Research Article

Dynamic Analysis and Optimization of a Production Control System under Supply and Demand Uncertainties

Qiankai Qing, 1, 2 Wen Shi, 3 Hai Li, 4 and Yuan Shao 5

1 School of Automation, Huazhong University of Science and Technology, Wuhan 430074, China
2 School of Automobile and Traffic Engineering, Wuhan University of Science and Technology, Wuhan 430080, China
3 School of Logistics and Engineering Management, Hubei University of Economics, Wuhan 430205, China
4 School of Business Administration, Zhongnan University of Economics and Law, Wuhan 430073, China
5 School of Management, Wuhan University of Science and Technology, Wuhan 430080, China

Correspondence should be addressed to Hai Li; hikee09@126.com

Received 12 May 2016; Revised 23 July 2016; Accepted 26 July 2016

Academic Editor: Francisco R. Villatoro

Copyright © 2016 Qiankai Qing et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This study investigates the dynamic performance and optimization of a typical discrete production control system under supply disruption and demand uncertainty. Two different types of uncertain demands, disrupted demand with a step change in demand and random demand, are considered. We find that, under demand disruption, the system's dynamic performance indicators (the peak values of the order rate, production completion rate, and inventory) increase with the duration of supply disruption; however, they increase and decrease sequentially with the supply disruption start time. This change tendency differs from the finding that each kind of peak is independent of the supply disruption start time under no demand disruption. We also find that, under random demand, the dynamic performance indicators (Bullwhip and variance amplification of inventory relative to demand) increase with the disruption duration, but they have a decreasing tendency as demand variance increases. In order to design an adaptive system, we propose a genetic algorithm that minimizes the respective objective function on the system's dynamic performance indicators via choosing appropriate system parameters. It is shown that the optimal parameter choices relate closely to the supply disruption start time and duration under disrupted demand and to the supply disruption duration under random demand.

1. Introduction

Recent years have witnessed various unforeseen disasters, such as terrorist acts, accidents, and natural calamities (e.g., earthquakes, floods, and hurricanes), implying that our world is becoming increasingly fragile and uncertain. Meanwhile, the popularity of industry trends and practices such as global sourcing, the division of the product process, the decreased number of supply sources, and increased dependence on critical suppliers makes supply chains more vulnerable. Apparently, all these factors may cause material flow in supply chain to interrupt and, hence, supply disruption may arise.

As a main type of supply uncertainty, supply disruption has received increasing attentions from both academic and industrial circles because it may cause significant downtime of production resources, considerable delay in customer delivery, financial loss, and, finally, a loss in a firm’s market value [1]. Examples of supply chain vulnerability are widespread. In March 2000, a sudden fire in a local Royal Philips Electronics plant caused the supply breakdown of microchips for Ericsson. Ericsson’s market share after the fire shrank remarkably, and the company subsequently lost its leadership in the mobile market [2]. Kobe earthquake in 1994 left many companies without parts [3]. The experimental study by Hendricks and Singhal [4] demonstrates that the average percentage decline of a firm’s market value of shares is 10.28% when the disruption of material supply lasts for just two days.

Demand uncertainty is another typical category of uncertainty in realistic supply chains. Demand uncertainty often arises when demand fluctuates under the impacts of some internal and external random factors or when demand
experiences a sudden change for various reasons [5]. Because uncertainties on both the supply and demand sides occur in a dynamic environment, their occurrences are usually accompanied by changes in the system’s dynamic performance. For instance, in a production control system, either supply or demand uncertainty will cause the production order rate and the system’s inventory level to fluctuate over time, which may generate considerable costs. In particular, such costs become even more remarkable if supply and demand uncertainties exist simultaneously.

In practice, the coexistence of supply and demand uncertainties is not uncommon because firms are often exposed to a highly uncertain environment in modern supply chains. Accordingly, coexistent supply and demand uncertainties have attracted much attention from researchers (e.g., [6–9]). However, the combined impacts of supply and demand uncertainties on the dynamic performance of production control systems have been generally overlooked by the literature. Apparently, this ignorance may cause a firm to significantly underestimate the negative effect of multiple sources of uncertainties and to undertake remarkable excessive production and inventory carrying costs.

By incorporating both supply disruption and demand uncertainty, this study investigates the dynamic performance of a typical production control system (automatic pipeline, inventory, and order based production control system, APIOBPCS). The APIOBPCS is representative of many realistic production control systems with an inherent feedback loop involved in manufacturers’ production and inventory decisions [10, 11] and has been extensively studied in the literature (e.g., [12, 13]). The APIOBPCS investigated in our study is a discrete-time system with system state variables updated at fixed time intervals. Our study differs from studies on production control systems because they generally focus on the dynamic analysis under a single source of uncertainty (e.g., [14–16]). To the best of our knowledge, none of the previous studies simultaneously considers supply and demand uncertainties in such a typical production control system.

Our main research objective is twofold. First, we analyze the impacts of supply disruption and demand uncertainty on the APIOBPCS’s dynamic performance. Second, we attempt to design an adaptive system with appropriate system parameters in the presence of uncertainties on both the supply and demand sides. We assume that the APIOBPCS has single supply source and, thereby, that the material flow towards the APIOBPCS is completely inoperative under supply disruption. Two categories of uncertain demands, disrupted demand with a sudden change in system demand and random demand obeying normal distribution, are investigated; in particular, we adopt step change in demand under disrupted demand. The desired inventory level of the product in the APIOBPCS is zero and the demand unmet by on-hand inventory can be backordered, as assumed in the literature (e.g., [12]).

