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Research Article

Backup Sourcing Decisions for Coping with Supply Disruptions under Long-Term Horizons

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This paper studies a buyer's inventory control problem under a long-term horizon. The buyer has one major supplier that is prone to disruption risks and one backup supplier with higher wholesale price. Two kinds of sourcing methods are available for the buyer: single sourcing with/without contingent supply and dual sourcing. In contingent sourcing, the backup supplier is capacitated and/or has yield uncertainty, whereas in dual sourcing the backup supplier has an incentive to offer output flexibility during disrupted periods. The buyer's expected cost functions and the optimal base-stock levels using each sourcing method under long-term horizon are obtained, respectively. The effects of three risk parameters, disruption probability, contingent capacity or uncertainty, and backup flexibility, are examined using comparative studies and numerical computations. Four sourcing methods, namely, single sourcing with contingent supply, dual sourcing, and single sourcing from either of the two suppliers, are also compared. These findings can be used as a valuable guideline for companies to select an appropriate sourcing strategy under supply disruption risks.

1. Introduction

With widespread applications of outsourcing, supply chains are becoming increasingly dependent upon suppliers, and supply interruption can obstruct the normal operations of the entire supply chain. The situation with Toyota in Japan serves as a perfect example. After the disasters hit Japan in March of 2011, Toyota and their part suppliers struggled to resume operations, which paralyzed the downstream supply chain; for example, General Motors was reported to be the first US auto maker to close a factory because of a short supply of a Japan-made part. Overall, the global auto industry is suffering losses in the production of hundreds of thousands of vehicles. Additionally, companies around the world are feeling the impacts of Japan's disasters as various supplies fall, because Japan accounts for roughly one-fifth of the world's supply of silicon wafers used to make semiconductors, is home to a large number of manufacturers for a key material in liquid-crystal-display panels, and supplies about 90% of the world's need of a chemical used in making circuit boards for telephone handsets (Supply Chain Asia [1]).

As the world is becoming increasingly volatile and uncertain, one important strategy and tactic for building supply chain resilience is to establish backup supplies, in the form of contingency supplies or dual sourcing, even if they cost more (Supply Chain Asia [1]). While the appropriate level of resilience depends heavily on the time horizon being considered (Swaminathan and Tomlin [2]), supply chain managers must not only consider all possible risks around, but also plan for sufficiently long term when making inventory and supply decisions. Most research to date, however, has studied the optimal solutions for finite-horizon situations under supply disruption risks. These solutions are not always optimal for infinite-horizon problems and could even lead to either incorrect financial investment or wrong partners. Therefore, it is important to examine decisions under long-term horizons so as to reduce the impact of supply risk. Unlike most of the existing research efforts on supply risk management that look for optimal decision parameters and strategies in finite horizon, especially in single period, this work aims to study the optimal backup sourcing methods and inventory management decisions from the viewpoint of

minimizing the buyer's long-term cost. Note we use the term, buyer, in the paper to generally refer to any firm procuring physical items from outside vendors.

We study the situation in which a buyer has two sources for the same product: a main source and a backup supply source, where the former is prone to supply disruptions during which it provides no supply of the critical component and the latter offers higher wholesale price. The buyer needs to decide whether to use the latter as a contingent source or a regular one, that is, single sourcing with contingent supply or dual sourcing. Under single sourcing with contingent supply, since the backup supplier replenishes inventory at unplanned moments when the main supplier is disrupted, its production may be insufficient to meet the buyer's order (Chen et al. [3]). Consequently, the buyer may suffer from the contingent supplier's yield uncertainty and/or limited capacity during disrupted periods. For example, when Japan earthquake occurred in March 2011, a plasma display panel maker in Anhui province of China was facing supply disruptions from its main supplier located in Japan. The company had to resort to a small domestic supplier for contingent supply of raw materials. As the contingent supplier's invested capacity was limited and its yield rate was uncertain, the panel maker suffered a great loss during that period. The dual sourcing method may avoid contingent supplier's problems but encounter new issues; because the buyer allocates a proportion of order to the more expensive supplier, its total purchasing costs during nondisrupted periods can be higher; on the other hand, the buyer would obtain more stable delivery from the backup supplier during disrupted periods.

Under each sourcing method, the buyer needs to control its inventory level to minimize the expected understock and overstock costs, as well as backup purchasing cost. He/she would estimate how much should be invested in supply chain resilience and prepare the stocks based on the likelihood of disruptions in next and more periods. It is reasonable to assume that the backup supplier provides no larger quantity than the cycle demand each time and that the buyer accepts the entire delivery from the backup supplier. We will analyze the impacts of the backup supply parameters on the sourcing method selection and inventory management decisions.

The paper is organized as follows. The related literature is reviewed briefly in the next section. Section 3 describes the problems and lists the assumptions used in our model. In Sections 4 and 5, the inventory decisions under single sourcing with contingent supply and dual sourcing are examined and analyzed separately. The comparisons between the two sourcing methods are explored in Section 6. Section 7 summarizes our work, discusses the model limitations, and suggests future research directions.

2. Literature Review

The issue of supply disruptions has received a great deal of attention lately in the literature. We concentrate on the papers most directly relevant to our research and refer the reader to the survey paper of Snyder et al. [4] for a comprehensive review of recent literature on the subject.

2.1. Backup Supply under Supply Disruptions. Perfectly reliable backup suppliers are considered by many researchers. For example, Yu et al. [5] evaluate the impacts of supply disruption risks on the choice between single and dual sourcing methods based on the assumption that the backup supplier is perfectly reliable. Hou et al. [6] focus on the backup contract between a buyer and their reliable backup supplier to mitigate supply disruptions. Some existing studies assume a spot market as a contingent supply that is totally reliable (e.g., Li et al. [7]), while in reality, the buyer often cannot receive the desirable amount due to high market uncertainty, and the accessible delivery thus may be limited or stochastic.

Different types of risks associated with a backup supplier or multiple suppliers are considered in some publications, including lead time uncertainty, random defaults, limited capacity, and asymmetrical information. Babich et al. [8] study the effects of disruption risk where one buyer deals with risky suppliers that not only compete against each other, but are subject to random defaults as well. Serel [9] examines the relationship between one buyer and one reliable manufacturer when there is competition from a second supplier that is prone to supply disruptions. Yang et al. [10] study a manufacturer's strategic use of a dual sourcing option when both suppliers have private, reliable information. In the work of Sajadieh and Eshghi [11], a dual sourcing model with constant demand and stochastic lead time is established. Sting and Huchzermeier [12] analyze how firms should contract with backup suppliers so that the latter would install responsive capacity to mitigate imminent mismatches of uncertain supply and demand. The recent work of Xu et al. [13] studies the contract between the buyer and an urgent supplier with private cost information.

In addition, there are a number of papers that focus on modeling supplier selection and order allocation decisions under supply risks. Berger et al. [14], Ruiz-Torres and Mahmoodi [15, 16], and Sarkar and Mohapatra [17] have addressed the problem of the optimal size of supply base under the situation where every supplier is prone to supply disruptions. The research conducted by Burke et al. [18] relies on the classic newsvendor framework to determine the optimal number of suppliers with the consideration of each supplier's capacity and reliability. Dada et al. [19] consider a newsvendor served by multiple suppliers, each of which delivers an amount strictly less than the amount desired with certain probabilities. The work of Sawik [20] deals with the selection of supply portfolio in the presence of supply chain disruption risks, and each supplier is assumed to have limited capacity, unique price, and different level of quality for the purchased parts.

The above studies consider different backup supply types and aim to derive optimal solutions in single-period models. In our study, we consider single and dual sourcing with capacitated and/or uncertain backup supply under long-term planning horizon. The impacts of the supplier's capacity and uncertainty on the buyer's decisions under two sourcing methods are examined, respectively.

2.2. Supply Risk Management under Long-Term Planning Horizon. There has been a group of works that considers multiperiod or long-term system optimization under supply

disruption risks. Parlar [21] and Parlar and Perry [22] consider stochastic inventory model with supply disruptions. By using the renewal reward theorem, the long-run average cost and reordering policies are derived. These models focus on deriving optimal multiperiod ordering policies but do not consider using a backup supplier.

In regard to long-term mitigation strategies under supply disruptions, Tomlin [23] focuses on the value of mitigation and contingency strategies for managing supply chain disruption risks under the situation where the reliable supplier may or may not possess volume flexibility. Tomlin and Snyder [24] characterize the optimal threat-dependent inventory levels in both infinite- and finite-horizon settings. The backup supplier in the sourcing mitigation is assumed to be totally reliable. The work of Serel [25] studies a multiperiod capacity reservation contract between a manufacturer and a long-term supplier when the random amount of supply available from the spot market is independently and identically distributed in each period. Schmitt et al. [26] develop a closed-form, approximate solution for a buyer who faces stochastic demand or supply yield. Schmitt and Snyder [27] compare the optimal inventory control solutions in single-period and infinite-horizon models. Their work assumes that the backup supplier is totally reliable.

This paper differs from the existing studies in the following two major ways. First, unlike most of the existing studies that investigate the optimal inventory system solutions under supply disruption risks with reliable or stochastic backup supply, we distinguish between the characteristics of backup supply under contingent sourcing and dual sourcing; specifically, under single sourcing with contingent supply, the backup supplier may have insufficient capacity and/or yield uncertainty, while under dual sourcing, the backup supplier's yield is dependent on both the buyer's order allocation and output flexibility. Second, both inventory control policies and backup sourcing method selection under long-term planning horizon are examined in our research. An in-depth analysis is also conducted to examine the impacts of the supply risks and the cost parameters. We show (1) how the buyer's optimal base-stock level and the corresponding expected cost are different under the two sourcing methods; (2) when single sourcing will outperform dual sourcing and vice versa; and (3) how the backup supply parameters affect the decisions in a long-term planning model.

