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Research Article

Mixed Synchronization of Chaotic Financial Systems by Using Linear Feedback Control

Ke Ding[1,2,3] and Xing Xu[4]

[1] *School of Information Technology, Jiangxi University of Finance and Economics, Nanchang 330013, China*
[2] *Jiangxi E-Commerce High Level Engineering Technology Research Centre, Jiangxi University of Finance and Economics, Nanchang 330013, China*
[3] *Higher Education Division, Central Queensland University, Rockhampton, QLD 4702, Australia*
[4] *School of Business Administration, Jiangxi University of Finance and Economics, Nanchang 330013, China*

Correspondence should be addressed to Ke Ding; keding@jxufe.edu.cn

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This paper deals with the mixed synchronization (coexistence of synchronization and antisynchronization) of two chaotic financial systems. Two mixed synchronization criteria for two chaotic financial systems are derived with a single controller and without external controls, respectively. In addition, the control method and synchronization criteria are applied to study the mixed synchronization of a class of modified chaotic financial systems. Three examples are used to illustrate the effectiveness of our derived results.

1. Introduction

Many dynamical behaviors of fluctuations for investments, prices, and interest rates can be described by a three-dimensional financial system [1]. By choosing some proper parameters, the chaotic behaviors have been generated by financial systems. The chaos results in the unpredictable evolution during the stabilization for financial systems. In order to control chaotic financial systems, synchronization of two chaotic financial systems has been studied. The classical synchronization of two chaotic systems refers to the variables of one system which achieve the consensus with the counterparts of the other system [2–12]. Besides the classical synchronization of two chaotic systems, antisynchronization can also be generated by two chaotic systems, which means the variables of one system achieve the consensus with the negative values of counterparts of the other system [13–15]. It should be pointed out that the mixed synchronization (coexistence of synchronization and antisynchronization) has been observed for some chaotic systems, which means that some variables of one chaotic system achieve synchronization with the counterpart variables of the other chaotic system, and the other variables of chaotic systems achieve antisynchronization simultaneously [16–18].

The synchronization of two chaotic financial systems has been widely investigated in [19–25]. However, to the best of author's knowledge, there are limited studies to investigate the coexistence of synchronization and antisynchronization of two chaotic financial systems [26, 27], in which the nonlinear control method (adaptive control) was used to achieve mixed synchronization. Within those existing studies of synchronization *(not mixed synchronization)* of two chaotic financial systems, the nonlinear control methods (active control and adaptive control) were utilized in [19–25]. Therefore, we focus on using the linear feedback control to achieve the coexistence of synchronization and antisynchronization of two chaotic financial systems.

In this paper, the mixed synchronization of two chaotic financial systems is studied. Two mixed synchronization criteria are derived with a single controller and without external controls for mixed synchronization of two chaotic financial systems, respectively. Those synchronization criteria and the control method are applied to investigate the mixed synchronization of a class of modified chaotic financial systems.

The effectiveness of our results is demonstrated by three simulation examples.

2. Preliminaries

Consider the following chaotic financial system:

$$\begin{aligned}
\frac{dx_1(t)}{dt} &= x_3(t) + (x_2(t) - a)\, x_1(t), \\
\frac{dx_2(t)}{dt} &= 1 - bx_2(t) - x_1^2(t), \\
\frac{dx_3(t)}{dt} &= -x_1(t) - cx_3(t),
\end{aligned} \tag{1}$$

where $x_1(t)$, $x_2(t)$, and $x_3(t)$ are the interest rate, the investment demand, and the price index, respectively. Constants a, b, and c are parameters which represent the saving amount, the cost per investment, and the elasticity of demand of commercial markets, receptively. $x_1(0) = x_{1_0}$, $x_2(0) = x_{2_0}$, and $x_3(0) = x_{3_0}$ are the initial conditions of (1).

