Research Article

Evolution of Cooperation with Moore Neighborhood and Self-Playing Rule

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Evolutionary spatial game is a promising way to unravel the mystery of cooperation, and it has been well recognized that spatial structure could enable cooperation to persist. Schweitzer et al.'s lattice model provides an innovative method to solve the problem. In this paper, we conduct simulations using the same von Neumann neighborhood as in Schweitzer et al.'s study (2002) and observe the effect of initial population and lattice size on the evolution of cooperation. Then, we extend the model with a more complicated Moore neighborhood and self-playing rule for each central player. Simulation results not only provide new evidence for the persistence of cooperation in the evolution with spatial structures, but also investigate critical conditions for the spatial coexistence or the invasion of cooperators and defectors with the more complicated neighborhood.

1. Introduction

Cooperation is one crucial driving force for human society to burgeon and flourish. Actually, it is a fundamental factor that impels nature to evolve unceasingly into the amazing world. As a ubiquitous phenomenon, cooperation can be found everywhere, from foraging ant colonies to workers in assembly lines, from evolving chromosomes in evolution of human to communicating nodes in ad hoc networks [1–3]. However, why can the cooperation be sustained and evolved?

To research into the evolution of cooperation, evolutionary game is a powerful framework [4, 5]. Since evolutionary stable strategy (ESS) was first proposed by Smith and Price [6], it has drawn a lot of attention from biologists, economists, and game theorists. Its striking advantages originate from the expansion of classic game theory in aspects of both bounded rationality and dynamics. As Axelrod remarked [7], “the evolutionary approach is based on a simple principle: whatever is successful is likely to appear more often in the future”; thus successful strategies spread widely and less successful ones are doomed to be wiped out by virtue of nature selection.

Prisoner's dilemma is a most famous paradigm to dig into the emergence and persistence of cooperation [8]. As

a two-person nonzero sum game, it reflects profound conflict between collective rationality and individual rationality. Everyone in this game has two pure strategies: cooperate and defect (resp., denoted by C and D), and they have to make decisions simultaneously. The source generating dilemma situation is that either prisoner can achieve maximum payoff through adopting D. In evolutionary contexts where population is well-mixed, defection is the only ESS. However, the iterated prisoner's dilemma introduced by Axelrod casts a light on this issue [7]. Axelrod conducted computer tournaments and identified that tit for tat (TFT) strategy, which simply cooperates at the first round and then takes whatever the opponent did in his last round, surprisingly performed best. The force behind this strategy that drives cooperation to evolve is kindness, forgiving, and retaliatory [2]. In fact, TFT is an ESS when the shadow of the future is large enough and can invade the population of defection through forming clusters.

In order to take a step towards realistic situations, spatial structure is further introduced into evolutionary prisoner's dilemma [9]. Axelrod seems to be the first one modeling this [2]. He considered that a territory is subdivided into

square cells, with each cell occupied by one player. Everyone only plays games with four nearest neighbors and updates his strategy by imitating the strategy used by the most successful neighbor in the last round. After simulations, he drew a conclusion that strategies in the spatial structure perform at least as good as mean-field structure, and he also revealed the process that one TFT invader spreads to the most population composed by the ALLD (all-defect) strategy.

In this paper, we focus on how to demonstrate various simulation results with respect to dynamics of lattice models. Schweitzer et al. [10] put forward an innovative way to comprehend the spatial effect. In Schweitzer et al.'s work [10], the evolution is based on the von Neumann neighborhood where each player has four nearest neighbors. However, different neighborhood structures may lead to different evolutionary patterns of the invasion or the coexistence of cooperator and defectors. Meanwhile, Schweitzer et al. [10] assumed each player could not play with itself. The assumption means each player cannot learn from itself, which is obviously inconsistent with the natural and especially social evolution of cooperation. Motivated by these observations, in this paper we first conduct simulations as Schweitzer et al. did to make further analysis on the evolution of cooperation with the von Neumann neighborhood and more importantly extend the model to a complicated level where the neighborhood turns into the structure of Moore and each cell can play games with itself. Based on our simulations, we analyze the effect of initial population and system size on the evolution of cooperation and investigate critical conditions for the spatial coexistence or the invasion of players with the Moore neighborhood and self-playing rule.

In Section 2, we give a brief account of current works on evolutionary spatial game. In Section 3, the evolution models with the von Neumann and Moore neighborhoods are, respectively, presented. In Section 4, we conduct simulations with different models and conditions to investigate the corresponding evolution patterns. Conclusions are drawn in Section 5, with recommendations on future studies.

2. Literature Review

Spatial structure can enable the persistence of cooperation in the evolutionary prisoner's dilemma game, which is a very positive and promising result and first derived by Nowak and May [11, 12]. Like Axelrod's model, players are located in a two-dimensional lattice, and everyone merely communicates with neighbors surrounding them. To observe the spatial effect, they pulled out the repeated effect by assuming that every player only has two choices in one generation: C and D, and they also adopted deterministic update rule: imitate-the-best. By the succinct model, Nowak and May found that C and D persist together indefinitely to reach variety of spatial and temporal dynamics dependent on the value of a polymorphic parameter b which includes stable, quasistable, dynamic fractal, and chaotically varying states. A decade later, this theoretical prediction was confirmed by biological experiments [13]. Huberman and Glance doubted the above model's usefulness for real-world systems [14] and proposed an asynchronous updating system where players

can be updated at any time. Besides the lattice-based models, more and more attention has been paid on network-based evolutionary games. Detailed advances regarding this could refer to recent works by Szabó and Fáth [15], Poncela et al. [16], Perc [17], and Vilone et al. [18].