3. Problem Description and Assumptions

We consider a buyer that has two supply options for a critical component: one is cheaper but subject to random disruptions (Supplier 1) and the other is perfectly reliable and responsive, but more expensive (Supplier 2). The demand at the buyer is certain and denoted as d. This assumption helps us focus on the impact of supply risks. Besides, it is reasonable to suppose stable demand for a lot of products, such as grocery and basic apparel (Lee [28]). The buyer operates by a periodic-review, base-stock policy. Unsatisfied demands are backordered with a unit penalty cost of p. The lead time is negligible, and the unit overstock cost is h. Under such a situation, the buyer needs to choose the sourcing method and determine the optimal base-stock level s by minimizing its expected long-term cost consisting of overstock and understock cost, as well as backup purchasing cost.

Two sourcing methods are available for the buyer: single sourcing with contingent backup supply and dual sourcing. Specifically, (1) under single sourcing with contingent backup supply, the buyer places an order to their major supplier referred to as Supplier 1 at the beginning of each cycle. When a disruption breaks down the main supplier's capacity, the backup supplier (Supplier 2) is available for meeting the buyer's demand but may have limited capacity and/or random yield; and (2) under dual sourcing, the buyer uses Supplier 2 as a second regular source. Previous studies (e.g., Yan and Liu [29], Chen et al. [3]) have found that splitting orders among multiple suppliers is usually an effective approach to mitigate supply disruptions. Let θ_i $(i = 1, 2)$ denote the proportion of demand allocated to supplier i where $\theta_1 + \theta_2 = 1$. In this case, as a regular supplier in a long-term relationship with the buyer, Supplier 2 has the incentive to increase its output with extra capacity related to its order proportion θ_2 in the event of Supplier 1 failure. Thus, the buyer has to determine both the order allocation and the optimal base-stock level s. Note that there is a possibility that the buyer single-sources from Supplier 2 which is completely reliable; in this case the buyer's optimal base-stock level can easily be deduced to be equal to demand d. Therefore, in what follows, we will first analyze the buyer's expected costs and optimal decisions under single sourcing with contingent backup supply and dual sourcing, respectively, and then compare these two sourcing methods.

To formulate the supply disruption process, we follow the assumptions made in Schmitt et al. [26] and Schmitt and Snyder [27]. Specifically, an infinite-state, discrete-time Markov chain is used to represent the disruption that lasts for a number of periods, with π_i representing the steady-state probability that the major supply has been disrupted for i consecutive periods, $i \in [0, \infty)$. In addition, denote α as the transition probability from the normal to the disrupted state and β as the recovery probability from the disrupted to the normal state. It is easy to see that the smaller the value of β is, the longer the disruption would last. Thus, the parameter set (α, β) represents the disruption probability and the average duration, respectively, and can be used to characterize each disruption. The notation π_i is used to denote the probability of being in state i where the state represents the number of consecutive disrupted periods. In addition, when a disruption occurs, since the buyer cannot assess the disruption duration, it is logical to assume that the buyer will start the backup supply from the first cycle of the disruption. A complete list of symbols used in the paper is provided in The List of Symbols Used in the Paper.

4. Single Sourcing with Contingent Backup Supply

During normal situations, the inventory position of the buyer is maintained at a level of s, while during disruptions the buyer can order only from the backup supplier (Supplier 2), whose capacity is limited and/or yield is uncertain.

Before the single sourcing with contingent supply method is analyzed, we first give the following results under single sourcing without contingent supply, as presented by Tomlin [23] and Schmitt et al. [26].

Theorem 1. *For a buyer with deterministic demand and supply disruptions,*

(a) *the expected cost per period is*

$$C_1(s) = \sum_{i=0}^{\infty} \pi_i \left[h(s-(i+1)d)^+ + p((i+1)d-s)^+ \right] \quad (1)$$

and is a convex function;

(b) $s_1^* = j^* d$, *where* j^* (>1) *is the smallest integer such that* $\sum_{i=0}^{j^*-1} \pi_i \geq p/(p+h)$.

4.1. Analysis of Contingent Backup Supply with Capacitated and Uncertain Yield. Under single sourcing with capacitated and uncertain backup supply, the backup capacity is limited to y $(y \leq d)$ in each period. That is, to satisfy the demand and maintain the inventory level of s, the maximum order quantity the buyer can place to the contingent supplier in each cycle during disruption periods is y. Moreover, the backup yield has an additive random quantity w, which is generally distributed with a normal distribution, $N(u_w, \sigma_w)$, and is independent of the order quantity and may be positive or negative (Schmitt and Snyder [27]). Thus the actual backup delivery is $y + w$ (we assume $F(-y) \approx 0$).

Under such assumption, the only difference between the buyer's expected cost with and without (as shown in Theorem 1) contingent supply is the added backup purchasing cost from the contingent supplier and larger inventory level per period; therefore, the buyer's cost can be written by modifying (1) as follows:

$$\begin{aligned} C_{cu} = {} & \pi_0 \left[h(s-d)^+ + p(d-s)^+ \right] + (y+u_w) c_2 \sum_{i=1}^{\infty} \pi_i i \\ & + \sum_{i=1}^{\infty} \pi_i \left[h \int_{(i+1)d-s-iy}^{\infty} (s+iy+w_i-(i+1)d) \right. \\ & \cdot f_i(w_i)\, dw_i \\ & \left. + p \int_{-\infty}^{(i+1)d-s-iy} ((i+1)d-s-iy-w_i) f_i(w_i)\, dw_i \right], \end{aligned} \quad (2)$$

where w_i is the total random quantity of w for i periods of disruption and follows a normal distribution with the density function of $f_i(w_i)$, mean of iu_w, and standard deviation of $\sqrt{i}\sigma_w$. The first term is the inventory holding cost and the penalty cost during normal periods. The second is the backup supply purchasing cost. The last is the holding and penalty cost when disruption occurs. Note that the purchasing cost from the major supplier would not affect the buyer's decision on inventory level s under single sourcing; we thus do not consider it in the cost function.

Using the standard normal loss function, $G(x)$, we can rewrite the cost function as

$$\begin{aligned} C_{cu}(s) = {} & \sum_{i=1}^{\infty} \pi_i \left[p((i+1)d-s-iy-iu_w) \right. \\ & \left. + (p+h)\sqrt{i}\sigma_w G\left(\frac{(i+1)d-s-iy-iu_w}{\sqrt{i}\sigma_w} \right) \right] \\ & + \pi_0 \left[h(s-d)^+ + p(d-s)^+ \right] + (y+u_w) c_2 \sum_{i=1}^{\infty} \pi_i i. \end{aligned} \quad (3)$$

The next proposition provides the buyer's optimal base-stock level in a long-term horizon.

Proposition 2. *Under single sourcing with a backup supplier that has capacitated yield and additive uncertainty, the buyer's optimal base-stock level under long-term horizon,* s_{cu}^*, *should satisfy*

$$\sum_{i=1}^{\infty} \pi_i \left(\Phi\left(\frac{(i+1)d-s_{cu}^*-iy-iu_w}{\sqrt{i}\sigma_w} \right) \right) = \frac{h}{h+p}, \quad \text{or } s_{cu}^* = d. \quad (4)$$

Proof. See Appendix A. □

The final optimal value of s_{cu} can be obtained by comparing $C_{cu}(d)$ and $C_{cu}(s_{cu}^*)$ with s_{cu}^* that satisfies $\sum_{i=1}^{\infty} \pi_i (\Phi(((i+1)d - s_{cu}^* - iy - iu_w)/\sqrt{i}\sigma_w)) = h/(h+p)$.

The formula in Proposition 2 suggests that the buyer would decrease their base-stock level s_{cu}^* if the backup supply capacity y increases. Since no close-form solution of s_{cu}^* can be obtained, we will rely on numerical examples to examine its properties in later part of this section. To examine the impacts of the backup capacity limitation and yield uncertainty, respectively, we consider two special cases below.

4.2. Two Special Cases

Case 1 (contingent supply with capacitated yield). Consider a backup supplier that can provide only a finite quantity y $(y \leq d)$ each period when disruption occurs; to satisfy the demand and maintain the inventory level, the buyer orders y units from the backup supplier in each cycle. Based on the assumptions above, the buyer's expected cost function is given by the following expression:

$$\begin{aligned} C_c(s) = {} & \sum_{i=0}^{\infty} \pi_i \left[h(s+iy-(i+1)d)^+ \right. \\ & \left. + p((i+1)d-s-iy)^+ + c_2 yi \right]. \end{aligned} \quad (5)$$

The buyer's optimal decision is presented in Proposition 3.

Proposition 3. *With a capacitated backup supply, the buyer's optimal base-stock level under the long-term horizon,* s_c^*, *should satisfy the following condition:*

$$s_c^* = j^* d - (j^* - 1) y, \quad (6)$$

where j^* (>1) *is the smallest integer such that* $\sum_{i=0}^{j^*-1} \pi_i \geq p/(p+h)$.

Proof. See Appendix B. □

It is clearly shown by Proposition 3 that (1) either higher understock cost or lower overstock cost implies higher base-stock level and (2) the higher the capacity the backup supplier has, the less the buyer stock should be to deal with supply disruption risks. Note that one special case exists when $y = 0$, the buyer single-sources from S1, and in this case, $s_1^* = j^* d$.

Case 2 (contingent backup supply with uncertain yield). In each period, the buyer observes the current inventory level (IL) and places an order of s − IL. The backup supplier's delivery then brings the buyer's IL to a value of $s+v$ during disrupted period, where v indicates the yield uncertainty which is independent of the order quantity. We use the notations u_v, σ_v, $m(v)$, and $M(v)$ to denote the mean, deviation, pdf, and cdf of v, respectively, and suppose $M(-s) \approx 0$. The buyer's expected cost in a long-term planning situation can be calculated below and the buyer's optimal base-stock level is presented in Proposition 4.

$$\begin{aligned} C_u(s) = \sum_{i=1}^{\infty} \pi_i \Bigg[& h \int_{d-s}^{\infty} (s+v-d)\, m(v)\, dv \\ & + p \int_{-\infty}^{d-s} (d-s-v)\, m(v)\, dv + c_2 (d+u_v) \Bigg] \\ & + \pi_0 \left[h (s-d)^+ + p (d-s)^+ \right]. \end{aligned} \tag{7}$$

Proposition 4. *With an uncertain backup supply, the buyer's optimal base-stock level under the long-term horizon,* s_u^*, *should satisfy the following condition:*

$$s_u^* = d - M^{-1}\left(\frac{(\alpha+\beta)h}{\alpha(h+p)}\right), \quad \text{if } \frac{(\alpha+\beta)h}{\alpha(h+p)} < 0.5 \tag{8}$$

or $s_u^* = d$.

