Research Article

Hybrid Dynamic Network Data Envelopment Analysis

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Conventional DEA models make no hypothesis concerning the internal operations in a static situation. To open the “black box” and work with dynamic assessment issues synchronously, we put forward a hybrid model for evaluating the relative efficiencies of a set of DMUs over an observed time period with a composite of network DEA and dynamic DEA. We vertically deal with intermediate products between divisions with assignable inputs in the network structure and, horizontally, we extend network structure by means of a dynamic pattern with unrelated activities between two succeeding periods. The hybrid dynamic network DEA model proposed in this paper enables us to (i) pry into the internal operations of DEA by another network structure, (ii) obtain dynamic change of period efficiency, and (iii) gain the overall dynamic efficiency of DMUs over the entire observed periods. We finally illustrate the calculation procedure of the proposed approach by a numerical example.

1. Introduction

Data envelopment analysis (DEA), originated by Charnes, Cooper, and Rhodes, is a nonparametric approach for measuring the relative efficiency of homogeneous decision making units (DMUs), which use similar inputs to produce similar outputs [1]. DEA evaluates the efficiency of each DMU relative to an estimated production possibility frontier constructed by all DMUs. Traditional DEA approach is based on the thinking of “black box,” which requires no assumption on the appearance of the frontier surface and it makes no hypothesis concerning the internal operations of a DMU. However, the actual transformation process is generally not modeled explicitly.

Network DEA allows us to look into these boxes and to evaluate organizational performance and its component performance (see, e.g., [2–5]). The development of network DEA model has come through four phases as follows: Firstly, Färe et al. [6] studied allocation of farmland to various crops, which allows for allocation of a (fixed) factor or input among alternative uses. This general structure could also be used to introduce allocation of a budget or allocation

of resources across units or branches. This is the rudiment of network DEA model. Secondly, Löthgren and Tambour [7] proposed a kind of network model that allows inclusion of customer satisfaction in efficiency and productivity measures. The network consists of a production node and a consumption node and offers flexibility in modelling the production and consumption process where a firm-specific allocation of input resources to production and customer oriented activities is allowed. In this phase, intermediate products were emphasized. Thirdly, Färe and Grosskopf [8] suggested dynamic DEA model which considered time factor. The dynamic DEA model introduced here is used to study the dynamic efficiency of APEC (Asian-Pacific Economic Community) countries. The network formulation is a dynamic DEA model in which some outputs at period t are inputs in the next period, $t + 1$. However, the third phase ignores the structure of network itself which is discussed in the second phase. Thus, it is a simple network model with time parameter, but not real dynamic network DEA model. Fourthly, Tone and Tsutsui [9] dealt with intermediate products formally to evaluate divisional efficiencies along with the overall efficiency of DMUs by a slack-based measure.

They handled any network structure in which nodes were connected and intermediates were directed.

Measurement of intertemporal efficiency change has long been an interesting subject of concern in DEA (called dynamic DEA). The dynamic DEA was explored to assess DMUs performance from a long-term perspective using carry-over variables. The dynamic DEA originated by Färe et al. [6] is the first innovative algorithm for dealing with interconnecting activities. It was further developed by several authors from different perspectives. In recent years, Chen and van Dalen [10] developed a dynamic DEA that incorporates dynamic factors via a linear parametric formulation. Soleimani-damaneh [11] provided a recursive enumeration algorithm for solving the nonconvex dynamic DEA models. Soleimani-damaneh [12] also utilized the envelopment models for estimating returns to scale in dynamic DEA. To cope with long time point of view, Tone and Tsutsui [13] put forward a dynamic DEA model that adds carry-over activities into the model to measure period specific efficiency based on the long time optimization during the whole period. Then, Tone and Tsutsui extended the model by a combination of network DEA and dynamic DEA [14]. However, sub-DMUs of their hybrid dynamic network DEA were connected by links by means of chain network structure. The intermediate products between sub-DMUs were neglected and inputs cannot be allocated to them.