Stochasticity is very important for finite population, so the scores obtained by the cells within the neighborhood do not fully determine which player will take over in the next generation but only specify the probability that a player will take over. Killingback and Doebeli extended their research object by studying what evolutionary dynamics Hawk-Dove game can bring. As a result, it has higher level of cooperation and the spatiotemporal chaos is quite different from prisoner's dilemma [9, 19]. Ellison concentrated on the coordination games in spatial structure and showed that they quickly converge to the risk-dominant equilibrium [20]. Szabó et al. studied the spatial evolutionary games with myopic players whose payoff interest is tuned from selfishness to other-regarding preference via fraternity [21, 22].

Recently, more social behaviors are considered in the evolutionary dilemma. Wang et al. [5] argued that the strategy choice of players should be affected by all their neighbors rather than the most successful neighbor and proposed an adaptive strategy-adoption rule to imitate the real-world social influence. Du and Fu [23] thought noise may be present in individual strategy learning and compared the impact of noise on spatial organization of cooperation. Gruji et al. [24] carried out comparative experiments and found that moody conditional cooperation is more supported than noninnovative game dynamics such as imitate-the-best or pairwise comparison rules. Cong et al. [25] introduced the reputation into the evolution model of the prisoner's dilemma and analyzed the effects of population density, mobility sensitivity, and reputation memory on the emergence and persistence of cooperation. Li et al. [26] considered the trust between each player and its neighbors to study the coevolution of quantum strategies on scale-free networks.

Moreover, a lot of works considering specific evolutionary mechanisms and interaction topologies have been contributed. In some recent reviews, corresponding advances as well as future directions have been well identified. The coevolutionary rules could be regarded as a natural upgrade of evolutionary games, but how kinds of rules influence the evolution of cooperation needs to be investigated. Motivated by this, Perc and Szolnoki [27] presented a detailed review on the effects of coevolutionary rules on dynamical interactions between players, population growth, teaching activity, mobility, and aging of players. Nowak suggested that there are five rules for the evolution of cooperation: kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection [28]. Specifically, Perc et al. [29] reviewed advances regarding the evolutionary dynamics of group interactions on structured populations and made a comparison with the results obtained on well-mixed populations. Interaction topology is another key affecting the evolution of cooperation. As summarized in [27], common interaction topologies include rock-paper-scissors game, the ultimatum game, and public goods game. Szolnoki et al. [30] presented a latest review on cyclic dominance in evolutionary games.

Although a lot of simulation models and findings have been achieved, there are few well-accepted theoretical works focusing on how to demonstrate various simulation results with respect to dynamics of lattice models. To the best of our knowledge, Szabó and Fáth [15] introduced the technique of generalized mean-field to analyze lattice system, and another fascinating method is pair approximation which considers the pair correlations between nearest neighbors [31]. Hutson and Vickers [32] used partial differential equations to build reaction-diffusion models and formulate two-dimensional grid dynamics. Schweitzer et al. [10] put forward an innovative way to comprehend the spatial effect. As mentioned in Section 1, in this work we conduct simulations using the same von Neumann neighborhood with [10] to observe the effect of initial population and system size on the evolution of cooperation. Then, we extend the model with a more complicated Moore neighborhood and self-playing rule for each central player, in order to investigate critical conditions for the spatial coexistence or the invasion of players in the extended evolution model.

3. The Model

Our model is built on the structure of a two-dimensional lattice. Each cell is occupied by one player, and everyone is only connected with the nearest neighbors. There are two kinds of neighborhood we will investigate. One is von Neumann neighborhood where only four nearest neighbors are considered, as Figure 1 shows. The other is Moore neighborhood where eight nearest neighbors are taken into account, as Figure 2 shows.

Instead of the mean-field structure in global models, individuals can only make local contacts by playing prisoner's dilemma with neighbors. Also, there are no iterated games, no complicated algorithms, and no memories. Every individual only plays games with neighbors once in every generation and is restricted to choose from two strategies: cooperate or defect. At the end of each generation, each player sums up its payoff against neighbors as its present fitness and decides what kind of strategy to use in the next generation. Players cannot determine what to do by collecting information to anticipate what others will do in the next generation but simply imitate the player whose strategy gives rise to the highest fitness. If there is a tie, every player always keeps its original strategy because switching strategy requires paying the price.

The above updating rule has twofold meanings. From the perspective of biology, strategies with higher fitness can survive and thrive; they will spread gradually to take over the majority of cells, while the ones with lower fitness are doomed to die out. From the perspective of sociology, people tend to adopt strategies proved to be more successful. As we can see, this is a deterministic and synchronous updating rule. Players may be regarded as cellular automata which change their states synchronously at discrete time steps, and the next state of each cell depends on the current states of the neighboring cells according to an update rule. All cells use the same rule, and the rule is applied to all cells at the same time [33]. In the model, the array has periodic boundaries, so the marginal

FIGURE 1: Von Neumann neighborhood where each cell has four nearest neighbors.

FIGURE 2: Moore neighborhood where each cell has eight nearest neighbors.

cells on one side of the array are regarded as the neighbors of the cells on the opposite margin of the lattice.

In the lattice with $n \times n$ cells, we denote a finite set of cells by $C = \{i, i = 1, 2, \dots, n \times n\}$ and a set of corresponding states by $S = \{s_i^t, i = 1, 2, \dots, n \times n, t = 1, 2, \dots\}$, where s_i^t means the state of cell i in the generation t . As mentioned above, every cell's state is only concerned with its neighbors, so, in Moore structure, each cell has a neighborhood with eight cells. We denote neighbors of cell i by $N^i = \{n_0^i, n_1^i, \dots, n_8^i\}$, in which n_0^i represents cell i itself. When cell i begins to play games with his neighborhood, it may receive the payoff $U_i = \sum_{j=0}^8 u_i(i, n_j^i)$, where $u_i(i, n_j^i)$ represents the payoff of cell i over its neighbor n_j^i . As a matter of fact, we count the payoff that cell i plays games with itself, which seems to be paradox but is very common in nature. We just need to regard every cell as a group, and then this consideration becomes reasonable. Last but not least, the force at the core in the dynamic system, that is, the updating rule, is $F : s_i^{t+1} = s_{n_j^i}^t, j = \arg \max_{j=0,1,\dots,8} U_{n_j^i}$.

