Research Article

Permanence for the Discrete Competition Model with Infinite Deviating Arguments

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Received 11 August 2016; Accepted 25 September 2016

Academic Editor: Zhan Zhou

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Sufficient conditions are obtained for the permanence of the following discrete model of competition: $x_1(k+1) = x_1(k) \exp \{r_1(k)[(K_1(k) + \alpha_1(k) \sum_{s=0}^{+\infty} J_2(s)x_2(k-s))/(1 + \sum_{s=0}^{+\infty} J_2(s)x_2(k-s)) - x_1(k - \delta_1(k))]\}$; $x_2(k+1) = x_2(k) \exp \{r_2(k)[(K_2(k) + \alpha_2(k) \sum_{s=0}^{+\infty} J_1(s)x_1(k-s))/(1 + \sum_{s=0}^{+\infty} J_1(s)x_1(k-s)) - x_2(k - \delta_2(k))]\}$, where r_i, K_i, α_i, J_i , and $\delta_i, i = 1, 2$, are nonnegative sequences bounded above and below by positive constants, and $K_i > \alpha_i, i = 1, 2$.

1. Introduction

Throughout this paper, for any bounded sequence $\{h(n)\}$, set $h^u = \sup_{n \in \mathbb{N}} \{h(n)\}$ and $h^l = \inf_{n \in \mathbb{N}} \{h(n)\}$.

Li and Xu [1] studied the following two-species integrodifferential model of mutualism:

$$\begin{aligned} N_1'(t) &= r_1(t) N_1(t) \\ &\cdot \left[\frac{K_1(t) + \alpha_1(t) \int_0^\infty J_2(s) N_2(t-s) ds}{1 + \int_0^\infty J_2(s) N_2(t-s) ds} \right. \\ &\quad \left. - N_1(t - \sigma_1(t)) \right], \\ N_2'(t) &= r_2(t) N_2(t) \end{aligned} \quad (1)$$

$$\begin{aligned} &\cdot \left[\frac{K_2(t) + \alpha_2(t) \int_0^\infty J_1(s) N_1(t-s) ds}{1 + \int_0^\infty J_1(s) N_1(t-s) ds} \right. \\ &\quad \left. - N_2(t - \sigma_2(t)) \right]. \end{aligned}$$

By applying the coincidence degree theory, they showed that system (1) admits at least one positive ω -periodic solution. Chen and You [2] argued that a general nonautonomous nonperiodic system is more appropriate, and for the general nonautonomous case, they showed that the system is permanent. For more background and biological adjustments of system (1), one could refer to [1–6] and the references cited therein. For more work on mutualism model, one could refer to [7–34] and the references cited therein.

Li and Yang [31] and Li [32] argued that the discrete time models governed by difference equations are more appropriate than the continuous ones when the populations have nonoverlapping generations; corresponding to system (1), they proposed the following two-species discrete model of mutualism with infinite deviating arguments:

$$\begin{aligned} x_1(k+1) &= x_1(k) \exp \left\{ r_1(k) \right. \\ &\cdot \left[\frac{K_1(k) + \alpha_1(k) \sum_{s=0}^{+\infty} J_2(s) x_2(k-s)}{1 + \sum_{s=0}^{+\infty} J_2(s) x_2(k-s)} \right. \\ &\quad \left. \left. - x_1(k - \delta_1(k)) \right] \right\}, \end{aligned}$$

$$x_2(k+1) = x_2(k) \exp \left\{ r_2(k) \cdot \left[\frac{K_2(k) + \alpha_2(k) \sum_{s=0}^{+\infty} J_1(s) x_1(k-s)}{1 + \sum_{s=0}^{+\infty} J_1(s) x_1(k-s)} - x_2(k - \delta_2(k)) \right] \right\}, \quad (2)$$

where $x_i(k)$, $i = 1, 2$, is the density of mutualism species i at the k th generation; $\{r_i(k)\}$, $\{K_i(k)\}$, $\{\alpha_i(k)\}$, $\{J_i(k)\}$, and $\{\delta_i(k)\}$, $i = 1, 2$, are bounded nonnegative sequences such that

$$\begin{aligned} 0 < r_i^l &\leq r_i^u, \\ 0 < \alpha_i^l &\leq \alpha_i^u, \\ 0 < K_i^l &\leq K_i^u, \\ 0 < \delta_i^l &\leq \delta_i^u, \end{aligned} \quad (3)$$

$$\sum_{j=1}^{+\infty} J_i(n) = 1,$$

$$\alpha_i > K_i.$$

They showed that, under the above assumption, system (2) is permanent.

