Adaptive Terminal Sliding Mode NDO-Based Control of Underactuated AUV in Vertical Plane

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The depth tracking issue of underactuated autonomous underwater vehicle (AUV) in vertical plane is addressed in this paper. Considering the complicated dynamics and kinematics model for underactuated AUV, a more simplified model is obtained based on assumptions. Then a nonlinear disturbance observer (NDO) is presented to estimate the external disturbance acting on AUV, and an adaptive terminal sliding mode control (ATSMC) based on NDO is applied to enhance the depth tracking performance of underactuated AUV considering both internal and external disturbance. Compared with the traditional sliding mode controller, the static error and chattering problem of the depth tracking process have been clearly improved by adopting NDO-based ATSMC. The stability of control system is proven to be guaranteed according to Lyapunov theory. In the end, simulation results imply that the proposed controller owns strong robustness and satisfied control effectiveness in comparison with the traditional controller.

1. Introduction

Autonomous underwater vehicle (AUV) is a kind of underwater vehicles with onboard power supply and intelligence. Due to its autonomous ability, AUV has become a very hot research topic in the past few decades for its wide applications in ocean scientific survey, emergency rescue, and military mission, such as gas and mine sources detection, marine environment research, and observation and manipulation task of military [1–3]. Traditionally, fully actuated AUVs are applied to underwater missions in the ocean. The trait of fully actuated AUVs is that the number of actuators equals degrees of freedom (DOF). However, with the development of ocean equipment, underactuated AUVs play more and more important role in marine tasks, whose degrees of freedom (DOF) are more than actual control inputs. In practical occasions, many AUVs are actually underactuated as they lack thrusters or other actuators in the heave or sway directions, and when the actuator failures of fully actuated AUVs occur, they will also transform into underactuated states. A significant advantage for underactuated AUV is that its own weight can be decreased due to less actuators and sensors. However, these underactuated AUVs bring a challenging control issue considering their lack of actuators. Furthermore, they own highly coupled and strongly nonlinear kinematic and dynamic models [4]. Therefore, various controllers for underactuated AUV need to estimate and deal with uncertainties, hydrodynamic effects, inherent nonlinearity, and so on.

Trajectory tracking in the vertical plane is one of the essential research problems for underactuated AUVs; many researchers have concentrated upon this control problem. Subudhi et al. [2] proposed an output feedback controller to complete AUV’s path tracking task in its vertical plane according to a linearized motion model. Cristi et al. [5] applied sliding mode approach to overcome dynamical uncertainties of AUV in a diving maneuver; Lapierre [6] presented a robust diving control technique based on adaptive backstepping technique and switching schemes, which could meet the robustness requirement; Naik and Singh [7] treated the suboptimal control issue of underactuated AUV by using SDRE technique, and AUV showed satisfactory performance in spite of system’s uncertainties and other constraints. Loc et al. [8] used an analytical method based on sliding mode control technique to obtain the optimal trajectory for unmanned underwater vehicle, which made the vehicle
follow the desired trajectory as well. Adhami-Mirhosseini et al. [9] put forward a controller combining Fourier series and pseudo-spectral thoughts to solve the bottom tracking problems for AUV. Lakhekar et al. constructed an enhanced dynamic fuzzy SMC to improve the control performance of AUV’s depth following mission [10].

Sliding mode control (SMC) is noticed for its strong robustness against various uncertainties and disturbances in the application of controllers [11]. Due to this, SMC has been widely used by many researchers in different systems [12–14], such as the inverted-pendulum control systems, and motion control systems for underwater vehicles. Jia et al. [15] built an iterative nonlinear sliding mode (INSM) controller to achieve bottom-following of underactuated AUV; Lakhekar and Saundarmal [16] introduced a novel fuzzy sliding mode controller for AUV’s depth control task. SMC can handle system’s uncertainties in a robust way. However, SMC is limited for its dependence on the plant model. To overcome this disadvantage, lots of controllers based on adaptive thoughts are introduced. Adaptive control algorithm is an efficient way to enhance the performance of AUV’s tracking controllers [17]. Antonelli et al. [18] proposed an adaptive controller for ODIN AUV; Yu et al. [19] used adaptive technique on the surge velocity, heave velocity, roll angular velocity, pitch angular velocity, and yaw angular velocity, respectively, in the body-fixed reference frame.

