Research Article

Composite Adaptive Antidisturbance Control for Discrete-Time Switched System

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A novel composite adaptive antidisturbance controller is developed for a class of discrete-time switched system. First, two composite adaptive observers are proposed to estimate the external disturbances and unknown parameters, respectively. Then, based on the estimation values, a composite adaptive antidisturbance controller is constructed, which can guarantee system has a good antidisturbance performance. A solvable sufficient condition is presented by using linear matrix inequalities (LMIs). Finally, a numerical example is shown to demonstrate the effectiveness of the proposed control approach.

1. Introduction

Disturbances widely exist in practical system and bring adverse effect of the control performance of the closed-loop system. To enhance the antidisturbance ability of the considered system, many effective antidisturbance control schemes have been proposed, for example, nonlinear H_∞ control [1, 2], nonlinear output regulation theory [3, 4], sliding mode control [5, 6], and disturbance observer based control (DOBC). DOBC technique has received more and more attention and has been applied to many kinds of control fields, such as hard disk drive systems [7, 8], robotic systems [9], grinding systems [10, 11], hypersonic vehicles [12, 13], spacecraft systems [14, 15], and general systems [16–18]. In the DOBC scheme, an observer is constructed to estimate external disturbances and the estimation is used for feedforward compensation. Then a composite controller is obtained based on disturbance observer and conventional feedback control law, which can guarantee system has a good disturbance rejection performance and desired stability or tracking performance. When systems are subject to multiple disturbances, composite hierarchical antidisturbance control

(CHADC) methods have been given [19] via DOBC method and conventional antidisturbance control technique, for example, $H_\infty(L_2 - L_\infty)$ control [20, 21], sliding mode control [22, 23], adaptive control [24, 25], and neural network [26]. The problem of robust stability analysis and robust controller design has been extensively investigated for discrete-time systems with uncertainty and/or disturbance. In [27, 28], two back-stepping controllers are proposed for the longitudinal dynamics of a generic hypersonic flight vehicle with neural networks based on discrete-time model. In [29], a composite antidisturbance controller has been developed for a class of discrete-time system with multiple disturbances. A composite hierarchical antidisturbance fault-tolerant controller has been developed for a class of discrete-time system with multiple disturbances and actuator fault via switching method in [30]. In [29, 30], they do not pay attention to estimating uncertain function and external disturbances for discrete-time system at the same time. Hence, it is necessary to develop a novel antidisturbance controller for discrete-time system with multiple mismatched disturbances, that is, single harmonic or constant disturbances and another unexpected nonlinear signal presented as a nonlinear function.

On the other hand, switched systems are a special class of hybrid system and can be used to describe many practical systems, for example, power electronics, embedded systems, chemical processes, computer-controlled systems, and automotive industries. Hence, the stability analysis and controller synthesis of switched system have received more and more attention during the last decade and many results have been developed [31–34]. By far, there are a number of methodologies on dealing with the stability analysis and control synthesis of switched systems, such as common Lyapunov function [31], multiple Lyapunov function [32], dwell time and average dwell time method [33], and switched Lyapunov function [34]. In [35], a CHADC method has been reported for a class of discrete-time switched system with multiple disturbances, but the disturbances are assumed to be described by an exogenous system or satisfy H_2 norm.

In this paper, the problem of adaptive antidisturbance control for discrete-time switched system with disturbances is addressed. The disturbances are described by not only single harmonic or constant disturbances but also another unexpected nonlinear signal presented as a nonlinear function. In order to improve the disturbance rejection and robustness performance of systems, an adaptive antidisturbance controller is proposed via disturbance observer technique and adaptive control method. First, the composite adaptive observers are constructed to estimate disturbances with known partial information and unknown parameters. Then, an adaptive antidisturbance controller is developed based on estimation value and conventional feedback control law. Combining switched Lyapunov function method and linear matrix inequalities, a sufficient condition is presented to obtain the controller and observer gains. Finally, a numerical example is given to demonstrate the effectiveness of the proposed scheme.