It brings to our attention the fact that the main results of [31, 32] deeply depend on the assumption $\alpha_i > K_i$, $i = 1, 2$. Now, an interesting issue is proposed: is it possible for us to investigate the persistent property of system (2) under the assumption $K_i > \alpha_i$, $i = 1, 2$?

From the point of view of biology, in the sequel, we shall consider (2) together with the initial conditions:

$$x_i(s) = \phi_i(s), \quad s = \dots, -k, -k+1, \dots, -2, -1, \quad i = 1, 2. \quad (4)$$

Then system (2) has a unique positive solution $(x_1(k), x_2(k))_{k=0}^{+\infty}$ satisfying the initial condition (4).

From now on, we assume that the coefficients of system (2) satisfy the following.

(A) $\{r_i(k)\}$, $\{K_i(k)\}$, $\{\alpha_i(k)\}$, $\{J_i(k)\}$, and $\{\delta_i(k)\}$, $i = 1, 2$, are bounded nonnegative sequences such that

$$\begin{aligned} 0 < r_i^l &\leq r_i^u, \\ 0 < \alpha_i^l &\leq \alpha_i^u, \\ 0 < K_i^l &\leq K_i^u, \\ 0 < \delta_i^l &\leq \delta_i^u, \end{aligned} \quad (5)$$

$$\sum_{j=1}^{+\infty} J_i(n) = 1,$$

$$K_i > \alpha_i.$$

We mention here that such an assumption implies that the relationship between two species is competition; indeed, under the assumption (A), the first equation in system (1) can be rewritten as follows:

$$\begin{aligned} x_1(k+1) &= x_1(k) \exp \left\{ r_1(k) \cdot \left[\frac{K_1(k) + \alpha_1(k) \sum_{s=0}^{+\infty} J_2(s) x_2(k-s)}{1 + \sum_{s=0}^{+\infty} J_2(s) x_2(k-s)} - x_1(k - \delta_1(k)) \right] \right\} \\ &= x_1(k) \exp \left\{ r_1(k) \left[K_1(k) - x_1(k - \delta_1(k)) - \frac{(K_1(k) - \alpha_1(k)) \sum_{s=0}^{+\infty} J_2(s) x_2(k-s)}{1 + \sum_{s=0}^{+\infty} J_2(s) x_2(k-s)} \right] \right\}. \end{aligned} \quad (6)$$

Similar to the above analysis, the second equation in system (2) can be rewritten as follows:

$$x_2(k+1) = x_2(k) \exp \left\{ r_2(k) \left[K_2(k) - x_2(k - \delta_2(k)) - \frac{(K_2(k) - \alpha_2(k)) \sum_{s=0}^{+\infty} J_1(s) x_1(k-s)}{1 + \sum_{s=0}^{+\infty} J_1(s) x_1(k-s)} \right] \right\}. \quad (7)$$

From (6) and (7), one could easily see that both species have negative effect on the other species; that is, the relationship between two species is competition.

Concerned with the persistent property of systems (2) and (4), we have the following result.

Theorem 1. *In addition to (A), assume further that*

$$\begin{aligned} r_1^l K_1^l &> r_1^u (K_1^u - \alpha_1^l) \frac{M_2}{1 + M_2}, \\ r_2^l K_2^l &> r_2^u (K_2^u - \alpha_2^l) \frac{M_1}{1 + M_1} \end{aligned} \quad (8)$$

holds, where

$$M_i = \frac{1}{r_i^l} \exp \{ r_i^u K_i^u (\delta_i + 1) - 1 \}, \quad i = 1, 2; \quad (9)$$

then system (2) is permanent; that is, there exist positive constants m_i , M_i , $i = 1, 2$ (M_i is defined by (9)), which are independent of the solution of system (2), such that, for any positive

solution $(N_1(t), N_2(t))$ of system (2) with initial condition (4), one has

$$m_i \leq \liminf_{k \rightarrow +\infty} x_i(k) \leq \limsup_{k \rightarrow +\infty} x_i(k) \leq M_i, \quad i = 1, 2. \quad (10)$$

2. Proof of the Main Result

Now we state several lemmas which will be useful in proving our main result.