As underactuated AUV is a highly nonlinear and coupled system, its kinematics and dynamics equations in the vertical plane are constituted of a few nonlinear equations, which can be expressed in the following forms [7]:

\[
\dot{z} = w \cos \theta - u \sin \theta,
\]

\[
(m - Z_0) \dot{w} = Z_q \dot{q} + m (uq + x_C \dot{q} + z_C \dot{q}^2) + Z_w \dot{q} - Z_{uu} w^2 \delta_s + M_w \dot{w} + (W_{eg} - B_{w}) \cos \theta + Z_{uu} u^2 \delta_s,
\]

wherein \(u, v, w, p, q,\) and \(r\) are the surge velocity, sway velocity, heave velocity, roll angular velocity, pitch angular velocity, and yaw angular velocity, respectively.

\[
(I_{yy} - M_q) \dot{q} = m \left( (x_W \dot{w} - uq) - z_c w q \right) + M_w \dot{w} + M_{uu} uq + M_{wuu} w^2 \left( |w| \right) + M_{q|q|} |q| + M_{wq} \dot{q} - z_w W_{eg} - z_B B_{w} \cos \theta - z_w W_{eg} - z_B B_{w} \sin \theta + M_{uu} u^2 \delta_s + D_{out},
\]

where \(I_{yy}\) is the inertial movement of AUV about the pitch axis \(y_B\); \(m\) is the mass of AUV, \((x_W, y_W, z_W, x_B, y_B, z_B)\) are the coordinates of AUV’s mass center and buoyancy center, respectively, in the body-fixed frame; \(o_{E-x_E-y_E-z_E}\) is the diving depth of AUV in the frame \(o_{E-x_E-y_E-z_E}; W_{eg}\) is the weight of AUV, \(B_{w}\) is the buoyancy of AUV in the water, \(D_{out}\) is the disturbance from the outside environment, \(\delta_s\) is the rudder input, \(M_{uu}, Z_{uu}\) are the coefficients of rudder, and \(Z_{w}, Z_{wq}, M_{wq}, M_{wuu}, M_{wq}, M_{wuu}, M_{wq}\) are the hydrodynamics coefficients of AUV.

Aiming at simplifying the controller design for underactuated AUV, the effect of heave velocity \(w\) is neglected by assuming \(w \approx 0, \dot{w} \approx 0\); according to the definition of body-fixed frame \(o_B-x_B-y_B-z_B\), it is obvious that \(x_B = y_B = z_B = 0\). It is assumed that the surge velocity \(u\) keeps a constant value as \(u = U_c, U_c > 0\) with the help of main thruster mechanism.

2. Mathematical Model of Underactuated AUV

The frame coordinate systems’ schematic drawing for the mathematical model of underactuated AUV is shown in Figure 1; the reference frame \(o_{E-x_E-y_E-z_E}\) is fixed on the earth, wherein \(\phi, \theta,\) and \(\varphi\) are the roll angle, pitch angle, and yaw angle, respectively, in this reference frame; the body-fixed reference frame \(o_B-x_B-y_B-z_B\) is fixed on the underactuated AUV; its origin \(o_B\) is located at AUV’s center of buoyancy, within the range of the desired trajectory as well. Adhami-Mirhosseini et al. [9] put forward a controller combining Fourier series and pseudo-spectral thoughts to solve the bottom tracking problems for AUV. Lakhekar et al. constructed an enhanced dynamic fuzzy SMC to improve the control performance of AUV’s depth following mission [10].

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\]

\[
(I_{yy} - M_q) \dot{q} = m \left( (x_W \dot{w} - uq) - z_c w q \right) + M_w \dot{w} + M_{uu} uq + M_{wuu} w^2 \left( |w| \right) + M_{q|q|} |q| + M_{wq} \dot{q} - z_w W_{eg} - z_B B_{w} \cos \theta - z_w W_{eg} - z_B B_{w} \sin \theta + M_{uu} u^2 \delta_s + D_{out},
\]

where \(u, v, w, p, q,\) and \(r\) are the surge velocity, sway velocity, heave velocity, roll angular velocity, pitch angular velocity, and yaw angular velocity, respectively.