Hence, this paper is a hybrid composite of dynamic and network DEA. However, our dynamic network DEA model deals with network structure with intermediate products between any sub-DMUs and copes with assignable inputs over observed dynamic periods. The hybrid model proposed in this paper enables us to open the “black box” to reveal the internal connections of DMUs as well as realizing dynamic evaluation of network DEA over the entire observed periods. The rest of this paper unfolds as follows. In Section 2, we formulate network DEA model with intermediates and assignable inputs. In Section 3, we propose a hybrid model called dynamic network DEA (DNDEA), combining dynamic DEA with network DEA. We also construct a new DEA unit to obtain the dynamic efficiency of network DEA. In Section 4, we exhibit an illustrative example to explain the operational process of the hybrid model. Some conclusions are presented in Section 5.

2. Network DEA Model

2.1. Introduction of DEA. DEA is a widely used operational programming approach for comparing the multiple inputs and outputs of a set of homogenous DMUs by measuring their relative efficiency. Suppose that there are n DMUs to be evaluated where each DMU _{j} , $j = 1, 2, \dots, n$, consumes m inputs, x_{ij} ($i = 1, 2, \dots, m$), producing s outputs, y_{rj} ($r = 1, 2, \dots, s$). The structure of conventional DEA can be viewed as a “black box” described in Figure 1.

The CCR (Charnes, Cooper, and Rhodes) model for evaluating the technical input-efficiency of j_0 th DMU (DMU₀) under constant returns to scale (CRS) is represented as

FIGURE 1: The structure of conventional DEA.

FIGURE 2: The structure of network DEA with chain links and nonassignable inputs.

$$P_{CCR} : \begin{cases} \max h_0 = \sum_{r=1}^s \mu_r y_{r0} \\ \sum_{i=1}^m \omega_i x_{i0} = 1 \\ \sum_{i=1}^m \omega_i x_{ij} - \sum_{r=1}^s \mu_r y_{rj} \geq 0; j = 1, 2, \dots, n \\ \omega_i, \mu_r \geq 0, \forall i, r. \end{cases} \quad (1)$$

As an extension of the “black box” model above, Färe and Grosskopf [8] introduced the network DEA model with chain links and nonassignable inputs illustrated in Figure 2. Tone and Tsutsui [13] developed the network structure by applying it in the dynamic network DEA using a slack-based measure.

2.2. Network DEA with Intermediate Products. In this paper, we propose a network structure with intermediate products between any two sub-DMUs. And the inputs can be assigned to sub-DMUs as well. We suppose that there are n homogeneous DMUs which involve s sub-DMUs P_l ($l = 1, 2, \dots, s$). The input and output of DMU _{j} are X_j^0 and Y_j^e , respectively. Inputs of sub-DMUs received from input node are X_j^{0l} and X_j^{0r} , which are assignable, and outputs of sub-DMUs exported to output node are Y_j^{le} and Y_j^{re} . There are input and output values between sub-DMUs which we call intermediate products; $\tilde{X}_j^{rl}$ is defined as input from P_r to P_l ; $\tilde{Y}_j^{lr}$ is defined as output from P_l to P_r , where $r, l \in \{1, 2, \dots, s\}$, $r \neq l$. We can deduce that $\tilde{X}_j^{rl} = \tilde{Y}_j^{rl}$ and $\tilde{X}_j^{lr} = \tilde{Y}_j^{lr}$. Figure 3 illustrates the structure of network DEA with intermediate products and assignable inputs.

FIGURE 3: The structure of network DEA with intermediate products and assignable inputs.