2. Problem Formulation and Preliminaries

Consider the following discrete-time switched systems with time-varying delays described by

$$\Sigma_0: \begin{aligned} x(k+1) &= A_{\sigma(k)} x(k) \\ &+ B_{\sigma(k)} (u(k) + d_1(k) + \phi^T(k) \theta(k)), \end{aligned} \quad (1)$$

where $x(k) \in \mathbb{R}^n$ is the states; $u(k)$ is the control input. $\phi(k)$ is a bounded function, which satisfies $\|\phi(k)\| \leq \phi$. Consider $\sigma(k) : \mathbb{Z}^+ \rightarrow \mathcal{N} = \{1, 2, \dots, N\}$ is the switching signal, which specifies which subsystem will be activated at a certain discrete-time instant and $A_i, B_i, i \in \mathcal{N}$, are constant matrices with appropriate dimensions. $d_1(k)$ is the external disturbances, which is described by Assumption 1. $\theta(k)$ describes a time-varying uncertain parametric vector to be updated or estimated.

Assumption 1 (see [29]). The disturbances $d_1(k)$ are generated by the exogenous system

$$\begin{aligned} \omega(k+1) &= W\omega(k), \\ d_1(k) &= V\omega(k), \end{aligned} \quad (2)$$

where $\omega(k) \in \mathbb{R}^m$. $W \in \mathbb{R}^{m \times m}$ and $V \in \mathbb{R}^{1 \times m}$ are known matrices.

Assumption 2. Unknown parametric vector θ in (1) can be supposed to satisfy the following formula:

$$\theta(k+1) = \Gamma\theta(k), \quad (3)$$

where $\Gamma \in \mathbb{R}^{K \times K}$ is a proper matrix and the initial condition is unknown.

Remark 3. Assumption 2 is a rational assumption condition. Because some unknown parameters can be described by a given model in practical system, for example, in spacecraft system, the inertial matrix can be defined by the given model via choosing appropriate parameters.

Here we are devoted to designing a state feedback controller by the following formula:

$$u(k) = K_{1\sigma(k)} x(k) - \hat{d}_1(k) - \phi^T(k) \hat{\theta}(k), \quad (4)$$

where $\hat{d}_1(k)$ and $\hat{\theta}(k)$ are the estimation of the disturbances $d_1(k)$ and $\theta(k)$. $\hat{d}_1(k)$ is obtained by the following disturbance observer:

$$\begin{aligned} \hat{\omega}(k) &= v(k) - Lx(k), \\ v(k+1) &= W\hat{\omega}(k) + L(A_{\sigma(k)} x(k) \\ &+ B_{\sigma(k)} (u(k) + \hat{d}_1(k) + \phi^T(k) \hat{\theta}(k))), \end{aligned} \quad (5)$$

$$\hat{d}_1(k) = V\hat{\omega}(k). \quad (6)$$

$\theta(k)$ is estimated by the following adaptive law:

$$\begin{aligned} \hat{\theta}(k) &= v_0(k) - L_0 x(k), \\ v_0(k+1) &= \Gamma\hat{\theta}(k) + L_0 (A_{\sigma(k)} x(k) \\ &+ B_{\sigma(k)} (u(k) + \hat{d}_1(k) + \phi^T(k) \hat{\theta}(k))). \end{aligned} \quad (7)$$

Remark 4. In this paper, we assume that the switching signal $\sigma(k)$ is not known a priori but its instantaneous value is available in real time [36]. Here we only consider the case of synchronous switching; that is, the controller switches just as the system Σ_0 does.

Defining $e_\omega(k) = \omega(k) - \hat{\omega}(k)$ and $e_\theta(k) = \theta(k) - \hat{\theta}(k)$ yields

$$\begin{aligned} e_\omega(k+1) &= (W + LV) e_\omega(k) + L\phi^T(k) e_\theta(k), \\ e_\theta(k+1) &= (\Gamma + L_0\phi^T(k)) e_\theta(k) + L_0 B_{\sigma(k)} e_\omega(k). \end{aligned} \quad (8)$$

Applying controller (4) to system Σ_0 and combining (2) and (6), we obtain

$$\begin{aligned} x(k+1) &= (A_{\sigma(k)} + B_{\sigma(k)}K_{\sigma(k)})x(k) \\ &\quad + B_{\sigma(k)}(d_1(k) - \hat{d}_1(k)) \\ &\quad + B_{\sigma(k)}(\phi^T(k)\theta(k) - \phi^T(k)\hat{\theta}(k)) \\ &= (A_{\sigma(k)} + B_{\sigma(k)}K_{\sigma(k)})x(k) \\ &\quad + B_{\sigma(k)}V_{\sigma(k)}e_\omega(k) + B_{\sigma(k)}\phi^T(k)e_\theta(k). \end{aligned} \quad (9)$$