Lemma 2 (see [11]). Assume that $\{x(k)\}$ satisfies $x(k) > 0$ and

$$x(k+1) \leq x(k) \exp\{a(k) - b(k)x(k)\} \quad (11)$$

for $k \in \mathbb{N}$, where $a(k)$ and $b(k)$ are nonnegative sequences bounded above and below by positive constants. Then

$$\limsup_{k \rightarrow +\infty} x(k) \leq \frac{1}{b^l} \exp(a^u - 1). \quad (12)$$

Lemma 3 (see [11]). Assume that $\{x(k)\}$ satisfies

$$x(k+1) \geq x(k) \exp\{a(k) - b(k)x(k)\}, \quad k \geq N_0, \quad (13)$$

$\limsup_{k \rightarrow +\infty} x(k) \leq x^*$ and $x(N_0) > 0$, where $a(k)$ and $b(k)$ are nonnegative sequences bounded above and below by positive constants and $N_0 \in \mathbb{N}$. Then

$$\liminf_{k \rightarrow +\infty} x(k) \geq \min \left\{ \frac{a^l}{b^u} \exp\{a^l - b^u x^*\}, \frac{a^l}{b^u} \right\}. \quad (14)$$

Lemma 4 (see [34]). Let $x : \mathbb{Z} \rightarrow \mathbb{R}$ be nonnegative bounded sequences, and let $H : \mathbb{N} \rightarrow \mathbb{R}$ be nonnegative sequences such that $\sum_{n=0}^{\infty} H(n) = 1$. Then

$$\begin{aligned} \liminf_{n \rightarrow +\infty} x(n) &\leq \liminf_{n \rightarrow +\infty} \sum_{s=-\infty}^n H(n-s)x(s) \\ &\leq \limsup_{n \rightarrow +\infty} \sum_{s=-\infty}^n H(n-s)x(s) \\ &\leq \limsup_{n \rightarrow +\infty} x(n). \end{aligned} \quad (15)$$

Now we are in the position to prove the main result of this paper.

Proof of Theorem 1. Set

$$\delta = \max\{\delta_1^u, \delta_2^u\}. \quad (16)$$

Let $(x_1(k), x_2(k))$ be any positive solution of system (2) with initial condition (4). From (6), it follows that

$$\begin{aligned} x_1(k+1) &\leq x_1(k) \exp\{r_1(k)K_1(k)\} \\ &\leq x_1(k) \exp\{r_1^u K_1^u\}. \end{aligned} \quad (17)$$

By using (17), one could easily obtain that

$$x_1(k - \delta_1(k)) \geq x_1(k) \exp\{-r_1^u K_1^u \delta\}. \quad (18)$$

Substituting (18) into (6), it follows that

$$\begin{aligned} x_1(k+1) &\leq x_1(k) \exp\{r_1^u K_1^u - r_1^l \exp\{-r_1^u K_1^u \delta\} x_1(k)\}. \end{aligned} \quad (19)$$

Thus, as a direct corollary of Lemma 2, according to (19), one has

$$\limsup_{k \rightarrow +\infty} x_1(k) \leq \frac{1}{r_1^l} \exp\{r_1^u K_1^u (\delta + 1) - 1\} \stackrel{\text{def}}{=} M_1. \quad (20)$$

By using (7), similar to the analysis of (17)–(20), we can obtain

$$\limsup_{k \rightarrow +\infty} x_2(k) \leq \frac{1}{r_2^l} \exp\{r_2^u K_2^u (\delta + 1) - 1\} \stackrel{\text{def}}{=} M_2. \quad (21)$$