We apply the systems dynamics method to model the APIOBPCS and investigate the system’s dynamic performance under supply and demand uncertainties. Systems dynamics, which was initially introduced by Forrester [17], is a powerful method suitable for studying complex economic and social systems with inherent feedback loops. In particular, it has been widely adopted for dynamic analysis of various production control systems (e.g., [15, 16, 18–20]). In the case of demand disruption, a step change in demand occurs in the beginning of the simulation time horizon, and we adopt the peaks of the order rate, production completion rate, and inventory level as the main dynamic performance indicators. The impacts of the supply disruption start time and duration on these peaks are investigated. In the case of random demand, we adopt Bullwhip, the ratio of the order rate variance and demand variance, and inventory fluctuation amplification relative to demand variance as the main dynamic performance indicators. The simulation is conducted in Matlab/Simulink.

We find that, under demand disruption, as the supply disruption duration increases, the order rate peak, the production completion rate peak, and inventory peak increase if the supply disruption start time is sufficiently small and otherwise first remain unchanged and then increase. However, given any supply disruption duration, each kind of peak first increases and then decreases to a constant as the supply disruption start time increases. This change tendency relates to the fact that, under demand disruption, the production completion rate first is less than the demand rate and then is larger than the demand rate. Subsequently, the inventory peak first increases and then decreases with the increase in the supply disruption start time. Given this result, the peaks of the production order rate and the completion rate peak, which have a similar change tendency, also increase and decrease with the supply disruption start time sequentially. Apparently, this change tendency is distinct from that in the case without demand disruption, in which these peak values are independent of the supply disruption start time.

We also discover that when demand is normally distributed, either the Bullwhip or inventory variance amplification (the ratio of inventory variance and demand variance) has a decreasing tendency as the demand variance enlarges for a given supply disruption duration. This outcome is related to the effects of risk pooling under which the changes in the variances of the order rate and inventory are less remarkable than those of the demand variance. Both the Bullwhip and inventory variance amplification increase strictly with the supply disruption duration under different values of demand variance. This result stems from the fact that the APIOBPCS becomes increasingly unstable when supply disruption lasts longer.

In order to design a more adaptive system under multiple uncertainties, we propose a genetic algorithm that optimizes the APIOBPCS’s dynamic performance by selecting proper system parameters (e.g., consumption averaging time, time to adjust inventory, and time to adjust work-in-progress). In particular, under the case of demand disruption, the weighted sum of the order rate peak and inventory peak is adopted as the objective function to minimize. We discover that both the supply disruption duration and start time have significant impacts on the selected optimal parameters. The optimization of the APIOBPCS is further investigated when the disruption start time and duration obey certain
distributions. Under the case of random demand, we choose the minimum weighted sum of Bullwhip and inventory error amplification as the objective function. The results show that the optimal inventory adjusting time is relatively stable, while the optimal consumption averaging time has a decreasing tendency as the disruption duration increases.

The rest of this study is organized as follows. Section 2 reviews some relevant literature. Section 3 introduces the simulation model. Section 4 investigates the impacts of supply and demand uncertainties on the system's dynamic performance. Section 5 focuses on the optimization problem of the system under supply and demand uncertainties. Section 6 provides concluding remarks.

2. Literature Review

There has been a growing body of literature on supply chain disruption risk in the operations management and related fields. Early studies have focused on the classification of difference types of risk and distinguished between them. Supply chain disruption risks can be categorized into supply-related, demand-related, and miscellaneous risks [21]. In particular, supply disruption occurs when a firm’s order cannot be met by its supplier, demand disruption is usually related to a sudden change of demand, and miscellaneous risks are those that may potently affect a firm’s business costs [22]. Sheffi and Rice Jr. [23] categorize risks based on the probability of disruption occurrence and the consequences of risk. Kleindorfer and Saad [24] classify supply chain risks into high-likelihood, low-impact risks and low-likelihood, high-impact risks.

Supply disruption has significant impacts on firms’ performance [25, 26]. Hendricks and Singhal [4] discover that short-term supply disruptions can lead to an obvious decrease in shareholder value. Furthermore, the average decrease in shareholder value approaches can reach up to almost 40% over a three-year period. Oke and Gopalakrishnan [21] argue that supply risks can be mitigated by the better planning and coordination of supply and demand, flexible capacity, identifying supply chain vulnerability points, and so forth. In addition, to cope with the negative effect of supply disruption, scholars propose different strategies, for example, multisourcing (e.g., [27–29]), alternative sources of materials (e.g., [30, 31]), and flexible supply (e.g., [32]). Our study differs from theirs because we focus on dynamic performance in a typical production control system under a single supply source when supply disruption and demand uncertainty coexist.

Demand disruption has also drawn increasing attentions from scholars. One stream of related research studies the supply chain coordination problem under demand disruption. Xiao et al. [33] propose a price-subsidy rate contract to coordinate the competing retailers’ investments under sales promotion and demand disruptions. Xiao and Qi [34] study how to coordinate a supply chain with one manufacturer and two competing retailers under demand disruption in the case of quantity discount schedule. Chen and Xiao [5] apply a linear quantity discount schedule and a wholesale price schedule to coordinate a two-stage supply chain with one dominant retailer and multiple fringe retailers after demand disruptions. Soleimani et al. [35] study the optimal pricing strategies in dual-channel supply chains under the coexistence of demand and production cost disruptions. Another stream of related research involves the dynamic analysis in production control systems when demand disruption arises in the form of step change in demand, as will be discussed later on.

