Research Article

Stochastic Predator-Prey System Subject to Lévy Jumps

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Received 6 January 2016; Accepted 13 March 2016

Academic Editor: Elvan Akin

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This paper investigates a new nonautonomous impulsive stochastic predator-prey system with the omnivorous predator. First, we show that the system has a unique global positive solution for any given initial positive value. Second, the extinction of the system under some appropriate conditions is explored. In addition, we obtain the sufficient conditions for almost sure permanence in mean and stochastic permanence of the system by using the theory of impulsive stochastic differential equations. Finally, we discuss the biological implications of the main results and show that the large noise can make the system go extinct. Simulations are also carried out to illustrate our theoretical analysis conclusions.

1. Introduction

Omnivory is considered as a common ecological phenomenon in the natural world. Omnivorous predator feeds on both animal prey and plant, so the intrinsic growth rate for predator should be positive. For example, the giant panda is omnivorous animal, since it can eat both meat and plant such as bamboo. With the development of the economy, pollution is becoming more and more serious. Thus pollution models have widely attracted the focus of the people [1–5]. A deterministic predator-prey system with omnivorous predator in an impulsive polluted environment takes the following form:

$$\begin{aligned} dx(t) &= x(t) (r_1(t) - d_1 c_0(t) - a_1(t) x(t) - \beta(t) y(t)) dt, \\ dy(t) &= y(t) (r_2(t) - d_2 c_0(t) - a_2(t) y(t) + \beta(t) x(t)) dt, \\ \dot{c}_0(t) &= k c_e(t) - g c_0(t) - m c_0(t), \\ \dot{c}_e(t) &= -h c_e(t), \end{aligned}$$

$$t \neq \tau_k,$$

$$\Delta x(\tau_k) = \Delta y(\tau_k) = \Delta c_0(\tau_k) = 0,$$

$$\Delta c_e(\tau_k) = u,$$

$$\tau_k = k\tau, \quad k = 0, 1, \dots, \quad (1)$$

where $\Delta x(\tau_k) = x(\tau_k^+) - x(\tau_k)$, $\Delta y(\tau_k) = y(\tau_k^+) - y(\tau_k)$, $\Delta c_0(\tau_k) = c_0(\tau_k^+) - c_0(\tau_k)$, $c_e(\tau_k) = c_e(\tau_k^+) - c_e(\tau_k)$, δ_i ($i = 1, 2$), k , g , m , h , and u are positive constants, $x(t)$ and $y(t)$ denote prey and omnivorous predator densities, and $c_0(t)$ and $c_e(t)$ denote the concentrations of the toxicant in the organism and in the environment, respectively. $r_i(t)$, $a_i(t)$ ($i = 1, 2$), and $\beta(t)$ are all positive bounded continuous functions on $R_+ := [0, +\infty)$. $r_1(t)$ and $r_2(t)$ represent the prey intrinsic growth rate and the predator intrinsic growth rate, respectively, $a_i(t)$ ($i = 1, 2$) are the density-dependent coefficients of the prey and the predator, $\beta(t)$ is the capturing rate of the predator, d_1 and d_2 are damage rates of the prey and predator by the toxicant, respectively, k represents environmental toxicant uptake rate per unit mass organism, g and m are organismal net ingestion and depuration rates of toxicant, respectively, h denotes the loss rate of toxicant from the environment itself by volatilization, and u is the amount of pulsed input concentration of the toxicant at each τ .

System (1) is a deterministic model, where all parameters in the model are deterministic. However, there are some limitations in mathematical model from a biological viewpoint. Therefore, it is significant to study the effects of noises on population systems [6–9]. There are many kinds of environmental noises. First, we assume that the toxicant uptake rates are disturbed by white noise. If we still let $d_i c_0(t)$ ($i = 1, 2$) represent the toxicant uptake rates, then $d_1 c_0(t)$ and $d_2 c_0(t)$ can be replaced by

$$\begin{aligned} d_1 c_0(t) &\longrightarrow d_1 c_0(t) + \sigma_1(t) \dot{B}_1(t), \\ d_2 c_0(t) &\longrightarrow d_2 c_0(t) + \sigma_2(t) \dot{B}_2(t), \end{aligned} \quad (2)$$

where $\dot{B}_i(t)$ ($i = 1, 2$) are white noises and $\sigma_i(t)$ ($i = 1, 2$) are the intensities of the white noises, which are bounded continuous functions on $[0, +\infty)$. Then we obtain the following stochastic system with impulsive toxicant input in a polluted environment:

$$\begin{aligned} dx(t) &= x(t) (r_1(t) - d_1 c_0(t) - a_1(t) x(t) - \beta(t) y(t)) dt \\ &\quad - \sigma_1(t) x(t) dB_1(t), \\ dy(t) &= y(t) (r_2(t) - d_2 c_0(t) - a_2(t) y(t) + \beta(t) x(t)) dt \\ &\quad - \sigma_2(t) y(t) dB_2(t), \\ \dot{c}_0(t) &= k c_e(t) - g c_0(t) - m c_0(t), \\ \dot{c}_e(t) &= -h c_e(t), \end{aligned} \quad (3)$$

$$t \neq \tau_k,$$

$$\Delta x(\tau_k) = \Delta y(\tau_k) = \Delta c_0(\tau_k) = 0,$$

$$\Delta c_e(\tau_k) = u,$$

$$\tau_k = k\tau, \quad k = 0, 1, \dots,$$

where $B_i(t)$ are mutually independent standard Brownian motions defined on a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathcal{P})$.

On the other hand, populations may be affected by sudden environmental fluctuations, such as severe weather, earthquakes, floods, and epidemics. Brownian motion cannot describe these phenomena better, so it is very important to introduce Lévy noise into the population system [10]. There are many researches about autonomous stochastic predator-prey system with Lévy jumps [11, 12].

Inspired by these, we focus on nonautonomous impulsive stochastic predator-prey system with white noises and Lévy jumps

$$\begin{aligned} dx(t) &= x(t) (r_1(t) - d_1 c_0(t) - a_1(t) x(t) - \beta(t) y(t)) dt \\ &\quad - \sigma_1(t) x(t) dB_1(t) \\ &\quad + \int_{\mathbb{Y}} x(t^-) \gamma_1(t, u) \tilde{N}(dt, du), \end{aligned}$$

$$\begin{aligned} dy(t) &= y(t) (r_2(t) - d_2 c_0(t) - a_2(t) y(t) + \beta(t) x(t)) dt \\ &\quad - \sigma_2(t) y(t) dB_2(t) \\ &\quad + \int_{\mathbb{Y}} y(t^-) \gamma_2(t, u) \tilde{N}(dt, du), \\ \dot{c}_0(t) &= k c_e(t) - g c_0(t) - m c_0(t), \\ \dot{c}_e(t) &= -h c_e(t), \end{aligned}$$

$$t \neq \tau_k,$$

$$\Delta x(\tau_k) = \Delta y(\tau_k) = \Delta c_0(\tau_k) = 0,$$

$$\Delta c_e(\tau_k) = u,$$

$$\tau_k = k\tau, \quad k = 0, 1, \dots, \quad (4)$$

where $x(t^-)$ and $y(t^-)$ represent the left limit of $x(t)$ and $y(t)$, respectively, $N(dt, du)$ is a Poisson counting measure with characteristic measure ν on a measurable bounded subset $\mathbb{Y}$ of $(0, \infty)$ with $\nu(\mathbb{Y}) < \infty$, and B_i ($i = 1, 2$) are independent of N . The Poisson counting measure is represented by $\tilde{N}(dt, du) := N(dt, du) - \nu(du)dt$; $\gamma_i > -1$, $\gamma_i : \mathbb{R}_+ \times \mathbb{Y} \rightarrow \mathbb{R}$ ($i = 1, 2$) are continuous functions on $[0, +\infty)$, which are assumed to be periodic with period $\tau > 0$. Other parameters are defined as in system (1).

