Research Article

Discrete-Time Dynamical Maximum Power Tracking Control for a Vertical Axis Water Turbine with Retractable Blades

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This paper addresses the power generation control system of a new drag-type Vertical Axis Turbine with several retractable blades. The returning blades can be entirely hidden in the drum, and negative torques can then be considerably reduced as the drum shields the blades. Thus, the power efficiency increases. Regarding the control, a Linear Quadratic Tracking (LQT) optimal control algorithm for Maximum Power Point Tracking (MPPT) is proposed to ensure that the wave energy conversion system can operate highly effectively under fluctuating conditions and that the tracking process accelerates over time. Two-dimensional Computational Fluid Dynamics (CFD) simulations are performed to obtain the maximum power points of the turbine's output. To plot the tip speed ratio curve, the least squares method is employed. The efficacy of the steady and dynamic performance of the control strategy was verified using Matlab/Simulink software. These validation results show that the proposed system can compensate for power fluctuations and is effective in terms of power regulation.

1. Introduction

With the development of economy and society, requirement for energy resource increases continuously. As a kind of ocean energy, ocean current energy is an abundant and renewable energy. For example, in UK, the government has the target of electricity demand from renewable sources, 15% by 2015 and 20% by 2020. In Scotland, the goal for 2020 is 50% [1]. Oceans cover 75% of the world's surface, so, the ocean energy is a global resource. Using ocean energy for electric power generation is an area of research interest, and, currently, most of our work focuses on the cost-effective utilization of this energy to ensure quality and reliability in electricity delivery. Over the last two decades, increasing varieties of wave energy systems have been designed, and different concepts have been developed and tested [2–4].

Characterized by their rotational axis orientation with regard to the water flow direction, water turbines can be classified as Horizontal Axis Turbines (HATs) and Vertical Axis Turbines (VATs). In addition, depending on the type of driving force that rotates the rotor, VATs can be classified into two categories: lift-type and drag-type. Darrieus turbines are the most efficient type of lift-type turbine; these turbines use airfoils to create the lift needed to maintain rotation. Darrieus turbines have been studied extensively, using CFD and experimental methods to optimize their performance [5, 6]. In comparison to lift-type turbines, drag-type turbines have better versatility and include Savonius turbines, Zephyr turbines, and Banki turbines. Savonius turbines have a number of advantages and have been studied experimentally and numerically regarding the effects of various design parameters, such as the rotor aspect ratio, the overlap, the number of buckets, the rotor endplates, and the influence of bucket stacking [7, 8].

In a drag-type VAT, such as the Savonius turbines, the blades will experience negative torque during the returning half cycle, which limits their power efficiency [9–11]. This paper describes a new drag-type VAT that consists of several retractable blades mounted in a drum. The axial position of each blade is controlled to achieve a high output performance. To reduce the negative torque in the returning half cycle and eventually increase the net driving torque, we adopt a blade control mechanism comprising a drum, a synchronous belt, and several links and shafts to reduce the negative torque
on the returning blade. The blades are mounted inside the

drum and controlled independently by the blade control

mechanism. In the advancing half cycle, the blades are pushed

out of the drum, generating positive torque to drive the rotor,

whereas in the returning half cycle, the blades are pulled into

the drum, reducing the negative torque [12].

For the control part, the system’s main control objective

is to maximize the power efficiency, which requires the

turbine tip speed ratio to be maintained at its optimum value

regardless of wave variations. Nevertheless, control does not

always aim to capture as much energy as possible. In fact,

the optimal turbine speed varies from one wave speed to

another. This paper’s contribution lies in its employment of an
effective MPPT control and optimal LQT control for varying

operating conditions. Optimal LQT control is one of the most

successful control algorithms and is widely used in handling
multivariables and constraints [13–17]. The designed control
discipline is based on a mathematic model of a controlled
object with the prescribed limit to acquire the optimal
performance index [18–20]. According to previous research
on PID control, the conventional control algorithm gives
good results at infinite steady state; the only difficulty occurs
when the reference trajectory is fluctuating [21, 22]. This
paper builds on the ideas presented in [23]. The performance
of a controller using optimal LQT control and PID will be
analyzed and discussed in the simulation section.