When we take intermediate products into consideration in DEA model, we can formulate network DEA as follows:

$$(D_{\text{Network}}): \begin{cases} \min \theta = V_D \\ \sum_{l=1}^s X_0^{0l} = \theta X_0^0, \sum_{j=1}^n \lambda_j^l X_j^{0l} + S^{0l-} = X_0^{0l}, \sum_{j=1}^n \lambda_j^l \tilde{X}_j^{rl} + \tilde{S}^{rl-} = \tilde{X}_0^{rl} \\ \sum_{j=1}^n \lambda_j^l Y_j^{le} - S^{le+} = Y_0^{le}, \sum_{j=1}^n \lambda_j^l \tilde{Y}_j^{lr} - \tilde{S}^{lr+} = \tilde{Y}_0^{lr}, l, r \in \{1, 2, \dots, s\}, l \neq r \\ \lambda_j \geq 0, j = 1, 2, \dots, n; X_0^{0l} \geq 0, S^{0l-} \geq 0, \tilde{S}^{rl-} \geq 0, S^{le+} \geq 0, \tilde{S}^{lr+} \geq 0. \end{cases} \quad (3)$$

3. Hybrid Dynamic Network DEA

3.1. Dynamic Network DEA Model. Network DEA has the structure with intermediate products between input node and output node, which shows a network form. However, it is a static modality essentially. On the basis of decision making space of network DEA, we introduce the time parameter t to formulate three-dimensional decision making issue, called dynamic network DEA (DN-DEA). During the t th time span, the input of DMU $_j$ is $X_{j(t)}^0$ and the output of DMU $_j$ is $Y_{j(t)}^e$, which can be formulated as

$$X_{j(t)}^0 = (x_{1(t)}^0, x_{2(t)}^0, \dots, x_{n(t)}^0) \\ = \begin{pmatrix} x_{11(t)}^0 & \cdots & x_{1j(t)}^0 & \cdots & x_{1n(t)}^0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{i1(t)}^0 & \cdots & x_{ij(t)}^0 & \cdots & x_{in(t)}^0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{m1(t)}^0 & \cdots & x_{mj(t)}^0 & \cdots & x_{mn(t)}^0 \end{pmatrix}_{m \times n},$$

$(P_{\text{Network}}):$

$$\begin{cases} \max (\mu^{eT} Y_0^e) = V_P \\ \omega^{0lT} X_j^{0l} + \sum_{r=1, r \neq l}^s \tilde{\omega}^{rlT} \tilde{X}_j^{rl} - \mu^{leT} Y_j^{le} - \sum_{r=1, r \neq l}^s \tilde{\mu}^{lrT} \tilde{Y}_j^{lr} \geq 0, \\ j = 1, 2, \dots, n; l = 1, 2, \dots, s; \omega^{0T} X_0^0 = 1, \omega^0 \geq \omega^{0l} \\ \tilde{\mu}^{rl} \geq \tilde{\omega}^{rl}, \omega^{0l} \geq 0, \tilde{\omega}^{rl} \geq 0, \mu^e \geq 0, \mu^{le} \geq 0, \tilde{\mu}^{lr} \geq 0. \end{cases} \quad (2)$$

The dual model of network DEA above can be transformed as (3), where θ is the network DEA efficiency of DMU $_0$, which reflects the overall efficiency of network DEA with intermediate products. θ is proportional to the performance of the evaluated DMU:

$$Y_{j(t)}^e = (y_{1(t)}^e, y_{2(t)}^e, \dots, y_{n(t)}^e)$$

$$= \begin{pmatrix} y_{11(t)}^e & \cdots & y_{1j(t)}^e & \cdots & y_{1n(t)}^e \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ y_{k1(t)}^e & \cdots & y_{kj(t)}^e & \cdots & y_{kn(t)}^e \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ y_{s1(t)}^e & \cdots & y_{sj(t)}^e & \cdots & y_{sn(t)}^e \end{pmatrix}_{s \times n}. \quad (4)$$

Inputs of sub-DMUs received from input node are $X_{j(t)}^{0l}$ and $X_{j(t)}^{0r}$, and outputs of sub-DMUs exported to output node are $Y_{j(t)}^{le}$ and $Y_{j(t)}^{re}$. There are input and output values between sub-DMUs in the t th time span; $\tilde{X}_{j(t)}^{rl}$ is defined as input from P_r to P_l ; $\tilde{Y}_{j(t)}^{lr}$ is defined as output from P_l to P_r , where $r, l \in \{1, 2, \dots, s\}$, $r \neq l$, $t = 1, 2, \dots, T$. We can also deduce that $\tilde{X}_{j(t)}^{rl} = \tilde{Y}_{j(t)}^{rl}$ and $\tilde{X}_{j(t)}^{lr} = \tilde{Y}_{j(t)}^{lr}$. We can describe the structure of DN-DEA in the t th time span as in Figure 4.

