Research Article

The Modelling and Control of a Singular Biological Economic System in a Polluted Environment

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Received 1 November 2015; Accepted 29 February 2016

Academic Editor: Viktor Avrutin

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A singular biological economic model with harvesting and stage structure is presented. The local stability of equilibriums of the system is investigated when the economic profit parameter is zero, and the conditions of the singularity induced bifurcation occurring at the positive equilibrium are obtained by the singular systems theory and bifurcation theory. In order to eliminate the singularity induced bifurcation, a state feedback controller is designed by controlling the fishing effort. At last, an application example is given to illustrate the validity of the theoretical results.

1. Introduction

The environmental pollution is increasing with the development of science and technology and the improvement of people's life. Pollution poses a threat to the survival of people and the development of human, which especially has great influence on biological economy. Some literatures have investigated the dynamic behavior of the biological systems which are affected by the environmental pollution [1–3]. In recent years, with the development of ecology and economic theory and application of general system theory in all fields, the results of singular biological models are continuously emerging, which include the following results. Chaotic phenomena of a class of singular biological economic systems are given in literatures [4–7]; bifurcations of a predator-prey model are obtained in literatures [8–11], which effected the stability of singular system model. Those authors propose different singular biological economic models from different views and design different controllers in line with corresponding bifurcation and impulsive behavior to eliminate those unstable phenomena.

The research of bifurcation problem of singular systems has obtained some results in power system, chemical industry, and biological system [12–16]. Further study on

bifurcation problems of the singular system has important theoretical significance and practical application value because it can reveal instability mechanism of the biological economy system and specify a reasonable control strategy.

Therefore, it is significant to model and investigate biological economic model. In this paper, a singular biological economic model is proposed by using the theory of singular system, and then the local stability of equilibriums and the existence of the bifurcation in the neighborhood of the positive equilibrium are analyzed when the economic profit is zero. Then, singularity induced bifurcation occurring at the positive equilibrium of the system is showed. A controller is designed to stabilize the system. At last, an example is given to illustrate the validity of the theoretical results.

2. Model Formulation

A single population model with stage structure is investigated in literatures [17–19]:

$$\begin{aligned}\dot{x}(t) &= ay(t) - bx(t) - r_1x(t), \\ \dot{y}(t) &= bx(t) - r_2y(t) - \beta y^2(t),\end{aligned}\tag{1}$$

where $x(t)$ and $y(t)$ are the densities of the immature population and mature population at time t , a denotes the birth rate of immature population, r_1, r_2 are the death rates of immature population and mature population, b denotes the conversion rate from immature population into mature population, β denotes the intraspecific effect coefficient, and all coefficients are positive.

A single population model in the polluted environment is investigated in [20]:

$$\begin{aligned}\dot{x}(t) &= rx(t) \left(1 - \frac{x(t)}{K}\right) - r_1 u(t) x(t), \\ \dot{u}(t) &= \theta - hu(t),\end{aligned}\quad (2)$$

where $x(t)$ is the population density, $u(t)$ is the concentration of environment pollutants, r denotes the intrinsic growth rate when there is no pollution, K denotes the capacity of the environment, $r_1 u(t)$ can be interpreted as the measuring response function of the reduction of populations because of the pollution factor, θ denotes the amount of pollutants that are inputted by the outside, and $hu(t)$ can be interpreted as the reduction of pollutant concentration because of other factors. Assume that endotoxin excretion rate and purification rate are relatively small in an organism body, which can be neglected.

Based on system (1) and system (2), a single population model with stage structure is given as follows:

$$\begin{aligned}\dot{x}(t) &= ay(t) - bx(t) - r_1 x(t) - \eta_1 u(t) x(t), \\ \dot{y}(t) &= bx(t) - r_2 y(t) - \beta y^2(t) - \eta_2 u(t) y(t), \\ \dot{u}(t) &= \theta - hu(t),\end{aligned}\quad (3)$$

where $\eta_1 u(t), \eta_2 u(t)$ can be interpreted as the measuring response function of the reduction of population because of the pollution factor and $hu(t)$ can be interpreted as the reduction of pollutant concentration because of other factors.