2. Ocean Energy Conversion System

Water turbines can be characterized by their rotational axis
orientation with regard to the water flow direction. VATs,
which are also known as cross-flow water turbines, rotate
around an axis perpendicular to the current. VATs are less
efficient than their horizontal counterparts, but they can
operate regardless of the flow direction and are more suitable
for small-scale, distributed power generation. VATs’ water
energy conversion system is composed of a hub, blades,
a nacelle, a gearbox, a generator, and a power regulator
controller (see Figure 1). The device’s operating principle is
similar to that of wind turbines. Each wave washes over
the streamlined blades at some speed and some angle,
leading to the rotation of the shaft. The hydrodynamic kinetic
energy is then converted into mechanical energy. The gearbox
transforms the low-speed, high-torque mechanical energy
and passes it to the generator, regulating the power and
simultaneously exporting the load. Therefore, these turbines
are also called “underwater windmills.”

2.1. Description of the VAT. The rotational period of a VAT

can be divided into the advancing half cycle and the returning
half cycle because the blades generate positive torque in the
former and negative torque in the latter. The VAT described
in this paper is quite different from previous designs. Here,
we adopted a blade control mechanism consisting of a drum,
a synchronous belt, and several links and shafts to reduce the
negative torque on the returning blade (see Figure 2). The
blades are mounted inside the drum and controlled inde-
pendently by the blade control mechanism. In the advancing

half cycle, the blades are pushed out of the drum, generating
positive torque to drive the rotor, whereas in the returning
half cycle, the blades are pulled into the drum, reducing the

negative torque.

The turbine works as follows: the blades are first pushed
out of the drum by the links as the turbine rotates, generating
positive torque to drive the rotor. Then, after fully extending,
the blades retract to the inside of the drum, allowing water to
flow past the turbine with minimum resistance. The resultant
torque drives the turbine and produces power.

Figure 3 presents a two-dimensional schematic of the tur-
bine mechanism and illustrates some important parameters: 

\( U \) is the velocity of the incident flow water, \( R \) is the diameter
of the drum, \( c \) is the eccentricity between the power shaft and
the eccentric shaft, \( r \) is the radius of the eccentric disc, \( l \) is
the length of the link, \( L \) is the length of the blade, and \( \theta \) is
Table 1: VAT characteristics.

<table>
<thead>
<tr>
<th>Rating</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of the drum</td>
<td>( R = 0.5 ) m</td>
</tr>
<tr>
<td>Eccentricity between the power and eccentric shaft</td>
<td>( c = 0.225 )</td>
</tr>
<tr>
<td>Radius of the eccentric disc</td>
<td>( r = 0.05 )</td>
</tr>
<tr>
<td>Length of the link</td>
<td>( l = 0.225 )</td>
</tr>
<tr>
<td>Length of the blade</td>
<td>( L = 0.45 )</td>
</tr>
</tbody>
</table>

Figure 3: Schematic of the turbine mechanism.

In the advancing half cycle (\( 0 \leq \theta < \pi \)), \( L_1 \) is maximized when \( \theta = \pi/2 \) for water impacting the blade perpendicularly, and the blade achieves a relatively high driving torque. In the returning half cycle (\( \pi \leq \theta < 2 \pi \)), \( L_1 \) remains \( r + L \) (constant). This finding confirms that the blades are entirely hidden in the drum and only rotate around the power shaft. Negative torques can thus be considerably reduced because the drum shields the blades.

Table 1 shows the main geometric characteristics of the VAT.

2.2. Kinetic Characteristics of the System. According to Betz theory [24], the hydrodynamic power \( P_a \) captured by the water turbine is given by

\[
P_a = \frac{1}{2} \rho R^2 C_p (\lambda) v^3. \tag{2}
\]

In this function, \( C_p \) represents the power conversion efficiency, which depends on the tip speed ratio \( \lambda \) and the blade pitch angle \( \beta \) in a pitch-controlled water turbine. It is a nonlinear function representing the ability of the water energy conversion system to maximize energy capture. \( \lambda \) is the ratio of the tip speed of the turbine blades to the wave speed and is defined as

\[
\lambda = \frac{R \omega}{v}. \tag{3}
\]

The power captured from the sea (hydrodynamic power) can also be defined by

\[
P_a = \omega T_a. \tag{4}
\]

Moreover,

\[
C_q (\lambda) = \frac{C_p (\lambda)}{\lambda}. \tag{5}
\]

From functions (2), (3), (4), and (5), we obtain

\[
T_a = \frac{1}{2} \rho R^2 C_q (\lambda) v^2. \tag{6}
\]

When the pitch angle \( \beta \) is fixed, the water energy conversion system can only operate at some special \( \lambda \) to capture the maximum power up to the rated speed. This special \( \lambda \) is called the optimum tip speed ratio: \( \lambda_{opt} \).