Then we have the following augmented switched system:

$$\begin{aligned} \Sigma_c: \quad x(k+1) &= (A_{\sigma(k)} + B_{\sigma(k)}K_{\sigma(k)})x(k) + B_{\sigma(k)}Ve_\omega(k) \\ &\quad + B_{\sigma(k)}\phi^T(k)e_\theta(k), \\ e_\omega(k+1) &= (W + LV)e_\omega(k) + L\phi^T(k)e_\theta(k), \\ e_\theta(k+1) &= (\Gamma + L_0\phi^T(k))e_\theta(k) + L_0B_{\sigma(k)}e_\omega(k). \end{aligned} \quad (10)$$

3. Main Results

In this section, we are devoted to developing sufficient condition to solve the control problem formulated in the previous section.

Theorem 5. Consider system Σ_c . System Σ_c is asymptotically stable if there exist matrices K_i , L , L_0 , $P_i > 0$, $S > 0$, $T > 0$, $i \in \mathcal{N}$, such that the following inequalities hold:

$$\Phi_i = \begin{bmatrix} \Phi_{11i} & 0 & 0 & \Phi_{15i} & 0 & 0 & \Phi_{15i} & 0 & 0 \\ * & -S & 0 & \Phi_{25i} & \Phi_{26i} & \Phi_{27i} & 0 & \Phi_{26i} & 0 \\ * & * & \Phi_{33} & 0 & 0 & \Phi_{37} & 0 & 0 & \Phi_{37} \\ * & * & * & -P_l^{-1} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -S^{-1} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -T^{-1} & 0 & 0 & 0 \\ * & * & * & * & * & * & -P_l^{-1} & 0 & 0 \\ * & * & * & * & * & * & * & -S^{-1} & 0 \\ * & * & * & * & * & * & * & * & -T^{-1} \end{bmatrix} \quad (11)$$

$$< 0,$$

$$P_i \leq \lambda_i I, \quad (12)$$

$$\begin{bmatrix} -\delta_2 I & L^T \\ L & -S^{-1} \end{bmatrix} < 0, \quad (13)$$

$$\begin{bmatrix} -\delta_3 I & L_0^T \\ L_0 & -T^{-1} \end{bmatrix} < 0, \quad (14)$$

where

$$\begin{aligned} \Phi_{11i} &= -P_i, \\ \Phi_{15i} &= (A_i + B_i K_i)^T, \\ \Phi_{26i} &= (W + LV)^T, \\ \Phi_{33} &= -T + 4\lambda_i \phi^2 \|B_i\|^2 + 3\delta_2 \phi^2 + 4\delta_3 \hat{\phi}^2, \\ &\quad l = 1, 2, \dots, N, \end{aligned} \quad (15)$$

$$\Phi_{37} = (\Gamma + L_0 E)^T,$$

$$\Phi_{25i} = (B_i V)^T,$$

$$\Phi_{27i} = (B_i^T L_0^T).$$

Proof. Choose a Lyapunov functional candidate as

$$V(k) = V_1(k) + V_2(k) + V_3(k), \quad (16)$$

where

$$\begin{aligned} V_1(k) &= x^T(k) P_{\sigma(k)} x(k), \\ V_2(k) &= e_\omega^T(k) S e_\omega(k), \\ V_3(k) &= e_\theta^T(k) T e_\theta(k). \end{aligned} \quad (17)$$

Without loss of generality, we assume that $\sigma(k+1) = l$, $\sigma(k) = i$, $\forall i, l \in \mathcal{N}$. Then taking the forward difference $V_1(k)$ yields

$$\begin{aligned} \Delta V_1(k) &= V_1(k+1) - V_1(k) \\ &= x^T(k+1) P_l x(k+1) - x^T(k) P_i x(k) \\ &= \xi^T(k) \mathcal{A}_i^T P_l \mathcal{A}_i \xi(k) - x^T(k) P_i x(k) \\ &= x^T(k) (A_i + B_i K_i)^T P_l (A_i + B_i K_i) x(k) \\ &\quad + e_\omega^T(k) (B_i V)^T P_l (B_i V) e_\omega(k) \\ &\quad + e_\theta^T(k) (B_i \phi^T(k))^T P_l B_i \phi^T(k) e_\theta \\ &\quad + 2x^T(k) (A_i + B_i K_i)^T P_l B_i V e_\omega \\ &\quad + 2x^T(k) (A_i + B_i K_i)^T P_l B_i \phi^T(k) e_\theta \\ &\quad + 2e_\omega^T(k) (B_i V)^T P_l (B_i \phi^T(k)) e_\theta - x^T(k) P_i x(k), \end{aligned} \quad (18)$$

