Research Article

Cross-Shareholdings Structural Characteristic and Evolution Analysis Based on Complex Network

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This study depicts the network morphology of firms which establish ties through cross-shareholdings by the theory of complex network analysis method. It calculates some complex network properties of the cross-shareholdings network and analyzes the evolution law of network structure in nearly 7 years. The network clearly displays small world properties and scale-free properties. The cross-shareholdings network average path length and clustering coefficient is with a small amplitude fluctuation; the network structure is relatively stable. Such a study is of practical importance and could provide opportunities for policy makers to improve the performance of the cross-shareholdings network.

1. Introduction

Nowadays, with the development of capital market, the corporate cross-shareholdings as a means of capital operation has been widespread in the mature capital market such as Japan and Germany. In recent years, the quantity of cross-shareholdings increases year by year in our country; cross-shareholdings phenomenon also attracted the attention of some scholars concerned.

With the influence of the development of macro economy and the company, cross-shareholding is the most popular in Japan and Germany as a relatively common phenomenon in capital markets. It is considered to be the important feature of corporate governance of “Japan and Germany model” [1]. Because of cross-shareholdings behaviour in Chinese enterprise appearing relatively late, the domestic study of cross-shareholding is relatively less introduced [2, 3]. Chu and Wang considered cross-shareholdings advantageous to the enterprise economic resources advantages combined [4]. Wang et al. found that the degree distribution obeys the power law distribution network structure, network structure evolution from the more dispersed industry as the centre to banks and other financial companies as the core [5]. Li and Ma constructed cross-shareholdings network and stock correlation network that make use of listed companies in Shanghai and Shenzhen stock markets and analyzed the complexity of cross-shareholdings behaviour [6]. Zhao thought the number of cross-shareholdings listed companies in our country stock market is increased, but the proportion of shareholding is generally low and institutional investors particularly fund shareholding that is very active and the regional and industry apparent clustering [7]. Liu and Zhang found that cross-shareholdings can reduce the risk of stock market, with continuous cross-shareholdings companies stock risk lower compared with the cross-shareholdings companies [8]. Sha and Zeng found that the position characteristics of network have no significant impact on corporate performance, but, compared with the state-owned enterprises, the network position characteristics of non-state-owned enterprises, especially private enterprises, have significant positive role on corporate performance [9].

We can find, from the perspective of research methods, that the study of cross-shareholdings is mainly limited in qualitative analysis phase of theoretical research in our
country; empirical research is relatively less. From the point of view of research, most research from the perspective of corporate governance studies the effects of cross-shareholdings and rarely focuses on cross-shareholdings from the perspective of network. Therefore, this article researches evolution law of cross-shareholdings based on complex network, and this study has certain reference significance in the field of investment in the securities market.

## 2. Sample Selection and Network Construction

Date for the cross-shareholding of listed firms was provided by the Securities Times (STCN) and the Wind Database. This paper selects the cross-shareholding of 300 index listed firms from 2007 to 2013 as research samples. The 300 index sample firms cover about sixty percent market value of the Shanghai and Shenzhen stock market and have well market representativeness. This article studies the inherent nature and evolution law of 300 index cross-shareholdings network that can realize the inner link of shareholding relationship in China stock market and provides some useful reference for the capital market investment.

The date for shareholding for each year was represented as a graph $G$ with $N$ nodes and $E$ edges, an associated adjacency matrix $X = [a_{ij}]$. The cross-shareholding network graph $G$ consists of nodes representing listed firms, and an edge between two nodes means that there is a shareholding relationship that links them. In other words, if a shareholding relationship $A$ consists of nodes $a_i$, that is, $A = \{a_1, a_2, \ldots, a_n\}$, then in the cross-shareholdings network the nearest neighbors of the node $a_i$ are $a_2, a_3, \ldots, a_n$. The node degree $k_i$ in this topology is the total number of nodes reachable using a single route and the distance can be interpreted as the number of transfers (plus one) one has to take to get from one stop to another. It is important to note that the objective of the study is not studying the underlying physical structure of the networks but of the movement of people between the different nodes. As such, when we say two nodes $i$ and $j$ are connected, $a_{ij} = 1$, we mean that there is cross-shareholding relation between firm $i$ and firm $j$ during the year. Such a representation has already been used to represent cross-shareholding between listed firms [9–12].

