Research Article

Dynamics of a Seasonally Forced Phytoplankton-Zooplankton Model with Impulsive Biological Control

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This paper investigates the dynamics of a seasonally forced phytoplankton-zooplankton model with impulsive biological control. It shows that the periodic eradicated solution is unstable. Further, the condition for permanence of the system is established by relations between the model parameters and the intensity of the impulses. The numerical analysis is performed to study the effect of seasonality and impulsive perturbations on plankton dynamics. The numerical results imply that the seasonal forcing can trigger more periodic mode and the impulsive period for control of the size of phytoplankton is more practicable to the system than the impulsive release of zooplankton. These conclusions provide a better understanding of controlling harmful algae blooms.

1. Introduction

Algae are very diverse and found almost everywhere on the planet. They play an important role in marine, freshwater, and some terrestrial ecosystems. On one hand, phytoplankton are critical to sustain most aquatic food chains and produce half of the world's oxygen in the process of photosynthesis [1]. On the other hand, phytoplankton populations can grow explosively and lead to severe oxygen depletion in the relevant waters when harmful algal blooms occur. As a consequence, human activities would be limited and economy would suffer. Therefore, in order to prevent and control harmful algae blooms a better understanding of the mechanisms that trigger the occurrence or explain the absence of a phytoplankton bloom is of considerable significance. Truscott and Brindley [2] were the first to offer a model considering the prey-predator system of phytoplankton and zooplankton as a nonlinear excitable system to explain the dynamics of harmful algal blooms. In the framework, the evolution of phytoplankton and zooplankton populations is formulated by

$$\begin{aligned}\dot{P} &= rP \left(1 - \frac{P}{K}\right) - R_m Z \frac{P^2}{\alpha^2 + P^2}, \\ \dot{Z} &= \gamma R_m Z \frac{P^2}{\alpha^2 + P^2} - \mu Z,\end{aligned}\tag{1}$$

where P and Z represent population densities of phytoplankton and zooplankton, respectively. The first term in the right side of the first equation in (1) is the logistic growth function. The term $P^2/(\alpha^2 + P^2)$ is called Hollings type III grazing function [3]. r is the maximum growth rate of phytoplankton when P is small, K is the environmental carry capacity of phytoplankton, R_m reflects the maximum specific predation rate, α governs how quickly that maximum is attained as prey densities increase, γ denotes the ratio of biomass consumed to biomass of new herbivores produced, and μ measures the zooplankton death rate. If we quote the typical parameter values [2, 4] for system (1), then the positive equilibrium of (1) is asymptotically stable. Assume that the initial phytoplankton concentration is fixed in the stationary value. Then, a phytoplankton bloom that is followed by a delayed zooplankton bloom is triggered by suppressing the initial zooplankton concentration sufficiently far below the stationary value [4].

As a classical predator-prey model, it has been extensively discussed by many researchers [5–9]. In fact in spite of noticing the remarkable impact of seasonal forcing on the phytoplankton birth-rate by Truscott and Brindley, there are few models explicitly taken into account. Even though Gao et al. [10] had considered the effect of seasonality and periodicity on the growth rate of phytoplankton in their

model, the growth rate taken as a sinusoidal forcing function of time could not reflect sufficiently the explosive growth of phytoplankton. Freund et al. [4] filled the gap, presenting and discussing simulation results of the seasonally forced Truscott-Brindley model with an exponential function depicting the explosive growth. Later, Luo [11] developed the seasonally forced Truscott-Brindley model by including the growth rate and the intrinsic carrying capacity of phytoplankton changing with respect to time and nutrient concentration. For simplicity we only consider the maximum growth rate r in the model (1) as periodically varying function of time due to seasonal variation. And, we adopt the same relation between the phytoplankton growth rate and seasonally varying temperature as Freund et al. [4] have suggested. The effect of changing temperature on growth rate of phytoplankton is