The paper is arranged as follows. In Section 2, we prove that system (4) has a global positive solution. Section 3 shows the main result; in Section 3.1 we prove the extinction of system (4). We also examine almost sure permanence in mean and the stochastic permanence of the system in Sections 3.2 and 3.3. Finally we present some simulations and conclusions to close the paper in Section 4.

2. Notations and Global Positive Solution

For the purpose of convenience, we introduce some notions and some lemmas which will be used for our main results. We throughout this paper assume that $x(t)$, $y(t)$, and $c_0(t)$ are continuous at $t = k\tau$, and $c_e(t)$ is left continuous at $t = k\tau$ and $c_e(k\tau^+) = \lim_{t \rightarrow k\tau^+} c_e(t)$ and let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathcal{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t \geq 0}$ satisfying the usual conditions (i.e., it is increasing and right continuous while $\mathcal{F}_0$ contains all $\mathcal{P}$ -null sets). Further assume that $B(t)$ is a scalar Brownian motion defined on the complete probability space Ω .

We denote $\mathbb{R}_+^2$ as the positive cone in $\mathbb{R}^2$; that is, $\mathbb{R}_+^2 = \{z = (z_1, z_2) \in \mathbb{R}^2 \mid z_i > 0, i = 1, 2\}$. If $f(t)$ is a bounded continuous function on $[0, +\infty)$, we define

$$\begin{aligned} \langle f(t) \rangle &= \frac{1}{t} \int_0^t f(t) dt, \\ f^M &= \max_{t \in [0, +\infty)} f(t), \end{aligned}$$

$$\begin{aligned}
f^L &= \min_{t \in [0, +\infty)} f(t), \\
f^* &= \limsup_{t \rightarrow +\infty} f(t), \\
f_* &= \liminf_{t \rightarrow +\infty} f(t).
\end{aligned} \tag{5}$$

Now we give some basic properties of the following subsystem of systems (1) and (4):

$$\begin{aligned}
dc_0(t) &= (kc_e(t) - gc_0(t) - mc_0(t)) dt, \\
dc_e(t) &= -hc_e(t) dt, \\
t &\neq k\tau, \quad k \in \mathbb{Z}^+, \\
\Delta c_0(t) &= 0, \\
\Delta c_e(t) &= u, \\
t &= k\tau, \quad k \in \mathbb{Z}^+.
\end{aligned} \tag{6}$$

Lemma 1 (see [3]). *System (6) has a unique positive τ -periodic solution $(c_0^*(t), c_e^*(t))^T$ which is globally asymptotically stable. Moreover, $c_0(t) > c_0^*(t), c_e(t) > c_e^*(t)$ for all $t \geq 0$ if $c_0(0) > c_0^*(0), c_e(0) > c_e^*(0)$, where*

$$\begin{aligned}
c_0^*(t) &= c_0^*(0) e^{-(g+m)(t-n\tau)} \\
&\quad + \frac{ku(e^{-(g+m)(t-n\tau)} - e^{-h(t-n\tau)})}{(h-g-m)(1-e^{-h\tau})}, \\
c_e^*(t) &= \frac{ue^{-h(t-n\tau)}}{1-e^{-h\tau}}, \\
c_0^*(0) &= \frac{ku(e^{-(g+m)\tau} - e^{-h\tau})}{(h-g-m)(1-e^{-(g+m)\tau})(1-e^{-h\tau})}, \\
c_e^*(0) &= \frac{u}{1-e^{-h\tau}},
\end{aligned} \tag{7}$$

for $t \in (k\tau, (k+1)\tau]$ and $k \in \mathbb{Z}^+$.

Lemma 2. *For any positive solution $(c_0(t), c_e(t))$ of system (1) or (4) with initial value $(c_0(0), c_e(0^+)) \in \mathbb{R}_+^2$, one has*

$$\lim_{t \rightarrow +\infty} \langle c_0(t) \rangle = \frac{ku}{h(g+m)\tau} \triangleq \bar{c}_0. \tag{8}$$

Proof. Through a simple calculation, we can get

$$\int_{n\tau}^{(n+1)\tau} c_0^*(t) dt = \frac{ku}{h(g+m)\tau}. \tag{9}$$

Moreover, since $c_0^*(t)$ is a periodic function, we have

$$\begin{aligned}
\lim_{t \rightarrow +\infty} \langle c_0(t) \rangle^* &\leq \lim_{n \rightarrow +\infty} \frac{1}{n\tau} \int_0^{(n+1)\tau} c_0^*(t) dt \\
&= \lim_{n \rightarrow +\infty} \frac{n+1}{n\tau} \int_{n\tau}^{(n+1)\tau} c_0^*(t) dt \\
&= \frac{ku}{h(g+m)\tau}, \\
\lim_{t \rightarrow +\infty} \langle c_0(t) \rangle_* &\geq \lim_{n \rightarrow +\infty} \frac{1}{(n+1)\tau} \int_0^{n\tau} c_0^*(t) dt \\
&= \lim_{n \rightarrow +\infty} \frac{n}{(n+1)\tau} \int_{(n-1)\tau}^{n\tau} c_0^*(t) dt \\
&= \frac{ku}{h(g+m)\tau}.
\end{aligned} \tag{10}$$

Hence one can observe that

$$\lim_{t \rightarrow +\infty} \langle c_0(t) \rangle = \frac{ku}{h(g+m)\tau} \triangleq \bar{c}_0. \tag{11}$$

□

Then we show an assumption which will be used in the following proof.

Assumption 3. There exists a bounded continuous function $c_i(t)$ such that

$$\int_{\mathbb{Y}} [\gamma_i - \ln(1 + \gamma_i)] \nu(du) \leq c_i(t), \quad (i = 1, 2). \tag{12}$$

Theorem 4. *For any given initial value $(x(0), y(0)) \in \mathbb{R}_+^2$, there is a unique solution $X(t) = (x(t), y(t))$ of (4) and the solution will remain in $\mathbb{R}_+^2$ with probability 1; that is, $X(t) \in \mathbb{R}_+^2$ for all $t \geq 0$ almost surely.*

Proof. The coefficient of (4) is locally Lipschitz continuous, and so, for any given initial value $(x(0), y(0)) \in \mathbb{R}_+^2$, there is a unique local solution $(x(t), y(t))$ for $t \in [0, \tau_e]$, where τ_e is the explosion time. To demonstrate that this solution is global, we need to show that $\tau_e = \infty$ a.s. Let $k_0 > 0$ be sufficiently large for $x_0 \in [1/k_0, k_0]$ and $y_0 \in [1/k_0, k_0]$. For each integer $k \geq k_0$, define the stopping time

$$\begin{aligned}
\tau_k &= \\
&= \inf \left\{ t \in [0, \tau_e] : x(t) \notin \left(\frac{1}{k}, k \right) \text{ or } y(t) \notin \left(\frac{1}{k}, k \right) \right\},
\end{aligned} \tag{13}$$

where we set $\inf \emptyset = \infty$ ($\emptyset$ denotes the empty set). Obviously, τ_k is increasing as $k \rightarrow +\infty$. Set $\tau_\infty = \lim_{k \rightarrow +\infty} \tau_k$; thus $\tau_\infty \leq \tau_e$ a.s., so we just need to demonstrate that $\tau_\infty = \infty$. If $\tau_\infty = \infty$ is not true, then there exist two constants $T > 0$ and $\varepsilon \in (0, 1)$ such that $P\{\tau_\infty \leq T\} > \varepsilon$. Thus there is an integer $k_1 \geq k_0$ such that $P\{\tau_k \leq T\} \geq \varepsilon$ for all $k \geq k_1$.