### 3. Topological Properties and Evolution

#### 3.1. Basic Properties

Figure 1 provides some computed network statistics, from basic network properties such as density and the degree of nodes to the more complex metrics such as clustering and path length.

The density of network, a measure of its closeness, is calculated using the following equation:

$$\delta = \frac{m}{n(n-1)}.$$  (1)

where $n$ is the total number of nodes in the network and $m$ is the number of edges in the network. In our case, the network density is defined as the closeness of holding association between cross-shareholding firms. The higher network density means that the more holding relationship between firms, the more powerful the functions of selecting, delivering, and processing information in network and the more the ways of gaining fund and information.

The network is a sparse network, and it possesses a low density of about 0.006 from 2007 to 2013, indicating that holding relationship between firms could be a large improved space. The network density is presented in Figure 1(a). Cross-shareholdings network density increases from 0.0087 to 0.0102 by 2009 and then decreased to 0.0054 by 2013. That is to say, cross-shareholding network density increases first and then decreases, indicating that the closeness of holding association increases first and then decreases. This result may be due to the reform of the shareholder structure in listed companies that was basically completed from 2007 to 2009; therefore, the ownership relationship between listed companies is unstable. But, as time goes on, network density in each period of time had no obvious change from 2010 to 2013 and the closeness of holding association restores stability. This result is similar to those researches for securities market network [13], which may be due to the sample firms having some industry differences, across different regions, which caused the low network density.

The degree of a node, a measure of its connectivity, is defined as the fraction of nodes with degree $k$ in the network. In our case, the degree is defined as the number of firms that can be reached from a given city via a single route. For a given node $i$, the degree can be represented using

$$k_i = \sum_{j} a_{ij}.$$  (2)

The average degree ($\langle k \rangle$) of the whole graph is average degree ($k_i$) over all nodes ($i$), and it can be obtained using the following equation:

$$\langle k \rangle = \frac{1}{N} \sum_{i} k_i = \frac{1}{N} \sum_{i} \sum_{j} a_{ij}.$$  (3)

The network possesses a low average degree form 2.977 in 2007, down to 2.113 in 2008; then from 2.797 in 2009 it decreases to 1.979 in 2013. The average degree of the network is presented in Figure 1(b). From the data, it is evident that the connection between companies gradually reduced in the network evolution process. But, on the whole, the average degree of network change is relatively stable, only on a small scale fluctuation, which shows that the shareholding relationship between companies is relatively stable. This can avoid enterprise management risk cause from frequent changes of shareholding relationship, guaranteeing the stability of corporate strategy and business model, promoting the company stable development.

The clustering coefficient of a node $i$ is defined as the ratio of the number of links shared by its neighboring nodes to the maximum number of possible links among them. Simply put, the clustering coefficient is a measure of cohesiveness around a given node $i$ and it is defined by the equation

$$C_i = \frac{2M_i}{k_i(k_i - 1)} = \frac{2}{k_i(k_i - 1)} \sum_{j,h} a_{ij}a_{ih}a_{jh}.$$  (4)
where $M_i$ is the number of edges between nodes $i$’s neighbors and $2/k_i(k_i - 1)$ is a normalization factor equal to the maximum number of possible edges among the neighbors. Because of this normalization, $C_i$ is in the interval $[0, 1]$ where 0 and 1 indicate that none or all node $i$’s neighbors are linked, respectively. The average clustering coefficient can thus be represented by the following mathematical expression:

$$\langle C \rangle = \frac{1}{N} \sum_{i=0}^{N} C_i. \quad (5)$$

Using the above equation, the change trend of average clustering coefficient ($C$) of the network presents “W” type. That is to say, the clustering coefficient decreases first then increases, and then decreases and at last increases. So the clustering coefficient of network has volatility in the evolution process. The average clustering coefficient of the network is presented in Figure 1(c). The date presents a large up from 2012, showing that shareholding relationship between the listed firms is increasing in recent years. The frequent circulation of capital, information technology sharing makes companies through ownership, transactions, and cooperation form alliance to promote the sustainable development of the company.