$$r_0(t) = (Q_{10})^{(T(t)-T)/10} r, \quad (2)$$

where r is the average value of the intrinsic growth rate of phytoplankton, Q_{10} is assumed to be a constant which asserts that a change of the temperature by 10°C will multiply the rate at mean temperature, t denotes time (days), $T(t) = T + \Gamma \sin(\theta t + \varphi)$ represents temperature on time t which is adapted from a fit by using average temperature data, T is the average temperature in one cycle, Γ denotes the amplitude of temperature, θ is the angular velocity, and φ is the initial phase. For the sake of simplification, the intraspecific competition of phytoplankton is not affected by seasonally varying temperature.

The use of impulsive control for ecological systems is proved to be one of the most effective methods and has received much attention from both ecologists and applied mathematicians [12–16]. However, almost all the work on the Truscott-Brindley models neglects the impulsive biological control of phytoplankton. Thus, we periodically release zooplankton in laboratories at a constant to reduce the population level of phytoplankton by grazing. Keeping these aspects in view, we establish the following model:

$$\begin{aligned} \dot{P} &= rP \left[(Q_{10})^{(T(t)-T)/10} - \frac{P}{K} \right] - R_m Z \frac{P^2}{\alpha^2 + P^2}, \\ \dot{Z} &= \gamma R_m Z \frac{P^2}{\alpha^2 + P^2} - \mu Z, \\ t &\neq n\tau, \quad (3) \end{aligned}$$

$$\Delta P = 0,$$

$$\Delta Z = Q,$$

$$t = n\tau,$$

where $\Delta P = P(t^+) - P(t)$, $\Delta Z = Z(t^+) - Z(t)$, τ is the period of the impulsive effect, $Q > 0$ denotes the concentration change of zooplankton by releasing which is determined by the maximum amount of zooplankton produced by laboratories and $n \in N$, and N is the set of all nonnegative integers. In system (3), all parameters are supposed to be positive constants.

This paper is organized as follows. In Section 2, we study the dynamics of system (3) without impulsive effect. In Section 3, we mainly focus on system (3) and obtain the stability of phytoplankton-eradication periodic solution and the condition of permanence of (3). In Section 4, the numerical analysis is performed to investigate the dynamics of (3). Finally, we close with a discussion in Section 5.

2. Dynamical Properties of (3) without Impulsive Effect

In order to analyze to the dynamical behavior of (3) without impulsive effect, we first consider a periodically logistic equation:

$$\dot{u} = u(\alpha(t) - \beta(t)u), \quad (4)$$

where $\alpha(t)$ and $\beta(t)$ are periodically continuous functions defined on R^+ with the common period $\omega > 0$. According to the results of [11, 17], we obtain the following conclusions.

Lemma 1 (see [17]). *If $\beta(t) \geq 0$ for $t \in R^+$ and $\int_0^\omega \beta(t)dt > 0$, then (4) has a unique nonnegative ω -periodic solution $u^*(t)$ which is globally asymptotically stable. That is, $\lim_{t \rightarrow \infty} u(t) = u^*(t)$, for each positive solution $u(t)$ of (4). Moreover, if $\int_0^\omega \alpha(t)dt > 0$, then $u^*(t) > 0$ for $t \in R^+$ and if $\int_0^\omega \alpha(t)dt \leq 0$, then $u^*(t) = 0$.*

Consider the extended logistic model:

$$\dot{P} = rP \left[(Q_{10})^{(T(t)-T)/10} - \frac{P}{K} \right]. \quad (5)$$

Since r , Q_{10} , and K are positive constants and $T(t)$ is ω -periodic, by means of Lemma 1, there are the following results.