According to literatures [21, 22], a singular biological economic model with environmental pollution factors is proposed as follows:

$$\begin{aligned}\dot{x}(t) &= ay(t) - bx(t) - r_1 x(t) - \eta_1 u(t) x(t), \\ \dot{y}(t) &= bx(t) - r_2 y(t) - \beta y^2(t) - \xi E(t) y(t) \\ &\quad - \eta_2 u(t) y(t), \\ \dot{u}(t) &= \theta - hu(t), \\ 0 &= E(t) (py(t) - c) - m,\end{aligned}\quad (4)$$

where $E(t)$ is the capture capability of mature population at time t , p denotes the unit price, c denotes the unit cost, and m denotes the economic profit. $pE(t)y(t)$ is the total revenue, $cE(t)$ is the total cost, and all the parameters are positive [23].

System (4) can be written as

$$A(t) \dot{X}(t) = G(x(t), y(t), u(t), E(t)), \quad (5)$$

where

$$\begin{aligned}X(t) &= (x(t), y(t), u(t), E(t))^T, \\ A(t) &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\ G(x(t), y(t), u(t), E(t)) &= \begin{pmatrix} ay(t) - bx(t) - r_1 x(t) - \eta_1 u(t) x(t) \\ bx(t) - r_2 y(t) - \beta y^2(t) - \xi E(t) y(t) - \eta_2 u(t) y(t) \\ \theta - hu(t) \\ E(t) (py(t) - c) - m \end{pmatrix}.\end{aligned}\quad (6)$$

Remark 1. All elements in the last line of $A(t)$ are 0 because the algebraic equation does not contain differential variable.

3. Stability Analysis

Based on singular system theory and bifurcation theory, the local stability of system (4) is investigated in this section. According to the theory of public resources due to Gordon [21], the system may show the phenomenon of ecological economic equilibrium when the economic interest $m = 0$.

If $ab - (r_2 + (\theta/h)\eta_2)(b + r_1 + (\theta/h)\eta_1) > 0$, $P_1(x_1, y_1, u_1, E_1)$ is an equilibrium point of system (4), where

$$\begin{aligned}x_1 &= \frac{a [ab - (r_2 + (\theta/h)\eta_2)(b + r_1 + (\theta/h)\eta_1)]}{\beta (b + r_1 + (\theta/h)\eta_1)^2}, \\ y_1 &= \frac{ab - (r_2 + (\theta/h)\eta_2)(b + r_1 + (\theta/h)\eta_1)}{\beta (b + r_1 + (\theta/h)\eta_1)}, \\ u_1 &= \frac{\theta}{h}, \\ E_1 &= 0.\end{aligned}\quad (7)$$

When $pab - (pr_2 + \beta c + p(\theta/h)\eta_2)(b + r_1 + (\theta/h)\eta_1) > 0$, $P_2(x^*, y^*, u^*, E^*)$ is the unique positive equilibrium point of system (4), where

$$\begin{aligned}x^* &= \frac{ac}{p(b + r_1 + (\theta/h)\eta_1)}, \\ y^* &= \frac{c}{p}, \\ u^* &= \frac{\theta}{h}, \\ E^* &= \frac{pab - (pr_2 + \beta c + p(\theta/h)\eta_2)(b + r_1 + (\theta/h)\eta_1)}{p\xi(b + r_1 + (\theta/h)\eta_1)}.\end{aligned}\quad (8)$$

In what follows, we will investigate the local stability of equilibriums.

The linearization matrix of any equilibrium point $P(\tilde{x}, \tilde{y}, \tilde{u}, \tilde{E})$ of system (4) is

$$J_P = \begin{bmatrix} -(b+r_1+\tilde{u}\eta_1) & a & -\eta_1\tilde{x} & 0 \\ b & -(r_2+2\beta\tilde{y}+\tilde{E}+\tilde{u}\eta_2) & -\eta_2\tilde{y} & -\xi\tilde{y} \\ 0 & 0 & -h\tilde{u} & 0 \\ 0 & p\tilde{E} & 0 & p\tilde{y}-c \end{bmatrix}. \quad (9)$$

The characteristic equation of system (4) at P_1 is

$$\det(\lambda A - J_{P_1}) = (c - py_1)(\lambda + hu_1) \cdot [(\lambda + r_2 + 2\beta y_1 + u_1\eta_2)(\lambda + b + r_1 + u_1\eta_1) - ab] = 0. \quad (10)$$

Obviously, $\lambda_1 = -hu_1 < 0$ is one root of (10), and the other two roots are determined by the following equation:

$$\lambda^2 + \left(\frac{ay_1}{x_1} + \frac{bx_1}{y_1} + \beta y_1\right)\lambda + \frac{a\beta y_1^2}{x_1} = 0. \quad (11)$$

It is obvious that $ay_1/x_1 + bx_1/y_1 + \beta y_1 > 0$, and $a\beta y_1^2/x_1 > 0$; then system (4) is stable at P_1 by the Routh-Hurwitz theorem [24].

