Research Article

Chaotic Synchronization of Modified Discrete-Time Tinkerbell Systems

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This paper studies chaotic synchronization of modified discrete-time Tinkerbell systems. By constructing the Lyapunov function and using the linear feedback control, some synchronization criteria for modified discrete-time Tinkerbell systems are derived. The conservativeness of those synchronization criteria is compared. The effectiveness of derived results is demonstrated by six examples.

1. Introduction

Many models in engineering can be mathematically described as a Tinkerbell system, which is a classical two-dimensional discrete-time system with Tinkerbell-like trajectories. The dynamical behaviors of Tinkerbell systems have been studied in some papers (see, e.g., [1–3]).

Within the dynamical evolution of a Tinkerbell system, chaotic behaviors have attracted many researchers' interests because chaos can increase the complexity of evolution of systems and even influence the stability of systems. Therefore, the chaotic synchronization of two discrete-time Tinkerbell systems was studied in [2, 4]. It should be pointed out that the synchronization methods of chaotic discrete systems are different from those of continuous systems (see, e.g., [5–14]) because of the difference of constructing of Lyapunov functions. Recently, the classical Tinkerbell system has been expected to extend to a modified format in order to well satisfy the requirement of the normalization. However, how to achieve the chaotic synchronization of two modified discrete-time Tinkerbell systems has not been studied yet. Furthermore, a question arises as to whether the linear feedback control can be used to achieve global synchronization for modified discrete-time Tinkerbell systems.

Motivated by these two points, the synchronization of modified discrete-time Tinkerbell systems is studied in this paper. Some synchronization results for modified discrete-time Tinkerbell systems are derived by using linear feedback control. The conservativeness of those derived synchronization criteria is compared. Six examples are used to illustrate the effectiveness of our derived results.

2. Preliminaries

Consider the following modified Tinkerbell system:

$$\begin{aligned}x_1(n+1) &= w(\sin^2(x_1(n)) - \sin^2(x_2(n))) + ax_1(n) \\ &\quad + bx_2(n), \\ x_2(n+1) &= p \sin(x_1(n)) \sin(x_2(n)) + cx_1(n) \\ &\quad + dx_2(n),\end{aligned}\tag{1}$$

where $x_1(n)$, $x_2(n)$ are state variables. Constants a, b, c, d, p, w are system parameters. The initial conditions of (1) are $x_1(0) = x_{1_0}$, $x_2(0) = x_{2_0}$.

One can construct the slave system associated with master system (1) as follows:

$$\begin{aligned} y_1(n+1) &= w(\sin^2(y_1(n)) - \sin^2(y_2(n))) + ay_1(n) \\ &\quad + by_2(n) + k_1(x_1(n) - y_1(n)) \\ &\quad + k_2(x_2(n) - y_2(n)), \\ y_2(n+1) &= p \sin(y_1(n)) \sin(y_2(n)) + cy_1(n) \\ &\quad + dy_2(n) + k_3(x_2(n) - y_2(n)) \\ &\quad + k_4(x_2(n) - y_2(n)), \end{aligned} \quad (2)$$

where $y_1(n), y_2(n)$ are the state variables, and the initial conditions of (2) are $y_1(0) = y_{10}, y_2(0) = y_{20}$. $k_1(x_1(n) - y_1(n)), k_2(x_2(n) - y_2(n))$ and $k_3(x_1(n) - y_1(n)), k_4(x_2(n) - y_2(n))$ are controls. The gains k_1, k_2, k_3, k_4 can be derived later. After defining two error variables

$$\begin{aligned} e_1(n) &= x_1(n) - y_1(n), \\ e_2(n) &= x_2(n) - y_2(n), \end{aligned} \quad (3)$$

one can have the following error system:

$$\begin{aligned} e_1(n+1) &= (a - k_1)e_1(n) + (b - k_2)e_2(n) \\ &\quad + w(\sin(x_1(n)) - \sin(y_1(n)))(\sin(x_1(n)) \\ &\quad + \sin(y_1(n))) - w(\sin(x_2(n)) - \sin(y_2(n))) \\ &\quad \cdot (\sin(x_2(n)) + \sin(y_2(n))), \\ e_2(n+1) &= (c - k_3)e_1(n) + (d - k_4)e_2(n) \\ &\quad + p(\sin(x_1(n)) \sin(x_2(n)) \\ &\quad - \sin(y_1(n)) \sin(y_2(n))). \end{aligned} \quad (4)$$

