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Research Article

The Optimal Multistage Effort and Contract of VC's Joint Investment

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If the venture project has a great demand of investment, venture entrepreneurs will seek multiple venture capitalists to ensure necessary funding. This paper discusses the decision-making process in the case that multiple venture capitalists invest in a single project. From the beginning of the project till the withdrawal of the investment, the efforts of both parties are long term and dynamic. We consider the Stackelberg game model for venture capital investment in multiple periods. Given the optimal efforts by the entrepreneurs, our results clearly show that as time goes, in every single period entrepreneurs will expect their share of revenue paid to shrink. In other words, they expect a higher ex ante payment and a lower ex post payment. But, in contrast, venture capitalists are expecting exactly the opposite. With a further analysis, we also design an optimal contract in multiple periods. Last but not the least, several issues to be further investigated are proposed as well.

1. Introduction

During the last decade, the development of venture capital captured scholarly attention. Venture capital structure models have developed into a variety of forms, and venture investment institutions have inclined to joint investment [1]. In financial markets, the latter is presented as the phenomenon below: more participants are from developed countries or regions; for example, about 60% of venture investments are joint investment in the United States and Canada, and about 30% in Europe [2]. From the geographical perspective, foreign VCs syndicate with a local VC in 57% of all cross-border deals [3]. More and more venture capitalists (later denoted as VCs) and venture entrepreneurs (later denoted as ENs) prefer joint investment; the reasons are as follows. (1) VCs only have limited investment, which is difficult to meet the necessary capital demand of the project. (2) Joint investment is a way for VCs to diversify risks. (3) ENs need varied specialized guidance from multiple VCs. (4) Joint investment may reduce costs and modify risk allocation, which would be a burden for foreign VCs if they invest alone [3]. (5) Through syndication, foreign VCs may obtain easier access to investment opportunities and may face lower information costs and risks [3]. (6) Venture capital syndication model for enterprises and investors would bring better performance. Therefore, when compared to separate investment, venture investment institutions tend to form a syndicate, resulting in a network connection among investing institutions that realizes the sharing of resources [4]. This explains the crucial and practical significance of the study on ENs' financing from multiple VCs (joint investment).

Two important tasks of venture investment management are to realize the value of venture capital and to ensure the success of venture projects. ENs' and VCs' efforts are very important to the success of venture projects, which is to say that two parties' efforts are equally indispensable to the daily running of the project. VCs play a crucial dual role, both in providing investment and in boosting firms to realize an efficiency gain. In the process of development, a joint effort of VCs and ENs is therefore needed to promote startups to develop and grow. Specifically speaking, ENs' efforts provide gains on the values of core technological innovations, while VCs make efforts to improve the qualities of corporate governance and market valuation and to mitigate operating cost by offering specialized service. However, both parties merely have partial residual claim. In the case of information asymmetry, especially in which their behaviors are not verifiable, a dual moral hazard might prevail.

As we would see later, virtually all of the studies in this specific area view venture capital as a short-term source of financing. VCs aim to exit once the firms grow into a sufficient size with credibility, this is, when VCs can cash out on their investment. Financing from VCs comes usually in several rounds, starting from seed financing and reaching the crest with an exit or in cash terms [5]. Thus venture capital management remains a long-term problem. In practice, NEs' and VCs' efforts are inserted in a number of stages; for example, we can denote the efforts of the first stage as e_1, the efforts of the second as e_2, and the efforts of the k-st as e_k. Obviously, e_1, e_2, and e_k can be different. At the same time, venture capital contracts can be different as well.

In this paper, efforts and contracts of the ENs and VCs are supposed to be resolved as a multistaged problem in a joint investment, and, therefore, a Stackelberg game model in a multistaged venture investment is established. Our study finds that a multistaged investment remains a considerable approach to deal with moral hazard. Optimal efforts of ENs are positively correlated with their future earnings and VCs' efforts, whereas their optimal efforts are negatively correlated with shares of sharing of VCs, fixed return, and effort; in every single stage, once VCs' efforts are given as a constant, expected payment of return sharing from ENs would decrease as time increases: this means that ENs expect to be paid more at early stages while less at later ones. Optimal Efforts of VCs are positively related to their share of the profits and also are positively related to the future revenue of ENs. Expected payment of return sharing from VCs would increase along with time increasing: this reveals that VCs hope to be paid less at early stages while more at later ones. In the meantime, we can have access to an optimal contract of venture investment.

2. Literature Review

There exist many important research results regarding moral hazard in venture investment management. See [6–12]. These studies focused substantially on three aspects.

First, venture capital contracts could be adopted by VCs to mitigate moral hazard of entrepreneurs (single moral hazard). Sahlman [13] considered that only when incentive restrictions are offered to ENs according to the observed information could the expected profits of the two parties converge. Shliefer and Vishy [14] found that if control rights were mainly assigned to minor shareholders, VCs' benefit, to some extent, could be protected from potential losses, but it simultaneously caused the issue of insider control. Ying and Zhao [15] designed an incentive mechanism to maximize the utilities of participants in venture capital, through integrating double principal-agent relationship terms of VCs.

The second aspect is to deal with the circumstance under which venture capital contracts could mitigate moral hazard on both VCs and ENs (double moral hazard). Guo et al. [16] considered the venture capital model on ENs' risk aversion when double moral hazard existed. Using principal-agent theory, Liu et al. [17] investigated the optimal contract between venture capital institutions and venture companies in light of double moral hazard, using principal-agent theory. They introduced VCs' default compensation on the basis of traditional equity contract, increasing the default costs of VCs as principals and thus reducing their tendency to default. Such approach avoided the moral hazard from venture capital institutions as principals, preventing them shirking the responsibilities in providing additional investment. In addition, the moral hazard's expenditure on inefficient employment by venture companies as agents was mitigated as well. Wu et al. [18] believed that personal bounded rationality, coupled with great uncertainty about returns, enabled the VCs and ENs to be exposed to double moral hazard. They divided the venture capital project into early days and the product market stage development and made descriptions of the properties of each stage. Jiang and Li [19] thought that serious problem of information asymmetry and multiple principal-agent relationship exists among the venture investors, VCs and ENs, which caused agency problems in venture capital more complicated than that in general corporations. How to design an incentive compatibility mechanism to mitigate severe moral hazard played a crucial role in the venture capital industry in searching for a sustainable and healthy development.

The third aspect is related to the emphasis on investigating entrepreneurs' past performances, reputations, and potentials. For instance, Ai et al. [20] considered that the relative performance evaluation, associated with the reputation incentive mechanism, had a benefit for solving the information asymmetry problem between the VCs and ENs and played a positive guiding role in designing a complete contract. Chan [21] thought that investigating ENs' past performance could only partially settle the adverse selection problem. However, a complete venture capital market and an objective performance evaluation system were needed. Xu et al. [22] emphasized that, like the control rights, reputation was an important component of the incentive mechanism in venture capitals as well. In their paper, they studied the impact of reputation effect on control rights from three aspects: equity, debt, and convertible preferred stock.

However, these researches were based on single period, while some scholars argued that multistaged financing was also a potent way to mitigate moral hazard [5]. Some scholars like Gompers and Lerner [23] and Admati and Peiderer [24] had made studies on this issue. Dahiya and Ray [25] viewed the staging as a mechanism for VCs. If the returns in early period were low, staged financing provided an option for the VCs to abandon the project in advance. By investigating the relationship between the VCs and the ENs, Elitzur and Gavious [5] provided a multistaged game model with moral hazard. Given the ENs' efforts in different periods, they designed a multistaged incentive contract to mitigate the moral hazard. It clearly showed that the VCs should give incentives to the ENs as late as possible, and the optimal contract was in the form of debt. Jin et al. [26] assumed that the output function in each period was the function of the effort in each period and investigated the two-staged financing problem of the ENs. Zhang and Wei [27] made some improvements for the model of Elitzur and Gavious [5]. In their model, double moral hazard was considered and two parties input different efforts in different periods.

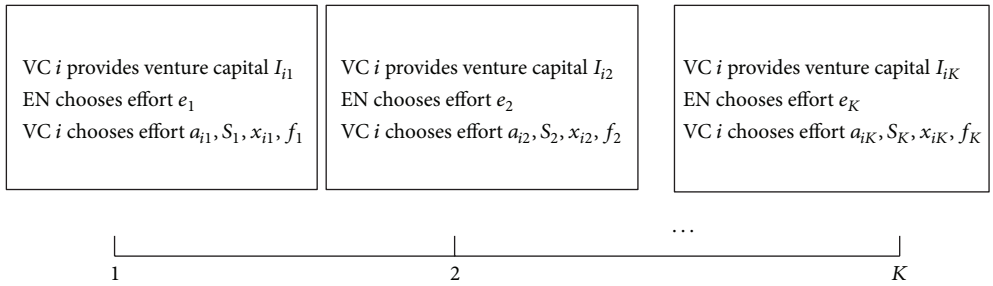

Figure 1: Multistage joint investment decision.