Theorem 2 (see [17]). *System (5) has a unique positive ω -periodic solution which is globally asymptotically stable; that is, there is a positive ω -periodic function $P^*(t)$ such that for each solution $P(t)$ of (5) with positive initial value $P(0)$, one has $\lim_{t \rightarrow \infty} P(t) = P^*(t)$.*

Consider the following system:

$$\begin{aligned} \dot{P} &= rP \left[(Q_{10})^{(T(t)-T)/10} - \frac{P}{K} \right] - R_m Z \frac{P^2}{\alpha^2 + P^2}, \\ \dot{Z} &= \gamma R_m Z \frac{x^2}{\alpha^2 + x^2} - \mu Z, \end{aligned} \quad (6)$$

where r , Q_{10} , R_m , α , γ , μ , and T are positive constants and $T(t)$ is a periodic continuous function on R^+ with the common period $\omega > 0$.

Definition 3. System (6) is said to be permanent if there exist constants T_0 , M , $m > 0$ satisfying $m \leq P(t) \leq M$ and $m \leq Z(t) \leq M$ when $t > T_0$, where $(P(t), Z(t))$ is any solution of system (6) with the initial values $P(0) > 0$ and $Z(0) > 0$.

Theorem 4 (see [11]). *System (6) is permanent provided that*

$$\frac{1}{\omega} \int_0^\omega \left[\gamma R_m \frac{(P^*)^2}{\alpha^2 + (P^*)^2} - \mu(t) \right] dt > 0, \quad (7)$$

where $P^*(t)$ is the uniquely periodic solution of (5) established by Theorem 2.

Theorem 5 (see [11]). *Assume that*

$$\frac{1}{\omega} \int_0^\omega \left[\gamma R_m \frac{(P^*)^2}{\alpha^2 + (P^*)^2} - \mu(t) \right] dt \leq 0; \quad (8)$$

then population of zooplankton tends to extinction; more precisely we have

$$\begin{aligned} \liminf_{t \rightarrow \infty} Z(t) &= 0, \\ \liminf_{t \rightarrow \infty} P(t) &> 0 \end{aligned} \quad (9)$$

for each solution of (6), where $P^*(t)$ is the uniquely periodic solution of (5) established by Theorem 2.

3. Extinction and Permanence of (3)

Definition 6. System (3) is regarded as permanent if there exist constants $T_0, M, m > 0$ satisfying $m \leq P(t) \leq M$ and $m \leq Z(t) \leq M$ when $t > T_0$, where $(P(t), Z(t))$ is any solution of system (3) with the initial values $P(0^+) > 0$ and $Z(0^+) > 0$.

Theorem 7. *Each solution of system (3) is ultimately bounded. That is, there exists a constant $M > 0$, such that $P(t) \leq M$ and $Z(t) \leq M$ for each solution $(P(t), Z(t))$ of system (3) with all t large enough.*

Proof. Suppose that $(P(t), Z(t))$ is a solution of system (3). Let $V(t) = \gamma P(t) + Z(t)$. Computing the upper right derivative of $V(t)$ for $t \neq n\tau$,

$$D^+ V(t) = \gamma r P \left[(Q_{10})^{(T(t)-T)/10} - \frac{P}{K} \right] - \mu Z. \quad (10)$$

Clearly, there exists $0 < s < \mu$ satisfying the following:

$$\begin{aligned} D^+ V(t) + sV(t) &\leq -\frac{\gamma r}{K} P^2 + \left[r(Q_{10})^{\Gamma/10} + s \right] \gamma P \\ &\leq L_0, \end{aligned} \quad (11)$$

where $L_0 = K\gamma[r(Q_{10})^{\Gamma/10} + s]^2/4r$.

For $t = n\tau$, $V(n\tau^+) = \gamma P(n\tau^+) + Z(n\tau^+) = \gamma P(n\tau) + Z(n\tau) + Q = V(n\tau) + Q$.