The DN-DEA model can be formed as follows:

$$(P_{\text{DN}}): \begin{cases} \max (\mu_{(t)}^{eT} Y_{0(t)}^e) = V_{P(t)} \\ \omega_{(t)}^{0lT} X_{j(t)}^{0l} + \sum_{r=1, r \neq l}^s \tilde{\omega}_{(t)}^{rlT} \tilde{X}_{j(t)}^{rl} - \mu_{(t)}^{leT} Y_{j(t)}^{le} - \sum_{r=1, r \neq l}^s \tilde{\mu}_{(t)}^{lrT} \tilde{Y}_{j(t)}^{lr} \geq 0, \\ j = 1, 2, \dots, n; l = 1, 2, \dots, s; t = 1, 2, \dots, T, \omega_{(t)}^{0T} X_{0(t)}^0 = 1, \omega_{(t)}^0 \geq \omega_{(t)}^{0l} \\ \tilde{\mu}_{(t)}^{rl} \geq \tilde{\omega}_{(t)}^{rl}, \omega_{(t)}^{0l} \geq 0, \tilde{\omega}_{(t)}^{rl} \geq 0, \mu_{(t)}^e \geq 0, \mu_{(t)}^{le} \geq 0, \tilde{\mu}_{(t)}^{lr} \geq 0. \end{cases} \quad (5)$$

FIGURE 4: The structure of DN-DEA in one time span.

The dual model of the above formulation can be transformed as (6), where $\theta^{(t)}$ is the network DEA efficiency of DMU₀

in the t th time span ($t = 1, 2, \dots, T$), showing dynamic efficiency of DN-DEA:

$$(D_{DN}): \begin{cases} \min \theta^{(t)} = V_{D(t)} \\ \sum_{l=1}^s X_{0(t)}^{0l} = \theta^{(t)} X_{0(t)}^0 \\ \sum_{j=1}^n \lambda_{j(t)}^l X_{j(t)}^{0l} + S_{(t)}^{0l-} = X_{0(t)}^{0l}, \sum_{j=1}^n \lambda_{j(t)}^l \bar{X}_{j(t)}^{rl} + \bar{S}_{(t)}^{rl-} = \bar{X}_{0(t)}^{rl} \\ \sum_{j=1}^n \lambda_{j(t)}^l Y_{j(t)}^{le} - S_{(t)}^{le+} = Y_{0(t)}^{le}, \sum_{j=1}^n \lambda_{j(t)}^l \bar{Y}_{j(t)}^{lr} - \bar{S}_{(t)}^{lr+} = \bar{Y}_{0(t)}^{lr}, \\ l, r \in \{1, 2, \dots, s\}, l \neq r; \lambda_{j(t)} \geq 0, j = 1, 2, \dots, n; t = 1, 2, \dots, T, \\ X_{0(t)}^{0l} \geq 0, S_{(t)}^{0l-} \geq 0, \bar{S}_{(t)}^{rl-} \geq 0, S_{(t)}^{le+} \geq 0, \bar{S}_{(t)}^{lr+} \geq 0. \end{cases} \quad (6)$$

3.2. Dynamic Evaluation of DN-DEA Efficiencies. $\theta_j^{(t)}$ is the DN-DEA efficiency of DMU_j in the t th time span. We define $\theta_j^{(t)}$ as the DN-DEA efficiency matrix, which means efficiencies of DN-DEA model in discrete time spans. The DN-DEA efficiency matrix can be formulated as follows:

$$\theta_j^{(t)} = (\theta_1^{(t)}, \theta_2^{(t)}, \dots, \theta_n^{(t)})$$

$$= \begin{pmatrix} \theta_1^1 & \dots & \theta_j^1 & \dots & \theta_n^1 \\ \dots & \dots & \dots & \dots & \dots \\ \theta_1^t & \dots & \theta_j^t & \dots & \theta_n^t \\ \dots & \dots & \dots & \dots & \dots \\ \theta_1^T & \dots & \theta_j^T & \dots & \theta_n^T \end{pmatrix}_{T \times n} \quad (7)$$