They deduced the optimal incentive contract and hence derived the optimal time to quit through analyzing the factors affecting the contract's design.

Furthermore, these researches were mainly involved with a single VC and a single EN, while, in practice, the EN may involve multiple VCs in their activities, which was actually a joint venture. Sah and Stiglitz [28] made the earliest research on joint venture and demonstrated that, in hierarchical organization, the financing decision of joint venture was superior to that of single investment. Aiming at the incentive contract of joint venture in the middle or late stages of the venture capital, Zhang and Yang [29] offered a model for joint venture contract, under the condition that the VC had made an initial investment and had obtained a better understanding of the project. They discussed the issue, under both the conditions of symmetric information and asymmetric information, of how to design an optimal contract so that two parties of joint venture could show their true signals. Casamatta [6] considered a situation in which leading VCs provided no initial investment and lacked an enough understanding of the project. In such a case, he made a research on how to design an optimal contract with other assistant VCs. Brander et al. [30] provided a model on incentive contracts, respectively, under both the conditions of selection and value-added hypothesis. They argued that, given the selection hypothesis, the quality of single investment was better than that of joint venture and hence the former had a better performance. While under the value-added hypothesis, joint venture performed better since multiple venture capitalists had advantages over single venture capital on providing a stronger management support, a higher reputation, and more kinds of contracts.

Existing researches above either focused on the decision-making issue on single period in venture capital, involved with a single VC and a single EN, or considered the decision-making in a single period under a joint venture. This left a space for our further study. What about multistaged financing? What about multistaged financing with the double effort of two parties? Therefore, based on the two perspectives above, this paper offers a multistaged Stackelberg game model by game theory. We focus on investigating several issues. First, we discuss the factors affecting the optimal effort of the VCs and EN in multiple periods. The second relates to the EN's time preference for investing: does it prefer an earlier investment or a later one? Finally, we come to the discussion of VCs' time preference to induce the EN to work extra hard, an earlier investment or a later one?

This paper proceeds as follows. In the next section, we put forward the model descriptions in Section 3. In Section 4, we consider the multistaged model and make solutions. In Section 5, we discuss the optimal contract in our model. Finally, Section 6 concludes the paper.

3. Model Descriptions

EN has an innovative project, which requires a total amount of financing as I from outer n VCs. EN and n VCs are risk neutral, which means that they are equivalent in risk taking for the potential outfit. EN's decision objective is the maximization of remuneration. VCs' decision objective is the maximization of capital profits.

If the n VCs and EN accomplished an investment intention, it would have been a long-term process for n VCs to invest risk capital in the venture enterprise. Assuming that the investment will be installed in k periods, VC i ($i = 1, 2, \ldots, n$) will provide external capital I_{ik} ($I_{ik} \geq 0$) for venture project at k period ($k = 1, 2, \ldots, K$); thus $\sum_{i=1}^{N} \sum_{k=1}^{K} = I$, obviously. See Figure 1.

The market average yield of external capital I_{ik} is r; namely, the capital cost of VCs is $(1 + r)I_{ik}$. The efforts given by EN in different stages are discrepant; naturally, there should also be stage-incentive for n VCs. Divide the long-term relationship among n VCs and EN into K periods, and the effort given by EN in k period is e_k, $k = 1, 2, \ldots, K$, and the costs of effort $c(e_k)$ can also be equivalent to the monetary costs. For simplicity, assuming the utility function of effort paid by EN is $c(e_k) = (1/2)e_k^2$, this means that, with the accumulation of efforts payment, the costs will also increase with an increasing speed; namely, $c'(e_k) > 0$ and $c''(e_k) > 0$. Equally, VC i will pay the effort of a_{ik}, $k = 1, 2, \ldots, K$, in k period, and also the costs of effort can be equivalent to the monetary costs. For simplicity, assuming the utility function of effort paid by VC i is $c(a_{ik}) = (1/2)a_{ik}^2$, it means that, with the accumulation of efforts payment, the cost will also increase with an increasing speed; namely, $c'(a_{ik}) > 0$ and $c''(a_{ik}) > 0$.

Given the success probability of K-period project p_k, $0 \leq p_k \leq 1$, the success of venture project is related to both the effort e_k paid by EN and the effort a_{ik}, $k = 1, 2, \ldots, K$, paid

by n VCs. The more effort they paid, the higher the probability is. However, VC's and EN's efforts on the success probability of the venture project are not the same role; due to the professional skills of n VCs, their efforts are bigger than the one paid by EN. Thus we can assume that $p_k = \theta e_k^{\mu} \prod_{i=1}^{n} a_{ik}^{1-\mu}, \theta > 0, 0 < \mu < 1/2, 0 \le p_k = \theta e_k^{\mu} \prod_{i=1}^{n} a_{ik}^{1-\mu} \le 1$, the successful profit in K period is R_k, the unsuccessful profit is 0, and the expected revenue is $U_k = p_k R_k + (1 - p_k)0 = p_k R_k$. For research's convenience, we assume that the expected revenue in various periods is equivalent, which is $U_k = U, k = 1, 2, \ldots, K$. The reasons for giving this hypothesis are showed as follows.

Firstly, if the success possibility p_k for venture project is so tiny that the return profit R_k will be great. On the contrary, if the success probability p_k is so big that the return profit R_k will be tiny. These would accord with the characteristic of the venture project.

Secondly, if the return in every period is increasing gradually, then the EN's and n VCs' decisions will require larger cash flow in the future; if the return in every period is decreasing gradually, then the ENs' and n VCs' decision will require more cash flow in the first place, which are meaningless to study. In spite of such a reason, these hypotheses are still very strict. Relaxing this hypothesis will also work, but the model will be more complicated.

After n VCs invest the capital, the venture project will be controlled by n VCs. If venture project succeeds, EN's profit share confirmed by n VCs is y_k $(0 \le y_k \le 1)$ in every stage; namely, the share gained by VC i $(i = 1, 2, \ldots, n)$ in stage k is x_{ik} $(0 \le x_{ik} \le 1)$, and then EN i's profit is $x_{ik} R_k$ and obviously needs $y_k + \sum_{i=1}^{n} = 1$; if the venture project in stage k fails, the fixed income that n VCs need to be paid for EN in every stage is f_k, and we call (S_k, x_{ik}, f_k) stock investment contract. Considering the time value of EN's profit, the discount factor is 1, and VC i's $(i = 1, 2, \ldots, n)$ discount factor is 1.

The sequence of events in the game is depicted in Figure 2.

4. The Multistage Decision Model and Solution

According to this time line, modeling process and its solving are as follows.

The K-period profit of VC i is shown as V_{ik}:

$$\begin{aligned} V_{ik} &= p_k \left(x_{ik} R_k + V_{i(k+1)}\right) - \left(1 - p_k\right) \frac{1}{n} f_k - c\left(a_{ik}\right) \\ &= \theta e_k^{\mu} \prod_{i=1}^{n} a_{ik}^{1-\mu} \left(x_{ik} R_k + V_{i(k+1)}\right) \\ &\quad - \left(1 - \theta e_k^{\mu} \prod_{i=1}^{n} a_{ik}^{1-\mu}\right) \frac{1}{n} f_k - \frac{1}{2} a_{ik}^2, \\ &\qquad k = 1, 2, \ldots, K. \end{aligned} \tag{1}$$

According to the recursive relationship in (1), the first-period profit of the venture investor is shown as

$$\begin{aligned} V_{i1} &= \theta e_1^{\mu} \prod_{i=1}^{n} a_{i1}^{1-\mu} x_{i1} R_1 - \left(1 - \theta e_1^{\mu} \prod_{i=1}^{n} a_{i1}^{1-\mu}\right) \frac{1}{n} f_1 - \frac{1}{2} \\ &\quad \cdot a_{i1}^2 + \sum_{k=2}^{K} \left\langle \left[\prod_{j=1}^{k-1} \theta e_j^{\mu} \prod_{i=1}^{n} a_{ij}^{1-\mu} \right] \right. \\ &\quad \cdot \left[\theta e_k^{\mu} \prod_{i=1}^{n} a_{ik}^{1-\mu} \left(x_{ik} R_k + V_{i(k+1)}\right) \right. \\ &\quad \left. \left. - \left(1 - \theta e_k^{\mu} \prod_{i=1}^{n} a_{ik}^{1-\mu}\right) \frac{1}{n} f_k - \frac{1}{2} a_{ik}^2 \right] \right\rangle. \end{aligned} \tag{2}$$