Then, if $t \in (n\tau, (n+1)\tau]$, we can get

$$\begin{aligned} V(t) &\leq \left(V(0^+) - \frac{L_0}{s} \right) \exp(-st) \\ &\quad + \frac{Q(1 - e^{-ns\tau})}{1 - e^{-s\tau}} e^{-s(t-n\tau)} + \frac{L_0}{s}. \end{aligned} \quad (12)$$

Hence, $\lim_{t \rightarrow \infty} V(t) \leq L_0/s + Q/(1 - e^{-s\tau})$. $V(t)$ is ultimately bounded by a constant and there exists a constant $M > 0$ such that $P(t) \leq M$ and $Z(t) \leq M$ for each solution $(P(t), Z(t))$ of system (3) with all t large enough. This completes the proof. $\square$

Theorem 8. *System (3) admits a positive periodic solution that corresponds to phytoplankton eradication:*

$$(\bar{P}(t), \bar{Z}(t)) = (0, Z^* e^{-\mu(t-n\tau)}), \quad (13)$$

where $Z^* = Q/(1 - e^{-\mu\tau})$. Furthermore, $\lim_{t \rightarrow \infty} |Z(t) - \bar{Z}(t)| = 0$.

Proof. If the phytoplankton population is absent, that is, $P(t) = 0$, system (3) reduces to

$$\begin{aligned} \dot{Z} &= -\mu Z, \quad t \neq n\tau, \\ \Delta Z &= Q, \quad t = n\tau, \\ Z(0^+) &= Z_0 > 0. \end{aligned} \quad (14)$$

Then it yields

$$Z(t) = Z(n\tau^+) e^{-\mu(t-n\tau)}, \quad t \in (n\tau, (n+1)\tau]. \quad (15)$$

Then, $Z((n+1)\tau^+) = Z((n+1)\tau) + Q = Z(n\tau^+) e^{-\mu\tau} + Q$, $t \in (n\tau, (n+1)\tau]$.

Imposing $Z((n+1)\tau^+) = Z(n\tau^+)$, we obtain the periodicity condition $Z^* = Q/(1 - e^{-\mu\tau})$, from which (13) is clear.

Note that the solution of (14) with initial value $Z_0 > 0$ is gotten by

$$Z(t) = (Z_0 - Z^*) e^{-\mu t} + \bar{Z}(t). \quad (16)$$

Hence, $\lim_{t \rightarrow \infty} |Z(t) - \bar{Z}(t)| = 0$. $\square$

Theorem 9. *Let $(P(t), Z(t))$ be a solution of (3). Then $(0, \bar{Z}(t))$ is unstable.*

Proof. To investigate the local stability of periodic solution $(0, \bar{Z}(t))$, we will use the method of small amplitude perturbations. To this purpose, define

$$\begin{aligned} P(t) &= u(t), \\ Z(t) &= \bar{Z}(t) + v(t), \end{aligned} \quad (17)$$

where $u(t)$ and $v(t)$ represent the small amplitude perturbations.

Therefore, the linearization of system (2) becomes

$$\begin{aligned} \dot{u} &= r_0(t) u, \\ \dot{v} &= -\mu v, \\ &\quad t \neq n\tau, \\ \Delta u &= 0, \\ \Delta v &= 0, \\ &\quad t = n\tau. \end{aligned} \quad (18)$$

FIGURE 1: Time series of system (6) without (a) and with (b) seasonal forcing of the phytoplankton growth rate when $(P(0), Z(0)) = (2, 5)$.

Then, it results in

$$\begin{pmatrix} u(t) \\ v(t) \end{pmatrix} = \Phi(t) \begin{pmatrix} u(0) \\ v(0) \end{pmatrix}, \quad 0 \leq t < \tau, \quad (19)$$

where $\Phi(t)$ satisfies

$$\dot{\Phi} = \begin{pmatrix} r_0(t) & 0 \\ 0 & -\mu \end{pmatrix} \Phi(t), \quad (20)$$

and $\Phi(0) = I$, the identity matrix.