Proof. See Appendix C. □

Given the value of σ_v, as $M(d - s_u^*) = (\alpha+\beta)h/\alpha(h+p)$, we see that $d - s_u^*$ increases with u_v; that is, s_u^* decreases with u_v. In the following analysis, we will further examine how the various input parameters impact the buyer's decision and expected cost.

4.3. Numerical Analysis of Single Sourcing with Contingent Supply. We now provide some numerical examples to investigate the properties of single sourcing with contingent supply. The proposed base values of the input parameters are listed below:

d: 100;

p: 18;

h: 2;

α: 0.1;

β: 0.5;

c_1: 8;

c_2: 11;

y: 50;

(u_w, σ_w): (−15, 5);

(u_v, σ_v): (0, 5).

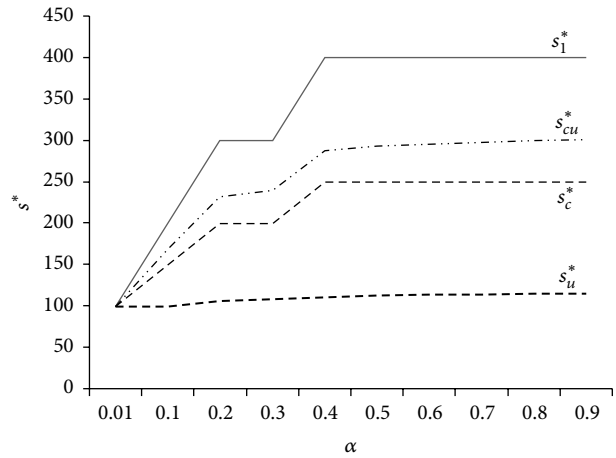


Figure 1: The buyer's optimal decision s^* under single sourcing versus α.

4.3.1. The Impact of Supply Disruptions under Single Sourcing. We first calculate the optimal base-stock level s^* by changing α from 0.01 to 0.9 with an increment of 0.1 ($\beta = 0.5$). Figure 1 shows how the buyer's optimal decisions are under single sourcing with (s_{cu}^*, s_c^*, s_u^*) and without the backup supply (s_1^*). It is seen that the buyer's base-stock level under single sourcing increases with the disruption probability and the use of a contingent backup supplier decreases the buyer's stock level. In addition, the line of s_u^* is flatter than the other two, meaning that the impact of disruption probability on s^* is highly affected by the type of backup supplier, which is large when the backup supply is capacitated (s_{cu}^* or s_c^*). In addition, the optimal value of s_{cu}^* under both capacitated and uncertain backup yield is larger than s_c^* (under capacitated yield) and s_u^* (under uncertain yield). This implies that either larger backup uncertainty or smaller backup capacity would increase the buyer's inventory level.

While the use of a contingent backup supplier would always lower the buyer's inventory level, the impacts of supply disruption on its expected costs vary for different types of backup supplier. Figure 2 plots the buyer's expected profits under single sourcing with (C_{cu}, C_c, C_u) and without (C_1) backup supplier.

According to Figure 2, the buyer's expected cost C increases with α. In particular, when the disruption risk is small enough ($\alpha \leq 0.1$ in this case), the buyer always benefits from using a contingent backup supplier, because $C_{cu}, C_c, C_u < C_1$. On the other hand, when the disruption risk becomes larger ($0.1 < \alpha \leq 0.6$ in this case), the buyer only benefits from using a contingent backup supplier with

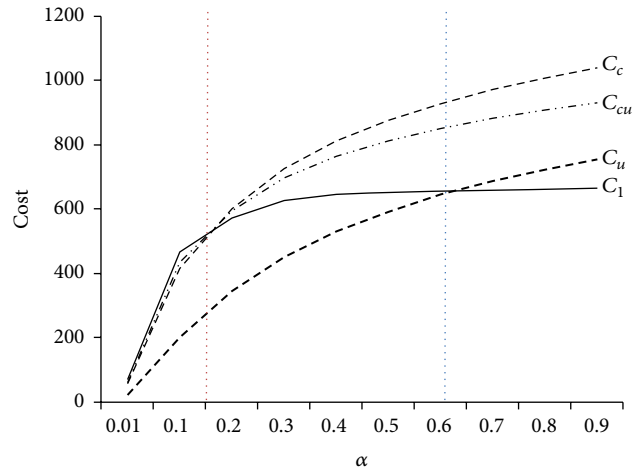


Figure 2: The buyer's expected cost C under single sourcing versus α.

no capacity limitation ($C_u < C_1 < C_{cu}, C_c$). Moreover, if the disruption risk is large enough ($\alpha \geq 0.6$ in this case), the buyer would single-source from Supplier 1 and stock more to mitigate disruptions.

4.3.2. The Impact of Contingent Backup Supply under Single Sourcing. To learn the impacts of backup capacity and uncertainty, which are also influenced by the disruption probability, we illustrate the buyer's decision and expected cost under single sourcing with capacitated or uncertain backup supply in Figures 3 and 4, respectively.

As shown in Figure 3, although larger backup capacity y would decrease the buyer's inventory level, only when the disruption probability is small ($\alpha \leq 0.1$ in this case), the buyer would benefit from larger backup capacity. Additionally, Figure 4 implies that the buyer always prefers lower backup uncertainty in order to reduce both inventory level and expected cost.

5. Dual Sourcing

Under dual sourcing, the buyer maintains the inventory level at s and receives d units of components in each nondisrupted period, with $\theta_1 d$ units from Supplier 1 and $\theta_2 d$ units from Supplier 2 ($\theta_1 + \theta_2 = 1$). In the event of Supplier 1 failure, Supplier 2, as a regular supplier, has an incentive to increase its output to $d\theta_2^k$ ($0 \leq k \leq 1$), where k indicates its output flexibility with k values close to 0 indicating high output flexibility and close to 1 suggesting low output flexibility (as in Ruiz-Torres and Mahmoodi [16]). It is worth mentioning that, compared with single sourcing with contingent backup supply discussed in Section 4, a regular supplier is likely more reliable or capable than a contingent one, which indicates that Supplier 2 output in disrupted periods under dual sourcing, $d\theta_2^k$, is larger than its delivery $y + w$ when it is used as a contingent source. In what follows, we will consider $d\theta_2^k \leq y + w$ as a possible scenario when contingent sourcing outperforms dual sourcing.

5.1. Analysis of Dual Sourcing. The buyer's long-term expected cost under dual sourcing can be calculated as

$$\begin{aligned} C_d(s,\theta_1,\theta_2) = \sum_{i=1}^{\infty}\pi_i \Big[& h\left(s + id\theta_2^k - (i+1)d\right)^+ \\ & + p\left((i+1)d - s - id\theta_2^k\right)^+ + dc_2\theta_2^k\Big] + \pi_0\left[h(s-d)\right. \\ & \left. + \Delta c\theta_2 d\right]. \end{aligned} \tag{9}$$

Note that $\Delta c\theta_2 d = (c_2 - c_1)\theta_2 d = (c_1\theta_1 d + c_2\theta_2 d) - c_1 d$ denotes the backup purchasing cost in nondisrupted period, which is essentially the additional cost incurred by sourcing from Supplier 2.

In this situation, the buyer needs to allocate their total order optimally and set the corresponding base-stock level by minimizing the expected total cost. We first derive the optimal base-stock level s_d^* under dual sourcing with a given order allocation (θ_1, θ_2) in the following proposition.

Proposition 5. *Under dual souring, given backup output flexibility k, the relationship between the buyer's optimal base-stock level s_d^* and the order allocation parameters (θ_1^*, θ_2^*) satisfies*

$$s_d^* = j^* d\left(1 - \theta_2^{*k}\right) + d\theta_2^{*k}, \tag{10}$$

where j^* (>1) is the smallest integer such that $\sum_{i=0}^{j^*-1}\pi_i \geq p/(p+h)$.

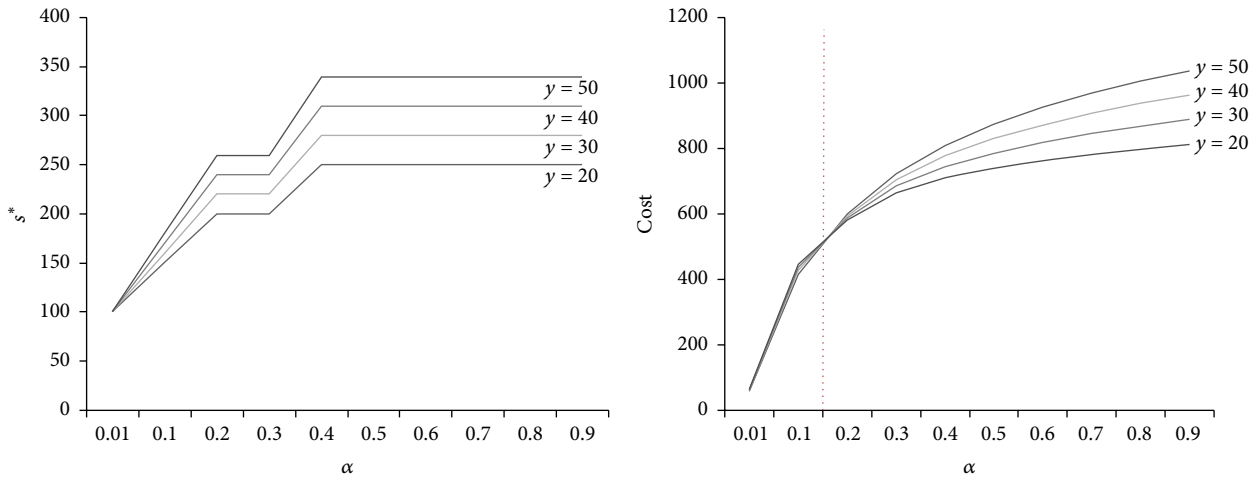
Proof. See Appendix D. □

It can be seen that if the buyer would lower their inventory level, he/she should order more from Supplier 2 and less from Supplier 1 with disruption risks. In contrast, if the buyer relies more on Supplier 1, he/she would stock more to mitigate the supply risks. With the allocation set, (θ_1, θ_2), the larger the output flexibility Supplier 2 has, the lower the inventory level the buyer should maintain.