The VC i $(i = 1, 2, \ldots, n)$ is more concerned about the whole project investment's returns; thus, the optimization problem (I) will be shown as

$$\begin{aligned} \max_{x_{ik}, f_k, e_{ik}} \quad & V_{i1} \\ &= \max_{x_{ik}, f_k, e_{ik}} \theta e_1^{\mu} \prod_{i=1}^{n} a_{i1}^{1-\mu} x_{i1} R_1 - \left(1 - \theta e_1^{\mu} \prod_{i=1}^{n} a_{i1}^{1-\mu}\right) \frac{1}{n} f_1 - \frac{1}{2} a_{i1}^2 \\ &\quad + \sum_{k=2}^{K} \left\langle \left[\prod_{j=1}^{k-1} \theta e_j^{\mu} \prod_{i=1}^{n} a_{ij}^{1-\mu} \right] \left[\theta e_k^{\mu} \prod_{i=1}^{n} a_{ik}^{1-\mu} \left(x_{ik} R_k + V_{i(k+1)}\right) - \left(1 - \theta e_k^{\mu} \prod_{i=1}^{n} a_{ik}^{1-\mu}\right) \frac{1}{n} f_k - \frac{1}{2} a_{ik}^2 \right] \right\rangle \\ \text{s.t.} \quad & x_{ik} R_k + V_{k+1} \ge 0, \\ & V_{ik} \ge 0, \\ & e_k \ge 0, \\ & a_{ik} \ge 0, \\ & k = 1, 2, \ldots, K, \ i = 1, 2, \ldots, n. \end{aligned} \tag{3}$$

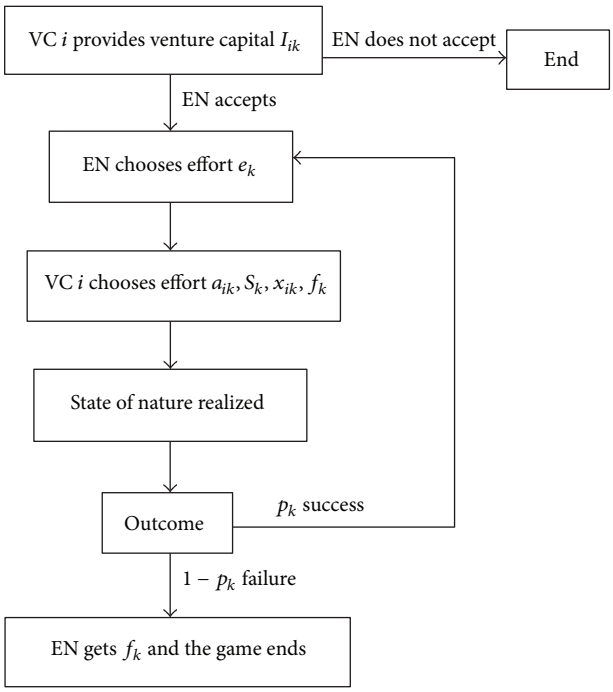

Figure 2: The time line.

The VC i ($i = 1, 2, \ldots, n$) chooses share x_i, and his K-period profit will be represented by E_k, which can be shown as the following equation:

$$\begin{aligned} E_k &= p_k\left(y_k R_k + E_{(k+1)}\right) - \left(1 - p_k\right) f_k - c\left(e_k\right) \\ &= \theta e_k^{\mu} \prod_{i=1}^{n} a_{ik}^{1-\mu}\left(y_k R_k + E_{(k+1)}\right) \\ &\quad - \left(1 - \theta e_k^{\mu} \prod_{i=1}^{n} a_{ik}^{1-\mu}\right) f_k - \frac{1}{2} e_k^2, \quad k = 1, 2, \ldots, K. \end{aligned} \tag{4}$$

It is obvious that $y_k + \sum_{i=1}^{n} x_{ik} = 1$.

In this condition, the K-period profit E_k will be transferred into

$$\begin{aligned} E_k &= p_k\left(y_k R_k + E_{(k+1)}\right) - \left(1 - p_k\right) f_k - c\left(e_k\right) \\ &= \theta e_k^{\mu} \prod_{i=1}^{n} a_{ik}^{1-\mu}\left[\left(1 - \sum_{i=1}^{n} x_{ik}\right) R_k + E_{(k+1)}\right] \\ &\quad - \left(1 - \theta e_k^{\mu} \prod_{i=1}^{n} a_{ik}^{1-\mu}\right) f_k - \frac{1}{2} e_k^2, \quad k = 1, 2, \ldots, K. \end{aligned} \tag{5}$$

It is assumed that the EN is shortsighted and that all he wants is to maximize the profit in this period, so we can arrive at the optimization problem (II) as follows:

$$\begin{aligned} \max_{e_k} \quad & E_k \\ &= \max_{e_k} \theta e_k^{\mu} \prod_{i=1}^{n} a_{ik}^{1-\mu}\left[\left(1 - \sum_{i=1}^{n} x_{ik}\right) R_k + E_{(k+1)}\right] \\ &\quad - \left(1 - \theta e_k^{\mu} \prod_{i=1}^{n} a_{ik}^{1-\mu}\right) f_k - \frac{1}{2} e_k^2 \\ \text{s.t.} \quad & \left(1 - \sum_{i=1}^{n} x_{ik}\right) R_k + E_{k+1} \geq 0, \\ & e_k \geq 0, \\ & k = 1, 2, \ldots, K. \end{aligned} \tag{6}$$

4.1. The Optimal Decision for the EN. The first-stage condition of the optimization problem (II) about e_k is shown as

$$\begin{aligned} &\theta \mu e_k^{(\mu-1)} \prod_{i=1}^{n} a_{ik}^{1-\mu}\left[\left(1 - \sum_{i=1}^{n} x_{ik}\right) R_k + E_{(k+1)}\right] \\ &\quad - \theta \mu e_k^{(\mu-1)} \prod_{i=1}^{n} a_{ik}^{(1-\mu)} f_k - e_k = 0, \quad k = 1, 2, \ldots, K. \end{aligned} \tag{7}$$

The optimal solution e_k solved by (7) is shown as

$$\begin{aligned} \left(e_k^*\right)^{2-\mu} &= \theta \mu \prod_{i=1}^{n} a_{ik}^{1-\mu}\left[\left(1 - \sum_{i=1}^{n} x_{ik}\right) R_k + E_{(k+1)}\right] \\ &\quad - \theta \mu \prod_{i=1}^{n} a_{ik}^{(1-\mu)} f_k, \quad k = 1, 2, \ldots, K. \end{aligned} \tag{8}$$

We can draw the first conclusion (marked as Conclusion 1) according to (8).

Conclusion 1. (1) VC's optimal effort is negatively related to his share. (2) VC's optimal effort is positively related to the future earnings. (3) VC's optimal effort is negatively related to the fixed income. (4) VC's optimal effort is positively related to VC's effort.

Proof. (1), (2), and (3) can be easily demonstrated from the coefficient of (4).

(4) can be demonstrated as follows.

According to the assumption 4 ($0 \leq \mu < 1/2$), it can be concluded that the character of e_k^* equals the counterpart

of $(e_k^*)^{2-\mu}$. So we can work with $(e_k^*)^{2-\mu}$ to simplify the problem

$$\begin{aligned}
&\frac{\partial (e_k^*)^{2-\mu}}{\partial a_{ik}} \\
&= \frac{1}{2-\mu}\left\{\theta\mu\prod_{i=1}^{n} a_{ik}^{1-\mu}\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k + E_{(k+1)}\right]\right. \\
&\left.-\theta\mu\prod_{i=1}^{n} a_{ik}^{(1-\mu)} f_k\right\}^{(\mu-1)/(2-\mu)} \\
&\cdot\frac{\partial}{\partial a_{ik}}\left\{\theta\mu\prod_{i=1}^{n} a_{ik}^{1-\mu}\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k + E_{(k+1)}\right]\right. \\
&\left.-\theta\mu\prod_{i=1}^{n} a_{ik}^{(1-\mu)} f_k\right\} \\
&= \frac{1}{2-\mu}\left\{\theta\mu\prod_{i=1}^{n} a_{ik}^{1-\mu}\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k + E_{(k+1)}\right]\right. \\
&\left.-\theta\mu\prod_{i=1}^{n} a_{ik}^{(1-\mu)} f_k\right\}^{(\mu-1)/(2-\mu)} \\
&\cdot\left\{\theta\mu\prod_{i=1,i\neq k}^{n} a_{ik}^{1-\mu}\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k + E_{(k+1)}\right]\right. \\
&\left.-\theta\mu\prod_{i=1,i\neq k}^{n} a_{ik}^{(1-\mu)} f_k\right\} \\
&= \frac{1}{2-\mu}\left\{\theta\mu\prod_{i=1}^{n} a_{ik}^{1-\mu}\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k + E_{(k+1)}\right]\right. \\
&\left.-\theta\mu\prod_{i=1}^{n} a_{ik}^{(1-\mu)} f_k\right\}^{(\mu-1)/(2-\mu)}\theta\mu\prod_{i=1,i\neq k}^{n} a_{ik}^{1-\mu} \\
&\cdot\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k + E_{(k+1)} - f_k\right] \\
&= \frac{1}{2-\mu}(\theta\mu)^{(\mu-1)/(2-\mu)} \\
&\cdot\left\{\prod_{i=1}^{n} a_{ik}^{1-\mu}\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k + E_{(k+1)}\right]\right. \\
&\left.-\prod_{i=1}^{n} a_{ik}^{(1-\mu)} f_k\right\}^{(\mu-1)/(2-\mu)}\theta\mu\prod_{i=1,i\neq k}^{n} a_{ik}^{1-\mu} \\
&\cdot\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k + E_{(k+1)} - f_k\right] \\
&= \frac{1}{2-\mu}(\theta\mu)^{(\mu-1)/(2-\mu)}\prod_{i=1}^{n} a_{ik}^{1-\mu} \\
&\cdot\left\{\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k + E_{(k+1)}\right]\right. \\
&\left.- f_k\right\}^{(\mu-1)/(2-\mu)}\theta\mu\prod_{i=1,i\neq k}^{n} a_{ik}^{1-\mu}\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k\right. \\
&\left.+ E_{(k+1)} - f_k\right] = \frac{1}{2-\mu}(\theta\mu)^{1/(2-\mu)}\prod_{i=1,i\neq k}^{n} a_{ik}^{1-\mu}\prod_{i=1}^{n} a_{ik}^{1-\mu} \\
&\cdot\left\{\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k + E_{(k+1)}\right]\right. \\
&\left.- f_k\right\}^{1/(2-\mu)} \geq 0.
\end{aligned} \tag{9}$$

□

Conclusion 2. In each period, EN expects the income share to decrease as the time goes by within the prescribed efforts. Specifically, he wants to pay more in early stages.