\[
(I_{yy} - M_q) \dot{q} = m \left( (x_W \dot{w} - uq) - z_c w q \right) + M_w \dot{w} + M_{uu} uq + M_{wuu} w^2 \left( |w| \right) + M_{q|q|} |q| + M_{wq} \dot{q} - z_w W_{eg} - z_B B_{w} \cos \theta - z_w W_{eg} - z_B B_{w} \sin \theta + M_{uu} u^2 \delta_s + D_{out},
\]

where \(I_{yy}\) is the inertial movement of AUV about the pitch axis \(y_B\); \(m\) is the mass of AUV, \((x_W, y_W, z_W, x_B, y_B, z_B)\) are the coordinates of AUV’s mass center and buoyancy center, respectively, in the body-fixed frame; \(o_{E-x_E-y_E-z_E}\) is the diving depth of AUV in the frame \(o_{E-x_E-y_E-z_E}; W_{eg}\) is the weight of AUV, \(B_{w}\) is the buoyancy of AUV in the water, \(D_{out}\) is the disturbance from the outside environment, \(\delta_s\) is the rudder input, \(M_{uu}, Z_{uu}\) are the coefficients of rudder, and \(Z_{w}, Z_{wq}, M_{wq}, M_{wuu}, M_{wq}, M_{wuu}, M_{wq}\) are the hydrodynamics coefficients of AUV.

Aiming at simplifying the controller design for underactuated AUV, the effect of heave velocity \(w\) is neglected by assuming \(w \approx 0, \dot{w} \approx 0\); according to the definition of body-fixed frame \(o_B-x_B-y_B-z_B\), it is obvious that \(x_B = y_B = z_B = 0\). It is assumed that the surge velocity \(u\) keeps a constant value as \(u = U_c, U_c > 0\) with the help of main thruster mechanism.
Define $x_1 = z$, $x_2 = \theta$, and $x_3 = q$; the previous equations would be replaced by a new one:
\[
\begin{align*}
\dot{x}_1 &= -U_c x_2 + D_1, \\
\dot{x}_2 &= x_3, \\
\dot{x}_3 &= M_1 x_3 + M_2 \sin x_2 + M_3 \delta_3 + M_4 [x_3] x_3 \\
&\quad + M_5 \cos x_2 + D_2,
\end{align*}
\] (2)
where $M_1 = M_{u1} U_c / (I_{yy} - M_q)$, $M_2 = -z_W W_{eg} / (I_{yy} - M_q)$, $M_3 = M_{u2} U_c^2 / (I_{yy} - M_q)$, $M_4 = M_{q1q} / (I_{yy} - M_q)$, $M_5 = -x_W W_{eg} / (I_{yy} - M_q)$, $D_1 = U_c (x_2 - \sin x_2)$, and $D_2$ is regarded as internal disturbance of state equations, $D_2 = D_{out} / (I_{yy} - M_q)$, and $D_3$ is regarded as external disturbance of state equations, which is also the main disturbance for the system.

3. Terminal Sliding Mode Control Based on NDO

3.1. Nonlinear Disturbance Observer Design. The main disturbance for the underactuated AUV is external disturbance $D_{out}$; for the purpose of eliminating its effect on the motion of AUV, it is necessary to predict the $D_{out}$ changing trend. This thought is transformed into estimating $D_2$ in the progress of constructing controller. Assume $\hat{D}_2$ is the estimation result of $D_2$, $\hat{D}_{out}$ is the estimation value of $D_{out}$, and, obviously, $\hat{D}_{out} = (I_{yy} - M_q) \hat{D}_2$. Define state variable vector $x = [x_1, x_2, x_3]^T$; according to (2), a vector-form state equation is obtained [24, 25]:
\[
\dot{x} = G(x) + E(x) \delta_1 + J(x) D_2,
\] (3)
where
\[
G(x) = \begin{bmatrix} -U_c x_2 + D_1 \\ x_3 \\ M_1 x_3 + M_2 \sin x_2 + M_4 [x_3] x_3 + M_5 \cos x_2 \end{bmatrix},
\]
\[
E(x) = \begin{bmatrix} 0 \\ 0 \\ M_3 \end{bmatrix},
\]
\[
J(x) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.
\] (4)
Define $f$ is the internal state variable of NDO; then a nonlinear disturbance observer can be constructed as below:
\[
\begin{align*}
f &= \hat{D}_2 - h(x) , \\
\dot{f} &= -Tf(G(x) + E(x) \delta_1 + J(x)(f + h(x))),
\end{align*}
\] (5)
where $h(x)$ can be designed as $h(x) = h_1 x_1 + h_2 x_2 + h_3 x_3$ under the condition that $h_1 > 0, h_2 > 0, h_3 > 0, T = [h_1, h_2, h_3]$.