Specifically speaking, there are only two states that every cell has $s_i = 0$, which means the cell defects, or $s_i = 1$, which means the cell cooperates. The interaction between

TABLE 1: Payoff matrix of prisoner's dilemma.

Player i	Player j	
	C	D
C	(R, R)	(S, T)
D	(T, S)	(P, P)

two cells is playing prisoner's dilemma game, and either of them does not have any prior knowledge about what the other will do. After all players choose their strategies from C and D, they will receive the payoffs according to the payoff matrix in Table 1. T means temptation payoff for defecting in a cooperative environment, S means sucker's payoff for cooperating in a defecting environment, R means reward payoff for cooperating in a likewise environment, and P means punishment payoff for defecting in a likewise environment [10].

They both get payoff R as reward if they both cooperate, while they receive payoff P as punishment if they both defect. A player using D obtains the payoff T against a C-player, and the counterpart has to suffer the minimum income S as for sucker, which signifies that selfish player can exploit the cooperator. The source leading to the dilemma lies in $T > R > P > S$. To promote cooperation and maintain this difference, assuming $2R > T + S$ is also necessary, which implies that the only Pareto efficient result is that they both cooperate.

4. The Spatial Dynamics and Analysis

In Schweitzer et al.'s work [10], they consider a cell and its nearest neighbors as a group, and what happens among them is regarded as an n -person game even though the actual behavior among each other in this group is independent. The group game can be decomposed as a combination of two-person games happening $n - 1$ times, and the payoff of the central cell in each generation is the sum of its payoffs by playing with every neighbor. From this perspective, the fitness of each cell is impacted intensely by the number of cooperators in the neighborhood. For instance, cell i has k neighbors who are cooperators under the structure of Moore neighborhood. Then it can get the payoff:

$$U_i = \sum_{j=0}^8 u_i(i, n_j^i) = (k+1)R + (8-k)S = (R-S)k + R + 8S. \quad (1)$$

If its own state $s_i = 1$, then it can get the payoff:

$$U_i = \sum_{j=0}^8 u_i(i, n_j^i) = kT + (9-k)P = (T-P)k + 9P. \quad (2)$$

If its own state $s_i = 0$, the game of cell i ends in the generation.

Due to $T > R > P > S$, payoff U_i is an increasing function of the number of cooperators. Thus, no matter what state a cell is in, the more cooperators there are in its neighborhood,

the higher fitness it will receive. Since the updating rule is determined by the cell with the highest score in the group, it is essential to take into account the whole configuration. Let the configuration Q_i^j indicate that the state of the central cell in its neighborhood is i and the number of cooperators in this group is j rather than standalone cell configuration. Apparently, the value of i can be 1 or 0, and $0 \leq j \leq 8$ in the structure of Moore neighborhood. Each Q_i^j corresponds to u_i^j that indicates the payoff of cell i against the j th cooperator in its group. We can get some simple summaries from the definition:

- (1) If $j > k$, then $u_0^j > u_0^k$ and $u_1^j > u_1^k$ (as we mentioned above);
- (2) $u_0^{j+1} > u_1^j$, $j = 0, 1, 2, \dots, 8$, because

$$\begin{aligned} u_0^{j+1} - u_1^j &= jT + (9-j)P - jR - (9-j)S \\ &= j(T-R) + (9-j)(P-S) > 0. \end{aligned} \quad (3)$$

Based on this theoretical foundation, Schweitzer et al. [10] did some simulations in the model of von Neumann structure. We extend the model to the Moore neighborhood and add the self-playing for every cell. The aim of this paper is to verify the accuracy of this theory and dig deeper into the application environment of the theory.

The flash point that makes the theory so special is that it transforms invasion from a macroscopic view into a microstructural one. There is a concept of invasion for every group when the value of i in Q_i^j changes. If i turns into 1 from 0, it can be regarded that configuration Q_0^j invades configuration Q_1^j . In our model, there are 18 different kinds of configurations, that is, $\{Q_0^0, Q_0^1, Q_0^2, Q_0^3, Q_0^4, Q_0^5, Q_0^6, Q_0^7, Q_0^8\}$ and $\{Q_1^0, Q_1^1, Q_1^2, Q_1^3, Q_1^4, Q_1^5, Q_1^6, Q_1^7, Q_1^8\}$. Since there is no room for invasion in the configuration Q_0^0 , we do not consider it. Configuration Q_1^0 is extremely unstable which rapidly turns into Q_0^0 after one generation, so we also rule it out.

Through the above discussion, we have

$$\begin{aligned} u_0^1 &< u_0^2 < u_0^3 < u_0^4 < u_0^5 < u_0^6 < u_0^7 < u_0^8, \\ u_1^1 &< u_1^2 < u_1^3 < u_1^4 < u_1^5 < u_1^6 < u_1^7 < u_1^8. \end{aligned} \quad (4)$$

So if the following condition is met, $u_1^8 < u_0^1$, the highest score a cooperator can obtain is still lower than the lowest score a defector can receive. Then, cooperators are doomed to be invaded, and defectors are able to invade cooperators completely no matter what initial distribution they have. However, there is no way by which cooperators completely invade defectors because $u_1^1 < u_1^7 < u_0^8$. Meanwhile, $u_1^1 < u_0^{j+1}$, $j = 1, 2, \dots, 7$ implies that structure Q_1^j must be invaded by structure Q_0^{j+1} . Since there is no limitation on u_1^{j+1} and u_0^{j+1} , if $u_1^j < u_0^{j+1} < u_1^{j+1}$ is satisfied, the power of aggression of Q_0^{j+1} will be crippled, and if $u_0^{j+1} < u_1^{j+1}$ is not satisfied, $u_0^{j+1} < u_1^{j+2}$ can serve as the second strict restrictive condition, and so forth. We may get several different patterns in lattice system with the relaxation of limitation on Q_0^j 's invasion.