Proof. Randomly, $0 < m < k \leq K$,

$$\begin{aligned}
\frac{\partial E_m}{\partial x_{ik}} &= \theta\mu(e_m^*)^{\mu-1}\frac{\partial (e_m^*)^{1-\mu}}{\partial x_{ik}}\prod_{i=1}^{n} a_{im}^{1-\mu}\left[\left(1-\sum_{i=1}^{n}x_{im}\right)\right. \\
&\left.\cdot R_m + E_{(m+1)}\right] + \theta\mu(e_m^*)^{\mu}\left[-R_m + \frac{E_{(m+1)}}{\partial x_{ik}}\right] \\
&- \theta\mu(e_m^*)^{\mu-1}\frac{\partial (e_m^*)^{1-\mu}}{\partial x_{ik}}\prod_{i=1}^{n} a_{im}^{1-\mu} f_m - ue_m^*\frac{\partial e_m^*}{\partial x_{ik}} \\
&= \frac{\partial e_m^*}{\partial x_{ik}}\left\{\theta\mu(e_m^*)^{\mu-1}\prod_{i=1}^{n} a_{im}^{1-\mu}\right. \\
&\cdot\left[\left(1-\sum_{i=1}^{n}x_{im}\right)R_m + E_{(m+1)}\right] - \theta\mu(e_m^*)^{\mu-1} \\
&\left.\cdot\prod_{i=1}^{n} a_{im}^{1-\mu} f_m - ue_m^*\right\} + \theta\mu(e_m^*)^{\mu}\left[\frac{\partial E_{(m+1)}}{\partial x_{ik}}\right].
\end{aligned} \tag{10}$$

According to (7)

$$\begin{aligned}
&\theta\mu(e_m^*)^{\mu-1}\prod_{i=1}^{n} a_{im}^{1-\mu}\left[\left(1-\sum_{i=1}^{n}x_{im}\right)R_m + E_{(m+1)}\right] \\
&\quad - \theta\mu(e_m^*)^{\mu-1}\prod_{i=1}^{n} a_{im}^{1-\mu} f_m - ue_m^* = 0.
\end{aligned} \tag{11}$$

The result of putting (11) into (10) is

$$\frac{\partial E_m}{\partial x_{ik}} = \theta(e_m^*)^{\mu}\prod_{i=1}^{n} a_{im}^{1-\mu}\left[\frac{\partial E_{m+1}}{\partial x_{ik}}\right]. \tag{12}$$

Optimization question (II) in

$$\begin{aligned}(e_k^*)^{2-\mu} &= \theta\mu\prod_{i=1}^{n}a_{ik}^{1-\mu}\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k+E_{(k+1)}\right]\\ &\quad-\theta\mu\prod_{i=1}^{n}a_{ik}^{(1-\mu)}f_k\end{aligned} \tag{13}$$

can be transferred into

$$\begin{aligned}\max_{e_k}\quad & E_k\\ &= \max_{e_k}\theta e_k^{\mu}\prod_{i=1}^{n}a_{ik}^{1-\mu}\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k+E_{(k+1)}\right]\\ &\quad-\left(1-\theta e_k^{\mu}\prod_{i=1}^{n}a_{ik}^{1-\mu}\right)f_k-\frac{1}{2}e_k^2\\ \text{s.t.}\quad & (e_k^*)^{2-\mu}\\ &= \theta\mu\prod_{i=1}^{n}a_{ik}^{1-\mu}\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k+E_{(k+1)}\right]\\ &\quad-\theta\mu\prod_{i=1}^{n}a_{ik}^{(1-\mu)}f_k,\quad k=1,2,\ldots,K.\end{aligned} \tag{14}$$

Because of $y_k+\sum_{i=1}^{n}x_{ik}=1$, (14) can be transferred into

$$\begin{aligned}\max_{e_k}\quad & E_k\\ &= \max_{e_k}\theta e_k^{\mu}\prod_{i=1}^{n}a_{ik}^{1-\mu}\left[y_kR_k+E_{(k+1)}\right]\\ &\quad-\left(1-\theta e_k^{\mu}\prod_{i=1}^{n}a_{ik}^{1-\mu}\right)f_k-\frac{1}{2}e_k^2\\ \text{s.t.}\quad & (e_k^*)^{2-\mu}\\ &= \theta\mu\prod_{i=1}^{n}a_{ik}^{1-\mu}\left[y_kR_k+E_{(k+1)}\right]-\theta\mu\prod_{i=1}^{n}a_{ik}^{(1-\mu)}f_k,\\ &\qquad k=1,2,\ldots,K.\end{aligned} \tag{15}$$

The partial derivative about y_k in (15) is

$$\begin{aligned}\frac{\partial E_k}{\partial y_k} &= \theta\mu e_k^{*\mu-1}\prod_{i=1}^{n}a_{ik}^{1-\mu}\left[y_kR_k+E_{(k+1)}\right]\\ &\quad+R_k\theta e_k^{*\mu}\prod_{i=1}^{n}a_{ik}^{1-\mu}-\theta f_k\mu e_k^{\mu-1}\frac{\partial e_k^*}{\partial y_k}\prod_{i=1}^{n}a_{ik}^{1-\mu}-e_k\frac{\partial e_k^*}{\partial y_k}\\ &= \frac{\partial e_k^*}{\partial y_k}\left\{\theta\mu e_k^{*\mu-1}\prod_{i=1}^{n}a_{ik}^{1-\mu}\left[y_kR_k+E_{(k+1)}\right]\right.\\ &\quad\left.+\theta f_k\mu e_k^{*\mu-1}\prod_{i=1}^{n}a_{ik}^{1-\mu}-e_k^*\right\}-R_k\theta e_k^{*\mu}\prod_{i=1}^{n}a_{ik}^{1-\mu}.\end{aligned} \tag{16}$$

The result of putting (11) into (16) is

$$\frac{\partial E_k}{\partial y_k}=R_k\theta e_k^{*\mu}\prod_{i=1}^{n}a_{ik}^{1-\mu}. \tag{17}$$

Using the recurrence relation in (12) and the given $0<m<k\le K$, there is

$$\begin{aligned}\frac{\partial E_m}{\partial y_k} &= \theta e_m^{*\mu}\prod_{i=1}^{n}a_{im}^{1-\mu}\left[\frac{\partial E_{m+1}}{\partial y_k}\right]\\ \frac{\partial E_{m+1}}{\partial y_k} &= \theta e_{m+1}^{*\mu}\prod_{i=1}^{n}a_{i(m+1)}^{1-\mu}\left[\frac{\partial E_{m+2}}{\partial y_k}\right]\\ &\cdots\cdots\\ \frac{\partial E_{k-1}}{\partial y_k} &= \theta e_{k-1}^{*\mu}\prod_{i=1}^{n}a_{i(k-1)}^{1-\mu}\left[\frac{\partial E_k}{\partial y_k}\right].\end{aligned} \tag{18}$$

Thus according to (18)

$$\frac{\partial E_m}{\partial y_k}=\prod_{j=m}^{k}\left(\theta e_j^{*\mu}\prod_{i=1}^{n}a_{ij}^{1-\mu}\right)\left[\frac{\partial E_k}{\partial y_k}\right]. \tag{19}$$

The result of putting (17) into (19) can be shown as follows:

$$\frac{\partial E_m}{\partial y_k}=R_k\theta e_k^{*\mu}\prod_{i=1}^{n}a_{ik}^{1-\mu}\prod_{j=m}^{k}\left(\theta e_j^{*\mu}\prod_{i=1}^{n}a_{ij}^{1-\mu}\right). \tag{20}$$

Identically

$$\frac{\partial E_m}{\partial y_{k+1}}=R_{k+1}\theta e_{k+1}^{*\mu}\prod_{i=1}^{n}a_{i(k+1)}^{1-\mu}\prod_{j=m}^{k+1}\left(\theta e_j^{*\mu}\prod_{i=1}^{n}a_{ij}^{1-\mu}\right). \tag{21}$$