Define $\hat{D}_2 = D_2 - \hat{D}_2$. In fact, there is seldom or even no prior knowledge about the changing law of $D_2$. Only when the changing speed of $D_2$ is slower compared with whole system’s dynamic traits, it is preferable to assume the derivative of $D_2$ satisfies $\dot{D}_2 = 0$.

From (5), we can obtain
\[
\begin{align*}
\ddot{\hat{D}}_2 &= \hat{D}_2 - \hat{D}_2 = -\hat{D}_2, \\
\hat{D}_2 &= f + \dot{h}(x) = TJ(x) \hat{D}_2.
\end{align*}
\] (6)

Then $\hat{D}_2$ can be gotten as
\[
\hat{D}_2 = -TJ(x) \hat{D}_2 = -h_3 \hat{D}_2.
\] (7)

From (7), the value of $\hat{D}_2$ converges to zero due to the fact that $h_3 > 0$.

3.2. Adaptive Terminal Sliding Mode Control. The overall schematic drawing for the NDO-based terminal sliding mode control method is presented in Figure 2. Considering $\hat{D}_2$ is available, (2) can be rewritten as
\[
\begin{align*}
\dot{x}_1 &= -U_c x_2 + D_1, \\
\dot{x}_2 &= x_3, \\
\dot{x}_3 &= M_1 x_3 + M_2 \sin x_2 + M_3 \delta_3 + M_4 [x_3] x_3 \\
&\quad + M_5 \cos x_2 + \hat{D}_2 + \Psi,
\end{align*}
\] (8)
where $\Psi = \hat{D}_2$.

Let us define the estimation of $\Psi$ is $\hat{\Psi}$, and $\Psi = \Psi - \hat{\Psi}$. According to the same principle of getting the derivative law of $\hat{D}_2$, assuming $\Psi$ varies more slowly compared to the system dynamics, then we have $\dot{\Psi} = 0$, and $\hat{\Psi} = \psi - \hat{\Psi} = -\hat{\Psi}$ in further step.

The following task is to design sliding surfaces for this controller construction based on the basic idea of backstepping strategy. In order to follow the desired depth trajectory, the first sliding surface is defined as
\[
S_1 = z - z_d = x_1 - z_d.
\] (9)

Considering that $D_1$ is another equivalent internal disturbance in the state equations, it is necessary to estimate its value as well. Define $\hat{D}_1$ as the estimation of $D_1$, and $\hat{D}_1 = D_1 - \hat{D}_1$. $\hat{D}_1$ can be obtained by introducing the following adaptive control law, as shown in (10). Consider
\[
\dot{\hat{D}}_1 = \varepsilon_1 S_1, \quad \varepsilon_1 > 0.
\] (10)

Aiming at eliminating the static error of depth tracking, an integral part relative to $S_1$ is necessarily included:
\[
\Omega_1 = \int_0^t S_1 dt.
\] (11)
Choose a Lyapunov function as
\[ V_1 = \frac{1}{2} S_1^2 + \frac{1}{2\varepsilon_1} \delta_1^2 + \frac{1}{2} K_{\Omega_i}^2, \quad K_{\Omega_i} > 0. \] (12)

Define a virtual input as
\[ \alpha_i = \frac{1}{U_c} \left( k_i S_1 + \hat{\vartheta} + K_{\Omega_i} - \dot{z}_d \right), \quad k_i > 0. \] (13)

Define the second sliding surface and choose a Lyapunov function separately as
\[ S_2 = \theta - \alpha_i = x_2 - \alpha_i, \] \[ V_2 = V_1 + \frac{1}{2} S_2^2. \] (14)

Define a virtual input and the tracking error of pitch angular velocity separately as
\[ \alpha_2 = -k_2 S_2 + \dot{\vartheta} + U_c e_1, \quad k_2 > 0, \] \[ S_3 = q - \alpha_2 = x_3 - \alpha_2. \] (15) (16)