Before the optimal order allocation under dual sourcing is obtained, we first compare the dual sourcing method and the single sourcing method with Supplier 1 as the major source. We write the buyer's expected cost by using (9):

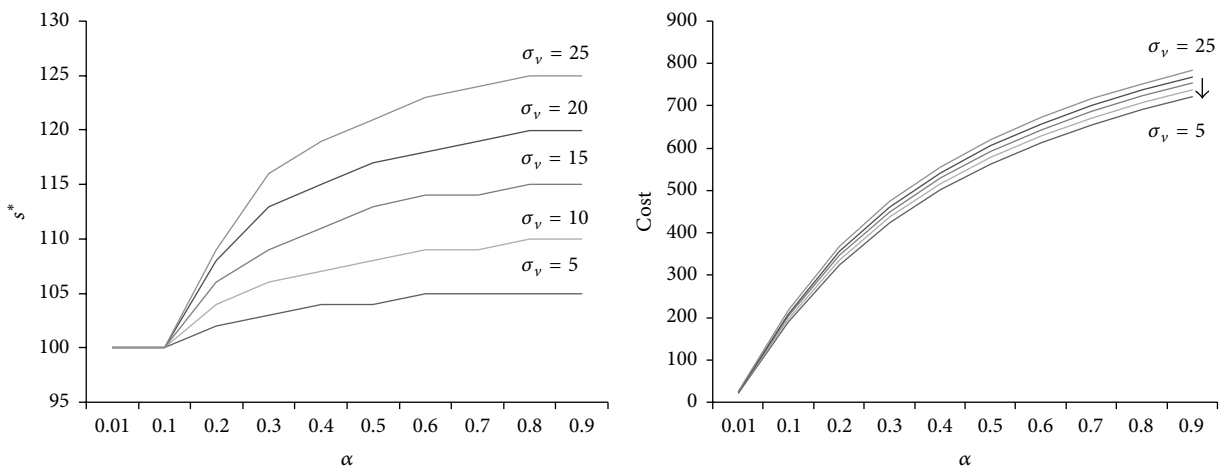
$$\begin{aligned} C_d = & \left(d - d\theta_2^k\right) \\ & \cdot \left[h\sum_{i=0}^{j^*-1}\pi_i\left(j^* - i - 1\right) + p\sum_{i=j^*}^{\infty}\pi_i\left(i + 1 - j^*\right)\right] \\ & + \sum_{i=1}^{\infty}\pi_i dc_2\theta_2^k + \pi_0\Delta c\theta_2 d, \end{aligned} \tag{11}$$

then the following result can be arrived for $0 < k < 1$.



(a) s^* versus y (b) Cost versus y

Figure 3: The impact of backup capacity under single sourcing with contingent supplier.



(a) s^* versus σ_v (b) cost versus σ_v

Figure 4: The impact of backup uncertainty under single sourcing with contingent supplier.

Proposition 6. *For $0 < k < 1$, if the backup supplier's unit wholesale price (c_2) satisfies the relationship*

$$c_2 \geq c_2^L = \frac{h\sum_{i=0}^{j^*-1}\pi_i\left(j^*-i-1\right)+p\sum_{i=j^*}^{\infty}\pi_i\left(i+1-j^*\right)}{1-\pi_0} \tag{12}$$

single sourcing from Supplier 1 (the one with disruption risks and lower wholesale price) then outperforms the dual sourcing method.

Proof. Given $s_d^* = j^*d(1-\theta_2^k)+d\theta_2^k$ we have $dC_d/d\theta_2 = dk\theta_2^{k-1}[c_2\sum_{i=1}^{\infty}\pi_i - h\sum_{i=0}^{j^*-1}\pi_i(j^*-i-1)-p\sum_{i=j^*}^{\infty}\pi_i(i+1-j^*)]+\pi_0 d(c_2-c_1)$. If $c_2\sum_{i=1}^{\infty}\pi_i - h\sum_{i=0}^{j^*-1}\pi_i(j^*-i-1)-p\sum_{i=j^*}^{\infty}\pi_i(i+1-j^*) \geq 0$, then C_d is a convex and increasing function of θ_2, and the optimal value $\theta_2^* = 0$; that is, the buyer would single-source from Supplier 1. The proof is complete. □

Next, we compare dual sourcing and single sourcing with Supplier 2 and arrive at the following result for decision-making.

Proposition 7. *For $0 < k < 1$, if the backup supplier's output flexibility (or wholesale price) meets the requirement*

$$k \geq k_L = \frac{\pi_0\Delta c}{h\sum_{i=0}^{j^*-1}\pi_i\left(j^*-i-1\right)+p\sum_{i=j^*}^{\infty}\pi_i\left(i+1-j^*\right)-c_2\sum_{i=1}^{\infty}\pi_i} \tag{13}$$

or $c_2 < c_2^S$

$$= \frac{k\left(h\sum_{i=0}^{j^*-1}\pi_i\left(j^*-i-1\right)+p\sum_{i=j^*}^{\infty}\pi_i\left(i+1-j^*\right)\right)+\pi_0 c_1}{\pi_0+k\left(1-\pi_0\right)} \tag{14}$$

using Supplier 2 (the one with larger wholesale price but no disruptions) as the only source is better than using both suppliers.

Proof. Given $dC_d/d\theta_2 = dk\theta_2^{k-1}[c_2\sum_{i=1}^{\infty}\pi_i - h\sum_{i=0}^{j^*-1}\pi_i(j^* - i-1) - p\sum_{i=j^*}^{\infty}\pi_i(i+1-j^*)] + \pi_0 d(c_2 - c_1)$, if $(dC_d/d\theta_2)|_{\theta_2=1} = dk[c_2\sum_{i=1}^{\infty}\pi_i - h\sum_{i=0}^{j^*-1}\pi_i(j^* - i - 1) - p\sum_{i=j^*}^{\infty}\pi_i(i+1-j^*)] + \pi_0 d(c_2 - c_1) < 0$, that is, $k \geq \pi_0\Delta c/(h\sum_{i=0}^{j^*-1}\pi_i(j^* - i - 1) + p\sum_{i=j^*}^{\infty}\pi_i(i+1-j^*) - c_2\sum_{i=1}^{\infty}\pi_i)$, then $c_2\sum_{i=1}^{\infty}\pi_i - h\sum_{i=0}^{j^*-1}\pi_i(j^* - i - 1) - p\sum_{i=j^*}^{\infty}\pi_i(i+1-j^*) < 0$ and $dC_d/d\theta_2$ increases with θ_2. Therefore, $dC_d/d\theta_2 \leq 0$ for all $\theta_2 \in [0,1]$, and the optimal value $\theta_2^* = 1$; that is, the buyer would single-source from Supplier 2. The proof is complete. □

The above results indicate that the backup supplier' wholesale price has a critical impact on the buyer's optimal decisions. Specifically, if the backup supplier's wholesale price is too high, the buyer would not use it even if it may have great output flexibility and no risks. But if the backup supplier's output flexibility is small enough or its wholesale price is small enough, the buyer would prefer to allocate the total order to it. Therefore, we could summarize the most profitable sourcing method and its corresponding decisions in the following proposition.

Proposition 8. *In making the optimal decisions of base-stock level s_d^* and order allocation proportion (θ_1, θ_2) for $0 < k < 1$ under dual sourcing, there are two critical values for Supplier 2 wholesale prices, (c_2^S, c_2^L), given by (10) and (13), respectively, which guide the decisions in the following way:*

(i) *if $c_2^S < c_2 < c_2^L$, then choose dual sourcing with*

$$\theta_2^* = \sqrt[k-1]{\frac{k_L}{k}},$$
$$\theta_1^* = 1 - \theta_2^*; \tag{15}$$
$$s_d^* = j^* d - d(j^* - 1)\left(\frac{k_L}{k}\right)^{k/(k-1)};$$

(ii) *if $c_2 > c_2^L$, then choose Supplier 1 as the sole source with $s_d^* = j^* d$;*

(iii) *if $c_2 < c_2^S$ (or $k \geq k_L$), then choose Supplier 2 as the sole source with $s_d^* = d$,*

where j^* (>1) is the smallest integer such that $\sum_{i=0}^{j^*-1}\pi_i \geq p/(p+h)$.

Proof. Using the results in Propositions 6 and 7 yields the conclusions in (ii) and (iii) directly. Besides, under dual sourcing, as

$$\frac{d^2C_d}{d\theta_2^2} = dk(k-1)\theta_2^{k-2}\left[c_2\sum_{i=1}^{\infty}\pi_i - h\sum_{i=0}^{j^*-1}\pi_i(j^* - i - 1) - p\sum_{i=j^*}^{\infty}\pi_i(i+1-j^*)\right], \tag{16}$$
$$c_2^S < c_2 < c_2^L,$$

C_d is a concave function of θ_2. The optimal decision should satisfy $dC_d/d\theta_2 = 0$; that is, $\theta_2^* = \sqrt[k-1]{k_L/k}$, $s_d^* = j^* d - d(j^* - 1)(k_L/k)^{k/(k-1)}$. The proof is complete. □

Proposition 9. *Under dual sourcing $(0 < k < 1)$, the buyer's optimal order allocation to backup Supplier S2, θ_2^*, is an increasing function of k if $k_L < 1$; or if $k_L > 1$, θ_2^* first increases until $k = k_0$, with k_0 satisfying $1 - 1/k_0 + \ln(k_L/k_0) = 0$, and then decreases.*