The optimum torque can be described by

\[
T_{opt} = k_{opt} \omega^2, \tag{7}
\]

where the torque constant \( k_{opt} \) is constant.

In the structural model in this paper, the pitch of the water turbines is manipulated using mechanical systems. Thus, the modified model considered here is a fixed-pitch turbine where \( \beta = 0 \).

So we have

\[
k_{opt} = \frac{1}{2 \lambda_{opt}^2 \rho \pi R^2 C_{q_{max}}} \tag{8}
\]

To study the kinetic properties of the generation system, the function of the power coefficient versus the tip speed ratio must be obtained. To obtain relatively accurate results for the rotating rotor, we first considered the hydrodynamic performance using the CFD method. Hence, two-dimensional numerical simulations were performed using the commercial code FLUENT 13.0. In this paper, we introduce a standard two-equation \( k-\epsilon \) model to predict the turbulence effects in the transient predictions [25], which is widely used and able to simulate many flow regimes. The model is based on the transport equations for the turbulence kinetic energy \( k \) and its dissipation rate \( \epsilon \). The coefficients of torque and power are given by

\[
C_m = \frac{M}{0.5 \rho U^2 R S}, \tag{9}
\]

\[
C_p = \frac{P}{0.5 \rho U^3 S} = \lambda C_m, \tag{10}
\]

\[ S = 2RH. \]

In these functions, \( M \) is the blade torque, \( S \) is the cross-sectional area, and \( H \) is the height of the blade. For 2D simulations, the unit height \( H = 1 \) m was used.
Figure 4: Velocity contours at a flow coefficient of $\lambda = 0.4$ and a rotational angle of $\theta = 60^\circ$.

Figure 5: Coefficients of the averaged torque and power versus the flow coefficient.

Table 2: Power coefficient ($C_p$) versus tip speed ratio $\lambda$.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_p$</td>
<td>0.15258</td>
<td>0.186044</td>
<td>0.1963</td>
<td>0.183111</td>
<td>0.130346</td>
</tr>
</tbody>
</table>

Figure 6: The power coefficient $C_p(\lambda)$ curve.

To obtain the specific output performance of the turbine, several cases were investigated for rotor angular velocities between 0.4 m/s and 1.2 m/s. Convergence is determined by the order of magnitude of the residuals. The drop of all scaled residuals below $10^{-5}$ is employed as the convergence criterion.

Figure 4 shows the contours of the absolute velocity in the flow field obtained at a flow coefficient of $\lambda = 0.4$ and a rotational angle of $\theta = 60^\circ$. Figure 5 indicates that velocity drop occurs when water passes the turbine, mainly caused by the advancing blade, which stretches to the outside of the drum and disturbs the flow. Because of the shielding of the drum, the flow inside the drum has a relatively small velocity and is less turbulent than that outside the drum. The flow velocity drops substantially when it passes the extended part of the blade, suggesting that the extended part of the blade contributes significantly to the total torque output, whereas the inside part of the blade hardly makes any contribution to the total torque.

The CFD simulations are presented in Table 2, which lists the different values of the power coefficient ($C_p$) as $\lambda$ varies. The values in Table 2 are utilized to obtain the power coefficient ($C_p$) versus the tip speed ratio curve using the least squares estimation method (see Figure 6). The quadratic fitting function is as follows:

$$C_p = 7.125\lambda^5 - 12.7396\lambda^3 + 7.955\lambda^2 - 1.0893\lambda + 0.1648.$$  \hspace{1cm} (10)

For a specific water flow velocity, the averaged power coefficient increases with the flow coefficient until reaching the maximum point; subsequently, further increases in the flow coefficient lead to a decrease in the averaged power coefficient. The maximum averaged power coefficient is obtained at $\lambda = 0.4$, with a value of 0.1963, which is competitive with current drag-type turbines.