From (20), (21), and Lemma 4, we have

$$\begin{aligned} \limsup_{k \rightarrow +\infty} \sum_{s=0}^{+\infty} J_i(s) x_i(k-s) &= \limsup_{k \rightarrow +\infty} \sum_{s=-\infty}^k J_i(k-s) x_i(s) \leq \limsup_{k \rightarrow +\infty} x_i(k) \\ &\leq M_i, \quad i = 1, 2. \end{aligned} \quad (22)$$

Condition (6) implies that, for enough small $\varepsilon > 0$, inequalities

$$\begin{aligned} r_1^l K_1^l &> r_1^u (K_1^u - \alpha_1^l) \frac{(M_2 + \varepsilon)}{1 + (M_2 + \varepsilon)}, \\ r_2^l K_2^l &> r_2^u (K_2^u - \alpha_2^l) \frac{(M_1 + \varepsilon)}{1 + (M_1 + \varepsilon)} \end{aligned} \quad (23)$$

hold. For above $\varepsilon > 0$, from (19)–(22), it follows that there exists $N_1 > 0$ such that, for all $k > N_1$ and $i = 1, 2$,

$$\begin{aligned} \sum_{s=0}^{+\infty} J_i(s) x_i(k-s) &< M_i + \varepsilon, \\ x_i(k) &< M_i + \varepsilon. \end{aligned} \quad (24)$$

By using the fact that $e^x \geq 1 + x$, $x \geq 0$, from (9), one could easily see that

$$\begin{aligned} M_1 &= \frac{1}{r_1^l} \exp\{r_1^u K_1^u (\delta + 1) - 1\} \geq \frac{1}{r_1^l} (r_1^u K_1^u (\delta + 1)) \\ &> K_1^u \geq \alpha_1^l. \end{aligned} \quad (25)$$

For $k \geq N_1 + \delta$, from (24) and (6), we have

$$\begin{aligned}
 x_1(k+1) &= x_1(k) \exp \left\{ r_1(k) \left[K_1(k) \right. \right. \\
 &\quad \left. \left. - x_1(k - \delta_1(k)) \right. \right. \\
 &\quad \left. \left. - \frac{(K_1(k) - \alpha_1(k)) \sum_{s=0}^{+\infty} J_2(s) x_2(k-s)}{1 + \sum_{s=0}^{+\infty} J_2(s) x_2(k-s)} \right] \right\} \\
 &\geq x_1(k) \exp \left\{ r_1(k) \left[K_1(k) - x_1(k - \delta_1(k)) \right. \right. \\
 &\quad \left. \left. - \frac{(K_1(k) - \alpha_1(k)) \sum_{s=0}^{+\infty} J_2(s) x_2(k-s)}{\sum_{s=0}^{+\infty} J_2(s) x_2(k-s)} \right] \right\} \\
 &= x_1(k) \exp \{ r_1(k) [K_1(k) - x_1(k - \delta_1(k)) \\
 &\quad - (K_1(k) - \alpha_1(k))] \} = x_1(k) \exp \{ r_1(k) [\alpha_1(k) \\
 &\quad - x_1(k - \delta_1(k))] \} \geq x_1(k) \exp \{ r_1^l \alpha_1^l - r_1^u (M_1 \\
 &\quad + \varepsilon) \};
 \end{aligned} \tag{26}$$

we mention here that, from (25), $r_1^l \alpha_1^l - r_1^u (M_1 + \varepsilon) < 0$. By using (26), we obtain

$$\begin{aligned}
 x_1(k - \delta_1(k)) \\
 \leq x_1(k) \exp \{ - [r_1^l \alpha_1^l - r_1^u (M_1 + \varepsilon)] \delta \}.
 \end{aligned} \tag{27}$$