From the above analysis, we have the following result.

Theorem 2. When $ab - (r_2 + (\theta/h)\eta_2)(b + r_1 + (\theta/h)\eta_1) > 0$, system (4) is stable at P_1 .

4. The Singularity Induced Bifurcation

We will consider the singularity induced bifurcation occurring at the positive equilibrium point of system (4).

Theorem 3. If $pab - (pr_2 + \beta c + p(\theta/h)\eta_2)(b + r_1 + (\theta/h)\eta_1) > 0$, singularity induced bifurcation (SIB) occurs at positive equilibrium point P_2 of system (4) when bifurcation value $m = 0$. That is, when m becomes from negative to positive, system (4) is unstable at $m = 0$.

Proof. System (4) is written as follows:

$$\begin{aligned} \dot{x} &= f(x(t), y(t), u(t), E(t)), \\ 0 &= g(x(t), y(t), u(t), E(t)), \end{aligned} \quad (12)$$

where

$$\begin{aligned} f(x(t), y(t), u(t), E(t)) &= \begin{pmatrix} ay(t) - bx(t) - r_1x(t) - \eta_1u(t)x(t) \\ bx(t) - r_2y(t) - \beta y^2(t) - \xi E(t)y(t) - \eta_1u(t)y(t) \\ \theta - hu(t) \end{pmatrix}, \quad (13) \\ g(x(t), y(t), u(t), E(t)) &= E(t)(py(t) - c) - m, \end{aligned}$$

where m is the bifurcation parameter. The following conditions are established when $m = 0$:

$$\begin{aligned} (1) \quad g_E(x^*, y^*, u^*, E^*) &= py^* - c = 0; \\ (2) \quad \text{trace}[f_E \text{adj}(g_E)(g_x \ g_y \ g_u)]|_{P_2} &= \text{trace} \begin{bmatrix} 0 & 0 & 0 \\ 0 & -p\xi y^* E^* & 0 \\ 0 & 0 & 0 \end{bmatrix} = -p\xi y^* E^*; \\ (3) \quad \begin{vmatrix} f_x & f_y & f_u & f_E \\ g_x & g_y & g_u & g_E \end{vmatrix}_{P_2} &= \begin{vmatrix} -(b+r_1+\eta_1u^*) & a & -\eta_1x^* & 0 \\ b & -(r_2+2\beta y^*+\xi E^*+\eta_2y^*) & -\eta_2y^* & -\xi y^* \\ 0 & 0 & -hu^* & 0 \\ 0 & pE^* & 0 & py^*-c \end{vmatrix} \\ &= ph\xi y^* u^* E^* (b+r_1+\eta_1u^*) \\ &\neq 0; \\ (4) \quad \Delta(x, y, u, E, m) &= \det g_E = py - c; \\ &\begin{vmatrix} f_x & f_y & f_u & f_E & f_m \\ g_x & g_y & g_u & g_E & g_m \\ \Delta_x & \Delta_y & \Delta_u & \Delta_E & \Delta_m \end{vmatrix}_{P_2} = \begin{vmatrix} -(b+r_1+\eta_1u^*) & a & -\eta_1x^* & 0 & 0 \\ b & -(r_2+2\beta y^*+\xi E^*+\eta_2y^*) & -\eta_2y^* & -\xi y^* & 0 \\ 0 & 0 & -hu^* & 0 & 0 \\ 0 & pE & 0 & py^*-c & -1 \\ 0 & p & 0 & 0 & 0 \end{vmatrix} \\ &= -hp\xi u^* y^* (b+r_1+\eta_1u^*) \\ &\neq 0. \end{aligned} \quad (14)$$

Thus, the basic conditions of the SIB theorem [25] are satisfied. Singularity induced bifurcation (SIB) occurs at

positive equilibrium point P_2 of system (4) when bifurcation value $m = 0$.