The column vector of $\theta_j^{(t)}$ reflects the dynamic efficiency of DN-DEA over the whole period of each DMU. Hence, measuring the dynamic network efficiency of DN-DEA is a typical multiple attributes decision making issue essentially. In addition to DEA model, many scholars have put forward some mathematical approaches to solve the problem, for example, analytic hierarchy process, grey correlation analysis, and fuzzy comprehensive evaluation, which have their own advantages and disadvantages. In this work, we apply DEA approach for the “second round” on the basis of hybrid DN-DEA model formulated above. In view of the structure of

DN-DEA efficiency matrix to be evaluated, $\theta_j^{(t)}$ represents the network DEA efficiency of DMU_j in the t th time span. However, the matrix itself has no network structure inside. Thus, we can build a new “black box” DEA to assess overall dynamic efficiency for DN-DEA. For DEA model, input and output index have their normal requirements: input is inversely proportional to the DEA efficiency; output is proportional to the DEA efficiency. From the analysis above we know that $\theta_j^{(t)}$ is proportional to the performance of DMU_j in period t ; thus we can treat it as output of the new DEA. Moreover, input can be formed as $1 \times n$ dimensional vector. In order to operate easily, we treat it as $1 \times n$ dimensional unit vector. The structure diagram of the new DEA is displayed as shown in Figure 5.

From the new constructed DEA as shown in Figure 5, we can build the dynamic evaluation model of DN-DEA efficiency, which can be formulated as follows:

$$(D^I \varepsilon): \begin{cases} \min [\sigma - \varepsilon (\bar{e}^T S^- + e^T S^+)] = \xi \\ \sum_{j=1}^n 1 \times \lambda_j + S^- = \sigma \times 1 \\ \sum_{j=1}^n \theta_j^{(t)} \lambda_j - S^+ = \theta_0^{(t)} \\ \lambda_j \geq 0, j = 1, 2, \dots, n; S^- \geq 0, S^+ \geq 0, \end{cases} \quad (8)$$

FIGURE 5: The constructed DEA for measuring dynamic efficiency of DN-DEA.

FIGURE 6: DN-DEA with 3 sub-DMUs in the one time span.

where $\hat{e} = (1, 1, \dots, 1)^T \in E^1$ and $e = (1, 1, \dots, 1)^T \in E^T$ are unit vectors, ε ($\varepsilon > 0$) is a non-Archimedean infinitesimal, and S^- and S^+ are slacks of DMUs. ξ is the dynamic efficiency of DMU_0 all over T period in DN-DEA model. We can further transform the model to

$$(D^I \varepsilon): \begin{cases} \min [\sigma - \varepsilon (\hat{e}^T S^- + e^T S^+)] = \xi \\ \sum_{j=1}^n \lambda_j = \sigma - S^-, \sum_{j=1}^n \theta_j^{(t)} \lambda_j = \theta_0^{(t)} + S^+ \\ \lambda_j \geq 0, j = 1, 2, \dots, n, S^- \geq 0, S^+ \geq 0. \end{cases} \quad (9)$$

4. Numerical Example

In this section, a numerical example is presented to illustrate DN-DEA model. To this end, ten DMUs with one input node, one output node, and three sub-DMUs are considered over four time periods. We can describe the DN-DEA structure of this example in Figure 6. It is worth mentioning that there are intermediate products between sub-DMU₁ and sub-DMU₂ and between sub-DMU₂ and sub-DMU₃. The data of inputs, outputs, and intermediate products for these ten DMUs over the observed four time periods have been listed in Tables 1–4.

By running (6), we can obtain the network DEA efficiency of ten DMUs in the t th time span ($t = 1, 2, 3, 4$), also

FIGURE 7: Dynamic fluctuation of network DEA efficiency.