2.3. Hydrodynamic Model of the System. The VAT drive train dynamics are given in Figure 7. In Figure 7, the hydrodynamic torque $T_a$ will lead the VAT to operate at speed $\omega_r$. The low-speed torque $T_{hs}$ exerts a braking torque on the rotor. The generator is driven by the high-speed torque $T_{hs}$ and braked by the generator electromagnetic torque $T_g$. The gearbox increases the rotor speed by the gearbox ratio $n_g$ to the generator speed $\omega_g$ and augments the low-speed torque.

The rotor dynamics equations are as follows:

$$J_r \ddot{\omega}_r = T_a - K_r \omega_r - B_r \dot{\theta}_r - T_{ls},$$

$$J_g \ddot{\omega}_g = T_{hs} - K_g \omega_g - B_g \dot{\theta}_g - T_{g}.$$  \hspace{1cm} (11)
The gearbox ratio $n_g$ is defined as

$$n_g = \frac{\omega_g}{\omega_r} = \frac{T_{hl}}{T_{hs}}. \quad (12)$$

After combining (11) and (12), we obtain

$$J_t \dot{\omega}_r = T_a - K_t \omega_r - B_t \dot{\theta}_r - T_g \quad (13)$$

with

$$J_t = J_r + n_g^2 J_g,$$
$$K_t = K_r + n_g^2 K_g,$$
$$B_t = B_r + n_g^2 B_g. \quad (14)$$

In the equation above, the combined inertia of the rotor and the generator dominates, and the external stiffness $B_t$ is low enough to be neglected. We represent the drive train using a single lumped mass for control purposes, and the following simplified model is proposed:

$$J_t \dot{\omega}_r = T_a - K_t \omega_r - T_g \quad (15)$$

We finally obtain the generated power by

$$P_g = T_g \omega_r. \quad (16)$$

Equation (10) indicates that the system is highly nonlinear in terms of the square wave speed and $C_p$ function. According to the linear model described in [26, 27], we performed modeling to determine the form used to represent optimal control as follows:

$$\Delta x = x - x_{\text{opt}}. \quad (17)$$

Near the optimal point, we define the hydrodynamic torque variation by

$$\Delta T_a = \gamma \Delta \omega_r + (2 - \gamma) \Delta v \quad (18)$$

with

$$\Delta T_a = T_a - T_a, \quad \Delta \omega_r = \omega_r - \omega_r, \quad \Delta v = v - v.$$ \quad (19)

The torque parameter $\gamma$ is defined as

$$\gamma = \gamma(\lambda_{\text{opt}}) = \frac{C_p(\lambda_{\text{opt}}) \lambda_{\text{opt}} - C_p(\lambda_{\text{opt}})}{C_p(\lambda_{\text{opt}})} . \quad (20)$$

The wave speed model is given by

$$v(t) = -\frac{1}{T_w} v(t) + \frac{1}{T_w} w(t), \quad (21)$$

where $T_w$ is the time constant of the filter and the turbulence component of the wave speed results from low-pass filtering of white noise $w(t)$.

After the linearization of (19), we have

$$J_t \dot{\omega}_r = \Delta T_a - K_t \omega_r - \Delta T_g \quad (22)$$

and the mechanical time constant $T_T = (\omega_r/T_a) I$. Finally, we can define the state space model as

$$\dot{x}(t) = A x(t) + B u(t) + L e(t), \quad (23)$$

where

$$A = \begin{bmatrix}
-\frac{K_t}{J_T} & \frac{1}{J_T} \\
\frac{\gamma}{T_w} & -\frac{\gamma}{T_w} K_t \\
\frac{1}{J_T} & \frac{1}{J_T} \\
\frac{2 - \gamma}{T_w} & -\frac{1}{T_w}
\end{bmatrix};$$

$$B = \begin{bmatrix}
-\frac{1}{J_T} \\
-\frac{\gamma}{J_T} \\
-\frac{1}{J_T} \\
-\frac{2 - \gamma}{T_w}
\end{bmatrix};$$

$$L = \begin{bmatrix}
0 \\
0 \\
0 \\
T_w
\end{bmatrix};$$

$$x(t) = \begin{bmatrix}
\Delta \omega_r(t) \\
\Delta T_a(t)
\end{bmatrix} ,$$

$$u(t) = \Delta T_g.$$ \quad (24)