A number of studies consider both supply and demand uncertainties in various supply chain contexts (e.g., [8, 9, 36–42]). Wang and Gerchak [36] apply stochastic dynamic programming to study periodic review production models in the presence of random yields and uncertain demand. Feng [39] focuses on an integrated decision-making process on pricing under both demand and supply uncertainties based on dynamic programming approach. Kouvelis and Li [40] build a dynamic programming model to investigate an offshore outsourcing arrangement for a buyer in the presence of uncertain supply yield and demand. Sting and Huchzermeier [9] study a manufacturer's dual sourcing decisions under supply and demand uncertainty when both high-margin offshore supply and reliable local supply are selectable. Peidro et al. [41] propose a mathematical programming model for supply chain planning in the presence of supply, demand, and process uncertainties. Yang and Ma [42] adopt a two-stage game model to study a two-part tariff contracting in a supply chain with two suppliers having uncertain supply and one retailer facing stochastic demand. All these studies apply mathematical analysis approaches.

Simulation approaches are also applied in the literature on supply and demand uncertainties. Petrovic et al. [43] study the modeling and discrete simulation of the fuzzy supply chains by considering both supply and demand uncertainties. Schmitt and Singh [44] develop a discrete event simulation model to assess firms’ vulnerability to supply chain disruption risk under stochastic demand. Using discrete event simulation approach, Schmitt and Singh [45] study the effect of inventory placement and back-up methodologies on reducing supply chain risk under supply and demand uncertainties. Jung et al. [46] build a discrete event simulation model to evaluate the performance functions in multistage supply chains under multiple sources of uncertainties. Mahnam et al. [47] use a hybridization approach based on simulation to optimize the supply chain performance with two criteria on cost and fill rate. Using a simulation approach, Mohbibi and Choobineh [48] investigate the combined effects of component commonality, demand uncertainty, and late procurement-order arrivals on the performance of a two-level assemble-to-order system. These papers do not involve the dynamic analysis of the APIOBPCS when supply and demand uncertainties arise, which is our research concern.

Our study is closely related to the literature on dynamic analysis of the APIOBPCS and relevant production control systems based on the systems dynamics simulation method. Towill [18] investigate the ability of an inventory and order based production control system (IOBPCS) to recover from disrupted demand (a step change in demand) based on systems dynamics modelling. The APIOBPCS, which is initially proposed by John et al. [10], is an improved version of IOBPCS which incorporates pipeline feedback.
Disney et al. [12] apply the systems dynamics method to study the dynamics of the APIOBPCS under demand disruption and design an adaptive system robust to change in lead time. The dynamic performances of the APIOBPCS under disrupted demand or stochastic demand are also studied in the literature (e.g., [15]). Wang et al. [49] investigate the stability of a constrained production control system with nonnegative order rate under different replenishment policies. The criteria for different types of dynamic behavior in a similar system are proposed by Wang et al. [16]. Wilson's study [50] is among the few studies that apply systems dynamics to study a system's dynamic performance under supply disruption and demand uncertainty. The research objective is comparing the effects of transportation disruption on system performance in a multistage supply chain between when VMI is implemented and when it is not. Our study is different because we concentrate on the combined impacts of supply disruption and demand uncertainty on the dynamic performance and optimization of the APIOBPCS.

### 3. Model Setting

In this section, we first briefly describe the APIOBPCS and then model the simulation under both supply disruption and demand uncertainty in this system.

#### 3.1. Description of APIOBPCS

The model chosen for our study is APIOBPCS, which is representative of many practical production control systems for manufacturers. In particular, the APIOBPCS can be described as an order-up-to (OUT) policy with the exponential smoothing forecasting and proportional feedback controllers [51]. To ensure normal production, the manufacturer needs one type of material, which is supplied by an external supply source, to make product in the APIOBPCS. The causal loop form of the APIOBPCS is constructed based on the systems dynamics method (see Figure 1). There are two feedback loops in the figure. The first is related to the recovery of the actual inventory (AINV) towards a desired inventory level (DINV) and customer demand satisfaction. The second relates to the recovery of the work-in-progress (WIP) to a desired work-in-progress (DWIP) level. Figure 1 shows how different system factors interact with each other in the feedback loops, based on which the manufacturer makes production and inventory decisions.

In Figure 1, ORATE denotes the manufacturer's order rate for the next period and ACON denotes the actual demand or consumption rate from the customer. ORATE is targeted to be equal to the sum of an exponentially smoothed ACON over \( T_a \) time period (denoted as "AVCON"), plus a fraction \( (T_i) \) of the error in AINV (denoted as "EINV"), plus a fraction \( (T_w) \) of the error in WIP (denoted as "EWIP") (John et al. [10]). In the APIOBPCS, EINV corresponds to the difference between DINV and AINV, and EWIP corresponds to the difference between DWIP and WIP. Let COMRATE denote the production completion rate, and COMRATE is a delayed version of ORATE. We assume that production lead time \( (T_p) \) is exponentially distributed. Let \( T_r \) denote the estimated pipeline lead time. Then, the desired work-in-progress DWIP equals the product of the average consumption rate AVCON and \( T_r \). Some notations are summarized as follows:

- **ACON**: actual consumption/demand rate
- **AVCON**: average consumption/demand rate
- **ORATE**: order rate
- **COMRATE**: production completion rate
- **AINV**: actual inventory level
- **DINV**: desired inventory level
- **EINV**: error in inventory level
- **WIP**: work-in-progress
- **DWIP**: desired work-in-progress
- **EWIP**: error in work-in-progress
- **\( T_a \)**: consumption/demand averaging time
- **\( T_i \)**: time to adjust inventory
- **\( T_w \)**: time to adjust work-in-progress
- **\( T_p \)**: production delay time
- **\( T_r \)**: estimated pipeline lead time
- **\( T_d \)**: duration of supply disruption
- **\( t \)**: period of the time
- **\( \Delta t \)**: simulation step size
- **\( \mu_{ACON} \)**: mean value of demand
- **\( \sigma^2_{ACON} \)**: variance of demand
- **\( \sigma^2_{ORATE} \)**: variance of ORATE
- **\( \sigma^2_{AINV} \)**: variance of inventory level
- **\( \mu_{EINV} \)**: expectation of square EINV
Some difference equations are given as follows:

\[
ACON_t = \begin{cases} 
0 & \text{if } t < 0 \\
1 & \text{if } t \geq 0 
\end{cases} 
\]

if demand undergoes a step change,

\[
ACON_t \sim N(\mu_{ACON}, \sigma^2_{ACON})
\]

if demand is normally distributed,

\[
AVCON_t = AVCON_{t-\Delta t} + \frac{1}{1 + T_a/\Delta t}(ACON_t - AVCON_{t-\Delta t}),
\]

\[
ORATE_t = \begin{cases} 
0 & T_s \leq t < T_s + T_d \\
\frac{EINV_t}{T_i} + \frac{EWIP_t}{T_w} & \text{else,}
\end{cases}
\]

\[
COMRATE_t = \begin{cases} 
0 & T_s \leq t < T_s + T_d \\
\frac{ORATE_t}{T_p} + \frac{COMRATE_{t-\Delta t}}{1 + T_p/\Delta t} & \text{else,}
\end{cases}
\]

\[
AINV_t = AINV_{t-\Delta t} + (COMRATE_t - ACON_t) \ast \Delta t,
\]

\[
EINV_t = DINV_t - AINV_t,
\]

\[
WIP_t = WIP_{t-\Delta t} + (ORATE_t - COMRATE_t) \ast \Delta t,
\]

\[
DWIP_t = AVCON_t \ast T_r,
\]

\[
EWIP_t = DWIP_t - WIP_t.
\]

(1)

### 3.2. Simulation Model.

We use Matlab/Simulink to develop the simulation model. Simulink is a window-oriented dynamics modeling software package that has been widely applied in the dynamics simulation (e.g., [52–55]). Figure 2 is the systems dynamics model for the APIOBPCS built in Simulink under supply and demand uncertainties. This simulation model is developed based on the conceptual model of the APIOBPCS described above and, hence, the above-mentioned difference equations directly apply to it. In general, the simulation model is composed of various variables (rate and state variables), one exponential smoothing block, one production delay block, two integrators, and several mathematical function blocks. The rate variable includes ACON, AVCON, ORATE, and COMRATE. The state variable contains the variables related to inventory (AINV, DINV, and EINV) and work-in-progress (WIP, DWIP, and EWIP); DINV is assumed to be zero without loss of generality. AINV is the integral of COMRATE minus ACON, and WIP is the integral of ORATE minus COMRATE. The simulated system is a discrete-time system updated periodically, and the simulation step size (time incremental) \( \Delta t \) equals 1. All system variables are initially set to zero.

The APIOBPCS’s dynamic performance under supply and demand uncertainties is investigated. We consider supply uncertainty in the form of supply disruption. The material flow towards the APIOBPCS is interrupted when supply disruption arises. Let \( T_s \) and \( T_d \) denote the start and end of the supply disruption period, respectively.

and duration of supply disruption, respectively. Once supply disruption occurs, the manufacturer’s production order before the disruption ends becomes invalid; in other words, $\text{ORATE}_t = 0$ for $T_s \leq t < T_s + T_d$. Such a change situation of ORATE in the presence of supply disruption is captured by a switch block of the simulation model in Figure 2. During supply disruption, COMRATE may not always be zero because an order placed before the disruption can still be completed. In order to capture various types of demand uncertainties, we consider two patterns of demand uncertainties. The first pattern is demand disruption with a step change in demand (the demand jumps from zero to one). We assume that the disrupted demand occurs at the beginning of the time horizon ($t = 0$). The second pattern is the random demand with normal distribution; we let $\mu_{\text{ACON}}$ and $\sigma^2_{\text{ACON}}$ denote the mean value and variance for customer demand, respectively.

For the case of a step change in demand, we adopt ORATE peak, COMRATE peak, and AINV peak (peak denotes the maximum value during the entire simulation time horizon) as the main investigated dynamics indicators; peak value is commonly adopted to evaluate a production control system’s dynamic performance when the system undergoes a step input in demand [56]. For the case of random demand, we choose Bullwhip and inventory fluctuation amplification as the main investigated dynamics indicators. Bullwhip corresponds to the variance amplification of ORATE relative to customer demand and has been extensively investigated in the literature (e.g., [57]). The mathematical definition of Bullwhip is as follows:

$$\text{Bullwhip} = \frac{\sigma^2_{\text{ORATE}}}{\sigma^2_{\text{ACON}}} \tag{2}$$

where $\sigma^2_{\text{ORATE}}$ represents the variance of ORATE. Inventory fluctuation amplification includes inventory variance amplification ($\text{VA}_t$) and inventory error amplification ($\text{EA}_t$), and

$$\text{VA}_t = \frac{\sigma^2_{\text{AINV}}}{\sigma^2_{\text{ACON}}} \tag{3}$$

$$\text{EA}_t = \frac{\mu_{\text{EINV}}^2}{\sigma^2_{\text{ACON}}}$$

where $\sigma^2_{\text{AINV}}$ represents the variance of AINV and $\mu_{\text{EINV}}^2$ represents the expectation of square EINV. Both $\text{VA}_t$ and $\text{EA}_t$ reflect the amplification of inventory fluctuation relative to demand variance, and the main difference between them is that the latter relates to inventory recovery towards the desired level.