Define a function $V : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$ as follows:

$$V(x, y) = x - 1 - \ln x + y - 1 - \ln y. \tag{14}$$

Let $T > 0$ be arbitrary. Applying Itô's formula and Assumption 3 leads to

$$\begin{aligned} dV(x, y) = & LVdt - (x-1)\sigma_1(t)dB_1(t) - (y-1) \\ & \cdot \sigma_2(t)dB_2(t) \\ & + \int_{\mathbb{Y}} [x(t^-)\gamma_1(t, u) - \ln(1 + \gamma_1(t, u))] \tilde{N}(dt, du) \\ & + \int_{\mathbb{Y}} [y(t^-)\gamma_2(t, u) - \ln(1 + \gamma_2(t, u))] \tilde{N}(dt, du), \end{aligned} \quad (15)$$

where

$$\begin{aligned} LV = & (x-1)(r_1(t) - d_1c_0(t) - a_1(t)x - \beta(t)y) \\ & + \frac{1}{2}\sigma_1^2(t) + (y-1)(r_2(t) - d_2c_0(t) - a_1(t)y) \end{aligned}$$

$$\begin{aligned} & + \beta(t)x + \frac{1}{2}\sigma_2^2(t) \\ & + \int_{\mathbb{Y}} [\gamma_1(t, u) - \ln(1 + \gamma_1(t, u))] \nu(du) \\ & + \int_{\mathbb{Y}} [\gamma_2(t, u) - \ln(1 + \gamma_2(t, u))] \nu(du) \leq -a_1(t) \\ & \cdot x^2 + (a_1(t) + r_1(t) - d_1c_0(t) - \beta(t))x - r_1(t) \\ & + d_1c_0(t) + \frac{1}{2}\sigma_1^2(t) + c_1(t) + [-a_2(t)y^2 \\ & + (a_2(t) + r_2(t) - d_2c_0(t) + \beta(t))y - r_2(t) \\ & + d_2c_0(t) + \frac{1}{2}\sigma_2^2(t) + c_2(t)] \leq K_1, \end{aligned} \quad (16)$$

where

$$K_1 = \max_{0 \leq t < +\infty} \sum_{i=1}^2 \frac{4a_i(t) [-r_i(t) + d_i c_0(t) + (1/2)\sigma_i^2(t) + c_i(t)] + [a_i(t) + r_i(t) - d_i c_0(t) + \beta(t)]^2}{4a_i(t)}. \quad (17)$$

So

$$\begin{aligned} dV(x, y) \leq & K_1 dt - (x-1)\sigma_1(t)dB_1(t) - (y-1) \\ & \cdot \sigma_2(t)dB_2(t) \\ & + \int_{\mathbb{Y}} [x(t^-)\gamma_1(t, u) - \ln(1 + \gamma_1(t, u))] \tilde{N}(dt, du) \\ & + \int_{\mathbb{Y}} [y(t^-)\gamma_2(t, u) - \ln(1 + \gamma_2(t, u))] \tilde{N}(dt, du). \end{aligned} \quad (18)$$

Integrating (18) from 0 to $\tau_k \wedge T$ and taking the expectations for both sides result in

$$\mathbb{E}[V(x(\tau_k \wedge T), y(\tau_k \wedge T))] \leq V(x_0, y_0) + K_1 T. \quad (19)$$

Let $\Omega_k = \{\tau_k \leq T\}$ for $k \geq k_1$; we have $P(\Omega_k) \geq \varepsilon$. Then

$$\begin{aligned} & V(x(\tau_k \wedge T, \omega), y(\tau_k \wedge T, \omega)) \\ & \geq [k-1 - \ln k] \wedge \left[\frac{1}{k} - 1 + \ln k \right]. \end{aligned} \quad (20)$$

It is inferred from (19) and (20) that

$$\begin{aligned} V(x_0, y_0) + K_1 T & \geq \mathbb{E}[1_{\Omega} V(x(\tau_k \wedge T), y(\tau_k \wedge T))] \\ & \geq \varepsilon [k-1 - \ln k] \wedge \left[\frac{1}{k} - 1 + \ln k \right], \end{aligned} \quad (21)$$

where 1_{Ω} is the indicator function of Ω_k . Let $k \rightarrow +\infty$; we have that

$$+\infty > V(x_0, y_0) + K_1 T \geq +\infty \quad (22)$$

is a contradiction; then we have $\tau_{\infty} = \infty$.

This completes the proof of Theorem 4. $\square$

3. Main Results

3.1. Extinction. For convenience, we prepare the following lemma.

Lemma 5. Suppose that $\eta(t) \in C(\Omega \times [0, \infty), \mathbb{R}_+)$ and let Assumption 3 hold.

(I) If there exist two positive constants T and δ_0 such that

$$\begin{aligned} \ln \eta(t) \leq & \int_0^t \delta(s) ds - \delta_0 \int_0^t \eta(s) ds + \alpha B(t) \\ & + \sum_{i=1}^2 \delta_i \int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_i(u)) \tilde{N}(ds, du) \end{aligned} \quad (23)$$

a.s.

for all $t \geq T$, where α , δ_1 , and δ_2 are constants, then

$$\langle \eta \rangle^* \leq \frac{\langle \delta \rangle^*}{\delta_0} \quad \text{a.s., if } \langle \delta \rangle^* \geq 0; \quad (24)$$

$$\lim_{t \rightarrow \infty} \eta(t) = 0 \quad \text{a.s., if } \langle \delta \rangle^* < 0.$$

(II) If there exist two positive constants T and δ_0 such that

$$\begin{aligned} \ln \eta(t) \geq & \int_0^t \delta(s) ds - \delta_0 \int_0^t \eta(s) ds + \alpha B(t) \\ & + \sum_{i=1}^2 \delta_i \int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_i(u)) \tilde{N}(ds, du) \end{aligned} \quad (25)$$

a.s.

for all $t \geq T$, then $\langle \eta \rangle_* \geq \langle \delta \rangle_* / \delta_0$ a.s. provided that $\langle \delta \rangle_* \geq 0$.

Lemma 5 is proved in Appendix by using a similar method in [13].

Now, we will prove the extinction of system (4).

Define

$$\begin{aligned} b_1(t) &= r_1(t) - \frac{1}{2}\sigma_1^2(t) \\ &\quad + \int_{\mathbb{Y}} (\ln(1 + \gamma_1(t, u)) - \gamma_1(t, u)) \nu(du), \\ b_2(t) &= r_2(t) - \frac{1}{2}\sigma_2^2(t) \\ &\quad + \int_{\mathbb{Y}} (\ln(1 + \gamma_2(t, u)) - \gamma_2(t, u)) \nu(du), \\ \bar{c}_0 &= \frac{ku}{h(g+m)\tau}. \end{aligned} \quad (26)$$

Theorem 6. *If*

$$\langle b_1 \rangle_* < d_1 \bar{c}_0, \quad (27)$$

$$\langle b_2 \rangle_* < d_2 \bar{c}_0, \quad (28)$$

then

$$\begin{aligned} \lim_{t \rightarrow \infty} x(t) &= 0, \\ \lim_{t \rightarrow \infty} y(t) &= 0 \end{aligned} \quad (29)$$

a.s.