The linearization of the third and the fourth equations of (3) becomes

$$\begin{pmatrix} u(n\tau^+) \\ v(n\tau^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u(n\tau) \\ v(n\tau) \end{pmatrix}. \quad (21)$$

The stability of periodic solution $(0, \bar{Z}(t))$ is determined by the eigenvalues of

$$M = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Phi(\tau), \quad (22)$$

which are

$$\begin{aligned} \lambda_1 &= \exp \int_0^\tau r_0(t) dt, \\ \lambda_2 &= e^{-\mu\tau}. \end{aligned} \quad (23)$$

Thus, according to Floquet theory [16], $(0, \bar{Z}(t))$ is locally stable if $|\lambda_1| < 1$ and $|\lambda_2| < 1$. With the assumption of (3), we know that $r_0(t) > 0$. So, $\lambda_1 > 1$. This establishes the theorem. $\square$

Theorem 10. System (3) is permanent if $2a\alpha\mu\tau > R_m Q$, where $a = (Q_{10})^{-\Gamma/10} r$.

FIGURE 2: Bifurcation diagrams of system (3) without (a) and with (b) seasonally varying phytoplankton growth rate when $0 \leq Q \leq 2$.

Proof. Suppose that $(P(t), Z(t))$ is a solution of (3) with (P_0, Z_0) . From Theorem 7, assume that $P(t) \leq M$ and $Z(t) \leq M$ for $t > 0$. From Theorem 8, we have $Z(t) \geq \bar{Z}(t) - \varepsilon_1$ for all t large enough and some $\varepsilon_1 > 0$. Let $m_2 = Qe^{-\mu\tau}/(1 - e^{-\mu\tau}) - \varepsilon_1$, so $Z(t) \geq m_2$ for t large enough. Therefore, we will next find m_1 such that $P(t) \geq m_1$ for t large enough. We will do it in the following two steps.

Step 1. Put $b = r/K$. Since $2a\alpha\mu\tau > R_m Q$, we can select $m_3 > 0$ and $\varepsilon_0 > 0$ small enough such that

$$\begin{aligned} m_3 &< \frac{a}{b}, \\ \delta &= \frac{\gamma R_m m_3}{\alpha^2} < \mu, \\ \sigma &= a\tau - bm_3\tau - \frac{R_m Q}{2\alpha(\mu - \delta)} - \frac{\varepsilon_0 R_m}{2\alpha}\tau > 0. \end{aligned} \quad (24)$$

We will show that there exists $t_1 \in (0, \infty)$ such that $P(t_1) \geq m_3$. Otherwise, using the above hypothesis, we get $\dot{Z} \leq Z(\delta - \mu)$. By Lemmas 2.2 in [18], we have $Z(t) \leq u(t)$ and $u(t) \rightarrow \bar{u}(t)$, $t \rightarrow \infty$, where $u(t)$ is the solution of the following equation:

$$\begin{aligned} \dot{u} &= u(\delta - \mu), \quad t \neq n\tau, \\ u(n\tau^+) &= u(n\tau) + Q, \quad t = n\tau, \\ u(0^+) &= Z(0) > 0 \end{aligned} \quad (25)$$

and $\bar{u}(t) = Qe^{(\delta-\mu)(t-n\tau)}/(1 - e^{(\delta-\mu)\tau})$, $t \in (n\tau, (n+1)\tau]$. Thus, there exists $t^* > 0$ such that

$$\begin{aligned} Z(t) &\leq u(t) \leq \bar{u}(t) + \varepsilon_0, \\ \dot{P} &\geq P\left(a - bm_3 - \frac{R_m}{2\alpha}(\bar{u} + \varepsilon_0)\right), \end{aligned} \quad (26)$$

for $t > t^*$.

FIGURE 3: Bifurcation diagrams of system (3) without (a) and with (b) seasonally varying phytoplankton growth rate when $2 \leq \tau \leq 180$.