It follows from the well-known mean value theory that

$$\sin(\alpha) - \sin(\beta) = \cos(\theta)(\alpha - \beta) \quad (5)$$

for $\theta \in [\min\{\alpha, \beta\}, \max\{\alpha, \beta\}]$. Then, error system (4) can be rewritten as

$$\begin{aligned} e_1(n+1) &= [a + w \cos(\theta_1)(\sin(x_1(n)) + \sin(y_1(n))) - k_1] \\ &\quad \cdot e_1(n) \\ &\quad + [b - w \cos(\theta_2)(\sin(x_2(n)) + \sin(y_2(n))) - k_2] \\ &\quad \cdot e_2(n), \\ e_2(n+1) &= [c + p \cos(\theta_4) \sin(y_2(n)) - k_3] e_1(n) \\ &\quad + [d + p \sin(x_1(n)) \cos(\theta_3) - k_4] e_2(n), \end{aligned} \quad (6)$$

where the initial conditions of (6) are $e_1(0) = x_{10} - y_{10}, e_2(0) = x_{20} - y_{20}$,

$$\begin{aligned} \theta_1, \theta_4 &\in [\min\{x_1(n), y_1(n)\}, \max\{x_1(n), y_1(n)\}], \\ \theta_2, \theta_3 &\in [\min\{x_2(n), y_2(n)\}, \max\{x_2(n), y_2(n)\}]. \end{aligned} \quad (7)$$

The stability of error system (6) is equivalent to the synchronization of (1) and (2). The purpose of this paper is to give some synchronization criteria for discrete-time systems described by (1) and (2).

3. Main Results

In this section, some synchronization criteria for discrete-time chaotic systems described by (1) and (2) will be provided.

Firstly, one can define the following Lyapunov function:

$$V(n) = |e_1(n)| + |e_2(n)|. \quad (8)$$

Theorem 1. For two given constants $0 < \eta, \mu < 1$ such that $|w| \leq \min\{\eta/2, \mu/2\}$ and $|p| \leq \min\{1 - \eta, 1 - \mu\}$, two discrete-time chaotic systems described by (1) and (2) can achieve global synchronization, if k_1, k_2, k_3, k_4 satisfy the following inequalities:

$$\begin{aligned} \max\{a + 2w, a - 2w\} - \eta &< k_1 \\ &< \eta + \min\{a + 2w, a - 2w\}, \end{aligned} \quad (9)$$

$$\begin{aligned} \max\{c + p, c - p\} - (1 - \eta) &< k_3 \\ &< 1 - \eta + \min\{c + p, c - p\}, \end{aligned} \quad (10)$$

$$\begin{aligned} \max\{b - 2w, b + 2w\} - \mu &< k_2 \\ &< \mu + \min\{b - 2w, b + 2w\}, \end{aligned} \quad (11)$$

$$\begin{aligned} \max\{d + p, d - p\} - (1 - \mu) &< k_4 \\ &< 1 - \mu + \min\{d + p, d - p\}. \end{aligned} \quad (12)$$

Proof. After computing the difference of $V(n)$ along with (6), one can obtain

$$\begin{aligned} V(n+1) - V(n) &\leq [|a \\ &\quad + w \cos(\theta_1)(\sin(x_1(n)) + \sin(y_1(n))) - k_1| + |c \\ &\quad + p \cos(\theta_4) \sin(y_2(n)) - k_3| - 1] |e_1(n)| + [|b \\ &\quad - w \cos(\theta_2)(\sin(x_2(n)) + \sin(y_2(n))) - k_2| + |d \\ &\quad + p \sin(x_1(n)) \cos(\theta_3) - k_4|] |e_2(n)|. \end{aligned} \quad (13)$$