The output $y(t)$ is defined as $y(t) = Cx(t)$ when

$$C = \begin{bmatrix}
2 & -1 \\
1 & -2 - \gamma
\end{bmatrix} . \quad (25)$$

In this paper, the output is simplified as the variation in the rotor speed, and matrix $C$ is given when $\gamma = 0$ by

$$C = \begin{bmatrix}
1 & 1 \\
1 & 1
\end{bmatrix} . \quad (26)$$

The controller strategy is presented in Figure 8.
2.4. Optimal LQT Control Description. Optimal control is a control concept that has been widely used for over 20 years, especially in the process industry [28–31]. With the development of optimization algorithms, researchers are now more interested in other areas of application. Optimal LQT control is a process model. Any process model capable of predicting future output signals based on initial values can be used to predict the future input within the prediction over the horizon time length. The predicted output signals are used to minimize the cost function based on the control purposes and applications. The dynamic equation and measured equation are given by

\[ X_{k+1} = A_k X_k + B_k U_k + W_k, \]

\[ Y_{k+1} = C_k X_k. \]  

(27)

The subscript \( k \) indicates the dimension of each matrix. Another output is

\[ M_k = D_k X_k. \]  

(28)

\( C_k \) is required to track the instructed matrix \( N_k \). One of the most popular cost functions uses the quadratic form for tracking references as follows:

\[ J = E \left\{ \sum_{k=1}^{N} e_k^T Q_k e_k + U_{k-1}^T R_{k-1} U_k \right\}, \]

(29)

where

\[ e_k = N_k - M_k. \]  

(30)

We assume that the instructed matrix \( N_k \) is created by a known system, which is described by

\[ Z_{k+1} = E_{k+1} Z_k + F_k \xi_k, \]

\[ N_k = G_k Z_k. \]  

(31)

In the function above, \( \xi_k \) is the white noise. To track the optimal output, we introduce the augmented state vector

\[ X^a_k = \begin{bmatrix} X_k \\ Z_k \end{bmatrix} \]  

(32)

and augmented noise vector

\[ F^a_k = \begin{bmatrix} W_k \\ \xi_k \end{bmatrix}. \]  

(33)

The dynamic equation of \( X^a_k \) then becomes

\[ X^a_{k+1} = A^a_{k+1} X^a_k + B^a_k U_k + F^a_k \xi^a_k, \]

(34)
where

$$A_{k+1}^a = \begin{bmatrix} A_{k+1} & 0 \\ 0 & E_{k+1} \end{bmatrix},$$

$$B_k^a = \begin{bmatrix} B_k \\ 0 \end{bmatrix},$$

$$F_k^a = \begin{bmatrix} I & 0 \\ 0 & F_k \end{bmatrix}.$$  (35)

The updated output equation is

$$Y_k^a = C_k^a X_k^a,$$  (36)

where

$$Y_k = \begin{bmatrix} Y_k \\ 0 \end{bmatrix},$$

$$C_k = \begin{bmatrix} C_k & 0 \\ 0 & G_k \end{bmatrix}.$$  (37)

The index functions indicated by the augmented vector and augmented matrix are

$$e_k = N_k - M_k = G_k Z_k - D_k X_k = \begin{bmatrix} -D_k & G_k \end{bmatrix} X_k^a,$$

$$J = E \left\{ \sum_{k=1}^{N} [(X_k^a)^T Q_k X_k^a + U_{k-1}^T R_{k-1} U_{k-1}] \right\}.$$  (38)

when

$$Q_k = \begin{bmatrix} -D_k & G_k \end{bmatrix} \begin{bmatrix} -D_k \\ G_k \end{bmatrix}.$$  (39)

$X_k^a$ is a matrix consisting of $\tilde{X}_k$ and $\tilde{Z}_k$, which constitute the feedback loop. Figure 9 is the construct schematic diagram.