Proof. As Proposition 8 showed, under dual sourcing, the buyer's optimal order allocation probability satisfies $\theta_2^* = \sqrt[k-1]{k_L/k}$, where $k_L = \pi_0\Delta c/(h\sum_{i=0}^{j^*-1}\pi_i(j^* - i - 1) + p\sum_{i=j^*}^{\infty}\pi_i(i+1-j^*) - c_2\sum_{i=1}^{\infty}\pi_i)$. Then we have $d\theta_2^*/dk = \sqrt[k-1]{k_L/k} = \sqrt[k-1]{k_L/k}(1/(1-k))(1/k + (1/(k-1))\ln(k_L/k)) = -\sqrt[k-1]{k_L/k}(1/k^2(1-k)^2)(1 - 1/k + \ln(k_L/k))$; therefore the sign of $d\theta_2^*/dk$ is opposite to $1 - 1/k + \ln(k_L/k)$. As $d(1 - 1/k + \ln(k_L/k))/dk = (1/k)(1/k - 1) > 0$, $\lim_{k\to 0^+}(1 - 1/k + \ln(k_L/k)) < 0$, and $\lim_{k\to 1}(1 - 1/k + \ln(k_L/k)) = \ln k_L$, if $k_L > 1$, $\ln k_L > 0$, then there must be $k = k_0$ within $(0,1)$ that satisfies $1 - 1/k_0 + \ln(k_L/k_0) = 0$, and when $k < k_0$, θ_2^* increases with k and when $k \geq k_0$, θ_2^* decreases with k; else if $k_L < 1$, $\ln k_L < 0$, then $1 - 1/k + \ln(k_L/k) < 0$, and $d\theta_2^*/dk > 0$, which means θ_2^* increases with k. The proof is complete. □

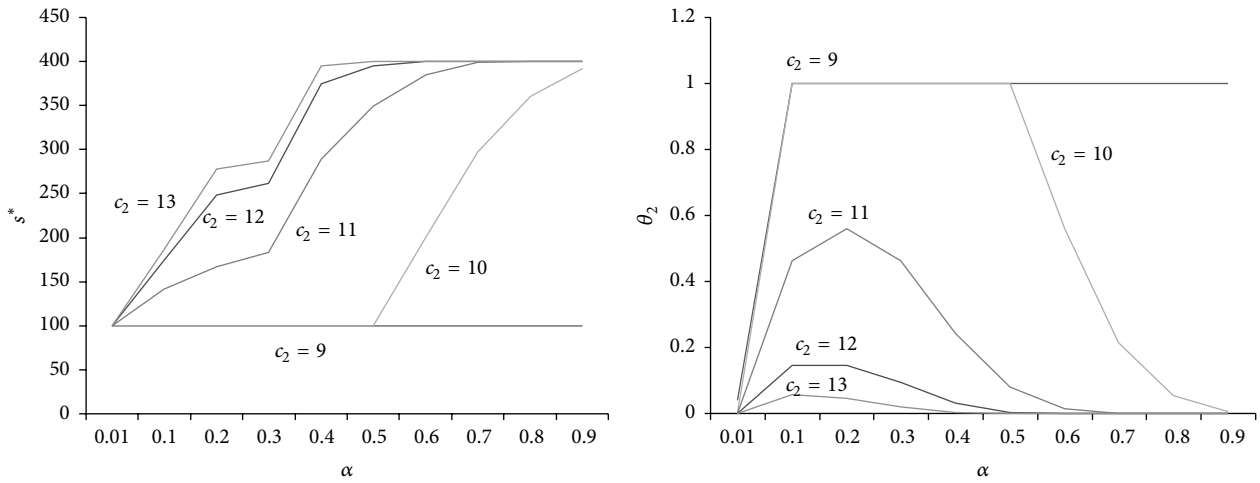
These results indicate that the buyer should not always decrease their order allocation to Supplier 2 if its output flexibility increases and that Supplier 2 should not always lower its output flexibility to obtain more orders. Both of them should evaluate the sign of $k_L - 1$ before making decisions.

Moreover, when $k = 0$, that is, Supplier 2 would satisfy the buyer's total demand during disrupted periods, the buyer would decrease the order proportion θ_2 to the smallest value that Supplier 2 could accept, denoted as $\theta_2 = \overline{\theta_2}$ for illustration purpose, and $s^* = d$.

When $k = 1$, that is, Supplier 2 would not expand its output during disrupted periods, then $(dC_d/d\theta_2)|_{k=1} = d(\sum_{i=1}^{\infty}\pi_i c_2 - h\sum_{i=0}^{j^*-1}\pi_i(j^* - i - 1) - p\sum_{i=j^*}^{\infty}\pi_i(i+1-j^*)) + \pi_0(c_2 - c_1)d$. If $c_2 > c_2^L(1-\pi_0) + \pi_0 c_1 = h\sum_{i=0}^{j^*-1}\pi_i(j^* - i - 1) + p\sum_{i=j^*}^{\infty}\pi_i(i+1-j^*) + \pi_0 c_1$, $C_d|_{k=1}$ increases with θ_2; thus $\theta_2^* = 0$; else $\theta_2^* = 1$. This implies that if Supplier 2 has no output flexibility, the buyer would single-source from either Supplier 1 or Supplier 2, depending on whether its wholesale price is smaller or larger than $c_2^L(1-\pi_0) + \pi_0 c_1$.

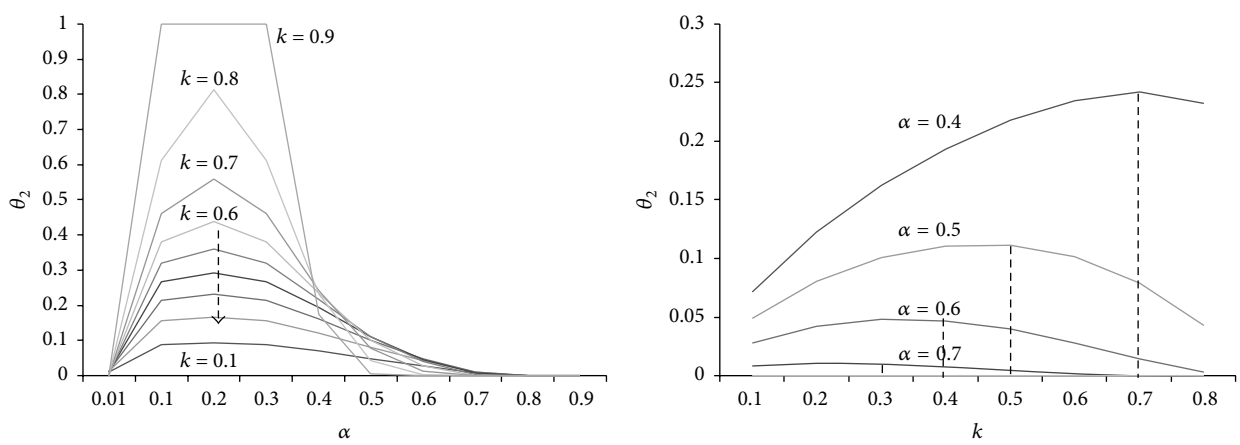
5.2. Numerical Analysis of Dual Sourcing. Similar to Section 4.3, the impacts of supply disruptions on the buyer's optimal decisions and expected cost are examined through numerical analysis, and the value of Supplier 2 output flexibility is also investigated with the basic parameters set in Section 4.3.

5.2.1. The Impact of Supply Disruptions under Dual Sourcing. Under dual sourcing, the buyer's decisions are order allocation proportion (θ_1, θ_2) and base-stock level s. Intuitively, from the buyer's perspective, higher disruption probability



(a) s^* versus α (b) θ_2 versus α

Figure 5: The buyer's optimal decisions s and θ_2 under dual sourcing versus α.



(a) θ_2 versus α (b) θ_2 versus k

Figure 6: The optimal value of θ_2 under dual sourcing versus (α, k).

may lead the buyer to allocate a larger order proportion θ_2; on the other hand, the larger the order proportion to Supplier 2 is, the higher the purchasing cost would be incurred to the buyer. Thus, we will examine the impacts of α on s and θ_2, respectively, under different values of wholesale price c_2 ($k = 0.7$).

It is indicated by Figure 5 that the increase of supply disruption risks raises the buyer's inventory level under dual sourcing, but the buyer would not always allocate more to Supplier 2. Specifically, when α is small ($\alpha \leq 0.2$ in this case), the buyer allocates more to Supplier 2 as disruption risks increase; but when α is large, the buyer decreases Supplier 2 order proportion θ_2. This result implies that if the disruption risk is large enough, the buyer prefers to order less from Supplier 2 and stock more to avoid large purchasing cost. In addition, the buyer's expected cost increases with disruption probability according to our data results.

It is also seen that the backup Supplier S2 wholesale price c_2 plays an important role; in particular, as c_2 increases, the buyer tends to order less from S2 and stock more with a result of larger expected cost. Furthermore, the impact of c_2 decreases as the value of c_2 increases.

5.2.2. The Impact of Output Flexibility under Dual Sourcing. From the results in Propositions 3 and 5, we can deduce that $s_d^* \leq s_c^*$ holds if $k \leq \log_{\theta_2^*}(y/d)$; that is, if Supplier 2 output flexibility is large enough, dual sourcing would reduce its inventory level compared to single sourcing with capacitated backup supply. For more implications concerning the impact of output flexibility, we will rely on numerical results shown in Figures 6-7.

Figure 6 provides the following interesting results: (1) larger output flexibility would bring down the order allocation proportion θ_2 to S2 under low disruption risks ($\alpha < 0.4$ in this case; except at $\alpha = 0.4$ when θ_2 is too small), indicating that backup Supplier S2 would decrease output flexibility to obtain a larger order; but from the buyer's perspective, he/she prefers to order more from S1 if S2 is very flexible;

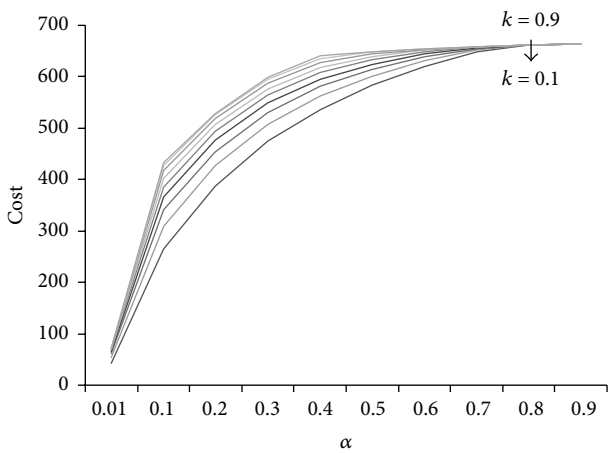


Figure 7: The buyer's expected cost C under dual sourcing versus (α, k).

and (2) when the disruption probability is large ($\alpha \geq 0.4$ in this case), the curve of θ_2 presents a concave function of k (as shown in Figure 6(b)); that is, θ_2 first increases with k and then decreases. This behavior suggests an optimal output flexibility k^* for S2 to maximize its order allocation proportion. Furthermore, the value of k^* tends to decrease with the disruption probability, implying that the larger the disruption risk, the higher the output flexibility S2 should have to maximize order allocation and the smaller the order the buyer should allocate to S2.