The outcome from both (20) and (21) is

$$\begin{aligned}&\frac{\partial E_m/\partial y_k}{\partial E_m/\partial y_{k+1}}\\ &\quad=\frac{R_k\theta e_k^{*\mu}\prod_{i=1}^{n}a_{ik}^{1-\mu}\prod_{j=m}^{k}\left(\theta e_j^{*\mu}\prod_{i=1}^{n}a_{ij}^{1-\mu}\right)}{R_{k+1}\theta e_{k+1}^{*\mu}\prod_{i=1}^{n}a_{i(k+1)}^{1-\mu}\prod_{j=m}^{k+1}\left(\theta e_j^{*\mu}\prod_{i=1}^{n}a_{ij}^{1-\mu}\right)}\\ &\quad=\frac{R_k\theta e_k^{*\mu}\prod_{i=1}^{n}a_{ik}^{1-\mu}}{R_{k+1}\theta e_{k+1}^{*\mu}\prod_{i=1}^{n}a_{i(k+1)}^{1-\mu}\theta e_{k+1}^{*\mu}\prod_{i=1}^{n}a_{i(k+1)}^{1-\mu}}\\ &\quad=\frac{R_k\theta e_k^{*\mu}\prod_{i=1}^{n}a_{ik}^{1-\mu}}{R_{k+1}\theta e_{k+1}^{*2\mu}\prod_{i=1}^{n}a_{i(k+1)}^{2(1-\mu)}}\\ &\quad=\frac{R_k\theta e_k^{*\mu}\prod_{i=1}^{n}a_{ik}^{1-\mu}}{R_{k+1}\theta e_{k+1}^{*\mu}\prod_{i=1}^{n}a_{i(k+1)}^{(1-\mu)}\theta e_{k+1}^{*\mu}\prod_{i=1}^{n}a_{i(k+1)}^{1-\mu}}\\ &\quad=\frac{R_kp_k+0\left(1-p_k\right)}{p_{k+1}\left[R_{k+1}p_{k+1}+0\left(1-p_{k+1}\right)\right]}=\frac{U_k}{p_{k+1}U_{k+1}}\\ &\quad\ge\frac{1}{p_{k+1}}\ge 1.\end{aligned} \tag{22}$$

Equally $\partial E_m/\partial y_k \ge \partial E_m/\partial y_{k+1}$; thus, Conclusion 2 is proved. □

4.2. The Optimal Decision for VC i. The following is the discussion on optimization decision made by VC i. According to (3), the first-order condition about effort a_{ik} is

$$\begin{aligned}\frac{\partial V_{i1}}{\partial a_{ik}} = {} & \left[\prod_{j=1}^{k-1}\theta e_j^{\mu}\prod_{i=1}^{n}a_{ij}^{1-\mu}\right]\left\{\left(\theta\frac{\partial e_k^{*\mu}}{\partial a_{ik}}\prod_{i=1}^{n}a_{ik}^{1-\mu}\right)x_{ik}R_k\right. \\ & + (1-\mu)\left(\theta e_k^{*\mu}\prod_{i=1,i\neq k}^{n}a_{ik}^{1-\mu}\right)x_{ik}R_k a_{ik}^{-\mu} \\ & + \left[\theta\frac{\partial e_k^{*\mu}}{\partial a_{ik}}\prod_{i=1}^{n}a_{ik}^{1-\mu}\right. \\ & \left.\left. + (1-\mu)\,\theta e_k^{*\mu}\left(\prod_{i=1,i\neq k}^{n}a_{ik}^{1-\mu}\right)a_{ik}^{-\mu}\right]\frac{1}{n}f_k - a_{ik}\right\} = 0.\end{aligned} \tag{23}$$

Thus VC i's optimal effort a_{ik} should meet the need of (24) as is shown in the following equation:

$$\begin{aligned}&\left(\theta\frac{\partial e_k^{*\mu}}{\partial a_{ik}}\prod_{i=1}^{n}a_{ik}^{1-\mu}\right)\left(x_{ik}R_k + \frac{1}{n}f_k\right) \\ &\quad + (1-\mu)\,a_{ik}^{-\mu}\theta e_k^{*\mu}\prod_{i=1,i\neq k}^{n}a_{ik}^{1-\mu}\left[x_{ik}R_k + \frac{1}{n}f_k - a_{ik}\right] \\ &\quad - a_{ik} = 0.\end{aligned} \tag{24}$$

Therefore

$$\begin{aligned}\frac{\partial e_k^{*\mu}}{\partial a_{ik}} = {} & \frac{\mu}{2-\mu}\left\{\theta\mu\prod_{i=1}^{n}a_{ik}^{1-\mu}\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k + E_{k+1}\right]\right. \\ & \left. - \theta\mu\left(\prod_{i=1}^{n}a_{ik}^{1-\mu}\right)f_k\right\}^{(2\mu-2)/(2-\mu)}\theta\mu(1-\mu) \\ & \cdot a_{ik}^{-\mu}\prod_{i=1,i\neq k}^{n}a_{ik}^{1-\mu}\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k + E_{k+1} - f_k\right].\end{aligned} \tag{25}$$

The result of putting (8) and (25) into (24) is

$$\begin{aligned}&\theta^2\mu^2\frac{(1-\mu)}{2-\mu}\left\{\theta\mu\prod_{i=1}^{n}a_{ik}^{1-\mu}\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k + E_{k+1}\right]\right. \\ &\quad \left.\cdot f_k\right\}^{(2\mu-2)/(2-\mu)}a_{ik}^{-\mu}\left(\prod_{i=1,i\neq k}^{n}a_{ik}^{1-\mu}\right)\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)\right. \\ &\quad \left.\cdot R_k + E_{k+1} - f_k\right]\left(\prod_{i=1}^{n}a_{ik}^{1-\mu}\right)(x_{ik})\,R_k + \frac{1}{n}f_k\,(1 \\ &\quad - \mu)\,a_{ik}^{-\mu}\theta\left\{\theta\mu\prod_{i=1}^{n}a_{ik}^{1-\mu}\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k + E_{k+1}\right]\right. \\ &\quad \left.\cdot f_k\right\}^{(2\mu-2)/(2-\mu)}\prod_{i=1,i\neq k}^{n}a_{ik}^{1-\mu}\left[x_{ik}R_k + \frac{1}{n}f_k\right] - a_{ik} \\ &\quad = 0.\end{aligned} \tag{26}$$

We can derive a_{ik}^* as follows:

$$\begin{aligned}a_{ik}^* = {} & \left[\mu(2-\mu)^{(\mu-2)/2} + \mu^{\mu/2}\right]\left(\prod_{i=1,i\neq k}^{n}a_{ik}^{1-\mu}\right) \\ & \cdot\theta\left((1-\mu)^{(2-\mu)/2}\right)\left(x_{ik}R_k + \frac{1}{n}f_k\right)^{(2-\mu)/2} \\ & \cdot\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k + E_{k+1} - f_k\right]^{\mu/2}.\end{aligned} \tag{27}$$

Conclusion 3. (i) VC i's optimal effort is positively related to the VC i's income share.

(ii) VC i's optimal effort is related to EN's future income.

Proof. According to (27), (i) the proof of Conclusion 3(i) is shown as follows:

$$\begin{aligned}\frac{\partial a_{ik}^*}{\partial x_{ik}} = {} & \theta\mu(2-\mu)^{(\mu-2)/2}(1-\mu)^{(2-\mu)/2}\frac{2-\mu}{2} \\ & \cdot R_k\left(\prod_{i=1,i\neq k}^{n}a_{ik}^{1-\mu}\right)\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)\left(x_{ik}R_k + \frac{1}{n}f_k\right)\right. \\ & \left.\cdot R_k + \left(x_{ik}R_k + \frac{1}{n}f_k\right)(E_{k+1} - f_k)\right]^{-\mu/2} \\ & \cdot\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k - \left(x_{ik}R_k + \frac{1}{n}f_k\right) + E_{k+1}\right. \\ & \left. - f_k\right]\left\{\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k + E_{k+1} - f_k\right]\right\}^{\mu-1} \\ & + \theta\mu R_k(2-\mu)^{(\mu-2)/2}(1-\mu)^{(4-\mu)/2}\left(\prod_{i=1,i\neq k}^{n}a_{ik}^{1-\mu}\right) \\ & \cdot\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)\left(x_{ik}R_k + \frac{1}{n}f_k\right)R_k\right. \\ & \left. + \left(x_{ik}R_k + \frac{1}{n}f_k\right)(E_{k+1} - f_k)\right]^{(2-\mu)/2} \\ & \cdot\left\{\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k + E_{k+1}\right]\right\}^{(\mu-2)/2} + (\theta\mu)^{\mu/2} \\ & \cdot[\theta(1-\mu)]^{(\mu-2)/2}\frac{2-\mu}{2}R_k\left(\prod_{i=1,i\neq k}^{n}a_{ik}^{1-\mu}\right)\left(x_{ik}R_k\right. \\ & \left. + \frac{1}{n}f_k\right)^{-\mu/2}\left\{\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k + E_{k+1} - f_k\right]\right\}^{\mu/2} \\ & - (\theta\mu)^{\mu/2}[\theta(1-\mu)]^{(\mu-2)/2}\frac{\mu}{2}R_k\left(\prod_{i=1,i\neq k}^{n}a_{ik}^{1-\mu}\right)\end{aligned}$$