System (1) can be regarded as a master system. One can construct the following slave system:

$$\begin{aligned}
\frac{dy_1(t)}{dt} &= y_3(t) + (y_2(t) - a)\, y_1(t) \\
&\quad - k_1(x_1(t) + y_1(t)), \\
\frac{dy_2(t)}{dt} &= 1 - by_2(t) - y_1^2(t) + k_2(x_2(t) - y_2(t)), \\
\frac{dy_3(t)}{dt} &= -y_1(t) - cy_3(t) - k_3(x_3(t) + y_3(t)),
\end{aligned} \tag{2}$$

where $y_1(t)$, $y_2(t)$, and $y_3(t)$ represent the state variables and k_1, k_2, and k_3 are gains. The initial conditions of (2) are given by $y_1(0) = y_{1_0}$, $y_2(0) = y_{2_0}$. $y_3(0) = y_{3_0}$.

The definition of synchronization of chaotic systems (1) and (2) refers to

$$\lim_{t\to\infty}(x_i(t) - y_i(t)) = 0, \quad \text{for } i = 1, 2, 3. \tag{3}$$

The antisynchronization of chaotic systems (1) and (2) is given as

$$\lim_{t\to\infty}(x_i(t) + y_i(t)) = 0, \quad \text{for } i = 1, 2, 3. \tag{4}$$

In this paper, the coexistence of antisynchronization and synchronization of chaotic systems (1) and (2) will be investigated; that is, some variables of chaotic systems (1) and (2) achieve antisynchronization, and the other variables of chaotic systems (1) and (2) achieve synchronization, which can be noted as the mixed synchronization of chaotic systems (1) and (2). In this paper, we focus on the case

$$\begin{aligned}
&\lim_{t\to\infty}(x_1(t) + y_1(t)) = 0, \\
&\lim_{t\to\infty}(x_2(t) - y_2(t)) = 0, \\
&\lim_{t\to\infty}(x_3(t) + y_3(t)) = 0.
\end{aligned} \tag{5}$$

Let $E_1(t) = x_1(t) + y_1(t)$, $e_2(t) = x_2(t) - y_2(t)$, and $E_3(t) = x_3(t) + y_3(t)$. Thus, one can derive the following error system:

$$\begin{aligned}
\frac{dE_1(t)}{dt} &= -(a + k_1)\, E_1(t) + E_3(t) + E_1(t)\, x_2(t) \\
&\quad - y_1(t)\, e_2(t), \\
\frac{de_2(t)}{dt} &= -(b + k_2)\, e_2(t) - E_1(t)\,(x_1(t) - y_1(t)), \\
\frac{dE_3(t)}{dt} &= -E_1(t) - (c + k_3)\, E_3(t).
\end{aligned} \tag{6}$$

The initial conditions of (6) are $E_1(0) = x_{1_0} + y_{1_0}$, $e_2(0) = x_{2_0} - y_{2_0}$, and $E_3(0) = x_{3_0} + y_{3_0}$.

3. Main Results

In this section, some mixed synchronization criteria for chaotic financial systems will be given.

One can define the following Lyapunov function:

$$V(t) = \frac{E_1^2(t) + e_2^2(t) + E_3^2(t)}{2}. \tag{7}$$

Theorem 1. *If the following inequalities hold:*

$$\max\{x_2(t),\ \forall t \ge 0\} < a + k_1, \tag{8}$$

$$0 < b + k_2, \tag{9}$$

$$0 < c + k_3, \tag{10}$$

$$\begin{aligned}
&\max\{|x_1^2(t)|,\ \forall t \ge 0\} \\
&\quad < 4(a + k_1 - \max\{x_2(t),\ \forall t \ge 0\})(b + k_2),
\end{aligned} \tag{11}$$

then two chaotic financial systems described by (1) and (2) can achieve global mixed synchronization.