Figure 7 presents that higher backup flexibility would help decrease the buyer's expected cost, but its impact is negligible when supply disruption risk is very small ($\alpha \leq 0.1$ in this case) or very large ($\alpha \geq 0.7$ in this case).

6. Comparative Studies of Single and Dual Sourcing

In this section, we will rely on numerical examples to find out the most appropriate method among the four sourcing options: single sourcing from S1 (parameters with subscript 1), single sourcing from S2 (subscript 2), dual sourcing (subscript d), and single sourcing from S1 with S2 as a capacitated and uncertain contingent supply (subscript cu).

First, based on the analysis in Section 5.1, the following results can be given with the consideration of the decision situations of dual sourcing and single sourcing.

Proposition 10. *In selecting the optimum sourcing method, the borderline between dual sourcing or single sourcing from S1 is not affected by S2 output flexibility, while the lower the output flexibility is, the more possible the buyer would select single sourcing from S2 over dual sourcing.*

Proof. As Proposition 8 stated, there are two critical values of Supplier 2 wholesale prices, (c_2^S, c_2^L), and $c_2 < c_2^S$ suggests the possibility of selecting single sourcing from S2 over dual sourcing. As the value of $c_2^L = (h\sum_{i=0}^{j^*-1}\pi_i(j^*-i-1) + p\sum_{i=j^*}^{\infty}\pi_i(i+1-j^*))/(1-\pi_0)$ does depend on k and $c_2^S = (k(h\sum_{i=0}^{j^*-1}\pi_i(j^*-i-1) + p\sum_{i=j^*}^{\infty}\pi_i(i+1-j^*)) + \pi_0 c_1)/(\pi_0 + k(1-\pi_0))$ is an increasing function of k, we can conclude that the possibility of $c_2 < c_2^S$ increases with k with a given c_2. The proof is complete. □

The optimal base-stock levels under four sourcing methods are plotted in Figure 8, with the base-stock level of selected sourcing methods shown in dotted line. We consider four cases: low or high output flexibility ($k = 0.2$ or 0.7) and low or high backup purchasing cost ($c_2 = 9$ or 12).

Two observations can be made from Figure 8: (1) $s_d^*, s_{cu}^* \in [s_1^*, s_2^*]$ holds, and the base-stock level under dual sourcing (s_d^*) can be higher or lower than that under single sourcing with contingent backup supply (s_{cu}^*); (2) the buyer does not always select the sourcing method with the lowest base-stock level and sometimes selects the one with the highest level ($c_2 = 12$, $\alpha \geq 0.6$). Note that the basic value of backup capacity y under single sourcing with contingent supply is set to be 50 in this example, and for the situations in which $y \geq d\theta_2^k$, single sourcing with contingent supply is supposed to outperform dual sourcing due to smaller purchasing cost.

By comparing the buyer's expected costs under the four sourcing methods with disruption parameter α changing from 0.01 to 0.9 and backup output flexibility k from 0.1 to 0.9, we can pinpoint the optimal sourcing method under different values of wholesale price c_2 as shown in Figure 9. As seen from Figure 9, if the difference between c_1 (=8) and c_2 is small ($\Delta c = 1$), the buyer should single-source from S2 except when their output flexibility is very large (dual sourcing) or the supply disruption probability is very small (single sourcing with contingent backup supply). As S2 wholesale price becomes larger, the range of single sourcing from S2 shrinks towards the upper-left corner until it disappears, and the range of single sourcing from S1 extends to the right. This indicates that if the difference between the two suppliers' wholesale price becomes larger, the possibility to single-source from S2 decreases, especially under large disruption risk or output flexibility, and the likelihood of single sourcing from S1 increases especially under large disruption risk.

In addition, single sourcing with contingent supply always outperforms the other methods at very low disruption risk ($\alpha < 0.1$) except when output flexibility is very high ($k \leq 0.2$), with its range slightly expanding with Δc. On the other hand, dual sourcing method always performs the best under small disruption risk and large backup output flexibility.

7. Concluding Remarks

In this paper, we have studied a buyer's decisions when their major supplier encounters supply disruption and backup suppliers are available for selection. The buyer's optimal base-stock level is obtained under single sourcing with contingent supply and dual sourcing, respectively, through minimizing the buyer's long-term cost, which includes overstock cost, penalty cost for unsatisfied demand, and backup purchasing cost. The impacts of three risk parameters, disruption probability, contingent capacity or uncertainty, and backup flexibility, are analyzed.

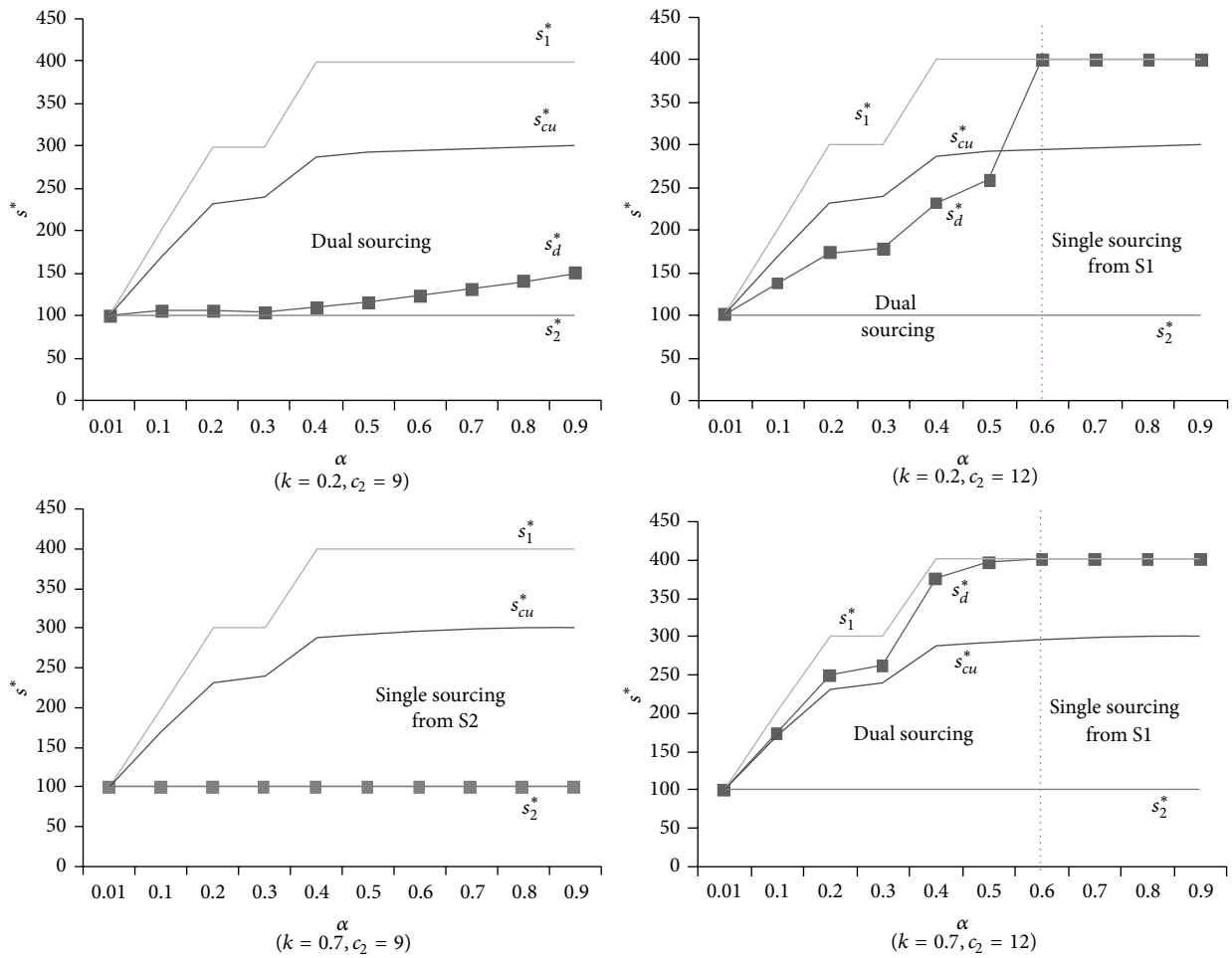


Figure 8: The buyer's optimal base-stock levels under the four sourcing methods.

Specifically, under single sourcing with contingent supply, we show (1) how backup capacity and uncertainty affect the buyer's decision and expected cost, respectively and (2) how the impacts of supply disruption on the buyer's expected costs differ for different types of contingent supplier. Table 1 summarizes our findings about the single sourcing with contingent supply.

Under dual sourcing, we identify the buyer's optimal base-stock level, the optimal order allocations, and also the critical values of wholesale price or output flexibility to determine whether the buyer should select dual sourcing or single sourcing. Our major findings about the impacts of supply disruptions and backup output flexibility are reported in Table 2.

Moreover, four sourcing methods, namely, single sourcing with contingent supply, dual sourcing, and single sourcing from either of the two suppliers, are compared. In choosing the optimal sourcing method, we find that (1) the lower the output flexibility is, the more possible the buyer would select single sourcing from S2 over dual sourcing; (2) single sourcing with contingent supply always outperforms the other methods at very low disruption risk except when output flexibility is very high; and dual sourcing method is always selected under small disruption risk and large backup output flexibility; and (3) if the difference between the two suppliers' wholesale prices is small enough, the buyer usually single-sources from S2; but when their difference becomes larger, single sourcing with S2 would be replaced by dual sourcing first and then single sourcing with S1 under large disruption risks.