**3. Simulation Results and Discussion**

Because of the volatility of the wave speed, the hydrodynamic torque changes proportional to the rapid fluctuation of the wave speed. Therefore, the wave power conversion system is a time-varying parameter system. Formally, the wave speed is measured as accurately in this system as in others; however, the current simulation can measure the waves with errors of less than 10%. The most widely used method is based on estimating the turbine shaft speed using Kalman filter. In our simulation, we describe the wave speed as a random signal with filtering, as shown in Figure 10, without considering large turbine effects. The wave velocity varies from 0.5 m/s to 3.0 m/s, and the average wave maintains a velocity of 2.0 m/s for 50 s; this time interval is sufficient to simulate the seas’ mild motion and observe the response of the controller.

The ocean power conversion system is designed to achieve two simple goals. Our primary goal was to capture the maximum wave power possible using the MPPT algorithm. Additionally, we aimed to determine how the system’s load tolerance could be increased. To achieve the second objective, we minimized the rate of change of the generator torque control input by integrating the cost function (38). The control part we built in Simulink is presented in Figure 11.
In the paper, we adopted two controllers—a PID controller and an optimal controller—and compared them, verifying that the optimal LQT controller is capable of better tracing accuracy. The PID gains are $K_p = 3$, $K_I = 5$, and $K_D = 0.05$. According to function (24), the subscript of dimension in optimal LQT controller is $k = 2$. By comparing the errors between the actual and optimal torques, the following results were found. As expected, the wave’s step changes substantially alter the optimal reference; the actual generator torque responses of the PID (blue) and optimal control (red) closely trace the optimal curve (black) in Figure 12(a). Thus, the optimal LQT controller provides better tracing stability and more accurate performance. We also plotted the absolute errors between the torque curves and the optimal curve in Figure 12(b) for additional clarity; again, the optimal controller clearly matches the ideal curve. Furthermore, the optimal LQT controller produces more accurate optimal values of the tip speed ratio, as shown in Figure 12(c).

The optimal LQT controller calculates the control input at each sampling time rather than simply producing the feedback gain once for each wave velocity. Figure 13 provides the output power and the maximum capture power. Figure 13(b) shows the small errors between the actual and optimal curves and the high efficiency of the optimal LQT control algorithm in maximizing power capture.
4. Conclusions

In this paper, we introduce a power generation control system for a new drag-type VAT with several retractable blades and describe how the turbine works under the repetitive wave disturbance model. To maximize power capture, we developed the model predictive controller using the linearized system to trace at the optimal tip speed ratio. Different wave speed types were investigated to solve the MPPT problem more efficiently. The simulation results show that the shaft follows the optimal curve perfectly, and, thus, the maximum power capture optimization occurs at convergence.

In the future, we will investigate other control algorithms and apply additional mathematical calculations to optimize their performance.

**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v )</td>
<td>Velocity of the water flow</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Water density</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Tip speed ratio</td>
</tr>
<tr>
<td>( C_p(\lambda) )</td>
<td>Power coefficient</td>
</tr>
<tr>
<td>( C_q(\lambda) )</td>
<td>Torque coefficient</td>
</tr>
<tr>
<td>( \omega_r )</td>
<td>Rotor speed</td>
</tr>
<tr>
<td>( \omega_g )</td>
<td>Generator speed</td>
</tr>
<tr>
<td>( T_g )</td>
<td>Generator electromagnetic torque</td>
</tr>
<tr>
<td>( T_{ls} )</td>
<td>Low-speed torque</td>
</tr>
<tr>
<td>( T_{hs} )</td>
<td>High-speed torque</td>
</tr>
<tr>
<td>( K_g )</td>
<td>Generator external damping</td>
</tr>
<tr>
<td>( K_r )</td>
<td>Rotor external damping</td>
</tr>
<tr>
<td>( K_t )</td>
<td>Turbine total external damping</td>
</tr>
</tbody>
</table>
Captured power using optimal LQT control and the absolute errors.

Figure 13: Captured power using optimal LQT control and the absolute errors.

\[ J_r: \text{Rotor inertia} \]
\[ J_g: \text{Generator inertia} \]
\[ J_t: \text{Turbine total inertia} \]
\[ B_r: \text{Rotor external stiffness} \]
\[ B_g: \text{Generator external stiffness} \]
\[ B_t: \text{Turbine total external stiffness} \]

### Competing Interests

The authors declare that no competing interests exist.

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