The payoff in classic prisoner's dilemma is $T = 5, R = 3, P = 1, S = 0$. To investigate varying degrees of limitation on the invasion of Q_0^j , we set the related parameters as follows: $R = 3, S = 0, u_0^j$ being a binary variable of T and P , and u_1^j as a constant. As a result, we could partition the P, T -plane according to the different rank ordering of R, S, T , and P .

4.1. The Spatial Dynamics of von Neumann Neighborhood without Self-Playing. In this section, like Schweitzer et al.'s work [10], each cell only communicates with its four nearest neighbors in von Neumann neighborhood and does not play game with itself. Here, u_0^j ($j = 1, 2, 3, 4$) provides four kinds of lines with different gradients, u_1^j ($j = 2, 3, 4$) stands for three different intercepts, and each gradient is matched with several intercepts according to the rule $u_0^j > u_1^j$. In the P, T -plane, there are six lines intersecting with each other and cutting apart the plane with eleven areas. The lines are, respectively, represented by

$$\begin{aligned} u_0^1 = u_1^4 &\implies T + 3P = 12; & u_0^1 = u_1^3 &\implies T + 3P = 9; \\ u_0^1 = u_1^2 &\implies T + 3P = 6; & u_0^2 = u_1^4 &\implies T + P = 6; \\ u_0^2 = u_1^3 &\implies T + P = 4.5; & u_0^3 = u_1^4 &\implies 3T + P = 12. \end{aligned} \quad (5)$$

The inside plane formed by two crossover points is regarded as a classification basis for six lines. Obviously, lines intersecting at the same point can be classified into the same category. Therefore we have three sets: $E_1 = \{l_{12}, l_{23}, l_{34}\}$, $E_2 = \{l_{13}, l_{24}\}$, and $E_3 = \{l_{14}\}$, in which l_{ij} represents straight line $u_0^i = u_1^j$. Each set also corresponds to a level of degree e_1 that indicates how intense it influences various configurations, and e_1 is the most intense class of all and so forth. Then we discuss distinct regions in accordance with simulation of spatial prisoner's dilemma.

Simulation I. The most stringent limitation for configuration Q_0^j which makes defectors too weak to recklessly invade cooperators is $u_0^i < u_1^{i+1}, i = 1, 2, 3$, namely,

$$u_0^1 < u_1^2, \quad u_0^2 < u_1^3, \quad u_0^3 < u_1^4. \quad (6)$$

From another point of view, this group of conditions enables cooperators with various sorts of configuration to invade the defectors of configuration in the process of evolution. Cooperation occupies the majority of domain in this situation as we expect, stabilizing above the average percentage of 78%. Defectors are clustered in linked curves, carving up the sea of cooperation as massive regions and sometimes accompanied by periodic oscillating cross, as presented in Figure 3.

In Figures 3, 4, and 5 red cells represent players who defect in this generation and in the previous generation, blue is cooperation following cooperation, green is cooperation following defection, and yellow is defection following cooperation.

FIGURE 3: Simulation results on a 50×50 square lattice with von Neumann neighborhood and random initial distribution of players. This pattern is performed by the parameters region: $u_0^1 < u_1^2, u_0^2 < u_1^3, u_0^3 < u_1^4$.

Simulation II. We loosen the restriction for configuration Q_0^j to change the sign of inequality of one condition, which produces two different feasible regions:

$$u_0^1 > u_1^2, \quad u_0^2 < u_1^3, \quad u_0^3 < u_1^4, \quad (7)$$

$$u_0^1 < u_1^2, \quad u_0^2 < u_1^3, \quad u_0^3 > u_1^4. \quad (8)$$

In the situation formed by (7), the number of defectors increases sharply and captures the most territory under both circumstances. Cooperation is found to exist in the form of big clusters, making up about 16% of players, and it is hard to find any periodically oscillating pattern, as Figure 4(a) shows.

The third situation is presented in Figure 4(b) where a peculiar phenomenon reveals itself: there is no terminal point for the evolution, and the pattern is changing all the time with high level of oscillation. This phenomenon can be referred to as spatial chaos: at first, defection dominates most parts, but cooperators cluster to compete with rivals; every time any clusters of cooperation are close enough to each other, the defectors between them will take advantage of them to grow; as the number of their alliances reaches to a point, all of them are facing an unfavorable payoff resulting in that cooperation has the better hand again; repetition of devouring of both sides brings about the phenomenon we have seen. The defectors outnumber cooperators in Figure 4(b) in the ratio of three to two.

Simulation III. We make further efforts to relax the restriction of defectors. The sign of inequality in two conditions has been changed, resulting in two different feasible regions:

$$u_0^1 > u_1^2, \quad u_0^2 > u_1^3, \quad u_0^3 < u_1^4, \quad (9)$$

$$u_0^1 < u_1^2, \quad u_0^2 > u_1^3, \quad u_0^3 > u_1^4. \quad (10)$$

Figure 5(a) shows the final state under conditions (9). Cooperation is suffering to survive as small clusters of crosses

FIGURE 4: Simulation results on a 50×50 square lattice with von Neumann neighborhood and random initial distribution of players. The pattern in (a) is performed by the parameters region: $u_0^1 > u_1^2$, $u_0^2 < u_1^3$, $u_0^3 < u_1^4$, and the pattern in (b) is by the region: $u_0^1 < u_1^2$, $u_0^2 < u_1^3$, $u_0^3 > u_1^4$.

FIGURE 5: Simulation results on a 50×50 square lattice with von Neumann neighborhood and random initial distribution of players. The pattern in (a) is performed by the parameters region $u_0^1 > u_1^2$, $u_0^2 > u_1^3$, $u_0^3 < u_1^4$, and the pattern in (b) is by the region $u_0^1 < u_1^2$, $u_0^2 > u_1^3$, $u_0^3 > u_1^4$.

reaching the average percentage of almost 7.5%, and the pattern is perfectly stable without oscillation. When we examine the matter from different angles, group of conditions (9) is apparently further relaxation for conditions (7), and the clusters have been transformed from big to small and from many to rare, as Figures 4(a) and 5(a) show.