Proof. Calculating the derivative of $V(t)$ along with (6) yields

$$\begin{aligned}
\frac{dV(t)}{dt} &= -(a + k_1)\, E_1^2(t) + E_1(t)\, E_3(t) \\
&\quad + E_1^2(t)\, x_2(t) - E_1(t)\, e_2(t)\, y_1(t) \\
&\quad - (b + k_2)\, e_2^2(t) - e_2 E_1(t)\,(x_1(t) - y_1(t)) \\
&\quad - E_1(t)\, E_3(t) - (c + k_3)\, E_3^2(t) \\
&= -(a + k_1 - x_2(t))\, E_1^2(t) - (b + k_2)\, e_2^2(t) \\
&\quad - x_1(t)\, E_1(t)\, e_2(t) - (c + k_3)\, E_3^2(t).
\end{aligned} \tag{12}$$

Condition (11) gives

$$x_1^2(t) < 4(a + k_1 - x_2(t))(b + k_2). \tag{13}$$

Inequalities $\max\{x_2(t),\ \forall t \geq 0\} < a+k_1, 0 < b+k_2$ in (8)-(9) and the inequality described by (13) indicate

$$-\left(a+k_1-x_2(t)\right)E_1^2(t)-\left(b+k_2\right)e_2^2(t) \\ -x_1(t)E_1(t)e_2(t)<0, \quad \forall t \geq 0. \tag{14}$$

The inequality described by (14) and $c+k_3 > 0$ in (10) imply that

$$\frac{dV(t)}{dt} < 0, \quad \forall E_1(t), e_2(t), E_3(t) \neq 0. \tag{15}$$

Using LaSalle invariant principle derives that the trajectories of (6) will be convergent to the largest invariant set in $dV(t)/dt = 0$ when $t \to \infty$. This ends the proof. □

Remark 2. Theorem 1 gives a mixed synchronization criterion for chaotic financial systems (1) and (2). Compared with existing synchronization results in [19–23, 25] for chaotic financial systems by using nonlinear feedback controls (the active control and adaptive control), the linear feedback control is used in Theorem 1.

Remark 3. In [19–23], hyperchaotic financial systems with four dimensions were studied. This paper mainly investigates the mix synchronization of three-dimensional chaotic financial systems (1). How to achieve mix synchronization of four-dimensional hyperchaotic financial systems by using the linear feedback control is our research work in the future.

If $c > 0$ and $k_3 = 0$, one can have the following corollary.

Corollary 4. *If $c > 0$, $k_3 = 0$, and the following inequalities hold:*

$$\begin{aligned} &\max\left\{x_2(t),\ \forall t \geq 0\right\} < a+k_1, \\ &0 < b+k_2, \\ &\max\left\{\left|x_1^2(t)\right|,\ \forall t \geq 0\right\} \\ &\quad < 4\left(a+k_1-\max\left\{x_2(t),\ \forall t \geq 0\right\}\right)\left(b+k_2\right), \end{aligned} \tag{16}$$

then two chaotic financial systems described by (1) and (2) can achieve global mixed synchronization.

If $c > 0, b > 0$, and $k_2 = k_3 = 0$, one can have the following theorem.

Theorem 5. *If $c > 0$, $b > 0$, $k_2 = k_3 = 0$, and the following inequalities hold:*

$$\begin{aligned} &\max\left\{x_2(t),\ \forall t \geq 0\right\} < a+k_1, \\ &\max\left\{\left|x_1^2(t)\right|,\ \forall t \geq 0\right\} \\ &\quad < 4\left(a+k_1-\max\left\{x_2(t),\ \forall t \geq 0\right\}\right)b, \end{aligned} \tag{17}$$

then two chaotic financial systems described by (1) and (2) can achieve global mixed synchronization.

Remark 6. Theorem 5 gives a mixed synchronization criterion, in which only a single controller $k_1E_1(t)$ is used.