Our main concern is twofold. First, we investigate the relationships between the APIOBPCS’s dynamics indicators and related impact factors. Second, we turn to the system optimization problem and develop a genetic algorithm that optimizes the system’s dynamic performance via parameter selection. The optimization procedure is programmed into Matlab and introduced in Section 5. We follow Sargent [58] to validate our simulation model. Two industrial engineers from a manufacturing firm worked closely with the authors throughout the project and verified the simulation model. In addition, our simulation model can be indirectly verified and validated because it is similar to the simulation model of the APIOBPCS studied in the literature (e.g., [11, 12]) except that we incorporate multiple sources of uncertainties.

### 4. Dynamic Analysis

In this section, we adopt the system parameters $T_a = 16$, $T_i = T_p = 8$, and $T_d = 8$ recommended for the APIOBPCS by John et al. [10]. The dynamic analyses for the two patterns of demand uncertainties under supply disruption are conducted, respectively.

#### 4.1. Demand Disruption

In this subsection, we first examine the impacts of $T_d$ and $T_i$ on the peaks of ORATE, COMRATE, and AINV under demand disruption and then compare such impacts with that under no demand disruption. The simulation time horizon is from 0 to 150 ($t \in [0, 150]$).

To investigate the impacts of $T_d$ and $T_i$ under demand disruption, we consider two different cases with and without supply disruption, respectively (see Figure 3). In the case without supply disruption (Figure 3(a)), AINV first decreases to a peak value and then returns to stability, while ORATE and COMRATE first increase to a peak value and then return to stability. Such a change tendency relates to the system’s feedback mechanism when demand disruption occurs. When demand jumps from 0 to 1 at the beginning ($t = 0$), AINV starts to decrease because ACON is larger than COMRATE. Then, in order to increase AINV to the desired level, the system needs to increase ORATE (see the related mathematical relationship between ORATE and AINV in the notations and difference equations given previously). Thus, as a delayed version of ORATE, COMRATE increases as well. As COMRATE increases, it can increase AINV when COMRATE becomes larger than ACON. Consequently, AINV decreases and increases sequentially. On the other hand, ORATE and ORATE will decrease when AINV increases; hence, either of them increases and decreases in sequence.

In the case with supply disruption (Figure 3(b); $T_d = 10$), ORATE experiences a stepwise decrease when supply disruption starts and then jumps from zero to the peak value when it ends. Accordingly, COMRATE first decreases and then increases to the peak value (note that production lead time is exponentially distributed and, thus, production does not necessarily equal zero during the disruption). For similar reasons to those stated above, after supply disruption, both ORATE and COMRATE will decrease and return to stability. Similar to the case without supply disruption, the AINV first decreases to a peak value and then returns to stability. By comparison, we find that each of the ORATE peak, COMRATE peak, and AINV peak increases compared with those in the case without supply disruption; to be more specific, the peaks of ORATE, COMRATE, and AINV equal 2.681, 1.728, and $-15.16$, respectively, in this case, whereas they equal 1.333, 1.231, and $-8.387$, respectively, in the former case.

In order to further illustrate the impacts of supply disruption, we let both the disruption start time $T_i$ and duration $T_d$...
vary continuously from 0 to 100 in the simulation (see Figure 4). The related peak values for \textsc{orate}, \textsc{comrate}, and \textsc{ainv} are shown in Figures 4(a), 4(b), and 4(c), respectively. From Figure 4, each kind of peak has an increasing tendency, with the increase of \( T_d \) for any given \( T_s \); meanwhile, each kind of peak has an increasing tendency and a decreasing tendency in sequence as \( T_s \) increases for any given \( T_d \). Such change tendencies of peak values can be observed more intuitively by letting one of \( T_d \) and \( T_s \) change continuously while fixing the other, which will be discussed in the following.

Figures 5(a), 5(b), and 5(c) depict the change tendencies of \textsc{orate} peak, \textsc{comrate} peak, and \textsc{ainv} peak as \( T_d \) changes for fixed \( T_s \), respectively. From these figures, each kind of peak strictly increases with \( T_d \) if \( T_s \) is relatively small (\( T_s = 0, 2, 8, 16 \)) but first remains constant and then increases with \( T_d \) if \( T_s \) is relatively large (e.g., \( T_s = 100 \)). This result relates to the comparison of \textsc{comrate} and \textsc{acon} (consumption rate) when the system undergoes a step input in demand. If \( T_s \) is relatively small, \textsc{comrate} is smaller than \textsc{acon} and, thus, \textsc{ainv} is still decreasing when the disruption starts. Given this case, as the disruption duration increases, the \textsc{ainv} peak will increase, which subsequently leads the peaks of \textsc{orate} and \textsc{comrate} to increase as well. If \( T_s \) is sufficiently large, the system recovers to stable state after the step input in demand. Therefore, \textsc{ainv} will not drop to the minimal value before the disruption unless the disruption duration is large enough. Therefore, the \textsc{ainv} peak amounts constantly to the minimal value of \textsc{ainv} before the disruption for relatively small \( T_d \) and increases with \( T_d \) if it is sufficiently large. Based on the change
tendency of the AINV peak, we can infer that ORATE peak and COMRATE peak have similar change tendencies as $T_d$ increases.