Group of conditions (10), which is the relaxation of the third, yields a sort of special pattern for von Neumann neighborhood. Cooperators grow as crosses, following the trace of the four directions, and there are always some separately oscillating crosses, as Figure 5(b) shows. In a nutshell, the ratio of T to P determines the stability level of the ultimate state. The larger the ratio is, the more oscillating the crosses will appear.

Simulation IV. Removing all the limitations for defectors in the first class, we have

$$u_0^1 > u_1^2, \quad u_0^2 > u_1^3, \quad u_0^3 > u_1^4. \quad (11)$$

The simulated pattern shows the domination of defection and the average percentage of cooperation does not exceed 0.6%. There is absolutely no cooperator under the assumption of initial random distribution especially when $u_0^2 > u_1^4$, $u_0^1 < u_1^3$.

Discussions. From the above simulation results, we have observed the phenomena of steady or oscillating coexistence between cooperators and defectors. These results verify that

cooperators and defectors could coexist in the evolution if the local spatial structure is considered. Meanwhile, different combinations of conditions on players have resulted in different evolutions. What the results indicate is somewhat similar with the underlying evolution of real-world nature and society. The neighborhood of each cell is like the relationship of each living being, and the parameters region is like the role of local environment setting.

We already know that cooperators are inevitable to die out while $u_0^1 > u_1^4$ no matter what the initial distribution is. As a matter of fact, we have observed huge differences of results from those in Schweitzer et al.'s work [10], especially in the regions $\{u_0^1 > u_1^2, u_0^2 < u_1^3, u_0^3 < u_1^4\}$ and $\{u_0^1 > u_1^2, u_0^2 > u_1^3, u_0^3 < u_1^4\}$. They showed that these regions stand for the case of unstable coexistence, which means the dynamics will always lead to an invasion of the defectors except for some special case. However, our simulations show that there are always stable clusters in the regions and sometimes even very big clusters.

After making a detailed comparison with [10], we find that the main differences of our simulations from Schweitzer et al.'s are the initial population of players and the lattice size. In [10], the initial distribution was randomly generated such that the number of defectors was equal to that of cooperators, and the lattice size was 40×40 ; In this work, we randomly assign each player on a 50×50 lattice as 0 (to defect) or 1 (to cooperate), so the number of initial defectors is basically equivalent to that of cooperators. Thus, the inconsistent results from [10] may be owing to the different initial structures of players and the lattice size.

In order to verify the impact of the randomness of initial structures on evolutionary coexistence, we conducted additional simulations, respectively, with regions $\{u_0^1 > u_1^2, u_0^2 < u_1^3, u_0^3 < u_1^4\}$ and $\{u_0^1 > u_1^2, u_0^2 > u_1^3, u_0^3 < u_1^4\}$. Like [10], we use the global frequency of cooperating players to show the evolution of these simulations, as Figure 6 shows.

From the results in Figure 6, we could find (i) for the two regions $\{u_0^1 > u_1^2, u_0^2 < u_1^3, u_0^3 < u_1^4\}$ and $\{u_0^1 > u_1^2, u_0^2 > u_1^3, u_0^3 < u_1^4\}$, all the conducted evolutions although with different random initial distributions finally came into the stable coexistence of cooperators and defectors; (ii) the frequency of cooperators in the initial distribution has direct impact on the ratio of players in the final coexistence.

In order to investigate the effect of population size on the evolution, we extend the simulations to square lattices in different sizes. As Figure 7 shows, we obtain the final frequencies of cooperators with lattice sizes ranging from 10×10 to 80×80 . Due to space limitations, we just give the results performed by the parameters region $\{u_0^1 > u_1^2, u_0^2 < u_1^3, u_0^3 < u_1^4\}$ and $\{u_0^1 > u_1^2, u_0^2 > u_1^3, u_0^3 < u_1^4\}$. From the results in Figure 7, we could have the following observations:

- (1) For the parameters region $\{u_0^1 > u_1^2, u_0^2 < u_1^3, u_0^3 < u_1^4\}$, the final states on all the lattices evolve into steady coexistence of cooperators and defectors, with the slightly fluctuated frequencies of cooperators in the final patterns.

- (2) For the parameters region $\{u_0^1 > u_1^2, u_0^2 > u_1^3, u_0^3 < u_1^4\}$, the pattern turns into the complete invasion by defectors when the square lattice is 10×10 , but all other patterns evolve into steady coexistence of cooperators and defectors.

To make more comprehensive investigation of the validity of our results, in the following subsection we extend the model to a more complicated level: the neighborhood turns into the formation of Moore and each cell can play games with itself. The self-playing rule is inspired by the inheritance of species in nature, which would have the side effect to improve the performance of cooperators.

4.2. The Spatial Dynamics of Moore Neighborhood with Self-Playing. As mentioned above, with Moore neighborhood, there are eight sorts of configuration Q_0^j which represent eight sorts of gradients of straight lines and eight sorts of configuration Q_1^j which imply eight sorts of intersects. Synthesizing various conditions presents thirty-six lines. According to whether they are intersecting at the same crossover point, these lines can be categorized into eight subsets:

$$\begin{aligned}
 E_1 &= \{l_{11}, l_{22}, l_{33}, l_{44}, l_{55}, l_{66}, l_{77}, l_{88}\}, \\
 E_2 &= \{l_{12}, l_{23}, l_{34}, l_{45}, l_{56}, l_{67}, l_{78}\}, \\
 E_3 &= \{l_{13}, l_{24}, l_{35}, l_{46}, l_{57}, l_{68}\}, \\
 E_4 &= \{l_{14}, l_{25}, l_{36}, l_{47}, l_{58}\}, \\
 E_5 &= \{l_{15}, l_{26}, l_{37}, l_{48}\}, \\
 E_6 &= \{l_{16}, l_{27}, l_{38}\}, \\
 E_7 &= \{l_{17}, l_{28}\}, \\
 E_8 &= \{l_{18}\}.
 \end{aligned} \tag{12}$$

E_1 embodies the relationship between two entities with homogeneous level (the same amount of cooperators). Once there is a limitation up to someone in this category, it will have the greatest strength to affect outcomes. Following the case of von Neumann, we create a degree e_k to measure the influence that E_k could exert on any configuration. We can easily deduce $e_k \geq e_j, k \geq j$. In contrast with absence of self-playing, the significant parts lie in $\{u_0^i < u_0^{i+1}, i = 1, 2, \dots, 7\}$ and the fact that Q_0^8 is no longer the invincible structure. Q_1^8 is reasonable to be superior to Q_0^8 here.