Then a novel terminal sliding mode surface is constructed as
\[ \sigma = S_3 + \int_0^t \left[ m_1 \text{sgn}^r S_2 + m_2 \text{sgn}^{r_1} S_3 \right] dt, \] (17)

where \( \text{sgn}^r(S_j) = |S_j|^r \text{sgn}(S_j), \quad \text{sgn}^{r_1}(S_j) = |S_j|^{r_1} \text{sgn}(S_j), \quad m_1, m_2, r_1 \geq 1, \quad 0 < r_2 < 1, \) all of them are positive constant, and the definition of \( \text{sgn}(S_3) = \text{sgn}(S_3) = 1, \quad S_3 > 0; \quad 0, \quad S_3 = 0; \quad -1, \quad S_3 < 0. \)

In order to obtain the change rule of \( \dot{\vartheta} \), the adaptive law of \( \dot{\vartheta} \) is built as
\[ \dot{\vartheta} = e_2 \sigma, \quad e_2 > 0. \] (18)

According to above adaptive laws and sliding mode surfaces design process, the control input of rudder is finally established as below:
\[ \delta_s = -\frac{1}{M_3} \left( M_1 x_3 + M_2 \sin x_2 + M_4 \left| x_3 \right| x_3 \right. \]
\[ + M_5 \cos x_2 + \hat{\vartheta} - \dot{\vartheta} + m_1 \text{sgn}^{r_1}(S_3) \]
\[ + \left. m_2 \text{sgn}^{r_2}(S_3) + L_{\text{weight}} \left| W_\sigma \right| \text{sgn} (\sigma) \right), \]

where \( W_\sigma = \int_0^t \left( L_{W_\sigma} W_\sigma + K_\sigma \sigma \right) dt, \quad L_{W_\sigma} < 0, \quad K_\sigma > 0, \) and this weighted integral section \( W_\sigma \) is designed to reduce the chattering phenomenon of traditional sliding mode control.

Define a Lyapunov function which includes the NDO part and terminal sliding surface as
\[ V_3 = \frac{1}{2} \sigma^2 + \frac{1}{2} \delta_1^2 + \frac{1}{2\varepsilon_1} \dot{\vartheta}^2. \] (19)

3.3. Stability Analysis of System. To study the convergence of depth trajectory, each sliding mode surface of controller should be analyzed. These positive definite Lyapunov functions constructed as \( V_i \) \( (i = 1, 2, 3) \) are discussed in detail to study sliding mode surfaces’ convergent issues, and their derivatives with respect to time can be obtained as
\[ \dot{V}_1 = S_1 \dot{S}_1 + \frac{1}{\varepsilon_1} \dot{\vartheta} \dot{\vartheta} + K_{\Omega_i} \Omega_i \dot{\Omega}_i \]
\[ = S_1 \left( -U_c x_2 + D_1 - \dot{z}_d \right) - \frac{1}{\varepsilon_1} \dot{\vartheta} \dot{\vartheta} + K_{\Omega_i} \Omega_i \dot{\Omega}_i \]
\[ = S_1 \left( -U_c x_2 - U_c \alpha_1 + D_1 - \dot{z}_d \right) - \frac{1}{\varepsilon_1} \dot{\vartheta} \dot{\vartheta} + K_{\Omega_i} \Omega_i \dot{\Omega}_i \]
\[ + K_{\Omega_3} \Omega_3 \dot{S}_3. \]
\[ S_1 (-U_c S_2 - k_1 S_1 + D_1 - K_1 \Omega_1) - \frac{1}{\epsilon_1} D_2 \dot{D}_1 \]
\[ + K_1 \Omega_1 S_1 = -k_1 S_1^2 - U_c S_1 S_2. \]  
(21)

If \( S_2 = 0 \), we can conclude that \( V_1 = -k_1 S_1^2 \leq 0 \), which means \( S_1 \) gradually converges to zero. Then the next step is to study \( V_2 = V_1 + S_2 S_2 = -k_1 S_1^2 - U_c S_1 S_2 + S_2 (S_3 + \alpha_2 - \alpha_1) \]
\[ = -k_1 S_1^2 - U_c S_1 S_2 + S_2 (S_3 - k_2 S_2 + \alpha_1 + U_c S_1 - \alpha_1) \]
\[ = -k_1 S_1^2 - k_2 S_2^2 + S_3 S_2. \]  
(22)

In the same way, if \( S_3 = 0 \), then we can conclude that \( V_2 = -k_1 S_1^2 - k_2 S_2^2 \leq 0 \), which means \( S_2 \) gradually converges to zero as well. From (17), it is easy to know that \( S_3 = 0 \) is equivalent to \( \sigma = 0 \); the key point is to verify the convergence of terminal sliding mode surface \( \sigma \).