Proof. By Assumption 3 and the strong law of large numbers for local martingales, one has

$$\begin{aligned} \lim_{t \rightarrow +\infty} \frac{1}{t} \int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_i(s, u)) \tilde{N}(ds, du) &= 0, \\ \lim_{t \rightarrow +\infty} \frac{\int_0^t \sigma_i(s) dB_i(s)}{t} &= 0 \end{aligned} \quad (30)$$

a.s., $i = 1, 2$.

Define

$$\begin{aligned} V_1(x) &= \ln x(t), \\ V_2(x) &= \ln y(t). \end{aligned} \quad (31)$$

Applying Itô's formula yields

$$\begin{aligned} d \ln x(t) &= (b_1(t) - d_1 c_0(t) - a_1(t) x(t) - \beta(t) y(t)) dt \\ &\quad - \sigma_1(t) dB_1(t) \\ &\quad + \int_{\mathbb{Y}} \ln(1 + \gamma_1(t, u)) \tilde{N}(dt, du), \end{aligned} \quad (32)$$

$$\begin{aligned} d \ln y(t) &= (b_2(t) - d_2 c_0(t) - a_2(t) y(t) + \beta(t) x(t)) dt \\ &\quad - \sigma_2(t) dB_2(t) \\ &\quad + \int_{\mathbb{Y}} \ln(1 + \gamma_2(t, u)) \tilde{N}(dt, du). \end{aligned} \quad (33)$$

Integrating both sides of (32) and (33) from 0 to t , respectively, we have

$$\begin{aligned} \ln x(t) - \ln x(0) &= \int_0^t (b_1(s) - d_1 c_0(s) - a_1(s) x(s) - \beta(s) y(s)) ds \\ &\quad - \int_0^t \sigma_1(s) dB_1(s) \\ &\quad + \int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_1(t, u)) \tilde{N}(dt, du), \end{aligned} \quad (34)$$

$$\begin{aligned} \ln y(t) - \ln y(0) &= \int_0^t (b_2(s) - d_2 c_0(s) - a_2(s) y(s) + \beta(s) x(s)) ds \\ &\quad - \int_0^t \sigma_2(s) dB_2(s) \\ &\quad + \int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_2(t, u)) \tilde{N}(dt, du). \end{aligned} \quad (35)$$

From (34), we can obtain that

$$\begin{aligned} \frac{\ln x(t) - \ln x(0)}{t} &\leq \frac{\int_0^t b_1(s) ds}{t} - d_1 \frac{\int_0^t c_0(s) ds}{t} - \frac{\int_0^t \sigma_1(s) dB_1(s)}{t} \\ &\quad + \frac{\int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_1(t, u)) \tilde{N}(dt, du)}{t}. \end{aligned} \quad (36)$$

Taking the limit superior results in

$$\left[\frac{\ln x(t)}{t} \right]^* \leq \limsup_{t \rightarrow \infty} \left[\frac{\int_0^t b_1(s) ds}{t} - d_1 \frac{\int_0^t c_0(s) ds}{t} \right] \quad (37)$$

a.s.

From (27) we can know that $[\ln x(t)/t]^* < 0$. Thus we have

$$\lim_{t \rightarrow \infty} x(t) = 0 \quad \text{a.s.} \quad (38)$$

Applying (35) leads to

$$\begin{aligned} & \frac{\ln y(t) - \ln y(0)}{t} \\ & \leq \frac{\int_0^t b_2(s) ds}{t} + \frac{\int_0^t \beta(s) x(s) ds}{t} - d_2 \frac{\int_0^t c_0(s) ds}{t} \\ & \quad - \frac{\int_0^t \sigma_2(s) dB_2(s)}{t} \\ & \quad + \frac{\int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_2(t, u)) \tilde{N}(dt, du)}{t}. \end{aligned} \quad (39)$$

Taking the limit superior, together with (28) and (38), we can see that

$$\left[\frac{\ln y(t)}{t} \right]^* < 0 \quad \text{a.s.} \quad (40)$$

Therefore,

$$\lim_{t \rightarrow \infty} y(t) = 0 \quad \text{a.s.} \quad (41)$$

This completes the proof of Theorem 6. $\square$

For simplicity, we define

$$R^* = \frac{\langle b_2 \rangle^* + \beta^M / a_1^L \cdot \langle b_1 \rangle^*}{(d_2 + d_1 (\beta^M / a_1^L)) \bar{c}_0}. \quad (42)$$

Theorem 7. If $\langle b_1 \rangle_* > d_1 \bar{c}_0$ and $R^* < 1$, then

$$\begin{aligned} & \frac{\langle b_1 \rangle_* - d_1 \bar{c}_0}{a_1^M} \leq \langle x(t) \rangle_* \leq \langle x(t) \rangle^* \leq \frac{\langle b_1 \rangle^* - d_1 \bar{c}_0}{a_1^L}, \\ & \lim_{t \rightarrow \infty} y(t) = 0 \end{aligned} \quad (43)$$

a.s.

Proof. According to (34), we can obtain that

$$\begin{aligned} & \ln x(t) - \ln x(0) \\ & \leq \int_0^t b_1(s) ds - d_1 \int_0^t c_0(s) ds - \int_0^t a_1(s) x(s) ds \\ & \quad - \int_0^t \sigma_1(s) dB_1(s) \\ & \quad + \int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_1(t, u)) \tilde{N}(dt, du) \\ & \leq \int_0^t b_1(s) ds - d_1 \int_0^t c_0(s) ds - a_1^L \int_0^t x(s) ds \\ & \quad - \int_0^t \sigma_1(s) dB_1(s) \\ & \quad + \int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_1(t, u)) \tilde{N}(dt, du). \end{aligned} \quad (44)$$

It follows from Lemma 5 that

$$\langle x(t) \rangle^* \leq \frac{\langle b_1 \rangle^* - d_1 \bar{c}_0}{a_1^L} \quad \text{a.s.} \quad (45)$$

By (35), we have

$$\begin{aligned} & \ln y(t) - \ln y(0) \leq \int_0^t b_2(s) ds + \int_0^t \beta(s) x(s) ds \\ & \quad - d_2 \int_0^t c_0(s) ds - \int_0^t a_2(s) y(s) ds \\ & \quad - \int_0^t \sigma_2(s) dB_2(s) \\ & \quad + \int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_2(t, u)) \tilde{N}(dt, du) \\ & \leq \left(\frac{\int_0^t b_2(s) ds}{t} + \beta^M \frac{\langle b_1 \rangle^* - d_1 \bar{c}_0}{a_1^L} \right. \\ & \quad \left. - d_2 \frac{\int_0^t c_0(s) ds}{t} \right) t - \int_0^t \sigma_2(s) dB_2(s) \\ & \quad + \int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_2(t, u)) \tilde{N}(dt, du), \end{aligned} \quad (46)$$

and then

$$\begin{aligned} & y(t) \leq y(0) \exp \left\{ \left(\frac{\int_0^t b_2(t) dt}{t} + \beta^M \frac{\langle b_1 \rangle^* - d_1 \bar{c}_0}{a_1^L} \right. \right. \\ & \quad \left. \left. - d_2 \frac{\int_0^t c_0(s) ds}{t} \right) t - \int_0^t \sigma_2(t) dB_2(t) \right. \\ & \quad \left. + \int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_2(t, u)) \tilde{N}(dt, du) \right\}. \end{aligned} \quad (47)$$