It follows from condition (9), condition (10), and $|w| \leq \min\{\eta/2, \mu/2\}, |p| \leq \min\{1 - \eta, 1 - \mu\}$ that

$$L_1 + L_2 < 1, \quad (14)$$

where

$$L_1 = |a + w \cos(\theta_1)(\sin(x_1(n)) + \sin(y_1(n))) - k_1|, \quad (15)$$

$$L_2 = |c + p \cos(\theta_4) \sin(y_2(n)) - k_3|.$$

Moreover, condition (11), condition (12), and $|w| \leq \min\{\eta/2, \mu/2\}$ and $|p| \leq \min\{1 - \eta, 1 - \mu\}$ imply that

$$L_3 + L_4 < 1, \quad (16)$$

where

$$L_3 = |b - w \cos(\theta_2)(\sin(x_2(n)) + \sin(y_2(n))) - k_2|, \quad (17)$$

$$L_4 = |d + p \sin(x_1(n)) \cos(\theta_3) - k_4|.$$

It follows from (13), (14), and (16) that

$$V(n+1) - V(n) < 0, \quad \forall e_1(n), e_2(n) \neq 0, \quad (18)$$

which means the stability of error system (6); that is, two discrete-time chaotic systems described by (1) and (2) can achieve global synchronization. This ends the proof. $\square$

Remark 2. η and μ can be any positive constants in $(0, 1)$ such that $|w| \leq \min\{\eta/2, \mu/2\}$ and $|p| \leq \min\{1 - \eta, 1 - \mu\}$.

Remark 3. In [2, 4], the active control and backstepping control were used to achieve synchronization for the classical discrete-time Tinkerbell systems. For the modified discrete-time Tinkerbell system (1), the linear control is used to achieve synchronization, which is the significant difference between Theorem 1 of this paper and results in [2, 4].

If $|b - w \cos(\theta_2)(\sin(x_2(n)) + \sin(y_2(n)))| + |d + p \sin(x_1(n)) \cos(\theta_3)| < 1$, one can have the following corollary with $k_2 = k_4 = 0$.

Corollary 4. For any given constant $0 < \eta < 1$ such that $|w| \leq \eta/2$, $|p| \leq 1 - \eta$, two discrete-time systems described by (1) and (2) can achieve global synchronization, if $k_2 = k_4 = 0$ and k_1, k_3 satisfy the following inequalities:

$$\begin{aligned} & \max\{a + 2w, a - 2w\} - \eta < k_1 \\ & < \eta + \min\{a + 2w, a - 2w\}, \\ & \max\{c + p, c - p\} - (1 - \eta) < k_3 \\ & < 1 - \eta + \min\{c + p, c - p\}, \\ & |b| + 2|w| + |d| + |p| < 1. \end{aligned} \quad (19)$$

If $|b - w \cos(\theta_2)(\sin(x_2(n)) + \sin(y_2(n)))| < 1$ and $|c + p \cos(\theta_4) \sin(y_2(n))| < 1$, one can have the following synchronization criterion with $k_2 = k_3 = 0$.

Corollary 5. Two discrete-time systems described by (1) and (2) can achieve global synchronization, if $k_2 = k_3 = 0$ and k_1, k_4 satisfy the following inequalities:

$$\begin{aligned} & \max\{a + 2w, a - 2w\} - \nu < k_1 \\ & < \nu + \min\{a + 2w, a - 2w\}, \\ & \max\{d + p, d - p\} - \delta < k_4 \\ & < \delta + \min\{d + p, d - p\}, \\ & |b| + 2|w| < 1, \\ & |c| + |p| < 1, \end{aligned} \quad (20)$$

where

$$\begin{aligned} \nu &= 1 - (|c| + |p|), \\ 2|w| &< \nu, \\ \delta &= 1 - (|b| + 2|w|), \\ |p| &< \delta. \end{aligned} \quad (21)$$

If $|b - w \cos(\theta_2)(\sin(x_2(n)) + \sin(y_2(n)))| + |d + p \sin(x_1(n)) \cos(\theta_3)| < 1$ and $|c + p \cos(\theta_4) \sin(y_2(n))| < 1$, one can have the following synchronization criterion with $k_2 = k_3 = k_4 = 0$.