$$
\begin{aligned}
&\cdot\left(x_{ik}R_k+\frac{1}{n}f_k\right)^{(2-\mu)/2}\\
&\cdot\left\{\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k+E_{k+1}-f_k\right]\right\}^{(\mu-2)/2}\\
&=\theta\mu(2-\mu)^{(\mu-2)/2}(1-\mu)^{(2-\mu)/2}\frac{2-\mu}{2}\\
&\cdot R_k\left(\prod_{i=1,i\neq k}^{n}a_{ik}^{1-\mu}\right)\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)\left(x_{ik}R_k+\frac{1}{n}f_k\right)\right.\\
&\left.\cdot R_k+\left(x_{ik}R_k+\frac{1}{n}f_k\right)(E_{k+1}-f_k)\right]^{-\mu/2}\\
&\cdot\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k-\left(x_{ik}R_k+\frac{1}{n}f_k\right)+E_{k+1}\right.\\
&\left.-f_k\right]\left\{\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k+E_{k+1}-f_k\right]\right\}^{\mu-1}\\
&+\theta\mu R_k(2-\mu)^{(\mu-2)/2}(1-\mu)^{(4-\mu)/2}\left(\prod_{i=1,i\neq k}^{n}a_{ik}^{1-\mu}\right)\\
&\cdot\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)\left(x_{ik}R_k+\frac{1}{n}f_k\right)R_k\right.\\
&\left.+\left(x_{ik}R_k+\frac{1}{n}f_k\right)(E_{k+1}-f_k)\right]^{(2-\mu)/2}\\
&\cdot\left\{\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k+E_{k+1}-f_k\right]\right\}^{(\mu-2)/2}\\
&+\left\langle\frac{2-\mu}{2}\left\{\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k+E_{k+1}\right]-f_k\right\}\right.\\
&\left.\cdot\left(x_{ik}R_k+\frac{1}{n}f_k\right)^{-1}\frac{\mu}{2}\right\rangle(\theta\mu)^{\mu/2}[\theta(1-\mu)]^{(\mu-2)/2}\\
&\cdot\left(\prod_{i=1,i\neq k}^{n}a_{ik}^{1-\mu}\right)\left(x_{ik}R_k+\frac{1}{n}f_k\right)^{-\mu/2}\\
&\cdot\left\{\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k+E_{k+1}-f_k\right]\right\}^{\mu/2}-(\theta\mu)^{\mu/2}\\
&\cdot[\theta(1-\mu)]^{(\mu-2)/2}\\
&\cdot\left\{\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k+E_{k+1}-f_k\right]\right\}^{(\mu-2)/2}\\
&\geq\theta\mu(2-\mu)^{(\mu-2)/2}(1-\mu)^{(2-\mu)/2}\frac{2-\mu}{2}\\
&\cdot R_k\left(\prod_{i=1,i\neq k}^{n}a_{ik}^{1-\mu}\right)\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)\left(x_{ik}R_k+\frac{1}{n}f_k\right)\right.\\
&\left.\cdot R_k+\left(x_{ik}R_k+\frac{1}{n}f_k\right)(E_{k+1}-f_k)\right]^{-\mu/2}\\
&\cdot\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k-\left(x_{ik}R_k+\frac{1}{n}f_k\right)+E_{k+1}\right.\\
&\left.-f_k\right]\left\{\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k+E_{k+1}-f_k\right]\right\}^{\mu-1}\\
&+\theta\mu R_k(2-\mu)^{(\mu-2)/2}(1-\mu)^{(4-\mu)/2}\left(\prod_{i=1,i\neq k}^{n}a_{ik}^{1-\mu}\right)\\
&\cdot\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)\left(x_{ik}R_k+\frac{1}{n}f_k\right)R_k\right.\\
&\left.+\left(x_{ik}R_k+\frac{1}{n}f_k\right)(E_{k+1}-f_k)\right]^{(2-\mu)/2}\\
&\cdot\left\{\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k+E_{k+1}-f_k\right]\right\}^{(\mu-2)/2}\\
&+\left\langle\left\{\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k+E_{k+1}\right]-f_k\right\}\right.\\
&\left.\cdot\left(x_{ik}R_k+\frac{1}{n}f_k\right)^{-1}\right\rangle(\theta\mu)^{\mu/2}[\theta(1-\mu)]^{(\mu-2)/2}\\
&\cdot\left(\prod_{i=1,i\neq k}^{n}a_{ik}^{1-\mu}\right)\left(x_{ik}R_k+\frac{1}{n}f_k\right)^{-\mu/2}\\
&\cdot\left\{\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k+E_{k+1}-f_k\right]\right\}^{\mu/2}-(\theta\mu)^{\mu/2}\\
&\cdot[\theta(1-\mu)]^{(\mu-2)/2}\\
&\cdot\left\{\left[\left(1-\sum_{i=1}^{n}x_{ik}\right)R_k+E_{k+1}-f_k\right]\right\}^{(\mu-2)/2}.
\end{aligned}
\tag{28}
$$

According to the early assumption and the real implication of $[(1-\sum_{i=1}^{n}x_{ik})R_k+E_{k+1}]\geq f_k$, each item on the right-hand side in the equation is greater than 0; thus $\partial a_{ik}^{*}/\partial x_{ik}\geq 0$.

We can easily prove that Conclusion 3(ii) is right. Obviously, because every coefficient of E_{k+1} in formula (27) is greater than 0, $\partial a_{ik}^{*}/\partial E_{k+1}\geq 0$. □

Conclusion 4. VC i's share of profits of expected payment is increased with the increase of time. Namely, he wants to pay less in early phase.

Proof. The first-order condition about x_{it} in (1) is

$$
\begin{aligned}
\frac{\partial V_{i1}}{\partial x_{it}}=&\left\{\theta\mu e_1^{\mu-1}\frac{\partial e_1^{*}}{\partial x_{it}}\prod_{i=1}^{n}a_{i1}^{*1-\mu}\right.\\
&\left.+\theta e_1^{*\mu}\sum_{m=1}^{n}\left[(1-\mu)a_{m1}^{*-\mu}\frac{\partial a_{m1}^{*}}{\partial x_{it}}\left(\prod_{i=1,i\neq m}^{n}a_{*ik}^{1-\mu}\right)\right]\right\}
\end{aligned}
$$

$$\cdot \left(x_{i1}R_1 + V_2\right) + \left(\theta e_k^{*\mu}\prod_{i=1}^{n} a_{i1}^{*1-\mu}\right)\frac{\partial V_2}{\partial x_{it}} - \left\{1 - \theta e_1^{*\mu}\sum_{m=1}^{n}\left[\left(1-\mu\right)a_{m1}^{*-\mu}\frac{\partial a_{m1}^{*}}{\partial x_{it}}\left(\prod_{i=1,i\neq m}^{n} a_{*ik}^{1-\mu}\right)\right]\right\}\frac{1}{n} \cdot f_1 - a_{*i1}\frac{\partial a_{i1}^{*}}{\partial x_{it}} = 0. \tag{29}$$

Given $K = 1$ in (1), (1) can be transferred into (30) as follows:

$$V_{i1} = \left(\theta e_1^{*\mu}\prod_{i=1}^{n} a_{i1}^{*1-\mu}\right)\left(x_{i1}R_1 + V_2\right) - \left(1 - \theta e_1^{*\mu}\sum_{i=1}^{n} a_{i1}^{*1-\mu}\right)\frac{1}{n}f_1 - \frac{a_i^{*2}}{2}. \tag{30}$$

According to formula (30), there is the consequence of V_{i2}:

$$V_{i2} = \frac{1}{\left(\theta e_1^{*\mu}\prod_{i=1}^{n} a_{i1}^{*1-\mu}\right)}V_{i1} + \left(\frac{1}{\left(\theta e_1^{*\mu}\prod_{i=1}^{n} a_{i1}^{*1-\mu}\right)} - 1\right)\frac{1}{n}f_1 + \frac{1}{\left(\theta e_1^{*\mu}\prod_{i=1}^{n} a_{i1}^{*1-\mu}\right)}\frac{a_i^{*2}}{2} - x_{i1}R_1. \tag{31}$$