Since $R^* < 1$, then we know that

$$\lim_{t \rightarrow \infty} y(t) = 0 \quad \text{a.s.}, \quad (48)$$

which combined with (34) and the property of the limit superior shows that, for any $\varepsilon > 0$, there exists a random number $T > 0$ for $t > T$ such that

$$\begin{aligned} & \ln \frac{x(t)}{x(0)} \geq \int_0^t (b_1(s) - d_1 c_0(s)) ds - \beta^M \varepsilon \\ & \quad - a_1^M \int_0^t x(t) dt - \int_0^t \sigma_1(t) dB_1(t) \\ & \quad + \int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_1(t, u)) \tilde{N}(dt, du). \end{aligned} \quad (49)$$

By Lemma 5, we have that

$$\langle x(t) \rangle_* \geq \frac{\langle b_1 \rangle_* - d_1 \bar{c}_0}{a_1^M} \quad \text{a.s.} \quad (50)$$

This completes the proof of Theorem 7. $\square$

3.2. Permanence in Mean. In this section, we need to show the permanence in mean. Note that $\langle b_1 \rangle_* < d_1 \bar{c}_0$ implies that productiveness of the prey is less than its death loss rate; then the prey can go extinct. Naturally, we assume $\langle b_1 \rangle_* > d_1 \bar{c}_0$ in the rest of this paper.

Define

$$\begin{aligned}\Delta_1 &= \beta^L \langle b_1 \rangle_* + a_1^M \langle b_2 \rangle_* - (\beta^L d_1 + a_1^M d_2) \bar{c}_0, \\ \Delta_2 &= \beta^L \beta^M + a_1^M a_2^M, \\ \Delta_3 &= \langle b_1 \rangle_* - d_1 \bar{c}_0 - \beta^L \frac{\Delta_1}{\Delta_2}, \\ \Delta_4 &= \langle b_1 \rangle_* - d_1 \bar{c}_0 - \beta^M \frac{\Delta_5}{a_2^L}, \\ \Delta_5 &= \langle b_2 \rangle_* - d_2 \bar{c}_0 + \beta^M \frac{\Delta_3}{a_1^L}, \\ R_* &= \frac{\langle b_2 \rangle_* + \beta^L / a_1^M \cdot \langle b_1 \rangle_*}{(d_2 + d_1 (\beta^L / a_1^M)) \bar{c}_0}.\end{aligned}\quad (51)$$

Theorem 8. *If*

$$\begin{aligned}\langle b_2 \rangle_* &> d_2 \bar{c}_0, \\ R_* &> 1, \\ \Delta_3 &> 0, \\ \Delta_4 &> 0,\end{aligned}\quad (52)$$

then

$$\begin{aligned}\frac{\Delta_4}{a_1^M} &\leq \langle x(t) \rangle_* \leq \langle x(t) \rangle^* \leq \frac{\Delta_3}{a_1^L}, \\ \frac{\Delta_1}{\Delta_2} &\leq \langle y(t) \rangle_* \leq \langle y(t) \rangle^* \leq \frac{\Delta_5}{a_2^L}.\end{aligned}\quad (53)$$

Proof. Define $u(t)$ such that $x(t) \leq u(t)$, where $u(t)$ satisfies

$$\begin{aligned}du(t) &= u(t) (r_1(t) - a_1(t) u(t)) dt \\ &\quad - \sigma_1(t) u(t) dB_1(t) \\ &\quad + \int_{\mathbb{Y}} u(t^-) \gamma_1(t, u) \tilde{N}(dt, du).\end{aligned}\quad (54)$$

In [9], we know that

$$\lim_{t \rightarrow \infty} \frac{\ln u(t)}{t} = 0, \quad \text{a.s.} \quad (55)$$

It can be inferred from comparison theorem that

$$\left(\frac{\ln x(t)}{t} \right)^* \leq \lim_{t \rightarrow \infty} \frac{\ln u(t)}{t} = 0. \quad (56)$$

From (34) and (35), we know that

$$\begin{aligned}\frac{\ln x(t) - \ln x(0)}{t} &= \frac{\int_0^t b_1(s) ds}{t} - \frac{\int_0^t d_1 c_0(s) ds}{t} - \frac{\int_0^t a_1(s) x(s) ds}{t} \\ &\quad - \frac{\int_0^t \beta(s) y(s) ds}{t} - \frac{\int_0^t \sigma_1(s) dB_1(s)}{t} \\ &\quad + \frac{\int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_1(t, u)) \tilde{N}(dt, du)}{t},\end{aligned}\quad (57)$$

$$\begin{aligned}\frac{\ln y(t) - \ln y(0)}{t} &= \frac{\int_0^t b_2(s) ds}{t} - \frac{\int_0^t d_2 c_0(s) ds}{t} - \frac{\int_0^t a_2(s) y(s) ds}{t} \\ &\quad + \frac{\int_0^t \beta(s) x(s) ds}{t} - \frac{\int_0^t \sigma_2(s) dB_2(s)}{t} \\ &\quad + \frac{\int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_2(t, u)) \tilde{N}(dt, du)}{t}.\end{aligned}\quad (58)$$

By $\beta_L \times (57) + a_1^M \times (58)$, we obtain that

$$\begin{aligned}\beta^L \times \frac{\ln x(t) - \ln x(0)}{t} + a_1^M \times \frac{\ln y(t) - \ln y(0)}{t} &\geq \Delta_1 - \Delta_2 \langle y(t) \rangle - \beta_L \frac{\int_0^t \sigma_1(s) dB_1(s)}{t} \\ &\quad + \beta_L \frac{\int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_1(t, u)) \tilde{N}(dt, du)}{t} \\ &\quad - a_1^M \frac{\int_0^t \sigma_2(s) dB_2(s)}{t} \\ &\quad + a_1^M \frac{\int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_2(t, u)) \tilde{N}(dt, du)}{t}.\end{aligned}\quad (59)$$

When $R_* > 1$ is used in (59), applying Lemma 5 and (56) leads to

$$\langle y(t) \rangle_* \geq \frac{\Delta_1}{\Delta_2}. \quad (60)$$

According to (57) and (60), we can have

$$\begin{aligned}\frac{\ln x(t) - \ln x(0)}{t} &\leq \langle b_1 \rangle^* - d_1 \bar{c}_0 - \beta^L \frac{\Delta_1}{\Delta_2} - a_1^L \langle x(t) \rangle \\ &\quad - \frac{\int_0^t \sigma_1(s) dB_1(s)}{t} \\ &\quad + \frac{\int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_1(t, u)) \tilde{N}(dt, du)}{t}.\end{aligned}\quad (61)$$