Remark 7. In [26], the control $u(t) = k(t)(x_1(t) + y_1(t))$ was added to the slave financial system

$$\begin{aligned} \frac{dy_1(t)}{dt} &= y_3(t)+\left(y_2(t)-a\right)y_1(t) \\ &\quad + k(t)\left(x_1(t)+y_1(t)\right), \\ \frac{dy_2(t)}{dt} &= 1-by_2(t)-y_1^2(t), \\ \frac{dy_3(t)}{dt} &= -y_1(t)-cy_3(t), \end{aligned} \tag{18}$$

where

$$\frac{dk(t)}{dt} = -r\left(x_1(t)+y_1(t)\right)^2. \tag{19}$$

In addition, a Lyapunov function was given as follows:

$$\widetilde{V}(t) = \frac{E_1^2(t)+e_2^2(t)+E_3^2(t)}{2} + \frac{\left(k(t)+L_3\right)^2}{2r}, \tag{20}$$

where $E_1(t)$, $e_2(t)$, and $E_3(t)$ are the same as those defined in (6). By using adaptive control technique and calculating the derivatives of $\widetilde{V}(t)$, two chaotic financial systems described by (1) and (18) can achieve mixed synchronization under the conditions such that

$$L_3 > M_3 \sup_{E_1(t)\neq 0} \frac{e_2(t)^2+E_1(t)^2+E_3(t)^2}{E_1(t)^2}, \tag{21}$$

where $M_3 = \max_{i=1}^{3} n_i$ and n_1, n_2, and n_3 were constants such that

$$\begin{aligned} &E_1(t)\left(-aE_1(t)+E_3(t)+E_1(t)x_2(t)-y_1(t)e_2(t)\right) \\ &\quad \leq n_1E_1(t)^2, \\ &e_2(t)\left(-be_2(t)-E_1(t)\left(x_1(t)-y_1(t)\right)\right) \leq n_2e_2(t)^2, \\ &E_3(t)\left(-E_1(t)-cE_3(t)\right) \leq n_3E_3(t)^2. \end{aligned} \tag{22}$$

It should be pointed out that it is difficult to obtain n_1, n_2, n_3, and L_3 which increases the complexity of using mixed synchronization in [26]. The similar method was used to achieve mixed synchronization for two four-dimensional hyperchaotic financial systems in [27].

Remark 8. Compared with the difficulty to access n_1, n_2, n_3, and L_3 in mixed synchronization results of [26, 27], Theorem 5 is easier to be used.

If $a > 0, b > 0, c > 0$, and $k_1 = k_2 = k_3 = 0$, one can have the following theorem.

Theorem 9. *If $c > 0$, $b > 0$, $c > 0$, $k_1 = k_2 = k_3 = 0$, and the following inequalities hold:*

$$\begin{aligned} &\max\left\{x_2(t),\ \forall t \geq 0\right\} < a, \\ &\max\left\{\left|x_1^2(t)\right|,\ \forall t \geq 0\right\} \\ &\quad < 4\left(a-\max\left\{x_2(t),\ \forall t \geq 0\right\}\right)b, \end{aligned} \tag{23}$$

then two financial systems described by (1) and (2) can achieve global mixed synchronization.

Remark 10. If parameters a, b, and c satisfy inequalities (23), two financial systems described by (1) and (2) can achieve global mixed synchronization without external controls.

4. An Application to Mixed Synchronization of Modified Chaotic Financial Systems

If $a = \hat{a} - d$, the chaotic financial system described by (1) can be transformed to the following system:

$$\begin{aligned}\frac{dx_1(t)}{dt} &= x_3(t) + (x_2(t) - \hat{a})\, x_1(t) + d x_1(t), \\ \frac{dx_2(t)}{dt} &= 1 - b x_2(t) - x_1^2(t), \\ \frac{dx_3(t)}{dt} &= -x_1(t) - c x_3(t),\end{aligned} \tag{24}$$

where d is a constant and $x_1(t)$, $x_2(t)$, $x_3(t)$, a, b, and c are the same as those defined in (1). The initial conditions of (24) are the same as those in (1). System (24) was noted as the modified chaotic financial system in [24].

It should be pointed out that Theorems 1, 5, and 9 and Corollary 4 are still valid for the mixed synchronization of modified chaotic financial systems described by (24) with $a = \hat{a} - d$.