Since overwhelming lines need to be analyzed individually, in Figure 8 we partition the P, T -plane according to the different patterns of spatial dynamics.

Simulation V. Cooperation holds the trump card in region A formed by $u_0^i < u_1^{i+2}$ ($i = 1, 2, \dots, 6$) with the average percentage above 90%, as shown in Figure 9. Defectors survive as single dot or oscillating cluster with the size of 3×3 , and sometimes there are a few inconsistent short lines.

FIGURE 6: Simulation results on a 50×50 square lattice with von Neumann neighborhood and different random initial distributions of players. The x -axis represents the number of generation, and y -axis represents the frequency of cooperators in the evolution. The results in (a) and (b) are, respectively, performed by the parameters region $\{u_0^1 > u_1^2, u_0^2 < u_1^3, u_0^3 < u_1^4\}$ and $\{u_0^1 > u_1^2, u_0^2 > u_1^3, u_0^3 < u_1^4\}$. $f(0)$ means the frequency of cooperators in the random initial distribution.

FIGURE 7: Simulation results on square lattices in different sizes with von Neumann neighborhood and $f(0) = 0.50$. The x -axis represents the lattice size, and y -axis represents the frequency of cooperators in the final pattern. The results in (a) are performed by the parameters region $\{u_0^1 > u_1^2, u_0^2 < u_1^3, u_0^3 < u_1^4\}$, and the results in (b) are by the region $\{u_0^1 > u_1^2, u_0^2 > u_1^3, u_0^3 < u_1^4\}$.

Simulation VI. While we make modifications to one of the inequalities $u_0^i < u_1^{i+2}$ ($i = 1, 2, \dots, 6$), only two groups of inequalities turn out to be feasible:

$$u_0^1 > u_1^3, \quad u_0^i < u_1^{i+2} \quad (i = 2, 3, \dots, 6), \quad (13)$$

$$u_0^6 > u_1^8, \quad u_0^i < u_1^{i+2} \quad (i = 1, 2, \dots, 5). \quad (14)$$

Under above two circumstances, defectors join up as integrated lines to partition the whole territory of cooperation into pieces, which manifests the enhancement of their strength of incursion. However, great distinctions still remain in the two regions. When conditions (13) are met, only steady clusters of defectors emerge, leaving oscillating clusters no place to live. Nevertheless, conditions (14) incubate various

FIGURE 8: Partitioning of the P, T -plane according to the different patterns of spatial dynamics.

FIGURE 9: Simulation results on a 50×50 square lattice with Moore neighborhood and self-playing. The initial distribution of cooperators and defectors is randomly generated. The pattern is performed by the parameters region $u_0^i < u_1^{i+2}$ ($i = 1, 2, \dots, 6$).

states with high level of oscillation, and every line or cluster formed by defectors is surrounded by oscillating cells with two periods. Figures 10(a) and 10(b) show the corresponding patterns.

Simulation VII. To make detailed exploration on how the dynamics changes gradually along with the relaxation of restriction of defection from $u_0^i < u_1^{i+2}$ ($i = 1, 2, \dots, 6$), we modify two inequalities to achieve two feasible regions:

$$u_0^1 > u_1^3, \quad u_0^2 > u_1^4, \quad u_0^i < u_1^{i+2} \quad (i = 3, 4, \dots, 6), \quad (15)$$

$$u_0^5 > u_1^7, \quad u_0^6 > u_1^8, \quad u_0^i < u_1^{i+2} \quad (i = 1, 2, \dots, 4). \quad (16)$$

These two regions are the expansion from conditions (13) and (14), respectively, and the dynamics are also in the wake of invasion degree of defection becoming worse in terms of cooperation, whose patterns are shown in Figures 11(a) and 11(b). It is reflected by the fact that there are more stable lines of defectors. Conditions (13) combined with conditions (15)

form region B in Figure 8, and conditions (14) and conditions (16) form the region C.

Simulation VIII. In the light of the law of development, three conditions need to be changed. Region $u_0^i > u_1^{i+2}$ ($i = 4, 5, 6$), $u_0^i < u_1^{i+2}$ ($i = 1, 2, 3$) assumes hybrid dynamics of spatial chaos and majority of cooperation. $u_0^5 < u_1^8$ (region D) holds the predominance of cooperation: clusters as the shape of tentacles are formed inside the picture and hardly touch the edges, as Figure 12(a) shows.

When it comes to $u_0^5 > u_1^8$ (region E), there comes the chaos, as presented in Figure 12(b), and we notice that the chaos pattern resulting from Moore neighborhood is quite different from that from von Neumann neighborhood. The former starts with a local fascinating pattern continuing migrating and slowly diffuses from small plots of cooperation in the territory of defectors until the whole is in the chaotic state that is never able to be restored again. Meanwhile, every plot of cooperation is engulfed repeatedly by defectors from inside. While the latter is relatively stable, as soon as cooperators get a firm foothold, they only stretch and shrink on the boundary and the percentage of cooperation stabilizes rapidly.