where $\xi^T(k) = [x^T(k) \quad e_\omega^T(k) \quad e_\theta^T(k)]$; $\mathcal{A}_i = [A_i + B_i K_i \quad B_i V \quad B_i \phi^T(k)]$. Note that

$$\begin{aligned} & e_\theta^T(k) (B_i \phi^T(k))^T P_l B_i \phi^T(k) e_\theta \\ & \leq \|e_\theta\|^2 \|\phi^T(k)\|^2 \|B_i\|^2 \|P_l\| \leq \phi^2 \|e_\theta\|^2 \|B_i\|^2 \|P_l\|, \end{aligned}$$

$$\begin{aligned} & 2x^T(k) (A_i + B_i K_i)^T P_l B_i \phi^T(k) e_\theta \\ & \leq x^T(k) (A_i + B_i K_i)^T P_l (A_i + B_i K_i) x(k) \\ & \quad + e_\theta^T \phi(k) B_i^T P_l B_i \phi^T(k) e_\theta \\ & \leq x^T(k) (A_i + B_i K_i)^T P_l (A_i + B_i K_i) x(k) \\ & \quad + \|e_\theta\|^2 \|\phi(k)\|^2 \|B_i\|^2 \|P_l\| \\ & \leq x^T(k) (A_i + B_i K_i)^T P_l (A_i + B_i K_i) x(k) \\ & \quad + \phi^2 \|e_\theta\|^2 \|B_i\|^2 \|P_l\|, \end{aligned} \quad (19)$$

$$\begin{aligned} & 2e_\omega^T (B_i V)^T P_l (B_i \phi^T) e_\theta \\ & \leq e_\omega^T (B_i V)^T P_l B_i V e_\omega + \phi^2 \|e_\theta\|^2 \|B_i\|^2 \|P_l\|. \end{aligned}$$

Combining (18) and (19) yields

$$\begin{aligned} \Delta V_1(k) & \leq 2x^T(k) (A_i + B_i K_i)^T P_l (A_i + B_i K_i) x(k) \\ & \quad + 2e_\omega^T(k) (B_i V)^T P_l (B_i V) e_\omega(k) \\ & \quad + 2x^T(k) (A_i + B_i K_i)^T P_l B_i V e_\omega \\ & \quad + 4\lambda_i \phi^2 \|e_\theta\|^2 \|B_i\|^2 - x^T(k) P_l x(k). \end{aligned} \quad (20)$$

Direct computation gives

$$\begin{aligned} \Delta V_2(k) & = e_\omega(k+1) S e_\omega(k+1) - e_\omega^T(k) S \omega(k) \\ & = e_\omega^T (W + LV)^T S (W + LV) e_\omega \\ & \quad + e_\theta^T \phi(k) L^T S L \phi^T(k) e_\theta \\ & \quad + 2e_\omega^T (W + LV)^T S L \phi^T(k) e_\theta \\ & \quad - e_\omega^T(k) S \omega(k). \end{aligned} \quad (21)$$

Note that

$$\begin{aligned} & e_\theta^T \phi(k) L^T S L \phi^T(k) e_\theta \leq \delta_2 \phi^2 \|e_\theta\|^2, \\ & 2e_\omega^T (W + LV)^T S L \phi^T(k) e_\theta \end{aligned}$$

$$\begin{aligned} & \leq e_\omega^T (W + LV)^T S (W + LV) e_\omega \\ & \quad + e_\theta^T \phi(k) L^T S L \phi^T(k) e_\theta \\ & \leq e_\omega^T (W + LV)^T S (W + LV) e_\omega + \delta_2 \phi^2 \|e_\theta\|^2. \end{aligned} \quad (22)$$

By substituting (22) into (21), we have

$$\begin{aligned} \Delta V_2(k) & \leq 2e_\omega^T (W + LV)^T S (W + LV) e_\omega \\ & \quad + 2e_\theta^T \phi(k) L^T S L \phi^T(k) e_\theta + 3\delta_2 \phi^2 \|e_\theta\|^2 \\ & \quad - e_\omega^T(k) S \omega(k). \end{aligned} \quad (23)$$