Remark 11. In [24], a nonlinear control method was used to achieve synchronization for modified chaotic financial systems described by (24). However, our mixed synchronization criteria (Theorems 1 and 5 and Corollary 4) are derived by the linear feedback control.

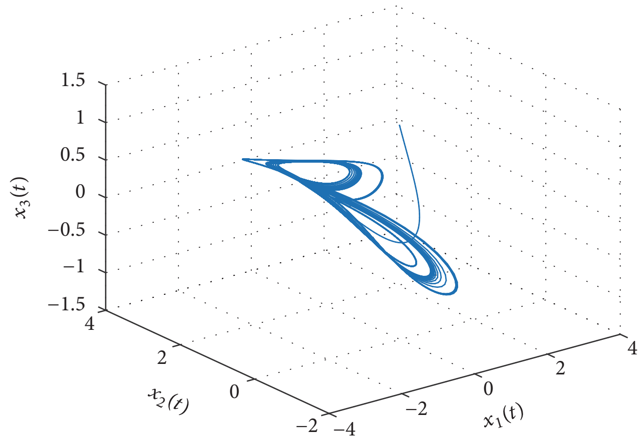

Figure 1: Chaotic attractor of system (1) with $a = 0.9$, $b = 0.2$, and $c = 1.5$.

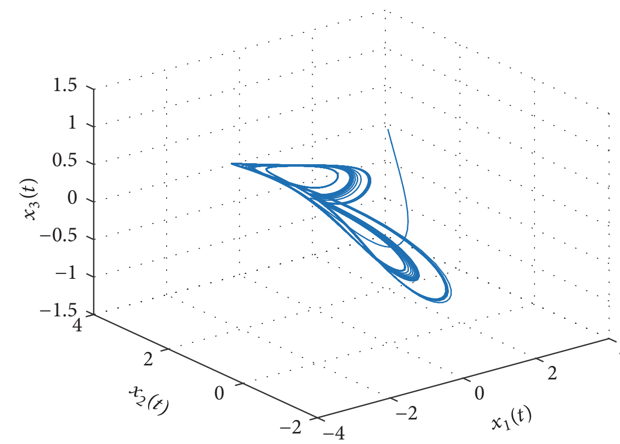

Figure 2: Phase figure of system (1) with $a = 0.9$, $b = 0.2$, and $c = 1.5$.

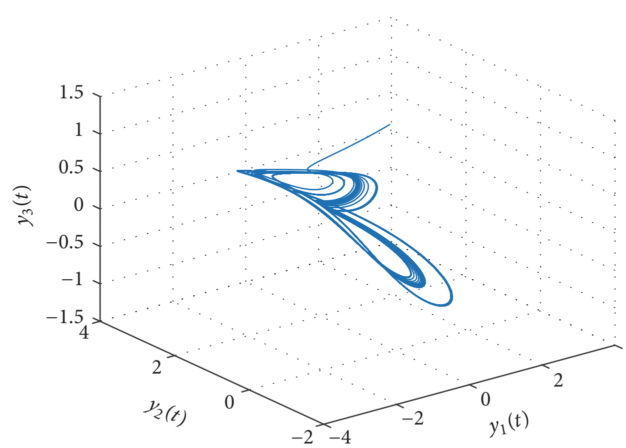

Figure 3: Phase figure of system (2) with $a = 0.9$, $b = 0.2$, $c = 1.5$, $k_1 = 9.6$, and $k_2 = k_3 = 0$.

5. Three Illustrated Examples

Example 1. Consider chaotic financial systems (1) and (2) with $a = 0.9$, $b = 0.2$, and $c = 1.5$. The initial conditions of (1) and (2) are $x_{1_0} = 1$, $x_{2_0} = 1$, $x_{3_0} = 1$, $y_{1_0} = 1.1$, $y_{2_0} = 1.2$, and $y_{3_0} = 1.1$, respectively. Figure 1 illustrates the chaotic attractor of chaotic system (1). From Figure 1, one can see that $-2.4 \le x_1(t) \le 2.4$ and $-0.3 \le x_2(t) \le 3.2$ for $t \ge 0$.