Integrating (26) on $(n\tau, (n+1)\tau]$ ($n\tau > t^*$), it gets

$$\begin{aligned}
 P((n+1)\tau) &\geq P(n\tau) \\
 &\cdot \exp\left(\int_{n\tau}^{(n+1)\tau} \left(a - bm_3 - \frac{R_m}{2\alpha}(\bar{u}(t) + \varepsilon_0)\right) dt\right) \quad (27) \\
 &= P(n\tau) \exp(\sigma).
 \end{aligned}$$

Then, $P((n+k)\tau) \geq P(n\tau) \exp(k\sigma) \rightarrow \infty, k \rightarrow \infty$, which is a contradiction to the boundedness of $P(t)$. Hence, there exists $t_1 > 0$ such that $P(t_1) \geq m_3$.

Step 2. If $P(t) \geq m_3$ for all $t > t_1$, then our goal is accomplished. If not, we set $t_2 = \inf_{t > t_1} \{P(t) < m_3\}$. Then, $P(t) \geq m_3$ for $t \in (t_1, t_2)$. Since $P(t)$ is continuous, we have $P(t_2) = m_3$ and $t_1 < t_2$. By Step 1, there also exists $t' > t_2$ such that $P(t') \geq m_3$. Put $t_3 = \inf_{t > t_1} \{P(t) > m_3\}$. Then, $P(t) < m_3$ for $t \in (t_1, t_2)$ and $P(t_2) = m_3$. Let this process continue by using Step 1. If the process is stopped in finitely many times, the theorem follows. Otherwise, define an interval sequence

$[t_{2k-1}, t_{2k}]$ ($k \in \mathbb{N}$) with $t_{2k-1} < t_{2k} < t_{2k+1}$ such that $P(t) < m_3, t \in (t_{2k-1}, t_{2k}]$ and $P(t_n) = m_3, n \in \mathbb{N}$. Put $\tau_0 = \sup\{t_{2k} - t_{2k-1} \mid k \in \mathbb{N}\}$. If $\tau_0 = \infty$, we can obtain a subsequence $\{t_{k_i}\}$ satisfying $t_{2k_i+1} - t_{2k_i} \rightarrow \infty, k_i \rightarrow \infty$. As shown in Step 1, it will lead to a contradiction because of the boundedness of $P(t)$. Thus, $\tau_0 < \infty$ is true. Note that $\liminf_{t \rightarrow \infty} \dot{P}(t) \geq P(t)A$, where $A = a - bm_3 - R_m M / 2\alpha$. Put $m'_3 = m_3 e^{A\tau_0}$ and $m_1 = \min\{m_3, m'_3\}$. It follows $P(t) \geq m_1$ for t sufficiently large. The proof is completed. $\square$

4. Numerical Analysis

In this section, we will study the influence of seasonal forcing and impulsive perturbations. We quote typical parameter values [4]: $r = 0.3/\text{day}$, $K = 108 \mu\text{g/L}$, $R_m = 0.7/\text{day}$, $\alpha = 5.7 \mu\text{g/L}$, $\gamma = 0.05$, $T = 10^\circ\text{C}$, $\Gamma = 6^\circ\text{C}$, $\theta = 2\pi/(365 \text{ days})$, $\varphi/(2\pi) = 0.59$, and $Q_{10} = 2$. Based on Theorems 2 and 4, we see that the sufficiently small value of μ leads to be permanent for (6). For enough large value of μ , in contrast, (6) possesses

FIGURE 4: Phase portraits of system (3) with initial values $(P(0), Z(0)) = (2, 5)$ when $\mu = 0.012$ and $Q = 0.5$. (a) $\tau = 5$, (b) $\tau = 15$, (c) $\tau = 25$, (d) $\tau = 35$, (e) $\tau = 45$, and (f) $\tau = 55$.