Computing the difference $V_3(k)$ yields

$$\begin{aligned} \Delta V_3(k) & = e_\theta^T(k+1) T e_\theta(k+1) - e_\theta^T(k) T e_\theta(k) \\ & = e_\theta^T (\Gamma + L_0 \phi^T(k))^T Q (\Gamma + L_0 \phi^T(k)) e_\theta \\ & \quad + 2e_\theta^T (\Gamma + L_0 \phi^T(k))^T Q L_0 B_i e_\omega(k) \\ & \quad + e_\omega^T B_i^T L_0^T Q L_0 B_i e_\omega - e_\theta^T(k) T e_\theta(k). \end{aligned} \quad (24)$$

Note that

$$\begin{aligned} & e_\theta^T (\Gamma + L_0 \phi^T(k))^T Q (\Gamma + L_0 \phi^T(k)) e_\theta \\ & = e_\theta^T (\Gamma + L_0 (E + \hat{\phi}^T(k)))^T \\ & \quad \cdot Q (\Gamma + L_0 (E + \hat{\phi}^T(k))) e_\theta = e_\theta^T (\Gamma + L_0 E)^T \\ & \quad \cdot Q (\Gamma + L_0 E) e_\theta + 2e_\theta^T (\Gamma + L_0 E)^T Q L_0 \hat{\phi}^T(k) e_\theta \\ & \quad + e_\theta^T \hat{\phi}^T(k) L_0^T Q L_0 \hat{\phi}^T(k) e_\theta, \\ & 2e_\theta^T (\Gamma + L_0 E)^T Q L_0 \hat{\phi}^T(k) e_\theta \leq e_\theta^T (\Gamma + L_0 E)^T \\ & \quad \cdot Q (\Gamma + L_0 E) e_\theta + e_\theta^T \hat{\phi}^T(k) L_0^T Q L_0 \hat{\phi}^T(k) e_\theta, \\ & 2e_\theta^T (\Gamma + L_0 \phi^T(k))^T Q L_0 B_i e_\omega(k) \\ & = 2e_\theta^T (\Gamma + L_0 E + L_0 \hat{\phi}^T(k))^T Q L_0 B_i e_\omega(k) \\ & = 2e_\theta^T (\Gamma + L_0 E)^T Q L_0 B_i e_\omega(k) + 2e_\theta^T (L_0 \hat{\phi}^T(k))^T \\ & \quad \cdot Q L_0 B_i e_\omega(k) \leq 2e_\theta^T (\Gamma + L_0 E)^T Q L_0 B_i e_\omega(k) \\ & \quad + e_\theta^T \hat{\phi}^T(k) L_0^T Q L_0 \hat{\phi}^T(k) e_\theta + e_\omega^T B_i^T L_0^T Q L_0 B_i e_\omega. \end{aligned} \quad (25)$$

By combining (24) and (25), we have

$$\begin{aligned}
\Delta V_3(k) &\leq 2e_\theta^T (\Gamma + L_0 E)^T Q (\Gamma + L_0 E) e_\theta \\
&\quad + 2e_\theta^T (\Gamma + L_0 E)^T Q L_0 B_i e_\omega(k) \\
&\quad + 2e_\omega^T B_i^T L_0 Q L_0^T B_i e_\omega + 4e_\theta^T \hat{\phi} L_0^T Q L_0 \hat{\phi}^T e_\theta \\
&\quad - e_\theta^T(k) T e_\theta(k) \\
&\leq 2e_\theta^T (\Gamma + L_0 E)^T Q (\Gamma + L_0 E) e_\theta \\
&\quad + 2e_\theta^T (\Gamma + L_0 E)^T Q L_0 B_i e_\omega(k) \\
&\quad + 2e_\omega^T B_i^T L_0 Q L_0^T B_i e_\omega + 4\delta_3 \|\hat{\phi}\|^2 \|e_\theta\|^2 \\
&\quad - e_\theta^T(k) T e_\theta(k).
\end{aligned} \tag{26}$$