We conclude by highlighting three research directions not considered in this paper. First, the backup supplier's profit and decision-making can be included in the models and it would be interesting to explore a Stackelberg game structure. Particularly, the backup supplier's decisions of wholesale price and capacities can be analyzed to maximize its expected profit. Second, the lead time of the backup suppliers can be an important factor for the buyer to consider when selecting a backup source. That is, how to select between single sourcing and dual sourcing when the lead times of the suppliers are different could be a worthwhile research problem. Finally, we can also examine the feasibility of using multiple backup suppliers in the event of supply disruption and figure out the optimal numbers of backup suppliers in long-term horizon decision-making. We hope that these directions for future research will further refine the insights provided in this paper.

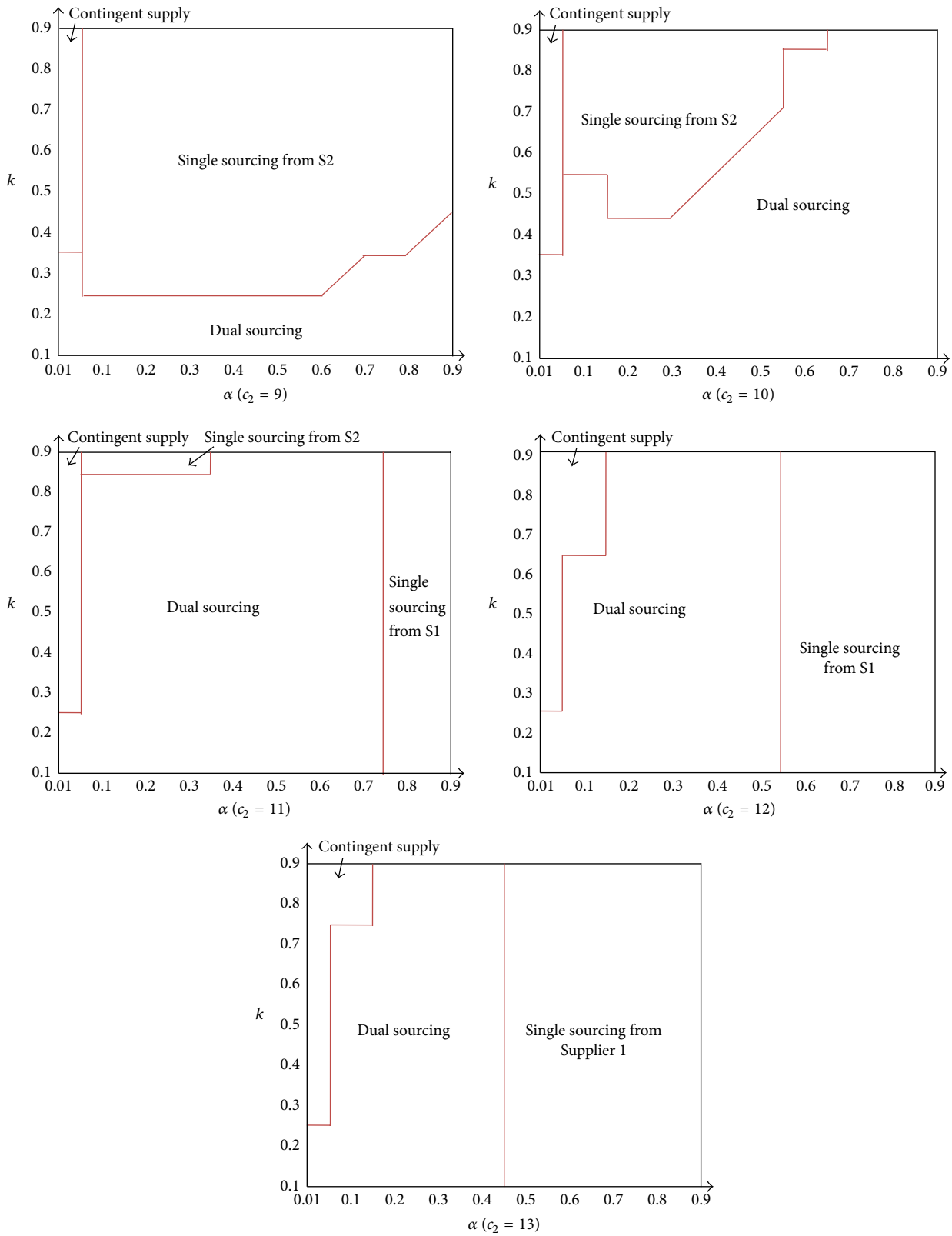


Figure 9: The buyer's optimal sourcing methods versus (α, k).

Table 1: Major results of single sourcing with contingent backup supply.

Properties	Results	
The value of contingent supply	Under small disruption risks, the buyer would benefit from the contingent supplier, especially the one with larger capacity.	Under large disruption risks, the buyer would stock more instead of using a contingent supplier.
The impact of disruption risks	Larger disruption probability increases both the buyer's optimal base-stock level and expected cost.	
The impact of backup capacity	Larger backup capacity increases the buyer's optimal base-stock level.	
The impact of backup uncertainty	Larger backup uncertainty increases both the buyer's optimal base-stock level and expected cost.	

Table 2: Major results of dual sourcing.

Properties	Results	
The impact of disruption risks	Larger disruption probability increases both the buyer's optimal base-stock level and expected cost.	
	Under small disruption risks, the buyer allocates more orders to the backup supplier as disruption risks increase. The backup supplier would decrease its output flexibility to obtain larger order.	Under large disruption risks, the buyer prefers to order less from the backup supplier and stock more inventory as disruption probability increases. The backup supplier would increase its output flexibility.
The impact of backup output flexibility	Under small disruption risks, larger output flexibility leads to smaller order proportion to the backup supplier.	Under large disruption risks, the buyer's optimal order proportion to the backup supplier becomes a concave function of the supplier's output flexibility.
	The impact of output flexibility is very limited when supply disruption risk is very small or too large.	

Appendix

A. Proof for Proposition 2

Since

$$
\begin{aligned}
&C_{cu}(s)\\
&= \sum_{i=1}^{\infty}\pi_i\left[p\left((i+1)d-s-iy-iu_w\right)+(p+h)\sqrt{i}\sigma_w G\left(\frac{(i+1)d-s-iy-iu_w}{\sqrt{i}\sigma_w}\right)\right]+\pi_0\left[h(s-d)^+ + p(d-s)^+\right]+c_1 d\pi_0\\
&\quad + (y+u_w)c_2\sum_{i=1}^{\infty}\pi_i i\\
&= \begin{cases}\displaystyle\sum_{i=1}^{\infty}\pi_i\left[p\left((i+1)d-s-iy-iu_w\right)+(p+h)\sqrt{i}\sigma_w G\left(\frac{(i+1)d-s-iy-iu_w}{\sqrt{i}\sigma_w}\right)\right]+\pi_0 h(s-d)+c_1 d\pi_0+(y+u_w)c_2\sum_{i=1}^{\infty}\pi_i i, & \text{for } s\ge d,\\ \displaystyle\sum_{i=1}^{\infty}\pi_i\left[p\left((i+1)d-s-iy-iu_w\right)+(p+h)\sqrt{i}\sigma_w G\left(\frac{(i+1)d-s-iy-iu_w}{\sqrt{i}\sigma_w}\right)\right]+\pi_0 p(d-s)+c_1 d\pi_0+(y+u_w)c_2\sum_{i=1}^{\infty}\pi_i i, & \text{for } s<d,\end{cases}
\end{aligned}
\tag{A.1}
$$

$$
\frac{dC_{cu}}{ds} = \begin{cases}\displaystyle -\sum_{i=1}^{\infty}\pi_i\left[p+(h+p)\left(\Phi\left(\frac{(i+1)d-s-iy-iu_w}{\sqrt{i}\sigma_w}\right)-1\right)\right]+\pi_0 h, & \text{for } s\ge d,\\ \displaystyle -\sum_{i=1}^{\infty}\pi_i\left[p+(h+p)\left(\Phi\left(\frac{(i+1)d-s-iy-iu_w}{\sqrt{i}\sigma_w}\right)-1\right)\right]-\pi_0 p, & \text{for } s<d,\end{cases}
$$

$$
\frac{d^2C_{cu}}{ds^2} = \sum_{i=1}^{\infty}\pi_i\frac{h+p}{\sqrt{i}\sigma_w}\phi\left(\frac{(i+1)d-s-iy-iu_w}{\sqrt{i}\sigma_w}\right)\ge 0,
$$

C_{cu} is a concave function of s. The optimal decision should satisfy

$$\sum_{i=1}^{\infty}\pi_i\left(\Phi\left(\frac{(i+1)d-s-iy-iu_w}{\sqrt{i}\sigma_w}\right)\right) = \begin{cases}\dfrac{h}{h+p}, & \text{for } s\geq d,\\ \dfrac{h}{h+p}-\pi_0=\dfrac{\alpha h-p\beta}{\alpha(h+p)}, & \text{for } s<d.\end{cases} \tag{A.2}$$

As $\alpha<\beta$ and $h<p$, $(\alpha h-p\beta)/\alpha(h+p)<0$; we can have that when $s<d$, C_{cu} decreases with s. Therefore, the optimal value of s^* either equals d or satisfies $\sum_{i=1}^{\infty}\pi_i(\Phi(((i+1)d-s-iy-iu_w)/\sqrt{i}\sigma_w))=h/(h+p)$. The proof is complete.

B. Proof for Proposition 3

Suppose $s^*-y=j(d-y)$, where j is an integer; then $s^*-y-i(d-y)=j(d-y)-i(d-y)=(j-i)(d-y)$. If $d-y\geq 0$, when $i+1\leq j$, $s^*-y-(i+1)(d-y)\geq 0$, and vice versa.

