) = x_{id} \left( t \right) + v_{id} \left( t + 1 \right),
\end{align*}
\]  

(7)  

(8)

where \( x_{id} \) and \( v_{id} \) are position and velocity of particle \( i \) in dimensionality \( d \). \( c_1 \) and \( c_2 \) are weight coefficients of the particle individual and \( r_1 \) and \( r_2 \) are two random functions whose values are between 0 and 1. \( p_{sid} \) represents the previous best local position of the particle \( i \). \( p_{gdi} \) represents the best global position found up by the whole swarm.
An important improvement in the basic PSO is the use of an inertia factor $\omega$ [10] that multiplies the velocity term in (7), as shown in (9). In general, $\omega$ starts large and is decremented from $w_{\text{ini}}$ to $w_{\text{end}}$ during the evolution of the algorithm as shown in (10), where $G_k$ is the largest generation of algorithm.

$$v_{sd}(t + 1) = \omega v_{sd}(t) + c_1 r_1 \times (p_{sd}(t) - x_{id}(t)) + c_2 r_2 \left( g_{id}(t) - x_{id}(t) \right),$$

$$\omega(t) = (w_{\text{ini}} - w_{\text{end}}) \times \frac{G_k - t}{G_k} + w_{\text{end}}.$$  

Clerc and Kennedy [11] proposed an alternative version of PSO in which the convergence factor is used, and the velocity adjustment is given as

$$v_{sd}(t + 1) = k \left[ \omega v_{sd}(t) + c_1 r_1 \times (p_{sd}(t) - x_{id}(t)) \right]$$

$$+ c_2 r_2 \left( g_{sd}(t) - x_{id}(t) \right),$$

where $k$ is the convergent factor, which is computed as (12), where $\phi = c_1 + c_2 > 4$

$$k = \frac{2}{2 - \phi - \sqrt{\phi^2 - 4\phi}}.$$  

PSO for constrained optimization problem finds optimal location of particle which has best fitness and satisfies the constraints. The approaches focus on several ways, such as adding penalty function to fitness function [12], valuing the infeasible solution by constraints violation [13, 14], adapting PSO parameters according to constraints violation [15, 16], generating new feasible solution [17], and adjusting parameters to make infeasible solution become feasible [18].

Referring to (13), when the rate of infeasible solution exceeds up limit of rIF, rIF of population po is computed as (14), where QIFpo is the number of infeasible solutions in po and Qpo is the size of po. The up limit of rIF is $\beta$ in this paper.

$$\text{rIF}_{po} = \frac{\text{QIF}_{po}}{Q_{po}}.$$  

### 4.2. Solving Procedure

Based on the above method, the flow for solving procedure is shown in Figure 3, where network loading includes load lines, stations, transfer relationship, and schedule.

In the procedure, fitness function is computed by simulating referencing [21–23]. During simulating, set the limit of inflow according to location of particle, update the number of passengers outside of stations, in stations, and on the trains with the train arriving and departing, and compute MTD and ADT at the end of simulating.

### 5. Numerical Experiments

#### 5.1. Background of Experiments

In order to demonstrate the key features of CPICN, experiments are carried out on a simple network with many-to-one OD flow shown in Figure 1. The inflows of $s_0, s_1, s_2, s_3$ are the same, which are time varying as shown in Table 1.

The safe limit of platform is 1800. The capacity of train is 1856. The headway of trains is 180 seconds and the time holding on station of trains is 30 seconds. The running time between two stations of trains is 300 seconds. The transfer time in transfer station is 120 seconds.

#### 5.2. Effectiveness of Algorithm

Algorithms are programed by Java and compiled by JDK 7.25 for Windows x64 in Eclipse 10. Experiments are tested on a personal computer with an Intel Core i5, 2.50GHz CPU, and 4 GB RAM. The parameters of algorithms are set as shown in Table 2.

The algorithm in this paper is named as Algorithm A. The algorithm proposed by Sun et al. [18] is named as
Algorithm B. 50 times experiments are carried out by two different algorithms with different sizes of population, respectively. The comparison of ATD, MTD, and average running times with different algorithms are shown in Table 3. The better optimal solution can be obtained by Algorithm A than Algorithm B.

The distribution of optimal solutions under Algorithm A ($Q_{po} = 80$) is shown in Figure 4.

Choose a running; the procedure of convergence of ADT and MTD are shown in Figures 5 and 6, respectively.

5.3. Analyzing the Results of Experiments. An optimal solution of model A is chosen, in which ATD is 194 seconds and MTD is 660 seconds. The detail of solution is shown in Table 4, where eight time segments during 7:30–8:50 are chosen.