Due to $c = 1.5 > 0$, $b = 0.2 > 0$, and $\max\{|x_1^2(t)|,\ \forall t \ge 0\} = 2.4^2 < 4(a - \max\{x_2(t),\ \forall t \ge 0\})b = 4(0.9 + k_1 - 3.2)0.2$, one can use Theorem 5 to derive that $k_1 > 9.5$ and $k_2 = k_3 = 0$. Let $k_1 = 9.6$. Figures 2 and 3 demonstrate the trajectories of systems (1) and (2), respectively. Figures 4, 5, and 6 illustrate trajectories $E_1(t)$, $e_2(t)$, and $E_3(t)$ of system (6), respectively, which clearly show that (1) and (2) achieve mixed synchronization. The unit of time t in Figures 4, 5, and 6 is the second.

Example 2. Consider two financial systems (1) and (2) with $a = 3.9$, $b = 0.5$, and $c = 1.5$ and $x_{1_0} = 1$, $x_{2_0} = -0.2$, $x_{3_0} = 1$, $y_{1_0} = 1.1$, $y_{2_0} = 1.2$, and $y_{3_0} = 1.1$. The bounds of $x_1(t)$ and $x_2(t)$ are $-0.1 \le x_1(t) \le 1$ and $-0.2 \le x_2(t) \le 2$ for $t \ge 0$.

Due to $a = 3.9 > 0$, $c = 1.5 > 0$, $b = 0.5 > 0$, and $\max\{|x_1^2(t)|,\ \forall t \ge 0\} = 1 < 4(a - \max\{x_2(t),\ \forall t \ge 0\})b = 3.8$, one can use Theorem 9 to derive that $k_1 = k_2 = k_3 = 0$.

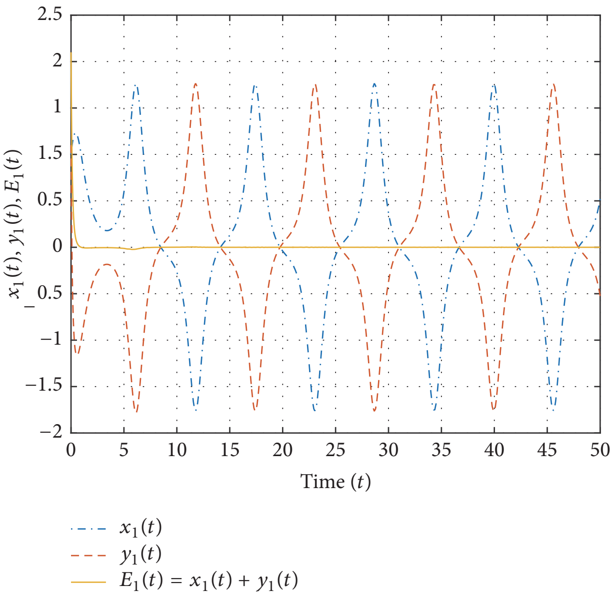

FIGURE 4: Trajectories $x_1(t)$ and $y_1(t)$ of system (1) and system (2) and $E_1(t)$ of system (6) with $a = 0.9$, $b = 0.2$, $c = 1.5$, $k_1 = 9.6$, and $k_2 = k_3 = 0$.