Corollary 6. Two discrete-time systems described by (1) and (2) can achieve global synchronization, if $k_2 = k_3 = k_4 = 0$ and k_1 satisfies the following inequalities:

$$\begin{aligned} & \max\{a + 2w, a - 2w\} - \nu < k_1 \\ & < \nu + \min\{a + 2w, a - 2w\}, \\ & |b| + 2|w| + |d| + |p| < 1, \\ & |c| + |p| < 1, \end{aligned} \quad (22)$$

where

$$\begin{aligned} \nu &= 1 - (|c| + |p|), \\ 2|w| &< \nu. \end{aligned} \quad (23)$$

If $|a + w \cos(\theta_1)(\sin(x_1(n)) + \sin(y_1(n)))| + |c + p \cos(\theta_4) \sin(y_2(n))| < 1$, one can derive the following synchronization result with $k_1 = k_3 = 0$.

Corollary 7. For given constant $0 < \mu < 1$ such that $|w| \leq \mu/2$ and $|p| \leq 1 - \mu$, two discrete-time systems described by (1) and (2) can achieve global synchronization, if $k_1 = k_3 = 0$ and k_2, k_4 satisfy the following inequalities:

$$\begin{aligned} & |a| + 2|w| + |c| + |p| < 1, \\ & \max\{b - 2w, b + 2w\} - \mu < k_2 \\ & < \mu + \min\{b - 2w, b + 2w\}, \\ & \max\{d + p, d - p\} - (1 - \mu) < k_4 \\ & < 1 - \mu + \min\{d + p, d - p\}. \end{aligned} \quad (24)$$

If $|a + w \cos(\theta_1)(\sin(x_1(n)) + \sin(y_1(n)))| + |c + p \cos(\theta_4) \sin(y_2(n))| < 1$ and $|b - w \cos(\theta_2)(\sin(x_2(n)) + \sin(y_2(n)))| < 1$, one can have the following synchronization criterion with $k_1 = k_2 = k_3 = 0$.

Corollary 8. Two discrete-time systems described by (1) and (2) can achieve global synchronization, if $k_1 = k_2 = k_3 = 0$ and k_4 satisfies the following inequalities:

$$\begin{aligned} |a| + 2|w| + |c| + |p| &< 1, \\ |b| + 2|w| &< 1, \\ \max\{d + p, d - p\} - \delta &< k_4 \\ &< \delta + \min\{d + p, d - p\}, \end{aligned} \quad (25)$$

where

$$\begin{aligned} \delta &= 1 - (|b| + 2|w|), \\ |p| &< \delta. \end{aligned} \quad (26)$$

Remark 9. The conservativeness of synchronization criteria is compared as follows. Corollary 6 is more conservative than Corollaries 4 and 5. Corollary 8 is more conservative than Corollaries 5 and 7. However, only one controller $k_1(x_1(n) - y_1(n))$ is used in Corollary 6. And the single controller $k_4(x_2(n) - y_2(n))$ is used in Corollary 8, which means that reducing of controllers results in the increasing of conservativeness.

4. Simulation Examples

Example 1. Consider modified discrete-time Tinkerbell systems (1) and (2) with $a = 0.9, b = -0.6, c = 0.9, d = 0.5, p = 0.2, w = 0.1$. The initial conditions are $x_1(0) = -0.64, x_2(0) = -0.72, y_1(0) = 0.1$, and $y_2(0) = 0.1$, respectively. A chaotic attractor can be generated by the discrete-time Tinkerbell system (1) which is revealed by Figure 1.