From the conditions of Theorem 8, we know that $\Delta_3 > 0$ and applying Lemma 5 results in

$$\langle x(t) \rangle^* \leq \frac{\Delta_3}{a_1^L}. \quad (62)$$

By (58) and (62), we obtain that

$$\begin{aligned} & \frac{\ln y(t) - \ln y(0)}{t} \\ & \leq \langle b_2 \rangle^* - d_2 \bar{c}_0 + \beta^M \frac{\Delta_3}{a_1^L} - a_2^L \langle y(t) \rangle \\ & \quad - \frac{\int_0^t \sigma_1(s) dB_1(s)}{t} \\ & \quad + \frac{\int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_1(t, u)) \tilde{N}(dt, du)}{t}. \end{aligned} \quad (63)$$

From the conditions of Theorem 8, we see that $\Delta_5 > 0$ and applying Lemma 5 leads to

$$\langle y(t) \rangle^* \leq \frac{\Delta_5}{a_2^L}. \quad (64)$$

By (58) and (64), we can have

$$\begin{aligned} & \frac{\ln x(t) - \ln x(0)}{t} \\ & \geq \langle b_1 \rangle_* - d_1 \bar{c}_0 - \beta^M \frac{\Delta_5}{a_2^L} - a_1^M \langle x(t) \rangle \\ & \quad - \frac{\int_0^t \sigma_1(s) dB_1(s)}{t} \\ & \quad + \frac{\int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_1(t, u)) \tilde{N}(dt, du)}{t}. \end{aligned} \quad (65)$$

According to the conditions of Theorem 8, we see that $\Delta_4 > 0$ and applying Lemma 5 yields

$$\langle x(t) \rangle_* \geq \frac{\Delta_4}{a_1^M}. \quad (66)$$

This completes the proof of Theorem 8. $\square$

3.3. Stochastic Permanence. For the sake of proving stochastic permanence, we should examine the p th moment boundedness firstly.

Define

$$\begin{aligned} \lambda_1 = & \max_{t \in [0, +\infty)} \left\{ pr_1(t) - pd_1 c_0(t) + \frac{p(p-1)}{2} \sigma_1^2(t) \right. \\ & \left. + c_1(t) \right\}, \end{aligned}$$

$$\begin{aligned} \lambda_1' = & \max_{t \in [0, +\infty)} \left\{ pr_2(t) - pd_2 c_0(t) + \frac{p(p-1)}{2} \sigma_2^2(t) \right. \\ & \left. + c_2(t) \right\}, \end{aligned}$$

$$\lambda_1'' = \max \{ \lambda_1, \lambda_1' \},$$

$$\lambda_2 = \min_{t \in [0, +\infty)} \left\{ a_1(t) p - \frac{p\beta(t)}{p+1} \right\},$$

$$\lambda_2' = \min_{t \in [0, +\infty)} \left\{ a_2(t) p - \frac{p^2\beta(t)}{p+1} \right\},$$

$$\lambda_2'' = \min \{ \lambda_2, \lambda_2' \}.$$

(67)

Theorem 9. *If there exists a constant p such that*

$$\int_{\mathbb{Y}} \left((1 + \gamma_i(t, u))^p - 1 - p\gamma_i(t, u) \right) \nu(du) < \infty, \quad (68)$$

$$\lambda_1'' > 0,$$

$$\lambda_2'' > 0,$$

(69)

then

$$\limsup_{t \rightarrow \infty} \mathbb{E} \left[\left(x^2(t) + y^2(t) \right)^{p/2} \right] \leq K(p) \quad \text{a.s.} \quad (70)$$

Proof. Define

$$V(x, y) = x^p(t) + y^p(t). \quad (71)$$

Applying Itô's formula and (68) yields

$$\begin{aligned} dV(x, y) & = LVdt - p\sigma_1(t) x^p(t) dB_1(t) \\ & \quad - p\sigma_2(t) y^p(t) dB_2(t) \\ & \quad + \int_{\mathbb{Y}} x^p(t^-) \left((1 + \gamma_1(t, u))^p - 1 \right) \tilde{N}(dt, du) \\ & \quad + \int_{\mathbb{Y}} y^p(t^-) \left((1 + \gamma_2(t, u))^p - 1 \right) \tilde{N}(dt, du), \end{aligned} \quad (72)$$

where

$$\begin{aligned} LV = & px^p(t) (r_1(t) - d_1 c_0(t) - a_1(t) x(t) - \beta(t) \\ & \cdot y(t)) + py^p(t) (r_2(t) - d_2 c_0(t) - a_2(t) x(t) \\ & + \beta(t) x(t)) + \frac{p(p-1)}{2} (\sigma_1^2(t) x^p(t) + \sigma_2^2(t) \\ & \cdot y^p(t)) + \int_{\mathbb{Y}} x^p(t^-) \\ & \cdot \left((1 + \gamma_1(t, u))^p - 1 - p\gamma_1(t, u) \right) \nu(du) \end{aligned}$$

$$\begin{aligned}
& + \int_{\mathbb{V}} y^p(t^-) \left((1 + \gamma_1(t, u))^p - 1 - p\gamma_1(t, u) \right) \\
& \cdot \nu(du) \leq \left(pr_1(t) - pd_1c_0(t) + \frac{p(p-1)}{2}\sigma_1^2(t) \right. \\
& \left. + c_1(t) \right) x^p(t) + \left(pr_2(t) - pd_2c_0(t) + \frac{p(p-1)}{2} \right. \\
& \cdot \sigma_2^2(t) + c_2(t) \left. \right) y^p(t) - a_1(t) px^{p+1}(t) - \beta(t) \\
& \cdot px^p(t)y(t) - a_2(t) py^{p+1}(t) + p\beta(t)x(t)y^p(t) \\
& \leq \left(pr_1(t) - pd_1c_0(t) + \frac{p(p-1)}{2}\sigma_1^2(t) + c_1(t) \right) \\
& \cdot x^p(t) - \left(a_1(t)p - \frac{p\beta(t)}{p+1} \right) x^{p+1}(t) + \left(pr_2(t) \right. \\
& \left. - pd_2c_0(t) + \frac{p(p-1)}{2}\sigma_2^2(t) + c_2(t) \right) y^p(t) \\
& - \left(a_2(t)p - \frac{p\beta(t)}{p+1} \right) y^{p+1}(t), \tag{73}
\end{aligned}$$

where $c_i(t)$ ($i = 1, 2$) is a bounded continuous function.

Integrating (72) from 0 to t and taking the expectations for both sides yield

$$\mathbb{E}[V(x, y)] = V(x(0), y(0)) + \mathbb{E}\left[\int_0^t LV dt\right], \tag{74}$$

and then

$$\begin{aligned}
& \frac{d\mathbb{E}[V(x, y)]}{dt} \\
& \leq \lambda_1\mathbb{E}[x^p] - \lambda_2\mathbb{E}[x^{p+1}] + \lambda'_1\mathbb{E}[y^p] \\
& \quad - \lambda'_2\mathbb{E}[y^{p+1}] \\
& \leq \lambda''_1\mathbb{E}[x^p + y^p] - \lambda''_2\mathbb{E}[x^{p+1} + y^{p+1}] \\
& \leq \lambda''_1\mathbb{E}[x^p + y^p] - \lambda''_2 \cdot 2^{-1/p} (\mathbb{E}[x^p + y^p])^{(p+1)/p} \\
& = \mathbb{E}[x^p + y^p] \left(\lambda''_1 - \lambda''_2 \cdot 2^{-1/p} (\mathbb{E}[x^p + y^p])^{1/p} \right). \tag{75}
\end{aligned}$$