The key indicators of the solutions are compared with the past research which is proposed by Zhao et al. [6]. The model in this paper is named as model A. The model proposed by Zhao et al. [6] is named as model B. Comparison of key indicators under different conditions is shown in Table 5. The key indicators become better under control than the one without control. The value of MDT under control of model
Table 3: Comparison of solution with different algorithms.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Number of populations</th>
<th>$Q_{po}$</th>
<th>Best</th>
<th>Average</th>
<th>Average running time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algorithm A</td>
<td>2</td>
<td>40</td>
<td>193</td>
<td>660</td>
<td>200  812</td>
</tr>
<tr>
<td>Algorithm B</td>
<td>1</td>
<td>80</td>
<td>195</td>
<td>780</td>
<td>432  1260</td>
</tr>
<tr>
<td>Algorithm A</td>
<td>2</td>
<td>80</td>
<td>192</td>
<td>660</td>
<td>195  775</td>
</tr>
<tr>
<td>Algorithm B</td>
<td>1</td>
<td>160</td>
<td>196</td>
<td>720</td>
<td>292  885</td>
</tr>
</tbody>
</table>

Table 4: Example of optimal solution.

<table>
<thead>
<tr>
<th>Station name</th>
<th>7:30–7:40</th>
<th>7:40–7:50</th>
<th>7:50–8:00</th>
<th>8:00–8:10</th>
<th>8:10–8:20</th>
<th>8:20–8:30</th>
<th>8:30–8:40</th>
<th>8:40–8:50</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>2100</td>
<td>1660</td>
<td>1290</td>
<td>1330</td>
<td>1490</td>
<td>1310</td>
<td>1770</td>
<td>950</td>
</tr>
<tr>
<td>$S_2$</td>
<td>1260</td>
<td>2010</td>
<td>1340</td>
<td>1700</td>
<td>292</td>
<td>1710</td>
<td>1810</td>
<td>700</td>
</tr>
<tr>
<td>$S_3$</td>
<td>1150</td>
<td>1430</td>
<td>1900</td>
<td>1670</td>
<td>2000</td>
<td>1770</td>
<td>1280</td>
<td>700</td>
</tr>
<tr>
<td>$S_0$</td>
<td>2100</td>
<td>1450</td>
<td>1240</td>
<td>1810</td>
<td>1510</td>
<td>1280</td>
<td>1300</td>
<td>1210</td>
</tr>
</tbody>
</table>

Table 5: Comparison of key indicators.

<table>
<thead>
<tr>
<th></th>
<th>ADT</th>
<th>MDT</th>
<th>$\max(W_i)$</th>
<th>$W_i \leq PL$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without control</td>
<td>178</td>
<td>900</td>
<td>7545</td>
<td>No</td>
</tr>
<tr>
<td>Under control of model A</td>
<td>194</td>
<td>660</td>
<td>1799</td>
<td>Yes</td>
</tr>
<tr>
<td>Under control of model B</td>
<td>162</td>
<td>900</td>
<td>7530</td>
<td>No</td>
</tr>
</tbody>
</table>

Table 6: Comparison of maximum number of passenger delayed.

<table>
<thead>
<tr>
<th></th>
<th>$S_0$</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without control</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>In station</td>
<td>6214</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Under control</td>
<td>1428</td>
<td>1316</td>
<td>1652</td>
<td>1752</td>
</tr>
<tr>
<td>In station</td>
<td>486</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

A is better than that of model B. The maximum number of waiting passengers under control of model A is less than the safety limitation of station.

The comparison of maximum number of passengers delayed under control against the one without control is shown in Table 6. It can be observed that the large number of passengers waiting next coming trains in $S_0$ is shared by other stations through inflow control.

6. Conclusion

In this paper, CPICN is proposed considering the safety of stations. The model of CPICN is built with the objective of minimizing ADT and MDT under constraint of capacity of station and train, O-D demand, schedule, and flow conservation. The PSO method for constrained problem is used to solving the model, which integrates the approaches about adjusting infeasible solution to feasible solution and crossing infeasible solution to feasible solution. The numerical experiments are carried out to prove the feasibility and effectiveness of the model and solving algorithm.

For the further research, the solving algorithm will be experimented and applied to the real network. Moreover the route choice about passengers under control measure will be the focus.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

Research is supported by National Key Technology Research and Development Program (2014BAG01B04) from the Ministry of Science and Technology of China, Scientific Research Project of Beijing Education Committee (PXM2015_014212_000023), and Project (I15H00010) from Beijing Municipal Science & Technology Commission.

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