Because $0.1 = |w| \leq \min\{\eta/2, \mu/2\}$ and $0.2 = |p| \leq \min\{1 - \eta, 1 - \mu\}$, one can choose $\eta = \mu = 0.5$. By using Theorem 1 with $\eta = \mu = 0.5$, one can obtain that $0.6 < k_1 < 1.2, -0.9 < k_2 < -0.3, 0.6 < k_3 < 1.2$, and $0.2 < k_4 < 0.8$. We choose $k_1 = 1, k_2 = -0.8, k_3 = 1$, and $k_4 = 0.7$. Figures 2 and 3 give the demonstration of master Tinkerbell system (1) and slave Tinkerbell system (2), respectively. The trajectories of error system (6) are illustrated by Figures 4 and 5, respectively, which show that master Tinkerbell system (1) and slave Tinkerbell system (2) achieve global synchronization.

Example 2. Consider modified discrete-time Tinkerbell systems (1) and (2) with $a = 0.9, b = -0.6, c = 0.9, d = 0.1, p = 0.1, w = 0.05$. The initial conditions are $x_1(0) = -0.64, x_2(0) = -0.72, y_1(0) = 0.1$, and $y_2(0) = 0.1$, respectively.

Due to $0.05 = |w| \leq \eta/2, 0.1 = |p| \leq 1 - \eta$, we can set $\eta = 0.5$. It is easy to see that $|b| + 2|w| + |d| + |p| = 0.9 < 1$. By using Corollary 4 with $\eta = 0.5$, one can derive that $k_2 = k_4 = 0, 0.5 < k_1 < 1.3$, and $0.5 < k_3 < 1.3$. We choose $k_2 = k_4 = 0, k_1 = 1.1$, and $k_3 = 1$. The trajectories of error system (6) are illustrated by Figure 6, which reveals that master Tinkerbell system (1) and slave Tinkerbell system (2) achieve global synchronization.

FIGURE 1: Chaotic attractor of discrete-time Tinkerbell system (1).

FIGURE 2: Master Tinkerbell system (1).

FIGURE 3: Slave Tinkerbell system (2).

FIGURE 4: Error variable $e_1(n)$ of error system (6) with $k_1 = 1$, $k_2 = -0.8$, $k_3 = 1$, and $k_4 = 0.7$.

FIGURE 5: Error variable $e_2(n)$ of error system (6) with $k_1 = 1$, $k_2 = -0.8$, $k_3 = 1$, and $k_4 = 0.7$.

Example 3. Consider modified discrete-time Tinkerbell systems (1) and (2) with $a = 0.9$, $b = -0.6$, $c = 0.6$, $d = 0.1$, $p = 0.1$, $w = 0.05$. The initial conditions are $x_1(0) = -0.64$, $x_2(0) = -0.72$, $y_1(0) = 0.1$, and $y_2(0) = 0.1$, respectively.

Due to $|b| + 2|w| = 0.7 < 1$, $|c| + |p| = 0.7 < 1$, one can set $\nu = 1 - (|c| + |p|) = 0.3 > 2|w| = 0.1$ and $\delta = 1 - (|b| + 2|w|) = 0.3 > |p| = 0.1$. By using Corollary 5 with $\nu = 0.3$ and $\delta = 0.3$, one can derive that $k_2 = k_3 = 0$, $0.7 < k_1 < 1.1$, and $-0.1 < k_4 < 0.3$. We choose $k_2 = k_3 = 0$ and $k_1 = 1$ and $k_4 = 0.2$. The trajectories of error system (6) are illustrated by Figure 7, which shows that master Tinkerbell system (1) and slave Tinkerbell system (2) achieve global synchronization.

Example 4. Consider modified discrete-time Tinkerbell systems (1) and (2) with $a = 0.9$, $b = -0.6$, $c = 0.5$, $d = 0.1$, $p = 0.1$, $w = 0.05$. The initial conditions are $x_1(0) = -0.64$, $x_2(0) = -0.72$, $y_1(0) = 0.1$, and $y_2(0) = 0.1$, respectively.

FIGURE 6: Error variables $e_1(n)$, $e_2(n)$ of error system (6) with $k_1 = 1.1$, $k_2 = 0$, $k_3 = 1$, and $k_4 = 0$.

FIGURE 7: Error variables $e_1(n)$, $e_2(n)$ of error system (6) with $k_1 = 1$, $k_2 = 0$, $k_3 = 0$, and $k_4 = 0.2$.