The first-order condition of (31) is

$$\frac{\partial V_{i2}}{\partial x_{it}} = -\frac{1}{\left(\theta e_1^{*\mu}\prod_{i=1}^{n} a_{i1}^{*1-\mu}\right)^2}\frac{\partial\left(\theta e_1^{*\mu}\prod_{i=1}^{n} a_{i1}^{*1-\mu}\right)}{\partial x_{it}}V_{i1} + \frac{1}{\left(\theta e_1^{*\mu}\prod_{i=1}^{n} a_{i1}^{*1-\mu}\right)^2}\frac{\partial V_{i1}}{\partial x_{it}} - \frac{1}{\left(\theta e_1^{*\mu}\prod_{i=1}^{n} a_{i1}^{*1-\mu}\right)^2}\frac{\partial\left(\theta e_1^{*\mu}\prod_{i=1}^{n} a_{i1}^{*1-\mu}\right)}{\partial x_{it}}\frac{1}{n}f_1 - \frac{1}{\left(\theta e_1^{*\mu}\prod_{i=1}^{n} a_{i1}^{*1-\mu}\right)^2}\frac{\partial\left(\theta e_1^{*\mu}\prod_{i=1}^{n} a_{i1}^{*1-\mu}\right)}{\partial x_{it}} - x_{i1}R_1. \tag{32}$$

There is $\partial(\theta e_1^{*\mu}\prod_{i=1}^{n} a_{i1}^{*1-\mu})/\partial x_{it} \geq 0$ and combining (32) and (29), there is also a transformation as follows:

$$\frac{\partial V_{i2}}{\partial x_{it}} = -\frac{1}{\left(\theta e_1^{*\mu}\prod_{i=1}^{n} a_{i1}^{*1-\mu}\right)^2}\frac{\partial\left(\theta e_1^{*\mu}\prod_{i=1}^{n} a_{i1}^{*1-\mu}\right)}{\partial x_{it}}V_{i1} - \frac{1}{\left(\theta e_1^{*\mu}\prod_{i=1}^{n} a_{i1}^{*1-\mu}\right)^2}\frac{\partial\left(\theta e_1^{*\mu}\prod_{i=1}^{n} a_{i1}^{*1-\mu}\right)}{\partial x_{it}}\frac{1}{n}f_1 - \frac{1}{\left(\theta e_1^{*\mu}\prod_{i=1}^{n} a_{i1}^{*1-\mu}\right)^2}\frac{\partial\left(\theta e_1^{*\mu}\prod_{i=1}^{n} a_{i1}^{*1-\mu}\right)}{\partial x_{it}} - x_{i1}R_1 \leq 0. \tag{33}$$

Given arbitrary t and l which can also fulfill $1 < t \leq l$, we can get the following from (3):

$$\max_{x_{ik},f_k,a_{ik}} V_{i2} = \max_{x_{ik},f_k,e_{ik}} \sum_{k=2}^{K}\left\{\left\langle\prod_{j=1}^{k-1}\left(\theta e_j^{\mu}\prod_{i=1}^{n} a_{ij}^{1-\mu}\right)\right\rangle\left[\left(\theta e_k^{\mu}\prod_{i=1}^{n} a_{ik}^{*1-\mu}\right)x_{ik}R_k - \left(1 - \theta e_k^{\mu}\prod_{i=1}^{n} a_{ik}^{1-\mu}\right)\frac{1}{n}f_1 - \frac{a_i^2}{2}\right]\right\}$$
$$\text{s.t:}\quad e_k^{*} = \left\{\theta\mu\prod_{i=1}^{n} a_{i1}^{*1-\mu}\left[\left(1 - \sum_{i=1}^{n} x_{ik}\right)R_k + E_{k+1}\right] - \theta\mu\prod_{i=1}^{n} a_{i1}^{*1-\mu} f_k\right\}^{1/(2-\mu)}. \tag{34}$$

The first-stage condition of x_{it} and x_{il} in (34) is

$$\frac{\partial V_{i2}}{\partial x_{it}} = \left[\prod_{j=1}^{t-1}\left(\theta e_j^{*\mu}\prod_{i=1}^{n} a_{it}^{*1-\mu}\right)\right]\left[\theta\mu e_t^{*\mu-1}\frac{\partial e_t^{*}}{\partial x_{it}}x_{it}\right) \cdot R_t\prod_{i=1}^{n} a_{it}^{*1-\mu} + \theta\mu e_t^{*\mu}R_t\prod_{i=1}^{n} a_{it}^{*1-\mu} + \theta\mu e_t^{*\mu}x_{it}R_t\left(1-\mu\right)a_{it}^{*-\mu}\frac{\partial a_{it}^{*}}{\partial x_{it}}\prod_{m=1,m\neq i}^{n} a_{mt}^{*1-\mu} + \frac{1}{n} \cdot f_t\theta\mu e_t^{*\mu-1}\frac{\partial e_t^{*}}{\partial x_{it}}\prod_{i=1}^{n} a_{it}^{*1-\mu} + \frac{1}{n}f_t e_t^{*\mu}\left(1-\mu\right)a_{it}^{*-\mu} \cdot \frac{\partial a_{it}^{*}}{\partial x_{it}}\prod_{m=1,m\neq i}^{n} a_{mt}^{*1-\mu} - a_{it}^{*}\frac{\partial a_{it}^{*}}{\partial x_{it}},$$

$$\frac{\partial V_{i2}}{\partial x_{il}} = \left[\prod_{j=1}^{l-1}\left(\theta e_j^{*\mu}\prod_{i=1}^{n} a_{il}^{*1-\mu}\right)\right]\left[\theta\mu e_l^{*\mu-1}\frac{\partial e_l^{*}}{\partial x_{il}}x_{il}\right) \cdot R_l\prod_{i=1}^{n} a_{il}^{*1-\mu} + \theta\mu e_l^{*\mu}R_l\prod_{i=1}^{n} a_{il}^{*1-\mu}$$

$$+ \theta\mu e_l^{*\mu} x_{il} R_l (1-\mu) a_{il}^{*-\mu} \frac{\partial a_{il}^*}{\partial x_{il}} \prod_{m=1, m\neq i}^{n} a_{ml}^{*1-\mu} + \frac{1}{n} \cdot f_l \theta\mu e_l^{*\mu-1} \frac{\partial e_l^*}{\partial x_{il}} \prod_{i=1}^{n} a_{il}^{*1-\mu} + \frac{1}{n} f_l e_l^{*\mu} (1-\mu) a_{il}^{*-\mu} \cdot \frac{\partial a_{il}^*}{\partial x_{il}} \prod_{m=1, m\neq i}^{n} a_{ml}^{*1-\mu} - a_{il}^* \frac{\partial a_{il}^*}{\partial x_{il}}. \tag{35}$$

In (35), the t and l are the same in other parts of stages according to arbitrary except that $[\prod_{j=1}^{t-1}(\theta e_j^{*\mu}\prod_{i=1}^{n} a_{it}^{*1-\mu})]$ and $[\prod_{j=1}^{l-1}(\theta e_j^{*\mu}\prod_{i=1}^{n} a_{il}^{*1-\mu})]$ are different.

At the same time, because $1 < t \leq l$,

$$\left[\prod_{j=1}^{l-1}\left(\theta e_j^{*\mu}\prod_{i=1}^{n} a_{ij}^{*1-\mu}\right)\right] = \left[\prod_{j=1}^{t-1}\left(\theta e_j^{*\mu}\prod_{i=1}^{n} a_{ij}^{*1-\mu}\right)\right]\left[\prod_{j=t}^{l-1}\left(\theta e_j^{*\mu}\prod_{i=1}^{n} a_{ij}^{*1-\mu}\right)\right]. \tag{36}$$

Thus one has the following according to (35):

$$\frac{\partial V_{i2}}{\partial x_{il}}\left[\prod_{j=1}^{l-1}\left(\theta e_j^{*\mu}\prod_{i=1}^{n} a_{ij}^{*1-\mu}\right)\right]\frac{\partial V_{i2}}{\partial x_{it}}. \tag{37}$$

According to the restriction of $\partial V_{i2}/\partial x_{it} \leq 0$ in (33) and the hypothesis of probability that $0 \leq \prod_{j=1}^{l-1}(\theta e_j^{*\mu}\prod_{i=1}^{n} a_{ij}^{*1-\mu}) \leq 1$, we can get

$$\frac{\partial V_{i2}}{\partial x_{il}}\left[\prod_{j=1}^{l-1}\left(\theta e_j^{*\mu}\prod_{i=1}^{n} a_{ij}^{*1-\mu}\right)\right]\frac{\partial V_{i2}}{\partial x_{it}} \geq \frac{\partial V_{i2}}{\partial x_{it}}. \tag{38}$$

Consequently, to any arbitrary t and l which meet with $1 < t \leq l$, we can always have $\partial V_{i2}/\partial x_{it} \leq \partial V_{i2}/\partial x_{il}$; thus Conclusion 4 is proved. □

5. Multistage Optimal Investment Contracts

In Section 3, we mainly research into the optimal effort level between VCs and EN, which naturally leads us to another question which is, according to optimal effort levels e_j^* and a_{ij}^*, what is the optimal investment contract? That is to say, we need to discuss optimal shared coefficient x_i and optimal fixed income f_i. The result is presented by Conclusion 5.