We know that the auxiliary equation

$$\dot{z}(t) = z(t) [\lambda_1 - \lambda_2 (z(t))^{1/p}] \tag{76}$$

has a globally asymptotically stable positive equilibrium $\bar{z} = (\lambda_1/\lambda_2)^p$. Let $z(t)$ be the solution of (76) with $z(0) = \mathbb{E}[(x(0))^p + (y(0))^p]$, and according to the comparison theorem we have $\mathbb{E}[x^p + y^p] \leq z(t)$, $t \geq 0$. Hence, we obtain

$$\limsup_{t \rightarrow +\infty} \mathbb{E}[V(x, y)] \leq 2^{1/p} \cdot \frac{\lambda''_1}{\lambda''_2} := K_1(p) \quad \text{a.s.} \tag{77}$$

We find that

$$(x^2 + y^2)^{p/2} \leq 2^{p/2} \cdot \max\{x^p, y^p\} \leq 2^{p/2} \cdot V(x, y), \tag{78}$$

so we have

$$\limsup_{t \rightarrow +\infty} \mathbb{E}[(x^2 + y^2)^{p/2}] \leq \sqrt{2} \cdot \frac{\lambda''_1}{\lambda''_2} := K(p) \quad \text{a.s.} \tag{79}$$

This completes the proof of Theorem 9. $\square$

Define

$$\begin{aligned}
r(t) &= \min\{r_1(t) - d_1c_0(t), r_2(t) - d_2c_0(t)\}, \\
\sigma(t) &= \max\{\sigma_1(t), \sigma_2(t)\}, \tag{80}
\end{aligned}$$

$$H(t) = r(t) - \sigma^2(t) - \sum_{i=1}^2 \int_{\mathbb{V}} \frac{\gamma_i^2}{1 + \gamma_i} \nu(du).$$

Theorem 10. *If*

$$\langle H(t) \rangle_* > 0, \tag{81}$$

then system (4) is stochastically permanent.

Proof. Define

$$V(x, y) = \frac{1}{x(t) + y(t)}. \tag{82}$$

By applying Itô's formula, we have

$$\begin{aligned}
d(V(x, y)) &= LVdt + \frac{1}{(x+y)^2} (\sigma_1(t)x(t)dB_1(t) \\
&+ \sigma_2(t)y(t)dB_2(t)) \\
&+ \int_{\mathbb{V}} \left(\frac{1}{x(t^-) + y(t^-) + x(t^-)\gamma_1(t, u) + y(t^-)\gamma_2(t, u)} \right. \\
&\left. - \frac{1}{x(t^-) + y(t^-)} \right) \tilde{N}(dt, du), \tag{83}
\end{aligned}$$

where

$$\begin{aligned}
LV &= -\frac{1}{(x+y)^2} (x(t)(r_1(t) - d_1c_0(t)) - a_1(t)x^2(t) \\
&+ (r_2(t) - d_2c_0(t))y(t) - a_2(t)y^2(t)) \\
&+ \frac{\sigma_1^2(t)x^2(t) + \sigma_2^2(t)y^2(t) + 2\sigma_1(t)\sigma_2(t)x(t)y(t)}{(x(t) + y(t))^3} \\
&+ \int_{\mathbb{V}} \left(\frac{1}{x+y + x\gamma_1(t, u) + y\gamma_2(t, u)} - \frac{1}{x+y} \right. \\
&+ \frac{1}{(x+y)^2} (x\gamma_1(t, u) + y\gamma_2(t, u)) \left. \right) \nu(du) \\
&\leq -\frac{1}{x(t) + y(t)} \left(r(t) - \sigma^2(t) \right. \\
&- \sum_{i=1}^2 \int_{\mathbb{V}} \frac{\gamma_i^2(t, u)}{1 + \gamma_i(t, u)} \nu(du) \left. \right) + a(t) = -V \cdot H(t) \\
&+ a(t), \tag{84}
\end{aligned}$$

where $a(t) = \max\{a_1(t), a_2(t)\}$.

Integrating (83) from 0 to t and taking the expectation on both sides yield

$$\mathbb{E}[V(t)] = V(0) + \mathbb{E}\left[\int_0^t LV(s) ds\right]. \quad (85)$$

Therefore,

$$\begin{aligned} d\mathbb{E}[V(t)] &= \mathbb{E}[LV(t)] dt \\ &\leq (-H(t)\mathbb{E}[V(t)] + a(t)) dt. \end{aligned} \quad (86)$$

By the variation of constants method, we can derive that

$$\begin{aligned} \mathbb{E}[V(t)] &\leq \int_0^t a(s) \exp\left(-\int_s^t H(\tau) d\tau\right) ds \\ &\quad + V(0) \exp\left(-\int_0^t H(s) ds\right), \end{aligned} \quad (87)$$

which together with (81) results in

$$\mathbb{E}[V(x, y)] \leq K, \quad (88)$$

where K is a positive constant. Then, for any given $\varepsilon > 0$ and constant $\delta(\varepsilon) = \varepsilon/K$, according to the Chebyshev inequality, we can know that

$$\begin{aligned} P\left\{\frac{1}{V(x, y)} < \delta\right\} &= P\left\{V(x, y) > \frac{1}{\delta}\right\} \\ &\leq \frac{\mathbb{E}[V(x, y)]}{1/\delta} = \delta \mathbb{E}[V(x, y)] \\ &\leq \varepsilon, \end{aligned} \quad (89)$$

and then

$$P\left\{\frac{1}{V(x, y)} \geq \delta\right\} \geq 1 - \varepsilon. \quad (90)$$

Therefore,

$$\liminf_{t \rightarrow \infty} P\{(x(t) + y(t)) \geq \delta\} \geq 1 - \varepsilon \quad \text{a.s.} \quad (91)$$

Then, we have

$$\liminf_{t \rightarrow \infty} P\left\{\left(\sqrt{x^2(t) + y^2(t)}\right) \geq \frac{\sqrt{2}}{2}\delta\right\} \geq 1 - \varepsilon \quad \text{a.s.} \quad (92)$$

Using Chebyshev inequality and Theorem 9, we can obtain

$$\limsup_{t \rightarrow \infty} P\left\{\left(\sqrt{x^2(t) + y^2(t)}\right) \leq \chi\right\} \geq 1 - \varepsilon \quad \text{a.s.}, \quad (93)$$

where χ is a constant. This completes the proof of Theorem 10. $\square$

4. Simulations and Conclusions

In this section, we show some numerical examples, which will demonstrate our results.

Choose the parameters in (4) as follows:

$$\begin{aligned} r_1(t) &= 1.8, \\ r_2(t) &= 1.3, \\ d_1 &= 0.35, \\ d_2 &= 0.4, \\ a_1(t) &= 0.3, \\ a_2(t) &= 0.4, \\ \beta(t) &= 0.1, \\ k &= 0.5, \\ g &= 0.2, \\ h &= 0.2, \\ m &= 0.3, \\ u &= 1, \\ \tau &= 1. \end{aligned} \quad (94)$$

In Figure 1, we choose $\sigma_1(t) = 0.3$, $\sigma_2(t) = 0.4$ and the values of $\gamma_1(t, u)$ and $\gamma_2(t, u)$ are different between (a) and (b).

In Figure 1(a), we choose $\gamma_1(t, u) = 0.1$, $\gamma_2(t, u) = 0.3$; then the conditions in Theorem 7 are satisfied. According to Theorem 7, we know that $x(t)$ is permanent and $y(t)$ is extinct.