Substituting (27) into (4), using (24), for $k \geq N_1 + \delta$, it follows that

$$\begin{aligned}
 x_1(k+1) &\geq x_1(k) \exp \left\{ r_1(k) \left[K_1(k) \right. \right. \\
 &\quad \left. \left. - x_1(k - \delta_1(k)) \right. \right. \\
 &\quad \left. \left. - \frac{(K_1(k) - \alpha_1(k)) \sum_{s=0}^{+\infty} J_2(s) x_2(k-s)}{1 + \sum_{s=0}^{+\infty} J_2(s) x_2(k-s)} \right] \right\} \\
 &\geq x_1(k) \exp \left\{ r_1^l K_1^l - r_1^u \right. \\
 &\quad \cdot \exp \{ - [r_1^l \alpha_1^l - r_1^u (M_1 + \varepsilon)] \delta \} x_1(k) \\
 &\quad \left. - \frac{(K_1^u - \alpha_1^l) \sum_{s=0}^{+\infty} J_2(s) x_2(k-s)}{1 + \sum_{s=0}^{+\infty} J_2(s) x_2(k-s)} \right\} \\
 &\quad \cdot \exp \left\{ r_1^l K_1^l - r_1^u \exp \{ - [r_1^l \alpha_1^l - r_1^u (M_1 + \varepsilon)] \delta \} \right. \\
 &\quad \cdot x_1(k) - \frac{r_1^u (K_1^u - \alpha_1^l) (M_2 + \varepsilon)}{1 + (M_2 + \varepsilon)} \left. \right\}.
 \end{aligned} \tag{28}$$

Thus, as a direct corollary of Lemma 3, according to (23) and (28), one has

$$\liminf_{k \rightarrow +\infty} x_1(k) \geq \min \{A_{1\varepsilon}, A_{2\varepsilon}\}, \tag{29}$$

$$\begin{aligned}
 A_{1\varepsilon} &= \frac{\Delta_1^\varepsilon}{r_1^u} \exp \{ [r_1^l \alpha_1^l - r_1^u (M_1 + \varepsilon)] \delta \}, \\
 A_{2\varepsilon} &= A_{1\varepsilon} \\
 &\quad \cdot \exp \{ \Delta_1^\varepsilon - r_1^u \exp \{ - [r_1^l \alpha_1^l - r_1^u (M_1 + \varepsilon)] \delta \} M_1 \}, \\
 \Delta_1^\varepsilon &= r_1^l K_1^l - \frac{r_1^u (K_1^u - \alpha_1^l) (M_2 + \varepsilon)}{1 + (M_2 + \varepsilon)}.
 \end{aligned} \tag{30}$$

Letting $\varepsilon \rightarrow 0$, it follows that

$$\liminf_{k \rightarrow +\infty} x_1(k) \geq \frac{1}{2} \min \{A_1, A_2\} \stackrel{\text{def}}{=} m_1 > 0, \tag{31}$$

where

$$\begin{aligned}
 A_1 &= \frac{\Delta_1}{r_1^u} \exp \{ [r_1^l \alpha_1^l - r_1^u M_1] \delta \}, \\
 A_2 &= A_1 \exp \{ \Delta_1 - r_1^u \exp \{ - [r_1^l \alpha_1^l - r_1^u M_1] \delta \} M_1 \}, \\
 \Delta_1 &= r_1^l K_1^l - \frac{r_1^u (K_1^u - \alpha_1^l) M_2}{1 + M_2}.
 \end{aligned} \tag{32}$$

Similar to the analysis of (26)–(29), by applying (24), from (7), we also have

$$\liminf_{k \rightarrow +\infty} x_2(k) \geq \frac{1}{2} \min \{B_1, B_2\} \stackrel{\text{def}}{=} m_2 > 0, \tag{33}$$

where

$$\begin{aligned}
 B_1 &= \frac{\Delta_2}{r_2^u} \exp \{ [r_2^l \alpha_2^l - r_2^u M_2] \delta \}, \\
 B_2 &= B_1 \exp \{ \Delta_2 - r_2^u \exp \{ - [r_2^l \alpha_2^l - r_2^u M_2] \delta \} M_2 \}, \\
 \Delta_2 &= r_2^l K_2^l - \frac{r_2^u (K_2^u - \alpha_2^l) M_1}{1 + M_1}.
 \end{aligned} \tag{34}$$

(20), (21), (31), and (33) show that system (2) is permanent. The proof of the theorem is completed. $\square$

Competing Interests

The author declares that there are no competing interests regarding the publication of this paper.

Acknowledgments

This work is supported by National Social Science Foundation of China (16BKS132), Humanities and Social Science Research Project of Ministry of Education Fund (15YJA710002), and the Natural Science Foundation of Fujian Province (2015J01283).

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