From (18)–(26), and by some manipulations, we obtain

$$\begin{aligned}
\Delta V(k) &\leq 2x^T(k) (A_i + B_i K_i)^T P_l (A + B_i K_i) x(k) \\
&\quad + 2e_\omega^T(k) (B_i V)^T P_l (B_i V) e_\omega(k) \\
&\quad + 2x^T(k) (A_i + B_i K_i)^T P_l B_i V e_\omega \\
&\quad + 4\lambda_l \phi^2 \|e_\theta\|^2 \|B_i\|^2 - x^T(k) P_i x(k) \\
&\quad + 2e_\omega^T (W + LV)^T S (W + LV) e_\omega \\
&\quad + 3\delta_2 \phi^2 \|e_\theta\|^2 - e_\omega^T(k) S \omega(k) \\
&\quad + 2e_\theta^T (\Gamma + L_0 E)^T Q (\Gamma + L_0 E) e_\theta \\
&\quad + 2e_\theta^T (\Gamma + L_0 E)^T Q L_0 B_i e_\omega(k) \\
&\quad + 2e_\omega^T B_i^T L_0 Q L_0^T B_i e_\omega + 4\delta_3 \|\hat{\phi}\|^2 \|e_\theta\|^2 \\
&\quad - e_\theta^T(k) T e_\theta(k) = \xi^T(k) \Phi_{1i} \xi(k),
\end{aligned} \tag{27}$$

where

$$\begin{aligned}
\Phi_{1i} &= \begin{bmatrix} \Phi_{11i} & \Phi_{12i} & 0 \\ * & \Phi_{22i} & \Phi_{23i} \\ * & * & \Phi_{33i} \end{bmatrix}, \\
\Phi_{11i} &= 2(A_i + B_i K_i)^T P_l (A + B_i K_i) - P_i, \\
\Phi_{12i} &= (A_i B_i K_i)^T P_l B_i V,
\end{aligned}$$

$$\begin{aligned}
\Phi_{22i} &= 2(W + LV)^T S (W + LV) + (B_i V)^T P_l (B_i V) \\
&\quad + B_i^T L_0^T Q L_0 B_i, \\
\Phi_{23i} &= B_i^T L_0^T Q (\Gamma + L_0 E), \\
\Phi_{33i} &= 2(\Gamma + L_0 E)^T Q (\Gamma + L_0 E) - Q + 4\lambda_l \phi^2 \|B_i\|^2 \\
&\quad + 3\delta_2 \phi^2 + 4\delta_3 \hat{\phi}^2.
\end{aligned} \tag{28}$$

By applying Schur complement formula, we obtain $\Delta V(k) < 0$ if (11) is true. Therefore, it is easy to see that the closed-loop system Σ_c is asymptotically stable by the Lyapunov function stability theorem. This completes the proof. $\square$

Remark 6. In Theorem 5, a sufficient condition is derived to guarantee system's asymptotical stability, but the condition is presented by some nonlinear matrix inequalities. In order to solve the controller and observer gains, we cast the inequalities (11)–(14) into linear matrix inequalities.

By premultiplying and postmultiplying $\text{diag}\{P_i^{-1}, I, I, I, S, T, I, S, T\} = \text{diag}\{X_i, I, I, I, S, T, I, S, T\}$ with (11), premultiplying and postmultiplying $\text{diag}\{I, S\}$ with (13), and premultiplying and postmultiplying $\text{diag}\{I, T\}$ with (14), we have

$$\begin{aligned}
&\bar{\Phi}_i \\
&= \begin{bmatrix} \bar{\Phi}_{11i} & 0 & 0 & \bar{\Phi}_{15i} & 0 & 0 & \bar{\Phi}_{15i} & 0 & 0 \\ * & -S & 0 & \Phi_{25i} & \bar{\Phi}_{26i} & \bar{\Phi}_{27i} & 0 & \bar{\Phi}_{26i} & 0 \\ * & * & \Phi_{33} & 0 & 0 & \bar{\Phi}_{37} & 0 & 0 & \bar{\Phi}_{37} \\ * & * & * & -X_l & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -S & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -T & 0 & 0 & 0 \\ * & * & * & * & * & * & -X_l & 0 & 0 \\ * & * & * & * & * & * & * & -S & 0 \\ * & * & * & * & * & * & * & * & -T \end{bmatrix} \tag{29} \\
&< 0,
\end{aligned}$$

$$X_i \geq \frac{1}{\lambda_i} I,$$

$$\begin{bmatrix} -\delta_2 I & \bar{L}^T \\ \bar{L} & -S \end{bmatrix} < 0,$$

$$\begin{bmatrix} -\delta_3 I & \bar{L}_0^T \\ \bar{L}_0 & -T \end{bmatrix} < 0,$$