In Figure 1(b), we choose $\gamma_1(t, u) = 0.3$, $\gamma_2(t, u) = 0.3$; by Theorem 6, we can obtain that both of $x(t)$ and $y(t)$ are extinct.

Then, we choose $\gamma_1(t, u) = 0.1$, $\gamma_2(t, u) = 0.1$, the values of $\sigma_1(t)$ and $\sigma_2(t)$ are different between Figures 1(c) and 1(d).

In Figure 1(c), we choose $\sigma_1(t) = 0.3$, $\sigma_2(t) = 0.9$; then the conditions in Theorem 7 are satisfied. According to Theorem 7, we know that $x(t)$ is permanent and $y(t)$ is extinct.

In Figure 1(d), we choose $\sigma_1(t) = 0.8$, $\sigma_2(t) = 0.9$; by Theorem 6, we get that both of $x(t)$ and $y(t)$ are extinct.

In Figure 2, we choose $\gamma_1 = 2.8$, $\gamma_2 = 2.3$, $\sigma_1(t) = 0.3$, $\sigma_2(t) = 0.4$, $\gamma_1(t, u) = 0.1$, and $\gamma_2(t, u) = 0.1$; other parameters are the same as in Figure 1; according to Theorems 8 and 10, we obtain that system (4) is permanent in mean and stochastically permanent.

In conclusion, permanence and extinction of the predator and the prey for the intensity of white noise and Lévy noise are given in Table 1.

In this paper, we consider a nonautonomous impulsive stochastic predator-prey system with Lévy jumps, which considers the predator is omnivorous. We prove that the system has a unique global positive solution. From Theorem 6 and Figure 1, we can know that if the intensities of white noise and Lévy noise are sufficiently large, then the species will be extinct (see Figures 1(b) and 1(d)). According to Theorems 8 and 10, we know that system (4) is permanent in mean and stochastically permanent under some conditions in Figure 2. The change of permanent conditions shows an

TABLE 1: Permanence and extinction for different noise parameters.

White noise σ_1	White noise σ_2	Lévy noise γ_1	Lévy noise γ_2	Permanence and extinction
0.3	0.4	0.1	0.3	Permanence for x and extinction for y
0.3	0.4	0.3	0.3	Extinction for x and y
0.3	0.9	0.1	0.1	Permanence for x and extinction for y
0.8	0.9	0.1	0.1	Extinction for x and y
0.3	0.4	0.1	0.1	Permanence for x and y

FIGURE 1: The red line and the black line represent $x(t)$ and $y(t)$, respectively. The initial value is $x(0) = 5$, $y(0) = 10$, $c_0(0) = 0.5$, and $c_e(0) = 1$. In (a), $x(t)$ is permanent and $y(t)$ is extinct; in (b), both of $x(t)$ and $y(t)$ are extinct. In (c), $x(t)$ is permanent and $y(t)$ is extinct; in (d), both of $x(t)$ and $y(t)$ are extinct.

FIGURE 2: The red line and the black line represent $x(t)$ and $y(t)$, respectively. The initial value is $x(0) = 5$, $y(0) = 10$, $c_0(0) = 0.5$, and $c_e(0) = 1$. Both $x(t)$ and $y(t)$ are stochastically permanent when $\sigma_1(t) = 0.3$, $\sigma_2(t) = 0.4$, $\gamma_1(t, u) = 0.1$, and $\gamma_2(t, u) = 0.1$.

important property; that is, the permanence of species has a close relationship with the intensity of white noise and Lévy noise. Obviously, white noises and Lévy noises are harmful to the permanence of populations.

There are some investigations for stochastic epidemic models [14, 15]. It is very significant for the natural world. Some interesting questions deserve further investigation. One could study more realistic but more complex models. Also it is interesting to investigate evolutionary dynamics of stochastic evolutionary model, and we leave these for future work.

Appendix

Proof of Lemma 5. (I) If $\langle \delta \rangle^* \geq 0$, $\int_0^t \delta(s) ds > 0$ for t large enough. By Assumption 3 and the strong law of large numbers for local martingales, one has

$$\lim_{t \rightarrow +\infty} \frac{1}{t} \sum_{i=1}^2 \delta_i \int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_i(u)) \tilde{N}(ds, du) = 0, \quad (A.1)$$

$$\lim_{t \rightarrow +\infty} \frac{B(t)}{t} = 0 \quad \text{a.s.}$$

Then, for arbitrary $\varepsilon > 0$, there exists a $T_1 > 0$ such that for $t > T_1$

$$\frac{1}{t} \left[\alpha B(t) + \sum_{i=1}^2 \delta_i \int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_i(u)) \tilde{N}(ds, du) \right] < \varepsilon. \quad (A.2)$$

Define

$$\varphi(t) = \int_0^t \eta(s) ds, \quad (A.3)$$

for $t \geq T_1$; then

$$\begin{aligned} \frac{d\varphi(t)}{dt} &= \eta(t), \\ \ln \frac{d\varphi(t)}{dt} &\leq \int_0^t (\delta(s) + \varepsilon) ds - \delta_0 \varphi(t), \end{aligned} \quad (A.4)$$

for $t \geq \max\{T, T_1\} = T_2$.

Integrating the above inequality from T_2 to t yields

$$\begin{aligned} \varphi(t) &\leq \frac{1}{\delta_0} \ln \left(e^{\delta_0 \varphi(T_2)} + \delta_0 \int_{T_2}^t e^{\int_0^s (\delta(s) + \varepsilon) ds} d\theta \right) \\ &\leq \frac{1}{\delta_0} \ln \left(e^{\delta_0 \varphi(T_2)} + \delta_0 e^{\int_0^t (\delta(s) + \varepsilon) ds} (t - T_2) \right) \\ &\leq \frac{1}{\delta_0} \int_0^t (\delta(s) + \varepsilon) ds \\ &\quad + \frac{1}{\delta_0} \ln \left(\delta_0 (t - T_2) + e^{\delta_0 \varphi(T_2)} \right). \end{aligned} \quad (A.5)$$

Dividing by t and taking the superior limit of both sides of (A.5) lead to

$$\langle \eta \rangle^* \leq \frac{\langle \delta \rangle^* + \varepsilon}{\delta_0} \quad \text{a.s.} \quad (A.6)$$

By the arbitrariness of ε , one has $\langle \eta \rangle^* \leq \langle \delta \rangle^* / \delta_0$, if $\langle \delta \rangle^* \geq 0$. Moreover, if $\langle \delta \rangle^* < 0$, we get

$$\limsup_{t \rightarrow +\infty} \frac{1}{t} \ln \eta(t) \leq \langle \delta \rangle^* + \varepsilon < 0, \quad (A.7)$$

for ε sufficiently small. Thus

$$\lim_{t \rightarrow \infty} \eta(t) = 0 \quad \text{a.s., if } \langle \delta \rangle^* < 0. \quad (A.8)$$

(II) The proof of (II) is similar to (I). Here we omits the proof. $\square$

Competing Interests

The authors declare that they have no competing interests.

Acknowledgments

The study was supported by the National Natural Science Foundation of China (11371230), the SDUST Research Fund (2014TDJH102), Shandong Provincial Natural Science Foundation, China (ZR2012AM012, ZR2015AQ001), Joint Innovative Center for Safe and Effective Mining Technology and Equipment of Coal Resources, Shandong Province, a Project of Shandong Province Higher Educational Science, and Technology Program of China (J13LI05).

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