Figures 6(a), 6(b), and 6(c), respectively, depict the change tendencies of the ORATE peak, COMRATE peak, and AINV peak as $T_s$ changes for fixed $T_d$. From these figures, each kind of peak first increases and then decreases with $T_s$. When $T_s$ is relatively small, for fixed $T_d$, COMRATE is much smaller than ACON before disruption, so there exists a value difference between the integral of COMRATE and the integral of ACON before the AINV peak arrives. When $T_s$ increases within a certain range, for fixed $T_d$, COMRATE will remain less than ACON before disruption, which leads to an increasing value difference between the integral of COMRATE and ACON before the AINV peak arrives. When $T_s$ increases beyond a certain range, for fixed $T_d$, COMRATE will exceed ACON before disruption, which results in the decreased value difference between the integral of COMRATE and ACON before the AINV peak arrives. When $T_s$ is sufficiently large, the system has recovered to a
4.2. Random Demand. As mentioned in Section 3.2, random demand obeys a normal distribution. We assume $\mu_{\text{ACON}} = 1$ and $\sigma^2_{\text{ACON}} \in [0,0.1]$ for the normally distributed demand. The simulation time horizon ranges from 0 to 400 ($t \in [0,400]$), and the simulation step size equals 1 ($\Delta t = 1$). We assume that supply disruption occurs when the system is in a relatively stable situation, which differs from the case of demand disruption in which supply disruption may occur alongside a step change in demand; without loss of generality, we let $T_d = 200$. Two dynamic performance indicators, Bullwhip (variance amplification of ORATE relative to demand) and VA_T (variance amplification of AINV relative to demand), are investigated; Bullwhip is measured for the time interval $[T_s + T_p, T_s + T_d + 100]$, and VA_T is measured for the time interval $[T_r, T_r + T_d + 100]$.

We first discuss the impacts of demand variance on Bullwhip and VA_T with a fixed disruption duration. Suppose three cases with $\sigma^2_{\text{ACON}}$ equalling 0.02, 0.04, and 0.08, respectively, and $T_d = 10$. By simulation, we derive the change situations of ORATE and AINV for these cases (see Figure 7). It is shown in each case that ORATE jumps to a very high value when the disruption ends and that AINV is decreasing during the disruption. Both ORATE and AINV gradually return to stability after the disruption finishes. In these cases, Bullwhip equals 2.40, 1.82, and 1.20, respectively, and VA_T equals 134.74, 77.57, and 71.73, respectively; this indicates that both Bullwhip and VA_T decrease with $\sigma^2_{\text{ACON}}$.

To further justify such a negative relationship, we let $\sigma^2_{\text{ACON}}$ vary continuously from 0.01 to 0.1 for different fixed values of $T_d$. The related change situations of Bullwhip and VA_T can be seen from Figure 8; it is clearly shown from the figures that either Bullwhip or VA_T has a decreasing tendency as $\sigma^2_{\text{ACON}}$ increases. This outcome can be attributed to risk pooling under which the changes in the variances of the order rate and inventory are less remarkable than those for the demand variance.

We now discuss the impacts of disruption duration on Bullwhip and VA_T. Similarly, we suppose three cases with $T_d = 10$, $T_d = 20$, and $T_d = 30$, respectively, and $\sigma^2_{\text{ACON}} = 0.05$. The change situations of ACON, ORATE, and AINV for these cases are shown as in Figure 9. In these cases, Bullwhip equals 1.63, 5.39, and 11.23, respectively, and VA_T equals 92.4, 438.9, and 1160.8, respectively; this indicates that both Bullwhip and VA_T have an increasing tendency as $T_d$ increases. Such an increasing tendency can be further justified in Figure 10, in which Bullwhip and VA_T change with $T_d$ for $T_d \in [0,20]$ under different values of $\sigma^2_{\text{ACON}}$.

5. Optimization of the APIOBPCS

In this section, we focus on the optimization of the system's dynamic performance in the APIOBPCS under supply and demand uncertainties by choosing appropriate system parameters $T_w$, $T_d$, and $T_w$. Without loss of generality, we assume $T_w = T_d = 8$. Under demand disruption, the weighted sum of the ORATE peak and AINV peak is selected as the manufacturer's objective function to minimize (COMRATE is a delayed version of ORATE and, thus, is not considered).
Let $F_d$ be the objective function, let $\text{ORATE}_p$ be the ORATE peak, and let $\text{AINV}_p$ be the AINV peak. Then, we obtain the manufacturer's minimization problem:

$$\min_{T_a, T_i, T_w} F_d = w_1 \text{ORATE}_p + w_2 \text{AINV}_p,$$

(4)

where $w_1$ and $w_2$ ($w_1 + w_2 = 1$) denote the weighting factors of $\text{ORATE}_p$ and $\text{AINV}_p$, respectively.

Under random demand, we choose the weighted sum of Bullwhip and $E_A$ ($E_A$ reflects the amplification of inventory error relative to demand variance as defined in Section 3.2) as the objective function to minimize. $E_A$, rather than $\text{VA}_I$, is adopted in the objective function because minimizing $E_A$ contributes to inventory recovery towards the desired level.