FIGURE 5: Phase portraits and time series of system (3) with initial values $(P(0), Z(0)) = (2, 5)$ when $\mu = 0.12$, $Q = 0.5$, and $\tau = 57$.

a periodic solution that corresponds to zooplankton eradication. Figure 1(b) illustrates our conclusion. By comparison with Figure 1, a seasonal forcing of the phytoplankton growth rate can trigger a bloom mode. This claim is pointed out to be important for considering the seasonal variation in study of the Truscott and Brindley model.

First, we analyze the effect of Q on the long-run dynamics of system (3). For fixed parameter values $\mu = 0.012/\text{day}$ and $\tau = 12$ days, the bifurcation diagrams with respect to Q have been plotted in Figure 2(b). It needs to highlight that with the change of the parameter value Q system (3) undergoes period-3 attractors and later enters into chaotic behaviors. We know that system (3) without impulsive release of zooplankton presents the harmful algae bloom mode (Figure 1(b)). However, it can be seen in Figure 2(b) that if the impulsive biological control is considered, then the nonbloom mode will occur. That is to say that the occurrence of phytoplankton blooms is successfully prevented. Besides, for the purpose of studying the influence of seasonal forcing of the phytoplankton growth rate on system (3), the bifurcation diagrams of system (3) without considering seasonality are shown in Figure 2(a). As we can see in Figure 2(a), system (3) without seasonal fluctuations in the growth rate of phytoplankton exhibits relatively simple dynamical behaviors: period-1 attractor. It can be observed in Figure 2 that adding seasonal forcing reduces the maximum density of phytoplankton and enhances the zooplankton population size. What is more is that adding seasonal forcing produces chaos. It is noted in Figure 2(b) that although system (3) shows chaotic dynamics, the phytoplankton population does not trigger bloom mode. Second, we will investigate the influence of impulsive period on the population dynamics of system (3). Thus, we set $\mu = 0.012/\text{day}$ and $Q = 0.5 \mu\text{g/L}$. It follows that system (3) shown in Figure 3(a) exhibits the only dynamical behavior: period-1 attractor when we do not consider seasonality. It is worth to be stressed that the max density of phytoplankton is monotonically increasing with

increase of τ , and the max density of zooplankton is monotonically decreasing. In Figure 3(a) we need to emphasize that changing of τ system (3) without seasonality does not exhibit the phytoplankton bloom. Conversely, if the seasonal factor is included, Figure 3(b) illustrates relatively complex dynamical behavior for system (3). Figures 4 and 5 display the detailed cases. It is worth pointing out that in Figure 4 when the time of releasing presents a 5-day delay, the n periodic solution inflates to $n+1$ until $n = 10$. If $\tau \geq 57$ days, the phytoplankton population of system (3) exhibits bloom dynamics (Figure 5). By comparison with Figure 4, it concludes that a seasonal forcing augments fluctuation of population. Further analysis shows that τ is more practicable to system (3) as a control parameter than Q . Thus, we suggest that in order to control the size of phytoplankton the impulsive period τ should be selected to be feasible.

5. Discussion

We have studied the dynamics of a seasonally forced phytoplankton-zooplankton model with impulsive biological control. By control of the releasing period of zooplankton, the size of phytoplankton could be reduced. However, the amount of Q is determined by laboratories and this kind of production is limited. Therefore, system (3) cannot be used for larger areas of water. We suggest that system (3) should be suitable for smaller areas of water such as urban lakes. In this paper we consider the dynamic behaviors of the system when periodic forcing is solely superimposed on the growth rate of the phytoplankton. In fact, there are a number of ways to apply periodic forcing in an ecological model [19]. Thus, in the future, applying periodic forcing to the Truscott and Brindley model with impulsive biological control will be considered. From the above, it is known that releasing zooplankton to control the size of phytoplankton is limited. Therefore, we need study other effective ways to reduce harmful

algae blooms such as introducing competitive population for phytoplankton and harvesting for phytoplankton.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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