Simulation IX. The conditions are as follows:

$$\begin{aligned} u_0^i > u_1^{i+2} \quad (i = 1, 2, 3), \quad u_0^i < u_1^{i+2} \quad (i = 4, 5, 6), \\ u_0^i > u_1^{i+2} \quad (i = 1, 2, \dots, 4), \quad u_0^i < u_1^{i+2} \quad (i = 5, 6), \\ u_0^i > u_1^{i+2} \quad (i = 1, 2, \dots, 5), \quad u_0^6 < u_1^8. \end{aligned} \quad (17)$$

These conditions (region F) are the watershed between the domination of cooperation and defection, forming the patterns similar with that in Figure 13. The force of cooperation has been deteriorated by opponents, reaching the state where cooperators survive in the shape of small rectangle clusters. This reflects another distinction between two structures of neighborhood. A crisscross pattern formed by cooperators is another choice for von Neumann neighborhood to cluster.

Other Simulations. The conditions are as follows:

$$\begin{aligned} u_0^1 < u_1^3, \quad u_0^2 < u_1^4, \quad u_0^i > u_1^{i+2} \quad (i = 3, 4, \dots, 6), \\ u_0^1 < u_1^3, \quad u_0^i > u_1^{i+2} \quad (i = 2, 3, \dots, 6). \end{aligned} \quad (18)$$

Defectors totally control the whole situation and fill up all the cells except when $u_0^5 < u_1^8$. $u_0^5 > u_1^8$ sometimes causes very few rectangle clusters of cooperators but always brings about the complete domination of defectors if the initial distribution is random. For some special initial distributions such that there is only one defector in the central cell, defection is not able to invade them all.

$u_0^i > u_1^{i+2}$ ($i = 1, 2, \dots, 6$) releases the force of incursion thoroughly on the basis of level of e_3 . This region is very much like the last one, which has few clusters when extra condition

FIGURE 10: Simulation results on a 50×50 square lattice with Moore neighborhood and self-playing. The initial distribution of cooperators and defectors is randomly generated. The pattern in (a) is performed by the parameters region $u_0^1 > u_1^3, u_0^i < u_1^{i+2} (i = 2, 3, \dots, 6)$, and the pattern in (b) is by the region $u_0^6 > u_1^8, u_0^i < u_1^{i+2} (i = 1, 2, \dots, 5)$.

FIGURE 11: Simulation results on a 50×50 square lattice with Moore neighborhood and self-playing. The initial distribution of cooperators and defectors is randomly generated. The pattern in (a) is performed by the parameters region $u_0^1 > u_1^3, u_0^2 > u_1^4, u_0^i < u_1^{i+2} (i = 3, 4, \dots, 6)$, and the pattern in (b) is by the region $u_0^5 > u_1^7, u_0^6 > u_1^8, u_0^i < u_1^{i+2} (i = 1, 2, \dots, 4)$.

$u_0^5 < u_1^8$ is satisfied. Together with inequations (18), the region G is regarded as almost complete defected area.

Once $u_1^8 < u_0^1$ (region H) is met, defection will completely dominate cooperation.

Discussions. Every group of straight lines in E_k divides the picture with similar presentation and the number of lines in E_k also decreases gradually. The area corresponding to E_1 is

$$P_1 = \left\{ (x, y) \mid y < \frac{27}{8} - \frac{1}{8}x \text{ or } y < 6 - 8x \right\} \quad (19)$$

which can be divided into three pieces:

$$\begin{aligned} P_{1_1} &= \left\{ (x, y) \mid y < \frac{27}{8} - \frac{1}{8}x, y < 6 - 8x \right\}, \\ P_{1_2} &= \left\{ (x, y) \mid y < \frac{27}{8} - \frac{1}{8}x, y > 6 - 8x \right\}, \\ P_{1_3} &= \left\{ (x, y) \mid y > \frac{27}{8} - \frac{1}{8}x, y < 6 - 8x \right\}. \end{aligned} \quad (20)$$

The level of cooperation in P_{1_1} certainly is highest among all the three. P_{1_2} turns out to be more oscillating and the

FIGURE 12: Simulation results on a 50×50 square lattice with Moore neighborhood and self-playing. The initial distribution of cooperators and defectors is randomly generated. The pattern in (a) is performed by the parameters region $u_0^i > u_1^{i+2}$ ($i = 4, 5, 6$), $u_0^i < u_1^{i+2}$ ($i = 1, 2, 3$), $u_0^5 < u_1^8$, and the pattern in (b) is by the region $u_0^i > u_1^{i+2}$ ($i = 4, 5, 6$), $u_0^i < u_1^{i+2}$ ($i = 1, 2, 3$), $u_0^5 > u_1^8$.

FIGURE 13: Simulation results on a 50×50 square lattice with Moore neighborhood and self-playing. The initial distribution of cooperators and defectors is randomly generated. The pattern is performed by the parameters region $u_0^i > u_1^{i+2}$ ($i = 1, 2, 3$), $u_0^i < u_1^{i+2}$ ($i = 4, 5, 6$); $u_0^i > u_1^{i+2}$ ($i = 1, 2, \dots, 4$), $u_0^i < u_1^{i+2}$ ($i = 5, 6$); $u_0^i > u_1^{i+2}$ ($i = 1, 2, \dots, 5$), $u_0^6 < u_1^8$.

clusters formed in this region are generally in the shape of curve, and P_{13} intends to be more stable without oscillation and the clusters are found in the shape of lump. The patterns forming the similar clusters like P_{13} are also different, and the lines in E_1 separate the level of cooperation in P_{13} into secondary classes changing along with the direction of radiation. The same law can be applied to other regions P_k ($k = 2, 3$). The extent of emerging of cooperation is diminishing from P_k to P_{k+1} ($k = 1, 2, \dots, 7$).