It is easy to obtain that

$$B = \text{trace} [f_E \text{adj}(g_E)(g_x \ g_y \ g_u)]|_{P_2} = -p\xi y^* E^*,$$

$$C = -(0 \ p \ 0 \ 0) \begin{bmatrix} -(b+r_1+\eta_1 u^*) & a & -\eta_1 x^* & 0 \\ b & -(r_2+2\beta y^*+\xi E^*+\eta_2 y^*) & -\eta_2 y^* & -\xi y^* \\ 0 & 0 & -hu^* & 0 \\ 0 & pE^* & 0 & py^*-c \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} \Big|_{P_2} = p; \quad (15)$$

$$\frac{B}{C} = -\xi y^* E^* < 0.$$

The above formulas meet the singularity induced bifurcation theorem. System (4) losses stability once bifurcation parameter changes from negative to positive. $\square$

5. Bifurcation Control

According to matrix A and the linearization matrix at positive equilibrium point, it is easy to obtain that

$$\text{rank}(J_{P^*}, AJ_{P^*}, A^2 J_{P^*}, A^3 J_{P^*}) = 4. \quad (16)$$

Based on theorem 2-2.1 of literature [13], system (4) is locally controllable at positive equilibrium point. That is, for any initial point in the neighborhood of the positive equilibrium P_2 , there exists a controller $u(t)$ that can make the trajectories of the system reach the equilibrium point.

Considering the system,

$$\begin{aligned} \dot{x}(t) &= ay(t) - bx(t) - r_1 x(t) - \eta_1 u(t) x(t), \\ \dot{y}(t) &= bx(t) - r_2 y(t) - \beta y^2(t) - \xi E(t) y(t) \\ &\quad - \eta_2 u(t) y(t), \\ \dot{u}(t) &= \theta - hu(t), \\ 0 &= E(t)(py(t) - c) + v(t). \end{aligned} \quad (17)$$

Denoting a state feedback controller $v(t) = k(E(t) - E^*)$, where k denotes the state feedback gain, system (17) is written as follows:

$$\begin{aligned} \dot{x}(t) &= ay(t) - bx(t) - r_1 x(t) - \eta_1 u(t) x(t), \\ \dot{y}(t) &= bx(t) - r_2 y(t) - \beta y^2(t) - \xi E(t) y(t) \\ &\quad - \eta_2 u(t) y(t), \\ \dot{u}(t) &= \theta - hu(t), \\ 0 &= E(t)(py(t) - c) + k(E(t) - E^*). \end{aligned} \quad (18)$$

Theorem 4. If the state feedback controller-gain k meets the following inequality,

$$k > \frac{pab - (pr_2 + \beta c + p(\theta/h)\eta_2)(b + r_1 + (\theta/h)\eta_1)}{\beta(b + r_1 + (\theta/h)\eta_1)}, \quad (19)$$

system (18) is stable at P_2 .

Proof. The Jacobi matrix of system (18) at P_2 is

$$\begin{bmatrix} -\frac{ay^*}{x^*} & a & -\eta_1 x^* & 0 \\ b & -\frac{bx^*}{y^*} - \beta y^* & -\eta_2 y^* & -\xi y^* \\ 0 & 0 & -u^* h & 0 \\ 0 & pE^* & 0 & k \end{bmatrix}. \quad (20)$$

The corresponding characteristic equation is

$$\begin{aligned} (\lambda + hu^*) \left[\lambda^2 + \left(\frac{ay^*}{x^*} + \frac{bx^*}{y^*} + \beta y^* - \frac{p\xi E^* y^*}{k} \right) \lambda \right. \\ \left. + \frac{a\beta y^{*2}}{x^*} - \frac{ap\xi E y^{*2}}{kx^*} \right] = 0. \end{aligned} \quad (21)$$

Then, system (18) is stable if

$$\begin{aligned} \frac{ay^*}{x^*} + \frac{bx^*}{y^*} + \beta y^* - \frac{p\xi E^* y^*}{k} &> 0, \\ \frac{a\beta y^{*2}}{x^*} - \frac{ap\xi E y^{*2}}{kx^*} &> 0. \end{aligned} \quad (22)$$

We can obtain that

$$\begin{aligned} k &> \frac{p\xi E^*}{\beta} \\ &= \frac{pab - (pr_2 + \beta c + p(\theta/h)\eta_2)(b + r_1 + (\theta/h)\eta_1)}{\beta(b + r_1 + (\theta/h)\eta_1)}. \end{aligned} \quad (23)$$

$\square$

6. Example

In recent years, environmental pollution accidents occur frequently in our country. Taking water pollution as an example, the total of the urban water pollution emergencies is 101 from 2001 to 2005 according to incomplete statistics. Since 2005, the sudden environmental pollution accidents are more and more prominent. Water pollution has both direct influence and indirect effects on the populations in aquatic ecosystems. The biological effects of pollutant on fish include death, slow growth, the decrease of production, the declination of increment rate, and lower yield. Since the fish resources with economic value can be damaged in the water containing a lot of aerobic organisms, many creatures are developed to adapt to the sewage environment. On June 4, 2011, the oil spill of Bohai Bay, Penglai 19-3 oil field, caused ecological damage and economic loss of fishermen.