From (8), (13), (15), (17), and (19), we can obtain
\[ \dot{\sigma} = \dot{S}_3 + m_3 \text{sign}^\tau (S_3) + m_2 \text{sign}^\tau (S_2) \]
\[ = \dot{x}_3 - \alpha_2 + m_3 \text{sign}^\tau (S_3) + m_2 \text{sign}^\tau (S_2) \]
\[ = M_1 x_3 + M_2 \sin x_2 + M_3 \delta_s + M_4 \left| x_3 \right| x_3 \]
\[ + M_5 \cos x_2 + D_2 + \Psi - \alpha_2 + m_1 \text{sign}^\tau (S_1) \]
\[ + m_2 \text{sign}^\tau (S_2) = -L_{\text{weight}} \left| W_\sigma \right| \text{sign} (\sigma) + \Psi. \]
(23)

From (7), (18), and (23), we can obtain
\[ V_3 = \sigma \dot{\sigma} + D_2 \dot{D}_2 + \frac{1}{\epsilon_2} \frac{\Psi}{\text{sign} (\sigma)} \]
\[ = \sigma \left( -L_{\text{weight}} \left| W_\sigma \right| \text{sign} (\sigma) + \Psi \right) - h_3 D_2^2 - \frac{\Psi}{\epsilon_2} \]
\[ = -L_{\text{weight}} \left| W_\sigma \right| \left| \sigma \right| - h_3 D_2^2 \leq 0. \]  
(24)

Considering the fact that \( V_3 \geq 0 \), \( V_3 \leq 0 \), it is easy to know that terminal sliding mode surface \( \sigma \) is reachable. Once \( \sigma = 0 \), we can know that \( S_1 \) will converge rapidly to zero in a finite period. Due to (22), \( S_2 \) converges to zero as well when \( S_3 = 0 \); and once \( S_2 = 0 \), it implies that \( S_1 \) converges to zero quickly. All above analysis proves that all closed-loop sliding mode surfaces will converge to the equilibrium points in finite time.

### 4. Simulation Experiment

In this section, simulations are presented to verify the effectiveness of designed controller, which is ATSMC with NDO. The method is applied into the depth control of REMUS AUV. REMUS is a small-size, low-cost moving platform applied into various oceanographic activities; it is developed by both Massachusetts Institute of Technology (MIT) and Woods Hole Oceanographic Institution. The parameters about REMUS can be obtained from [7]; here some parameters which will be used in this paper are shown in Tables 1 and 2.

**Table 1: Hydrodynamic parameters of REMUS AUV.**

<table>
<thead>
<tr>
<th>Variables</th>
<th>Values</th>
<th>Units</th>
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<tbody>
<tr>
<td>( M_1 )</td>
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<td>kg/m²/rad</td>
</tr>
<tr>
<td>( M_{\text{cyl}} )</td>
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<td>( M_{\text{ww}} )</td>
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<tr>
<td>( Z_{\text{ww}} )</td>
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<td>kg/(m rad)</td>
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</tbody>
</table>

**Table 2: Physical parameters of REMUS AUV.**

<table>
<thead>
<tr>
<th>Variables</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_\text{ww} )</td>
<td>0</td>
<td>m</td>
</tr>
<tr>
<td>( z_\text{ww} )</td>
<td>299</td>
<td>N</td>
</tr>
<tr>
<td>( m_\text{ww} )</td>
<td>30.48</td>
<td>kg</td>
</tr>
<tr>
<td>( y_\text{ww} )</td>
<td>0</td>
<td>m</td>
</tr>
<tr>
<td>( x_\text{ww} )</td>
<td>299</td>
<td>N</td>
</tr>
<tr>
<td>( z_\text{ww} )</td>
<td>0</td>
<td>m</td>
</tr>
<tr>
<td>( x_\text{ww} )</td>
<td>0</td>
<td>m</td>
</tr>
<tr>
<td>( z_\text{ww} )</td>
<td>306</td>
<td>N</td>
</tr>
<tr>
<td>( I_\text{yy} )</td>
<td>3.45</td>
<td>Kg m²</td>
</tr>
</tbody>
</table>