where

$$\begin{aligned}\Phi_{11i} &= -X_i, \\ \Phi_{15i} &= (A_i X_i + B_i \bar{K}_i)^T, \\ \Phi_{26i} &= (SW + \bar{L}V)^T, \\ \Phi_{33} &= -T + 4\lambda_i \phi^2 \|B_i\|^2 + 3\delta_2 \phi^2 + 4\delta_3 \hat{\phi}^2, \\ &\quad l = 1, 2, \dots, N, \quad (30)\end{aligned}$$

$$\Phi_{37} = (T\Gamma + \bar{L}_0 E)^T,$$

$$\Phi_{25i} = (B_i V)^T,$$

$$\Phi_{27i} = (B_i^T \bar{L}_0^T)^T,$$

with $K_i = \bar{K}_i X_i^{-1}$, $L = S^{-1} \bar{L}$, and $L_0 = T^{-1} \bar{L}_0$.

Remark 7. Although we first design a disturbance observer and adaptive control law to estimate the disturbance and unknown parameter, respectively, and then the composite controller is constructed based on the estimation values and feedback control law, the disturbance observer and the composite controller are solved simultaneously in Theorem 5. By solving conditions (29), we can get the values of controller and observer gains K_i , L_0 , and L .

Now, we consider the case: switched system with Assumptions 1 and 2 has one subsystem ($N = 1$):

$$x(k+1) = Ax(k) + B(u(k) + d_1(k) + \phi^T(k)\theta(k)), \quad (31)$$

with

$$\omega(k+1) = W\omega(k), \quad (32)$$

$$d_1(k) = V\omega(k),$$

$$\theta(k+1) = \Gamma\theta(k). \quad (33)$$

And the composite adaptive antidisturbance controller is designed:

$$\begin{aligned}u(k) &= Kx(k) - \hat{d}_1(k) - \phi^T(k)\hat{\theta}(k), \\ \hat{\omega}(k) &= v(k) - Lx(k), \\ v(k+1) &= W\hat{\omega}(k) + L(A_{\sigma(k)}x(k) \\ &\quad + B_{\sigma(k)}(u(k) + \hat{d}_1(k) + \phi^T(k)\hat{\theta}(k))), \\ \hat{d}_1(k) &= V\hat{\omega}(k), \\ \hat{\theta}(k) &= v_0(k) - L_0x(k), \\ v_0(k+1) &= \Gamma\hat{\theta}(k) + L_0(A_{\sigma(k)}x(k) \\ &\quad + B_{\sigma(k)}(u(k) + \hat{d}_1(k) + \phi^T(k)\hat{\theta}(k))),\end{aligned} \quad (34)$$

where K , L , and L_0 are controller and observer gains to be determined later.

For such a case, the closed-loop system becomes a class of discrete-time nonlinear system effectively operating at one of the subsystems all the time, and it can be described by

$$\begin{aligned}x(k+1) &= (A + BK)x(k) + BVe_\omega(k) \\ &\quad + B\phi^T(k)e_\theta(k), \\ e_\omega(k+1) &= (W + LV)e_\omega(k) + L\phi^T(k)e_\theta(k), \\ e_\theta(k+1) &= (\Gamma + L_0\phi^T(k))e_\theta(k) + L_0B_{\sigma(k)}e_\omega(k).\end{aligned} \quad (35)$$

Corollary 8. Consider system (35). System (35) is asymptotically stable if there exist matrices K , L , L_0 , $P > 0$, $S > 0$, and $T > 0$ such that the following inequalities hold:

$$\Phi = \begin{bmatrix} \Phi_{11} & 0 & 0 & \Phi_{15} & 0 & 0 & \Phi_{15} & 0 & 0 \\ * & -S & 0 & \Phi_{25} & \Phi_{26} & \Phi_{27} & 0 & \Phi_{26} & 0 \\ * & * & \Phi_{33} & 0 & 0 & \Phi_{37} & 0 & 0 & \Phi_{37} \\ * & * & * & -P^{-1} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -S^{-1} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -T^{-1} & 0 & 0 & 0 \\ * & * & * & * & * & * & -P^{-1} & 0 & 0 \\ * & * & * & * & * & * & * & -S^{-1} & 0 \\ * & * & * & * & * & * & * & * & -T^{-1} \end{bmatrix} \quad (36)$$

$$< 0,$$

$$P \leq \lambda I,$$

$$\begin{bmatrix} -\delta_2 I & L^T \\ L & -S^{-1} \end{bmatrix} < 0,$$

$$\begin{bmatrix} -\delta_3 I & L_0^T \\ L_0 & -T^{-1} \end{bmatrix} < 0,$$

where

$$\begin{aligned}\Phi_{11} &= -P, \\ \Phi_{15} &= (A + BK)^T, \\ \Phi_{26} &= (W + LV)^T, \\ \Phi_{33} &= -T + 4\lambda_1 \phi^2 \|B\|^2 + 3\delta_2 \phi^2 + 4\delta_3 \hat{\phi}^2 \\ \Phi_{37} &= (\Gamma + L_0 E)^T, \\ \Phi_{25} &= (BV)^T, \\ \Phi_{27} &= (B^T L_0^T)^T.\end{aligned} \quad (37)$$