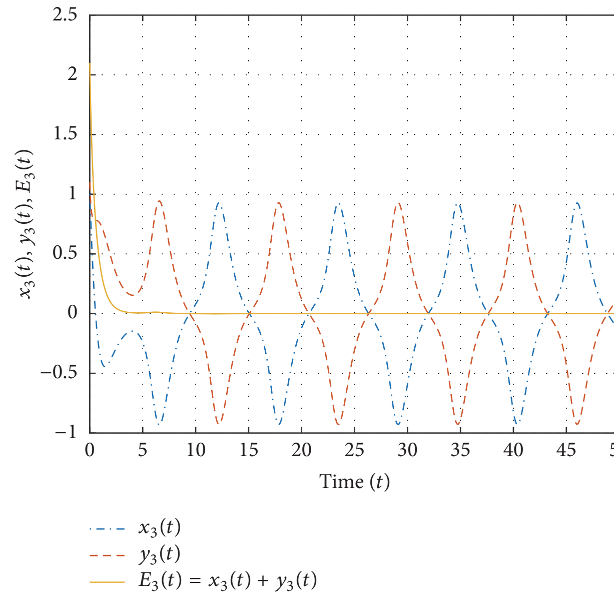

FIGURE 6: Trajectories $x_3(t)$ and $y_3(t)$ of system (1) and system (2) and $E_3(t)$ of system (6) with $a = 0.9$, $b = 0.2$, $c = 1.5$, $k_1 = 9.6$, and $k_2 = k_3 = 0$.

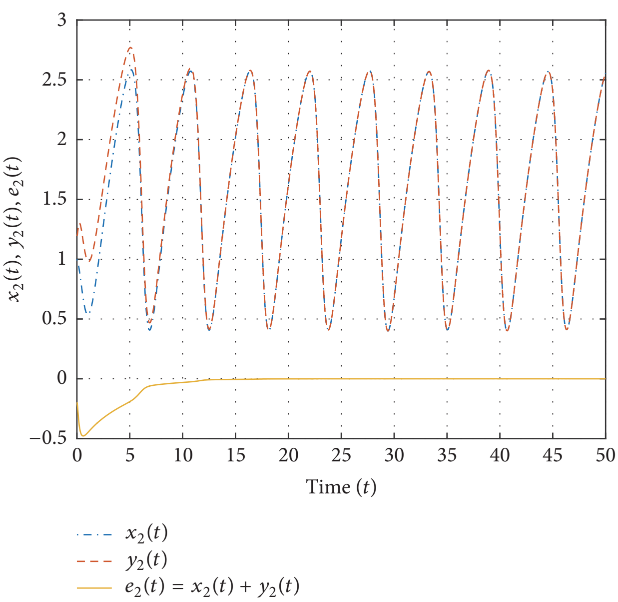

FIGURE 5: Trajectories $x_2(t)$ and $y_2(t)$ of system (1) and system (2) and $e_2(t)$ of system (6) with $a = 0.9$, $b = 0.2$, $c = 1.5$, $k_1 = 9.6$, and $k_2 = k_3 = 0$.

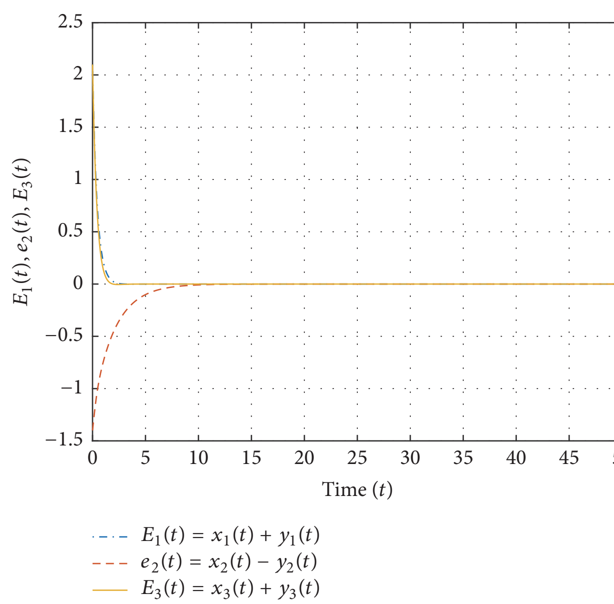

FIGURE 7: Trajectories of $E_1(t)$, $e_2(t)$, and $E_3(t)$ of system (6) with $a = 3.9$, $b = 0.5$, $c = 1.5$, and $k_1 = k_2 = k_3 = 0$.