FIGURE 8: Dynamic network DEA efficiency of 10 DMUs.

summarized in Tables 1–4. Thus, we can gain DN-DEA efficiency matrix as shown in

$$\theta_j^{(t)} = \begin{pmatrix} 1 & 0.76 & 0.79 & 0.83 & 0.92 & 0.94 & 0.79 & 1 & 0.90 & 0.95 \\ 1 & 0.79 & 0.76 & 0.89 & 0.87 & 0.96 & 0.70 & 1 & 0.93 & 0.91 \\ 1 & 0.80 & 0.77 & 0.90 & 0.89 & 0.97 & 0.74 & 1 & 0.95 & 0.92 \\ 1 & 0.71 & 0.74 & 0.86 & 0.82 & 0.90 & 0.79 & 0.83 & 0.80 & 0.85 \end{pmatrix}. \quad (10)$$

Figure 7 reflects the dynamic fluctuation of network DEA efficiency for each DMU over the four time periods, from which we can conclude that the network DEA efficiency of the observed time periods vibrates irregularly. We apply model (9) for the sake of evaluating the overall dynamic network efficiency of DN-DEA on the basis of the DEA structure proposed in Figure 5. The overall dynamic network efficiency and ranking sequence of DN-DEA can be finally provided in Table 5, which can be compared in Figure 8. Table 5 also provides slacks from which we can get improved

TABLE 1: Evaluation data of DN-DEA ($t = 1$).

DMU _j	Efficiency	Input	Sub-DMU ₁			Sub-DMU ₂			Sub-DMU ₃	
	θ_j^1	X_j^0	X_j^{01}	$\bar{X}_j^{12}$	Y_j^{1e}	X_j^{02}	$\bar{X}_j^{23}$	Y_j^{2e}	X_j^{03}	Y_j^{3e}
1	1	4170	2412	4310	195	1066	722	198	692	910
2	0.76	6349	4662	7710	191	963	414	189	724	492
3	0.79	8046	6395	7939	183	936	510	183	715	510
4	0.83	4679	2522	5533	188	1252	568	177	905	599
5	0.92	5938	4211	5194	177	1012	621	199	715	814
6	0.94	4442	3148	4426	195	891	204	193	403	355
7	0.79	7036	5338	6623	185	956	493	187	742	493
8	1	5122	3606	6490	194	906	681	196	610	858
9	0.90	5740	4253	5285	185	775	289	192	712	355
10	0.95	4680	3087	3839	189	780	580	193	813	630

TABLE 2: Evaluation data of DN-DEA ($t = 2$).

DMU _j	Efficiency	Input	Sub-DMU ₁			Sub-DMU ₂			Sub-DMU ₃	
	θ_j^1	X_j^0	X_j^{01}	$\bar{X}_j^{12}$	Y_j^{1e}	X_j^{02}	$\bar{X}_j^{23}$	Y_j^{2e}	X_j^{03}	Y_j^{3e}
1	1	3820	2308	4209	207	932	677	213	580	996
2	0.79	6082	4564	7608	211	840	408	198	678	512
3	0.76	8199	6432	7966	168	997	518	143	770	448
4	0.89	4457	2443	5411	190	1130	550	187	884	625
5	0.87	6144	4321	5233	141	1136	664	178	687	905
6	0.96	4290	3028	4297	212	870	189	197	392	392
7	0.70	8307	6408	6667	176	1102	535	168	797	445
8	1	4929	3503	6370	209	886	656	208	540	967
9	0.93	5583	4105	5123	194	770	248	197	708	377
10	0.91	4867	3134	3899	170	884	598	168	849	628

TABLE 3: Evaluation data of DN-DEA ($t = 3$).