Remark 9. To the best of the authors' knowledge, this is the first time that the disturbance observer based on adaptive control strategy is applied to the control problem for discrete time with multiple disturbances.

4. A Numerical Example

Now, we provide an example to show the effectiveness of the main result in this paper.

Consider discrete-time switched system with parameters as follows:

$$\begin{aligned} A_1 &= \begin{bmatrix} -0.3 & -0.1 \\ 0.5 & -0.1 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -0.1 & 0.2 \\ 0 & -0.5 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} 0.01 \\ -0.2 \end{bmatrix}, \\ B_2 &= \begin{bmatrix} -0.02 \\ 0.2 \end{bmatrix}. \end{aligned} \quad (38)$$

The disturbance model is presented by the following parameters:

$$\begin{aligned} W &= \begin{bmatrix} 0.8776 & 0.4794 \\ -0.4794 & 0.8776 \end{bmatrix}, \\ V &= [10 \ 0]. \end{aligned} \quad (39)$$

The parameter of model (33) is given as

$$\Gamma = \begin{bmatrix} 0.7071 & -0.7071 \\ 0.7071 & 0.7071 \end{bmatrix}. \quad (40)$$

The controller gains and observer gain are listed as follows:

$$\begin{aligned} K_1 &= [2.5738 \ -0.4718], \\ K_2 &= [2.5777 \ -0.5513], \\ L &= \begin{bmatrix} 0.0708 & -0.3454 \\ -0.0361 & 0.2504 \end{bmatrix}, \\ L_0 &= \begin{bmatrix} 0.0612 & 0.0235 \\ 0.0155 & -0.0189 \end{bmatrix}. \end{aligned} \quad (41)$$

The initial value of the states is chosen as $\phi(s) = [-2 \ 0.5]^T$. In order to illustrate the effectiveness of the proposed method, we consider two kinds of switching signal: determinate switching signal and stochastic switching signal.

(i) *Determinate Switching Signal.* Suppose the switching sequence as 121212... In Figure 1, system states are shown, in which we can see the proposed method can achieve a good disturbance rejection performance in spite of external disturbance and unknown parameters. Figure 2 depicts the curves of control input. In order to demonstrate the effectiveness of the proposed observer, curves of disturbances and disturbance estimation are presented in Figure 3 and curves of unknown parameters and parameters estimation

FIGURE 1: Response curves of system states $x(k)$.

FIGURE 2: Curves of system input.

FIGURE 3: Curves of disturbances and disturbances estimation.

are described in Figure 4. From these figures, we can see the disturbance observer and adaptive control law can effectively estimate external disturbance and unknown parameters, respectively. According to the simulation results, we can conclude that the proposed scheme can guarantee system has a satisfactory performance in presence of external disturbances and unknown parameters.

(ii) *Stochastic Switching Signal.* The simulation results are presented as follows. From Figure 5, we can see the proposed method can obtain a good performance in spite of stochastic switching signal. The switching signal is presented in Figure 6.

Remark 10. In this paper, the problem of composite adaptive antidisturbance control is considered for a class of switched systems with arbitrary switching signal; namely, if we design different switching signal, the system can be stable under the switching signal. The simulation results demonstrate this point.

FIGURE 4: Curves of unknown parameters and parameters estimation.

FIGURE 5: Response curves of system states $x(k)$ with stochastic switching signal.

FIGURE 6: Curves of stochastic switching signal.

5. Conclusions

The problem of adaptive antidisturbance control for discrete-time switched systems subject to disturbances has been investigated. The composite adaptive observers have been constructed to estimate disturbance and unknown parameters. A

composite antidisturbance adaptive controller has been given based on the output of observer and conventional feedback control law. Some sufficient conditions have been developed via linear matrix inequalities technique. A numerical example has been provided to show the effectiveness of the proposed algorithm.

Competing Interests

The authors declare that they have no competing interests.

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