Figure 7 demonstrates the trajectories of (6), which clearly illustrates that systems (1) and (2) achieve mixed synchronization. The unit of time t in Figure 7 is the second.

Example 3. Consider the modified financial system described by (24) with $\hat{a} = 0.6$, $b = 0.2$, $c = 0.9$, and $d = 0.5$ which can be transformed to (1) with $a = 0.1$, $b = 0.2$, and $c = 0.9$. Consider (1) and (2) with $a = 0.1$, $b = 0.2$, and $c = 0.9$ and $x_{1_0} = 1$, $x_{2_0} = 1$, $x_{3_0} = 1$, $y_{1_0} = 1.1$, $y_{2_0} = 1.2$, and $y_{3_0} = 1.1$. The bounds of $x_1(t)$ and $x_2(t)$ are $-3 \le x_1(t) \le 3$ and $-1.7 \le x_2(t) \le 3.1$ for $t \ge 0$.

Due to $a = \hat{a} - d = 0.1 > 0$, $c = 0.9 > 0$, $b = 0.2 > 0$, and $\max\{|x_1^2(t)|,\ \forall t \ge 0\} = 3^2 < 4(\hat{a} - d + k_1 - \max\{x_2(t),\ \forall t \ge 0\})b$, one can use Theorem 5 to derive that $k_1 > 14.25$ and $k_2 = k_3 = 0$. One can choose $k_1 = 14.26$ and $k_2 = k_3 = 0$. Figure 8 demonstrates the trajectories of (6), which clearly shows that systems (1) and (2) achieve mixed synchronization. The unit of time t in Figure 8 is the second.

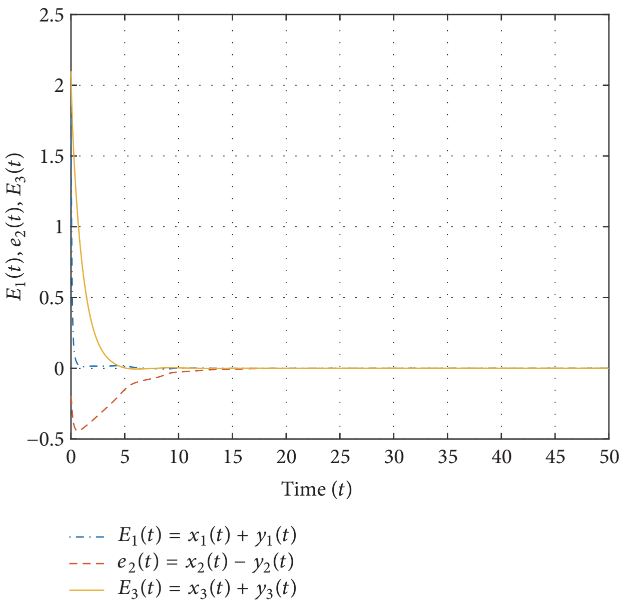

FIGURE 8: Trajectories of $E_1(t)$, $e_2(t)$, and $E_3(t)$ of system (6) with $a = 0.1$, $b = 0.2$, $c = 0.9$, $\hat{a} = 0.6$, $d = 0.5$, $k_1 = 14.26$, and $k_2 = k_3 = 0$.

6. Conclusions and Future Works

We have derived some mixed synchronization criteria for chaotic financial systems, in which the synchronization and antisynchronization coexist in chaotic financial systems by using linear feedback control, rather than nonlinear controls in the previous results. Moreover, we have obtained two mixed synchronization criteria with a single controller and without external controls, respectively. In addition, we have applied the mixed synchronization criteria and the control method to study the mixed synchronization for a class of modified chaotic financial systems. We have used three examples to illustrate the effectiveness of our derived results. In this paper, the synchronization and antisynchronization coexist in two identical financial systems with three dimensions. How to achieve the coexistence of synchronization and antisynchronization in two four-dimensional financial systems with mismatched parameters by using linear feedback control or nonlinear control is our research interest in the future.

Competing Interests

The authors declare that they have no competing interests.

Acknowledgments

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