Conclusion 5. During the multistage venture capital joined by multiple VCs, if the venture project fails in the stage k, the optimal contract offered by VC will go as $x_{ik}^* = 1$, $y_k^* = 0$, and $f_k^* = 0$; if it succeeds, there would be a stage $k_0 \in \{1, 2, \ldots, K\}$.

When $k \geq k_0$, there follows the optimal contract offered by VC:

$$x_{i(k+1)}^* = \frac{R_{k+1}^* + V_{i(k+2)} + (1/n) f_k^* + c\left(a_{i(k+1)}^*\right)}{R_{k+1}^*} > 0, \quad k = k_0+1, \ldots, K,$$
$$y_k^* = 1 - \sum_{i=1}^{n}\left[\frac{R_{k+1}^* + V_{i(k+2)} + (1/n) f_k^* + c\left(a_{i(k+1)}^*\right)}{R_{k+1}^*}\right], \quad k = k_0+1, \ldots, K. \tag{39}$$

When $k < k_0$, the optimal contract offered by VC goes to $x_{ik}^* = 1$, $y_k^* = 0$.

Proof. Firstly, we demonstrate the case when the project fails.

When $(\partial V_{ik}/\partial x_i(k-1))|_{e_k^*, a_{ik}^*} = 0$, $k = 1, 2, \ldots, K$, there would be

$$(1 - x_{ik}) R_k + V_{ik} + \frac{1}{n} f_k^* + c\left(a_{ik}^*\right) = 0. \tag{40}$$

It leads to

$$V_{ik} = \left(\theta e_k^{*\mu}\right)\left[\left(1 - x_{ik}^*\right) R_k + V_{i(k+1)} + \frac{1}{n} f_k^* + c\left(a_{ik}^*\right)\right] + c\left(a_{ik}^*\right) - \frac{1}{n} f_k^* - c\left(a_{ik}^*\right) = -\frac{1}{n} f_k^*. \tag{41}$$

Because $V_{ik} \geq 0$, there is $f_k^* = 0$. Taking $f_k^* = 0$ into the optimal profit, $x_{ik}^* = 1$, $y_k^* = 0$ must exist.

Then we demonstrate the case when the project succeeds.

When it succeeds, there would be $(\partial V_{i1}/\partial x_{ik})|_{e_k^*, a_{ik}^*} = 0$, $k = 1, 2, \ldots, K$. It is not the optimal strategy when $x_{ik}^* = 0$. If it is, then $R_{k+1} + V_{i(k+2)} > 0$, $(1/n) f \geq 0$, which means $(\partial V_{i1}/\partial x_{ik})|_{e_k^*, a_{ik}^*} > 0$, $k = 1, 2, \ldots, K$, exists. But clearly that strategy of venture investor is optimal when $x_{ik}^* = 0$ is untenable.

When $k > k_0$, there follows

$$x_{i(k+1)}^* > 0, \quad \left.\frac{\partial V_{i1}}{\partial x_{i(k+1)}}\right|_{e_k^*, a_{ik}^*} = 0, \quad k = 1, 2, \ldots, K, \tag{42}$$

$$(1 - x_{ik}) R_k + V_{ik} + \frac{1}{n} f_k^* + c\left(a_{ik}^*\right) = 0. \tag{43}$$

According to (43),

$$x_{i(k+1)}^* = \frac{R_{k+1}^* + V_{i(k+2)} + (1/n) f_k^* + c\left(a_{i(k+1)}^*\right)}{R_{k+1}^*} > 0, \quad k = k_0+1, \ldots, K. \tag{44}$$

And there is

$$\sum_{i=1}^{n} x_{i(k+1)}^{*} + y_k^{*} = 1. \tag{45}$$

So,

$$y_k^{*} = 1 - \sum_{i=1}^{n} \left[\frac{R_{k+1}^{*} + V_{i(k+2)} + (1/n) f_k^{*} + c\left(a_{i(k+1)}^{*}\right)}{R_{k+1}^{*}} \right]. \tag{46}$$

When $x_{ik}^{*} = 1, y_k^{*} = 0$, there is $(\partial V_{i1}/\partial x_{ik})|_{e_k^{*}, a_{ik}^{*}} \leq 0,\ k = 1, 2, \ldots, K$.

Because $R_k + V_{i(k+1)} > 0$, we can know that $(\partial V_{i1}/\partial x_{i(k-1)})|_{e_k^{*}, a_{ik}^{*}} \leq 0,\ k = 1, 2, \ldots, K$. As a consequence, when $x_{i(k-1)}^{*} = 1,\ y_{k-1}^{*} = 0$, there is k_0.

When $k < k_0$, there is the result that $x_{i(k-1)}^{*} = 1,\ y_{k-1}^{*} = 0,\ k = 1, 2, \ldots, k_0$.

The practical significance of Conclusion 5 lies in the following.

During a multistage united venture capital investment, there exists a period of k_0. Before this period, EN only has a fixed income without later profit sharing; however, after k_0, VCs and EN share project proceeds together. Instinctively, for the EN, the shorter the period k_0 is the better, while for VCs, the longer the period k_0 is the better. This is because the success of the later stage depends on that of the earlier stages, which stimulate ENs to some extent in the earlier stages when return is paid later. As a consequence, the return should be paid to ENs as late as possible, which will serve as the most convenient and inexpensive way of arranging incentive contract. Now we come across another problem: whether we can find out the optimal period k_0. To complete the reasoning, we need further research because of the model's complexity.

During multistage united venture capital investments, there exists a period of k_0. Before this period, ENs only have a fixed income without later profit sharing while after, VCs and ENs share project proceeds. Instinctively, for ENs, the shorter is the better when considering period k_0, while the longer is the better for VCs, for success of the late stage depends on that of the early one, which stimulate ENs in the early stage to some extent when return is paid later. As a consequence, the return should be paid to ENs as late as possible, which will serve as the most convenient and inexpensive way of incentive contract arrangement. Now we come across another problem: whether we can find out the optimal period k_0. To complete the reasoning, we need further researches because of the complexity of the model. □

6. Conclusions

This paper focuses on a venture investment problem in which several VCs make an alliance to invest in the same venture project in a venture capital market. Whatever financing pattern we take, once what to invest on has already been decided by VCs, how to achieve the venture project and how to realize its added value will become the common goals for both VCs and ENs. This requires them to positively participate in the development of the venture project and to play dual roles both in providing financing funds and in promoting added value of the project's output. In the process of development, the startup company needs joint effort of VCs and ENs to promote growth. However, improving the managerial capabilities of this venture company as well as lowering company operating costs requires ENs' efforts to promote the creative value of core technology and requires as well VCs' offering specialized services, which will obviously result in the information asymmetry of two parties and therefore further bilateral moral hazard of investment.

Further, in the operation practice of venture capital, from entering the venture enterprise to its exit, there remains a long-term process [5, 29]. When we built the game model, the ENs' and VCs' efforts, the sharing contract, and the project's operation are all dispersed into the multistaged problem. Multistage dynamic game model is thus constructed, and its significance is revealed as follows. (1) The model is consistent with the practice of venture capital. (2) Multistaged investment is a way to deal with this sort of moral hazard.

In terms of the considerations mentioned above, this paper separates the efforts of venture investment, VCs and EN, into multiple phases and then establishes the Stackelberg game model in a multistaged venture investment. By analyzing our model, dynamic subgame perfect Nash equilibrium (dynamic SPNE) is given, and ENs' and VCs' optimization efforts are therefore resolved. We have also collected some significant results, specifically as follows.

(1) Optimal efforts of ENs are positively correlated with their future earnings and effort put out by VCs, while they negatively correlate with shares of sharing of VCs and the fixed return.

(2) Venture investment is regarded as a multistage investment; due to the uncertainty of future earnings, ENs expect earlier cash and thus want to obtain more revenue in earlier stages and less revenue in later stages. This is to say that ENs hope to be paid more at early stages and less at later ones.

(3) Optimal efforts of VCs are positively correlated with their shares of sharing and the future earnings of ENs.

(4) The goal of VCs is precisely opposite to ENs. VCs are expecting more profits in the future. Therefore, in earlier stages VCs hope to get less benefits, while ENs expect more profits.

However, our research is still limited with some problems that need further discussion, especially the following.

Firstly, the paper has studied issues about joint investment which requires equality among multiple VCs. But in practice, some of them may gain more advantages because of their stronger companies and larger capitals. This is in fact what we called syndication market, and, therefore, in the market we should look into multiple-staged investments.

Secondly, the paper has studied the optimal strategies of both VCs and ENs and in the meantime leaves out further considerations about the optimal exit point of VCs.

Thirdly, among multiple strategies, strategies of the later stage would be affected by the earlier ones, and this sluggish relationship needs a further discussion.

Fourthly, the amount of financing needed by ENs and capital held by VCs are both limited. Then How many venture capitalists should be financed under these constraints?

Lastly, we have only studied venture capital investment problems with a discrete multistaged model. We might also need to explore deeply the venture investment with a continuous time model. This remains an aspect that needs a further discussion.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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