Let $F_r$ denote the manufacturer's objective function. Then, we obtain the manufacturer's minimization problem:

$$\min_{T_a, T_i, T_w} F_r = \alpha_1 \text{Bullwhip} + \alpha_2 E_A,$$

(5)

where $\alpha_1$ and $\alpha_2$ ($\alpha_1 + \alpha_2 = 1$) are, respectively, the weighting factors of Bullwhip and $E_A$. Apparently, solving the above maximization problems in (4) and (5) analytically is highly impractical because of the high complexity of the system under multiple uncertainties. Departing from this, we propose a genetic algorithm that approximately minimizes these objective functions. It is worth noting that the genetic algorithm (GA), an artificial intelligent algorithm based on Darwin's Theory of Evolution, is applied broadly in various research fields. In particular, it has also been applied in the
5.1. Optimization under Demand Disruption. In this subsection, we apply the above-described GA to choose the optimal system parameters under supply disruption and demand disruption. The superscript “*” represents the optimal values under the GA. As an example to illustrate the GA, we assume equal weights for ORATE, and AINV, (w1 = w2 = 0.5) in the objective function. In the following, we first discuss the case where the disruption start time and duration have a single fixed value and then present the case where either or both of them obey certain distributions.

5.1.1. Disruption Start Time and Duration with a Single Value. Based on the optimization procedure in Figure II, we can derive the optimal parameters when both T_s and T_d with a single value are located in the value range [1, 15]; the related results are summarized in Table 1. It is seen from Table 1 that the optimal system parameters are significantly impacted by T_s and T_d. T^*_a, which is nondecreasing with T_d for any given T_s. In particular, it is optimal to make T^*_a as small as possible (T^*_a = 1) for relatively small T_s (1 ≤ T_s ≤ 8); T^*_a first remains unchanged and then increases as T_d increases for relatively large T_s (9 ≤ T_s ≤ 15). T^*_a changes differently with T_d under different values of T_s. More specifically, T^*_a is nondecreasing with T_d if T_s is relatively small (1 ≤ T_s ≤ 3), is nonincreasing with T_d if T_s is relatively large (6 ≤ T_s ≤ 15), and is nonincreasing and nondecreasing with T_d sequentially otherwise (4 ≤ T_s ≤ 5). T^*_w has a nondecreasing tendency as T_d increases if T_s is relatively large (6 ≤ T_s ≤ 15), but the relationship between T^*_w and T_d is not apparent if T_s is relatively small (1 ≤ T_s ≤ 5).

5.1.2. Disruption Start Time and Duration with Certain Distributions. We first consider the case in which either T_s or T_d...
**Table 1:** Optimal parameters and objective function values for fixed $T_s$ and $T_d$.

<table>
<thead>
<tr>
<th>$T_d$</th>
<th>$T_s$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>$T_i^*$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

$T_d^*$, $T_i^*$, $T_w^*$, $F_d^*$
follows a certain distribution. Without loss of generality, we let $T_s$ obey a uniform distribution in the value range $[0, 16]$ with the expectation of 8. $T_d$ has a fixed value and equals 8, 16, or 24. Under the GA, we can obtain the optimal parameters (see Table 2). From Table 2, the optimal system parameters have different change tendencies; more specifically, as $T_d$ increases, $T_s^*$ remains unchanged, $T_i^*$ increases, and $T_w^*$ increases and decreases sequentially.

We now turn to the case where both $T_s$ and $T_d$ have certain distributions. Let $T_s$ follow the identical uniform distribution with the expectation of 8 in the value range $[0, 16]$. $T_d$ is equally likely to be one of 8, 16, and 24. Then, we can derive the optimal parameters as shown in Table 3.

From Table 3, we observe that the optimal values of the system parameters and objective function (except for $T_w^*$) are equal to or very close to that for $T_d = 16$ under the former case (see Table 2). This result occurs because the distributions

\[ \begin{array}{c|c|c|c|c} T_d & T_s^* & T_i^* & T_w^* & F^* \\ \hline 8 & 1 & 2 & 10 & 8.84 \\ 16 & 1 & 3 & 256 & 17.04 \\ 24 & 1 & 16 & 5 & 26.06 \end{array} \]

\[ \begin{array}{c|c|c|c|c} T_s^* & T_i^* & T_w^* & F^* \\ \hline 1 & 3 & 27 & 17.65 \end{array} \]
Table 4: Optimal parameters and objective function values for fixed $\sigma_{ACON}^2$ and \( T_d \).