It is easy to see that $|b| + 2|w| + |d| + |p| = 0.9 < 1$ and $|c| + |p| = 0.6 < 1$. Let $\nu = 1 - (|c| + |p|) = 0.4 > 2|w| = 0.1$. By using Corollary 6, one can derive that $k_2 = k_3 = k_4 = 0$ and $0.6 < k_1 < 1.2$. We choose $k_2 = k_3 = k_4 = 0$ and $k_1 = 1$. The trajectories of error system (6) are illustrated by Figure 8, which demonstrates that master Tinkerbell system (1) and slave Tinkerbell system (2) achieve global synchronization.

Example 5. Consider modified discrete-time Tinkerbell systems (1) and (2) with $a = 0.4$, $b = -0.6$, $c = 0.1$, $d = 0.5$, $p = 0.2$, $w = 0.05$. The initial conditions are $x_1(0) = -0.64$, $x_2(0) = -0.72$, $y_1(0) = 0.1$, and $y_2(0) = 0.1$, respectively.

FIGURE 8: Error variables $e_1(n), e_2(n)$ of error system (6) with $k_1 = 1$, $k_2 = 0$, $k_3 = 0$, and $k_4 = 0$.

FIGURE 10: Error variables $e_1(n), e_2(n)$ of error system (6) with $k_1 = 0$, $k_2 = 0$, $k_3 = 0$, and $k_4 = 0.6$.

FIGURE 9: Error variables $e_1(n), e_2(n)$ of error system (6) with $k_1 = 0$, $k_2 = -0.9$, $k_3 = 0$, and $k_4 = 0.75$.

Because $0.05 = |w| \leq \mu/2$ and $0.2 = |p| \leq 1 - \mu$, one can choose $\mu = 0.5$. It is easy to see that $|a| + 2|w| + |c| + |p| = 0.8 < 1$. By using Corollary 7 with $\mu = 0.5$, one can obtain that $k_1 = k_3 = 0$, $-1 < k_2 < -0.2$, and $0.2 < k_4 < 0.8$. We choose $k_1 = k_3 = 0$, $k_2 = -0.9$, and $k_4 = 0.75$. The trajectories of error system (6) are illustrated by Figure 9, which shows that master Tinkerbell system (1) and slave Tinkerbell system (2) achieve global synchronization.

Example 6. Consider modified discrete-time Tinkerbell systems (1) and (2) with $a = 0.4$, $b = -0.6$, $c = 0.1$, $d = 0.5$, $p = 0.1$, $w = 0.05$. The initial conditions are $x_1(0) = -0.64$, $x_2(0) = -0.72$, $y_1(0) = 0.1$, and $y_2(0) = 0.1$, respectively.

Notice that $|a| + 2|w| + |c| + |p| = 0.7 < 1$ and $|b| + |p| = 0.7 < 1$. Let $\delta = 1 - (|b| + |p|) = 0.3 > 2|w| = 0.1$. We can also use Corollary 8 to derive that $k_1 = k_2 = k_3 = 0$ and $0.3 < k_4 < 0.7$. We choose $k_1 = k_2 = k_3 = 0$ and $k_4 = 0.6$. The trajectories of error system (6) are demonstrated by Figure 10, which reveals that master Tinkerbell system (1) and slave Tinkerbell system (2) achieve global synchronization.

Remark 10. Corollaries 6 and 8 fail to derive conclusions for Examples 2 and 3, respectively, which can illustrate the effectiveness of the statement of Remark 9.

5. Conclusions and Future Works

We have studied synchronization of modified discrete-time Tinkerbell system. We have obtained some synchronization criteria by using linear feedback control. We have compared the conservativeness of those synchronization criteria. Six examples have been provided to illustrate the effectiveness of our results. In this paper, we only consider the linear feedback control. It should be pointed out that the control process could suffer time delays. How to use time-delayed control to achieve synchronization of modified discrete-time Tinkerbell system is our research interest in the future.

Competing Interests

The authors declare that there are no competing interests regarding the publication of this paper.

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