DMU _j	Efficiency	Input	Sub-DMU ₁			Sub-DMU ₂			Sub-DMU ₃	
	θ_j^1	X_j^0	X_j^{01}	$\bar{X}_j^{12}$	Y_j^{1e}	X_j^{02}	$\bar{X}_j^{23}$	Y_j^{2e}	X_j^{03}	Y_j^{3e}
1	1	3500	2150	4100	230	884	654	222	466	1033
2	0.80	5762	4406	7499	234	792	385	207	564	549
3	0.77	7879	6274	7857	191	949	495	152	656	485
4	0.90	4137	2285	5302	213	1082	527	196	770	662
5	0.89	5824	4163	5124	164	1088	641	187	573	942
6	0.97	3970	2870	4188	235	822	166	206	278	429
7	0.74	7987	6250	6558	199	1054	512	177	683	482
8	1	4609	3345	6261	232	838	633	217	426	1004
9	0.95	5263	3947	5014	217	722	225	206	594	414
10	0.92	4547	2976	3790	193	836	575	177	735	665

information between inefficient DMUs and optimum values by means of projection. We can know that DMU₁ performs best and DMU₇ displays worst. The ranking sequence shows that DMU₁ > DMU₈ > DMU₆ > DMU₉ > DMU₁₀ > DMU₅ > DMU₄ > DMU₂ > DMU₃ > DMU₇.

5. Conclusions

In this paper, we put forward a hybrid dynamic network DEA model, taking dynamic assessment and network structure in

DEA into consideration simultaneously. The hybrid system measures the relative efficiencies of a set of DMUs with network structure of intermediate products between sub-DMUs with assignable inputs over observed time periods. The dynamic efficiency reflects overall network efficiency during the same discrete time spans, showing more practical meaning. What is more, improved information between inefficient DMUs and optimum values can be provided to decision makers by means of non-Archimedean infinitesimal and slacks of DMUs. Notably, different dynamic structure of

TABLE 4: Evaluation data of DN-DEA ($t = 4$).

DMU _j	Efficiency	Input	Sub-DMU ₁			Sub-DMU ₂			Sub-DMU ₃	
	θ_j^1	X_j^0	X_j^{01}	$\bar{X}_j^{12}$	Y_j^{1e}	X_j^{02}	$\bar{X}_j^{23}$	Y_j^{2e}	X_j^{03}	Y_j^{3e}
1	1	4509	2531	4523	178	1202	809	183	776	947
2	0.71	6688	4781	7923	174	1099	501	174	808	529
3	0.74	8385	6514	8152	166	1072	597	168	799	547
4	0.86	5018	2641	5746	171	1388	655	162	989	636
5	0.82	6277	4330	5407	160	1148	708	184	799	851
6	0.90	4781	3267	4639	178	1027	291	178	487	392
7	0.79	7375	5457	6836	168	1092	580	172	826	530
8	0.83	5461	3725	6703	177	1042	768	181	694	895
9	0.80	6079	4372	5498	168	911	376	177	796	392
10	0.85	5019	3206	4052	172	916	667	178	897	667

TABLE 5: Evaluation results of DN-DEA.

DMU _j	ξ_j	S_1^-	S_1^+	S_2^+	S_3^+	S_4^+	Ranking
1	1	0	0	0	0	0	1
2	0.80	0	0.04	0.01	0	0.09	8
3	0.79	0	0	0.03	0.02	0.05	9
4	0.90	0	0.07	0.01	0	0.04	7
5	0.92	0	0	0.05	0.03	0.10	6
6	0.97	0	0.03	0.01	0	0.66	3
7	0.78	0	0	0.09	0.05	0	10
8	0.99	0	0	0	0	0.17	2
9	0.95	0	0.05	0.02	0	0.15	4
10	0.94	0	0	0.04	0.03	0.10	5

network DEA such as carry-over dynamic type or continuous one may need diverse models for measuring corresponding dynamic efficiency of DN-DEA. The dynamic evaluation results of DN-DEA will depend on the constructed dynamic model related to the defined dynamic structure. The “black box” DEA model set up above adapts to the structure of DN-DEA efficiency matrix. However, other DEA models give more importance to more recent years. For instance, super-efficiency DEA will help to solve unsatisfactory differentiation of overall dynamic network efficiencies. Further work based on the framework proposed in this paper could extend the hybrid dynamic network model to include other dynamic patterns as well as different network structures.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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