**Case 1.** In order to undergo the simulation, due to Section 3, the parameters of NDO are chosen as \( h_1 = h_2 = h_3 = 1 \); the parameters of ATSMC are set as \( k_1 = 1.2, k_2 = 1.8, k_3 = 1.1, e_1 = 0.1, e_2 = 0.01, m_1 = m_2 = 1.1, r_1 = 1.2, r_2 = 0.5, L_\text{ww} = -1, K_\sigma = 1, L_{\text{weight}} = 1, \) and \( K_{\Omega_1} = 0.5 \). The initial values of state variables are \([u(0), \theta(0), z(0), p(0)] = [0, 0, 4, 0] \) for the REMUS AUV. The external disturbance exposed on REMUS is assumed to be in the following form \( D_{\text{ext}} = 20 \sin(\pi t/10) + 10 \sin(\pi t/15) + 6 \cos(\pi t/20 + \pi/2) \) during the time period \( 20 \leq t \leq 40 \). The surge velocity \( u \) holds constant with \( U_s = 1.54 \) m/s. The desired depth of AUV is defined as a time varying function \( z_d = 5 + 1.5 \sin(\pi t/10) \). The task of the designed ATSMC based on NDO is to achieve the depth trajectory tracking of REMUS AUV in reference to desired depth \( z_d \).

Figure 3 shows the depth tracking process of under-actuated AUV, and the changing trend of rudder control input \( \delta_r \). We can obtain that the tracking error of depth
Almost as close to zero in the overall process, which satisfies the assumption that $\omega \approx 0$, $\dot{\omega} \approx 0$.

The main disturbance for AUV is external disturbance $D_{\text{out}}$. To reduce external disturbance’s effect on AUV, the proposed controller introduces the NDO to observe and predict $D_{\text{out}}$. Another important relationship is $D_{\text{out}} = (I_{yy} - M_q)D_2$; it is convenient to estimate $D_2$ based on NDO in the controller design. We can have the observation value $\hat{D}_{\text{out}}$ due to NDO from Figure 5. Another part of disturbance is the constructed internal disturbance $D_1$ which is established for convenience of designing proposed controller. An adaptive law for $D_1$ is also introduced to accelerate the convergent speed of controller; its estimation process is shown in Figure 6.

The traits of sliding mode surfaces in the controller building procedure are vital to the convergence of the whole control system. The internal relationships between each sliding mode surface can be known from Section 3.3. The
change laws of sliding mode surfaces are shown in Figure 7; we can know that sliding mode surfaces $S_1, S_2, S_3$ converge to equilibrium points. Another important aspect is the terminal sliding mode surface $\sigma$, which can be reachable from Figure 8. $\sigma$ guarantees the convergence of $S_i$ $(i = 1, 2, 3)$. From (19) and Figure 9, we can understand that weighted value $W_\sigma$ is used to weaken the chattering issue which occurs frequently in the traditional sliding mode control, and its value changes with $\sigma$, and apparently no strong oscillation happens to the rudder in the tracking procedure from Figure 3.

**Case 2.** In order to compare with the proposed method, a traditional sliding mode controller and a proposed controller which lacks integer part $\Omega_1$ in (11) are separately applied into accomplishing the depth tracking task. Their control performances are shown, respectively, in Figures 10 and 11. Among them, the traditional sliding mode control law [26] can be expressed with the expression of (19) when $m_1 = m_2 = 0$, $W_\sigma = 1$. Through comparing the performance shown in Figure 3 with Figure 10, it is obvious that $m_1 \text{sig}^\nu(S_3)$, $m_2 \text{sig}^\nu(S_3)$ and the change rule of $W_\sigma$ play an important role in reducing chattering issue of rudder input; when $K_{\Omega_1} = 0$, the rudder input of the ATSMC based on NDO can make AUV track the desired depth, but a small tracking error always exists by contrasting the control effect of Figure 3 with Figure 11; this illustrates that integer part $\Omega_1$ is necessary to achieve the expected tracking effect.

**5. Conclusion**

A methodology combined with ATSMC and NDO was presented in this paper. For the convenience of design, both the kinematics and dynamics equations of underactuated AUV in the vertical plane were simplified firstly based on some assumptions. Then, a nonlinear disturbance observer (NDO) was built, and another form of AUV’s state variables equation was reconstructed on the basis of NDO. Furthermore, an ATSMC with NDO was designed, and the
controller’s stability was verified according to Lyapunov’s theory and backstepping technique. Through the simulation experiments of AUV, the designed controller was proved to make the underactuated AUV track the expected time-varying depth trajectory, and it showed strong robustness to external disturbance.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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