By selecting some related data from China environment protection database and doing the appropriate treatment, the following parameters can be obtained [26, 27]:

$$\begin{aligned}
 a &= 2, \\
 b &= 1, \\
 c &= 1, \\
 p &= 1, \\
 \theta &= 1, \\
 \eta_1 &= 0.3, \\
 \eta_2 &= 0.2, \\
 h &= 1, \\
 \beta &= 0.2, \\
 r_1 &= 0.2, \\
 r_2 &= 0.2, \\
 \xi &= 0.5.
 \end{aligned} \tag{24}$$

System (18) can be written as follows:

$$\begin{aligned}
 \dot{x}(t) &= 2y(t) - x(t) - 0.2x(t) - 0.3u(t)x(t), \\
 \dot{y}(t) &= x(t) - 0.3y(t) - 0.2y^2(t) - 0.5E(t)y(t) \\
 &\quad - 0.2u(t)y(t), \\
 \dot{u}(t) &= 1 - u(t), \\
 0 &= E(t)(y(t) - 1) - k(E(t) - E^*),
 \end{aligned} \tag{25}$$

$P_2(1.3333, 1.000, 1.000, 1.4667)$ is the unique positive equilibrium of system (25). By Theorem 3, the singularity induced bifurcation occurs at the positive equilibrium P_2 of system (25). In order to eliminate singularity induced bifurcation, we

FIGURE 1: The dynamic response shows that all species will be die out when the parameter values are given, $k = 4$, $E^* = 1.8$.

design a controller $v(t) = k(E(t) - 1.4667)$. Thus, system (25) can be written as follow:

$$\begin{aligned}
 \dot{x}(t) &= 2y(t) - x(t) - 0.2x(t) - 0.3u(t)x(t), \\
 \dot{y}(t) &= x(t) - 0.3y(t) - 0.2y^2(t) - 0.5E(t)y(t) \\
 &\quad - 0.2u(t)y(t), \\
 \dot{u}(t) &= 1 - u(t), \\
 0 &= E(t)(y(t) - 1) + k(E(t) - 1.4667).
 \end{aligned} \tag{26}$$

According to Theorem 4, when $k > 3.6667$, system (26) is stable at P_2 ; that is, the bifurcation at P_2 of system (26) is eliminated. Figure 1 shows that the species face extinction gradually along with excessive capture. In order to maintain the sustainable development of population, the capture capability is needed to control. When $E^* = 1 < 1.4667$, the state response of system (26) is shown by Figure 2, and the population and economic profits develop sustainably.

7. Conclusions

Based on the mathematical biology theory, the bifurcation theory of nonlinear system, and the singular system theory, this paper considers a singular biological economic system with stage structure which is affected by the environmental pollution. The bifurcation occurring at the positive equilibrium causes ecological imbalance. The population may immensely change and even be extinct if the fishing effort is not reasonable. In order to eliminate the bifurcation, the feedback controller is presented. When the capture revenue is appropriate, the bifurcation at positive equilibrium can be eliminated by controlling the fishing effort. Combined with practical example, such as water pollution, this paper shows that the stability of the population is maintained and

FIGURE 2: Simulation shows that all species coexist when some parameters of the singular biological economic system are controlled by selecting reasonable values, $k = 4$, $E^* = 1$.

the maximum economic benefits are obtained by controlling the capture capability when the water is polluted by oil. At the same time, in order to guarantee the continued growth of population in the pollutant environment, we must take effective measures to reduce the environmental pollution.

Competing Interests

The authors declare that there are no competing interests regarding the publication of this paper.

Acknowledgments

This work was supported by National Natural Science Foundation of China under Grant no. 61273008, National Natural Science Foundation of Liaoning Province under Grant no. 2015020007, Science and Technology Research Fund of Liaoning Education Department (L2013051), and Jiangsu Planned Projects for Postdoctoral Research Funds no. 1401044.

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