<table>
<thead>
<tr>
<th>( T_d )</th>
<th>( \sigma_{ACON}^2 )</th>
<th>0.01</th>
<th>0.02</th>
<th>0.03</th>
<th>0.04</th>
<th>0.05</th>
<th>0.06</th>
<th>0.07</th>
<th>0.08</th>
<th>0.09</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>33</td>
<td>48</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>31</td>
<td>38</td>
<td>16</td>
<td>32</td>
<td>4</td>
<td>44</td>
<td>7</td>
<td>7</td>
<td>2</td>
<td>38</td>
<td>42</td>
</tr>
<tr>
<td>3</td>
<td>33</td>
<td>31</td>
<td>10</td>
<td>32</td>
<td>3</td>
<td>43</td>
<td>11</td>
<td>49</td>
<td>36</td>
<td>44</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>30</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>37</td>
<td>12</td>
<td>49</td>
<td>2</td>
<td>43</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>32</td>
<td>33</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>40</td>
<td>13</td>
<td>49</td>
<td>36</td>
<td>42</td>
<td>14</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>27</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>38</td>
<td>17</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>23</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>38</td>
<td>20</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>14</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>38</td>
<td>22</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>41</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>16</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>39</td>
<td>20</td>
<td>2</td>
<td>1</td>
<td>42</td>
<td>14</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>17</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>51</td>
<td>23</td>
<td>1</td>
<td>1</td>
<td>40</td>
<td>14</td>
</tr>
<tr>
<td>11</td>
<td>1</td>
<td>18</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>39</td>
<td>22</td>
<td>1</td>
<td>1</td>
<td>38</td>
<td>14</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>18</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>38</td>
<td>21</td>
<td>1</td>
<td>1</td>
<td>36</td>
<td>14</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>19</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>38</td>
<td>22</td>
<td>1</td>
<td>1</td>
<td>38</td>
<td>14</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>16</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>39</td>
<td>27</td>
<td>1</td>
<td>1</td>
<td>38</td>
<td>14</td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>20</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>40</td>
<td>26</td>
<td>1</td>
<td>1</td>
<td>32</td>
<td>14</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>7</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

\( T_a \), \( T_i \), and \( T_w \) represent different time intervals or factors affecting the parameters, and \( F_r \) is a function related to these parameters.
5.2. Optimization under Random Demand. In this subsection, we use the GA to determine the optimal system parameters under supply disruption and normally distributed demand. The mean value of the demand, $\mu_{\text{ACON}}$, is assumed to be 1 without loss of generality. In the objective function in (5), Bullwhip is measured for the time interval $[T_i + T_d, T_i + T_d + 100]$, and $E_A$ is measured for the time interval $[T_i, T_i + T_d + 100]$. Based on the GA, we derive the optimal parameters and objective function values for $T_d \in [1, 1.5]$ and $\sigma^2_{\text{ACON}} \in [0.01, 0.1]$ (see Table 4).

From Table 4, the values of $T_d^*$ are sufficiently small and remain stable (equaling 1 or 2) for almost all combinations of $T_d$ and $\sigma^2_{\text{ACON}}$. This result relates to the fact that $E_A$ is relatively larger than Bullwhip for relatively large $T_i$. Because Bullwhip is equally weighted in the objective function to minimize, it is optimal to decrease $E_A$ by choosing sufficiently small $T_i$, while Bullwhip increases with $T_i$. Overall, the value of $T_d^*$ has a decreasing tendency, as $T_d^*$ increases for most values of $\sigma^2_{\text{ACON}}$ (except for $\sigma^2_{\text{ACON}} = 0.07$ or sufficiently small $T_d$). This outcome stems from the fact that $E_A$ increases more rapidly than Bullwhip as $T_d$ increases for given $T_d$. Therefore, it is optimal to decrease $T_d^*$ to achieve smaller $E_A$, given larger $T_d$. The values of $T_w^*$ are relatively small (no more than 10) for most combinations of $T_d$ and $\sigma^2_{\text{ACON}}$. This result occurs because, given that $E_A$ is larger than Bullwhip, it is optimal to choose smaller $T_w^*$ to decrease $E_A$ and the objective function. The objective function value $F_r^*$ increases strictly with $T_d$ for given $\sigma^2_{\text{ACON}}$ and has a decreasing tendency as $\sigma^2_{\text{ACON}}$ increases for given $T_d$.

6. Conclusion

This study investigates the dynamic performance of the API-OBPCS and the optimization of the system in the presence of supply disruption and demand uncertainty. In the case of disrupted demand with a step change in demand, we find that the order rate peak, production complete rate peak, and inventory peak increase with the supply disruption duration if the disruption start time is relatively small but otherwise first remain unchanged and then increase with the disruption duration. Meanwhile, given any supply disruption duration, each kind of peak first increases and then decreases gradually to a constant as the supply disruption start time increases. Thus, the impacts of the supply disruption start time on the system's dynamic performance under demand disruption differ from that under no demand disruption because the start time does not affect the dynamic performance if no demand disruption arises. When demand is random and obeys a normal distribution, we find that the higher demand variance has a decreasing effect on both Bullwhip and inventory variance amplification for a given supply disruption duration. In contrast, as the supply disruption duration increases, Bullwhip and inventory variance amplification increase for a given demand variance.

In order to improve the API-OBPCS's dynamic performance, we propose a well-designed genetic algorithm, based on which the optimal system parameters (consumption averaging time, time to adjust the inventory, and time to adjust work-in-progress) are selected. Under a step change in demand, we choose the weighted sum of the order rate peak and inventory peak as the objective function to minimize. It is shown that the optimal consumption averaging time is nondecreasing with the supply disruption duration, the optimal time to adjust inventory has different change tendencies with an increase in the supply disruption duration, and the optimal time to adjust work-in-progress is nondecreasing with the supply disruption duration for a relatively large duration; the optimal time to adjust inventory is nondecreasing, nonincreasing, and nondecreasing with the supply disruption start time in sequence. In addition, the system optimization under multiple possible values of the supply disruption start time and duration is also studied. Under random demand, the weighted sum of Bullwhip and inventory error amplification is chosen as the objective function to minimize. The results demonstrate that the correlation between the optimal parameter selection and demand variance is nonsignificant, while the correlation between the optimal parameter selection and the supply disruption duration is significant.

Competing Interests

The authors declare that they have no competing interests.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (nos. 71401129, 71501147, 71402048, and 71502176) and China Postdoctoral Science Foundation (nos. 2016M592340 and 2015M582228).

References


---