The above discussion is subject to the dynamics on the basis of e_3 , rather than the case of von Neumann which we start with e_1 . The reason is that the dynamics of Moore

structure are far more complicated than that of von Neumann structure, and the influence of e_3 sufficiently brings the majority of cooperation to the system. Since each group of lines in E_k corresponds to a set of constraint conditions, e_1 obviously has the strictest constraint on configuration Q_0^j which is $\{u_0^i < u_1^i, i = 1, 2, \dots, 6\}$. Just as we expected, defectors suffer as independent dots and the number is rare. Every time we convert an inequality to unleash a level of constraint for Q_0^j , e_2 would put on some opposite strength to prevent Q_0^j from inflating excessively. These forces of expansion and restriction result in various amazing patterns.

Our modification is made from both sides of E_k ($k = 1, 2, 3$), so we come to the conclusion that $u_0^1 < u_1^{1+(k-1)}$ has less power to undermine the defectors than $u_0^6 < u_1^{6+(k-1)}$. The same rule works for $\{u_0^1 < u_1^{1+(k-1)}, u_0^2 < u_1^{2+(k-1)}\}$, $\{u_0^6 < u_1^{6+(k-1)}, u_0^5 < u_1^{5+(k-1)}\}$, and so forth. Simulation results show that the regions dominated by cooperators in Moore neighborhood are larger than those in von Neumann, and the clusters of defectors in the former structure are in diversified shapes which are much more than those in the later. All the phenomena demonstrate the fact that the more neighbors surround the central cell, the more cooperators would be produced.

With regard to the state of occupation of cooperation, the procedure of evolution is relatively slow and regular: at the beginning defectors rule the most part and cooperators form the clusters of local and scattered dots; then cooperators develop gradually. The cooperated state with oscillation is generated by major and small clusters; the less cooperated state with stable defectors is usually formed by minor and big clusters. The spatial chaos reveals the range that promotes the situation in both neighborhood structures is generally the same. When the value of T/P is over a threshold, the chaos appears; after it reaches some limit, the chaos will vanish again.

FIGURE 14: Simulation results on square lattices in different sizes with Moore neighborhood and $f(0) = 0.50$. The x-axis represents the lattice size, and y-axis represents the frequency of cooperators in the final pattern. The results in (a)–(f) are, respectively, performed by the parameters region $u_0^i < u_1^{i+2}$ ($i = 1, 2, \dots, 6$); $u_0^1 > u_1^3$, $u_0^i < u_1^{i+2}$ ($i = 2, 3, \dots, 6$); $u_0^6 > u_1^8$, $u_0^i < u_1^{i+2}$ ($i = 1, 2, \dots, 5$); $u_0^i > u_1^{i+2}$ ($i = 4, 5, 6$), $u_0^i < u_1^{i+2}$ ($i = 1, 2, 3$), $u_0^5 < u_1^8$; $u_0^i > u_1^{i+2}$ ($i = 4, 5, 6$), $u_0^i < u_1^{i+2}$ ($i = 1, 2, 3$), $u_0^5 > u_1^8$, and $u_0^i > u_1^{i+2}$ ($i = 1, 2, 3$), $u_0^i < u_1^{i+2}$ ($i = 4, 5, 6$).

In order to test the above results against the population size, we select six typical parameters regions to conduct simulations with Moore neighborhood and self-playing on square lattices in different sizes. As Figure 14 shows, we obtain the final frequencies of cooperators with lattice sizes ranging from 10×10 to 80×80 . Note that the two curves in the oscillating patterns like Figure 14(a) and chaotic patterns like Figure 14(e), respectively, represent the maximal and minimal frequencies of cooperators. From the results in Figure 14, we can find the following.

- (1) The steady patterns performed by parameters regions $u_0^1 > u_1^3$, $u_0^i < u_1^{i+2}$ ($i = 2, 3, \dots, 6$) and $u_0^i > u_1^{i+2}$ ($i = 1, 2, 3$), $u_0^i < u_1^{i+2}$ ($i = 4, 5, 6$) almost keep the coexistence of cooperators and defectors on square lattices in different sizes with Moore neighborhood and self-playing, as Figures 14(b) and 14(f) show. However, the evolution with Moore neighborhood may also turn into the complete invasion of defectors if the population size is not big enough, as Figure 14(f) shows. Together with the extinction phenomenon in Figure 7(b), we find that it is easy to produce complete invasion of defectors in the evolution with loose constraints on defectors and small population size.
- (2) The population size has the impact on the evolution of oscillating patterns such as Figures 14(a), 14(c), and 14(d), but the impact is not big enough to turn the oscillating patterns into complete invasion of cooperators or defectors. The similar impact is observed for the chaotic patterns, as Figure 14(e) shows.
- (3) Comparing the results with those on the 50×50 lattice with Moore neighborhood and self-playing, we basically test the effectiveness of the results against population size. The results on lattices in different sizes provide further evidence on that the Moore neighborhood and self-playing rule could promote the cooperation in evolutionary games.

5. Conclusions

Evolutionary game is a powerful framework to study the evolution of cooperation, and different spatial structures may bring about different patterns of the invasion or the coexistence of cooperator and defectors. In this work, we conducted lattice-based simulations with von Neumann neighborhood as Schweitzer et al. did and found some inconsistent results. In order to find the cause, the impact of initial population distribution and the lattice size on the evolution has been investigated. What's more, we have extended the model to a complicated level where the neighborhood turns into the structure of Moore and each cell can play games with itself.

Simulation results provided new evidence for the persistence of cooperation in the evolution with spatial structures and obtain some observations: (i) both the randomness of initial population distribution and the size of lattice have direct impact on the ratio of players in the final patterns; (ii) it is easy to produce complete invasion of defectors in the evolution with loose constraints on defectors and

small population size; and (iii) comparing with the model with von Neumann neighborhood, our extended model with Moore neighborhood and self-playing rule could promote the cooperation in evolutionary games.

There are still further works which we will perform. Considerations will be given to conduct simulations in heterogeneous structures where the initial distribution of players is not random. Specific real-world evolution situations need be considered in order to observe more practical rules for promoting the cooperation behavior.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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