Thus

$$\begin{aligned}C_c(s)&=\sum_{i=0}^{\infty}\pi_i\left[h\left(s-y-(i+1)(d-y)\right)^+\right.\\&\quad\left.+p\left((i+1)(d-y)-s+y\right)^+\right]+c_2y\sum_{i=1}^{\infty}\pi_i i=\sum_{i=0}^{j-1}\pi_i\\&\quad\cdot h\left(s-y-(i+1)(d-y)\right)+\sum_{i=j}^{\infty}\pi_i\\&\quad\cdot p\left((i+1)(d-y)-s+y\right)+c_2y\sum_{i=1}^{\infty}\pi_i i,\\\frac{dC_c}{ds}&=h\sum_{i=0}^{j-1}\pi_i-p\sum_{i=j}^{\infty}\pi_i.\end{aligned} \tag{B.1}$$

If $dC_c/ds=h\sum_{i=0}^{j-1}\pi_i-p\sum_{i=j}^{\infty}\pi_i\geq 0$, C_c increases with s. Therefore, s^* takes the smallest value; that is, j takes the smallest integer so that $dC_c/ds=h\sum_{i=0}^{j-1}\pi_i-p\sum_{i=j}^{\infty}\pi_i=h\sum_{i=0}^{j-1}\pi_i-p+p\sum_{i=0}^{j-1}\pi_{i-1}=(h+p)\sum_{i=0}^{j-1}\pi_i-p\geq 0$, which is also $\sum_{i=0}^{j-1}\pi_i\geq p/(p+h)$. Else if $dC_c/ds=h\sum_{i=0}^{j-1}\pi_i-p\sum_{i=j}^{\infty}\pi_i\leq 0$, j takes the largest integer so that $\sum_{i=0}^{j-1}\pi_i\leq p/(p+h)$. Because $\sum_{i=0}^{j-1}\pi_i$ increases with j, then the two values of j are the same for $dC_c/ds\geq 0$ and $dC_c/ds\leq 0$.

Thus the proof is complete.

C. Proof for Proposition 4

Because

$$\begin{aligned}C_u(s)&=\sum_{i=1}^{\infty}\pi_i\\&\quad\cdot\left[h\int_{d-s}^{\infty}(s+v-d)m(v)\,dv+p\int_{-\infty}^{d-s}(d-s-v)m(v)\,dv\right]\\&\quad+\pi_0\left[h(s-d)^++p(d-s)^+\right]+c_2(d+u_v)\sum_{i=1}^{\infty}\pi_i,\\\frac{dC_u}{ds}&=\begin{cases}\sum_{i=1}^{\infty}\pi_i\left[h-(h+p)\int_{-\infty}^{d-s}m(v)\,dv\right]+\pi_0h & \text{for } s\geq d\\ \sum_{i=1}^{\infty}\pi_i\left[h-(h+p)\int_{-\infty}^{d-s}m(v)\,dv\right]-\pi_0p & \text{for } s<d\end{cases}\\&=\begin{cases}\dfrac{\alpha}{\alpha+\beta}\left[h-(h+p)\int_{-\infty}^{d-s}m(v)\,dv\right]+\dfrac{\beta}{\alpha+\beta}h & \text{for } s\geq d\\ \dfrac{\alpha}{\alpha+\beta}\left[h-(h+p)\int_{-\infty}^{d-s}m(v)\,dv\right]-\dfrac{\beta}{\alpha+\beta}p & \text{for } s<d\end{cases}\\&=\begin{cases}h-\dfrac{\alpha}{\alpha+\beta}(h+p)\int_{-\infty}^{d-s}m(v)\,dv & \text{for } s\geq d\\ \dfrac{\alpha h-\beta p}{\alpha+\beta}-\dfrac{\alpha}{\alpha+\beta}(h+p)\int_{-\infty}^{d-s}m(v)\,dv & \text{for } s<d,\end{cases}\\\frac{d^2C_u}{ds^2}&=\sum_{i=1}^{\infty}\pi_i(h+p)m(d-s)\geq 0,\end{aligned} \tag{C.1}$$

C_u is a concave function of s. And the optimal decision should satisfy

$$\int_{-\infty}^{d-s}m(v)\,dv = \begin{cases}\dfrac{h}{h+p}\dfrac{1}{1-\pi_0}=\dfrac{(\alpha+\beta)h}{\alpha(h+p)}, & \text{for } s\geq d,\\ \dfrac{h-(h+p)\pi_0}{(h+p)(1-\pi_0)}=\dfrac{\alpha h-p\beta}{\alpha(h+p)}, & \text{for } s<d.\end{cases} \tag{C.2}$$

As $\alpha<\beta$ and $h<p$, $(\alpha h-p\beta)/\alpha(h+p)<0$, C_u decreases with s when $s<d$.

Therefore, the optimal base-stock level should satisfy $\int_{-\infty}^{d-s^*}m(v)dv=(\alpha+\beta)h/\alpha(h+p)$ or $s^*=d$. The proof is complete.

D. Proof for Proposition 5

Suppose $s^*-d\theta_2^k=j(d-d\theta_2^k)$, where j is an integer; then when $i+1\leq j$, $s^*-d\theta_2^k-(i+1)(d-d\theta_2^k)\geq 0$, and vice versa. Furthermore,

$$\begin{aligned}C_d(s)&=\sum_{i=1}^{\infty}\pi_i\left[h\left(s+id\theta_2^k-(i+1)d\right)^+\right.\\&\quad\left.+p\left((i+1)d-s-id\theta_2^k\right)^++dc_2\theta_2^k\right]\\&\quad+\pi_0\left[h(s-d)+\Delta c\theta_2d\right]=\sum_{i=1}^{j-1}\pi_ih\left(s-d\theta_2^k\right.\end{aligned}$$

$$- (i+1)\left(d - d\theta_2^k\right)\Big) + \sum_{i=j}^{\infty}\pi_i p\left((i+1)\left(d - d\theta_2^k\right)\right.$$

$$\left. - s + d\theta_2^k\right) + \sum_{i=1}^{\infty}\pi_i d c_2 \theta_2^k + \pi_0\left[h(s-d) + \Delta c\theta_2 d\right],$$

$$\frac{dC_d}{ds} = h\sum_{i=1}^{j-1}\pi_i - p\sum_{i=j}^{\infty}\pi_i + \pi_0 h = h\sum_{i=0}^{j-1}\pi_i - p\sum_{i=j}^{\infty}\pi_i = (h + p)\sum_{i=0}^{j-1}\pi_i - p \tag{D.1}$$

increases with j (which also means dC_d/ds increases with s; i.e., $d^2C_d/ds^2 > 0$).

If $dC_d/ds = (h+p)\sum_{i=0}^{j-1}\pi_i - p \geq 0$, then C_d increases with s, so s^* takes the smallest value; that is, j takes the smallest integer so that $dC_d/ds = (h+p)\sum_{i=0}^{j-1}\pi_i - p \geq 0$, which is also $\sum_{i=0}^{j-1}\pi_i \geq p/(p+h)$. Else if $dC_d/ds = (h+p)\sum_{i=0}^{j-1}\pi_i - p < 0$, j takes the largest integer so that $\sum_{i=0}^{j-1}\pi_i < p/(p+h)$. Because $\sum_{i=0}^{j-1}\pi_i$ increases with j, then the two values of j are the same for $dC_d/ds \geq 0$ and $dC_d/ds \leq 0$.

Thus the proof is complete.

The List of Symbols Used in the Paper

α: The disruption probability or the transition probability from the normal to the disrupted state
β: The recovery probability from the disrupted to the normal state
π_i: The steady-state probability that the major supply has been disrupted for i consecutive periods, $i \in [0, \infty)$
d: The average demand per cycle (unit)
p: Unit understock cost of the buyer (\$/unit) ($p > c_2 + h$)
h: Unit overstock cost of the buyer (\$/unit)
s: The buyer's optimal base-stock level, a decision variable (unit)
L: The customer service-level
C: The buyer's expected total cost per period
$\Phi(\cdot)$: The standard normal distribution function
$G(\cdot)$: The standard normal loss function
y: Supplier 2 average backup capacity during disrupted period under single sourcing ($y \leq d$)
w: Supplier 2 additive random quantity w under single sourcing with capacitated and uncertain yield (it follows a normal distribution $N(u_w, \sigma_w)$, with $F(\cdot)$ as its distribution function and $f(\cdot)$ as its density function)
v: Supplier 2 yield uncertainty under single sourcing with uncertain yield (it follows a normal distribution, $N(u_v, \sigma_v)$, with $G(\cdot)$ as its distribution function and $g(\cdot)$ as its density function)
θ_i: The proportion of demand allocated to supplier i under dual sourcing ($i = 1, 2$, and $\theta_1 + \theta_2 = 1$)
k: Supplier 2 output flexibility during disrupted period under dual sourcing ($0 \leq k \leq 1$).

Competing Interests

The authors declare that they have no competing interests.

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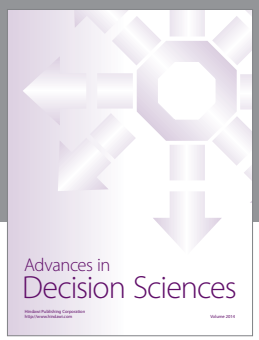

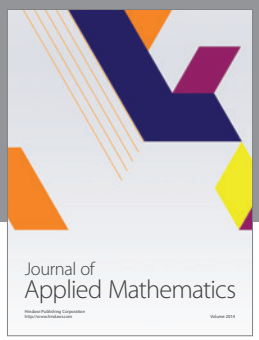

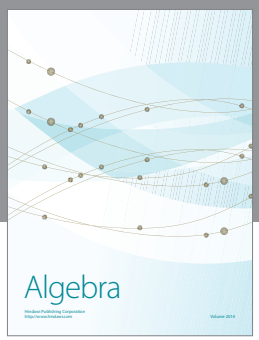

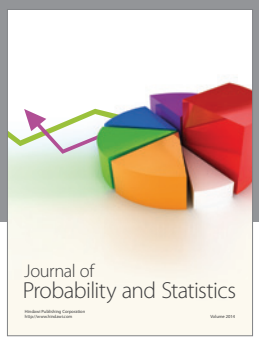

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