The average shortest path length (the minimum number of edges passed through to get from one node to another) between one node and all other nodes of the network is calculated using the following equation:

$$l = \frac{1}{N(N - 1)} \sum_{i \neq j} d(i, j), \quad (6)$$

where $d(i, j)$ is the shortest path from $i$ to $j$, and $N$ is the total number of nodes in the network.

A small average path length, for example, $l = 1.409$, in 2013, means that there is shareholding relationship between almost all listed firms of 300 index. Table 1 has a very good analysis of cross-shareholdings network path length. The company shareholding minimum distance is one in 2013, appearing 187 times, and the maximum is three, but the distance is there rarely, only 7.5%. The most appearing time is $L = 1$ from 2007 to 2013; the proportion is 45.8%, 45.6%, 60.3%, 49.7%, 53.6%, 51.1%, and 66.5%. That means that most companies can be directly set up as a link in cross-shareholdings.
Table 1: Path length distribution of cross-shareholdings network.

<table>
<thead>
<tr>
<th>Average shortest path length</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L = 1 ) F</td>
<td>256</td>
<td>169</td>
<td>193</td>
<td>229</td>
<td>196</td>
<td>211</td>
<td>187</td>
</tr>
<tr>
<td>P</td>
<td>45.8%</td>
<td>45.6%</td>
<td>60.3%</td>
<td>49.7%</td>
<td>53.6%</td>
<td>51.1%</td>
<td>66.5%</td>
</tr>
<tr>
<td>( L = 2 ) F</td>
<td>227</td>
<td>92</td>
<td>94</td>
<td>125</td>
<td>105</td>
<td>139</td>
<td>73</td>
</tr>
<tr>
<td>P</td>
<td>40.6%</td>
<td>24.8%</td>
<td>29.4%</td>
<td>271%</td>
<td>28.7%</td>
<td>33.7%</td>
<td>26%</td>
</tr>
<tr>
<td>( L = 3 ) F</td>
<td>66</td>
<td>79</td>
<td>28</td>
<td>75</td>
<td>53</td>
<td>57</td>
<td>21</td>
</tr>
<tr>
<td>P</td>
<td>11.8%</td>
<td>21.3%</td>
<td>8.7%</td>
<td>16.3%</td>
<td>14.5%</td>
<td>13.8%</td>
<td>7.5%</td>
</tr>
<tr>
<td>( L = 4 ) F</td>
<td>10</td>
<td>26</td>
<td>5</td>
<td>32</td>
<td>12</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>1.8%</td>
<td>7%</td>
<td>1.6%</td>
<td>6.9%</td>
<td>3.3%</td>
<td>1.4%</td>
<td></td>
</tr>
<tr>
<td>( L = 5 ) F</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>1.3%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

network. The separation between companies is very small, and the network transmission performance is good. There is part of companies’ path length that is five in 2008, because most companies’ shareholding relationship is not stable at the beginning of the reform of nontradable shares. In 2008, some isolated companies which are entering the network often possess a large shareholding path length, and the number of companies which have \( L = 3 \) rapid growth reached 21.3%. A small average path length of the cross-shareholdings network means that any two listed companies can be easily connected in the network. This will help the listed company enrich information, technology, and resources and promote the diversification and differentiation of information and knowledge. At the same time, the volatility of the local nodes causes a great influence on the network structure.

The average shortest path length of the network is presented in Figure 1(d). The average path length of cross-shareholdings network form 1.696 in 2007 increases to 1.938 in 2008 and then decreased to 1.409 in 2013. As a whole, the average path length of network is slow to reduce, and network transmission performance and efficiency are improving, apart from 2007 to 2009, because most companies’ shareholding relationship is not stable at the beginning of the reform of nontradable shares. Most companies of the cross-shareholding networks can be connected to the other companies by means of less transit times. That is to say, the network has a good accessibility.

3.2. Small Word Properties. Albert and Barabási [14] found that the actual network does not always completely conform to the characteristics of rule network and random network, and the small world network relatively conforms to the real situation. Watts and Strogatz [15] proposed a model of small world network in the context of various social and biological networks. We apply the same method to see if the small world properties are present in cross-shareholding network.

Compared with the same size random network, if the network has a small average path length and larger clustering coefficient, then the network with small world properties also is a small world network [14]. Therefore, average path length and clustering coefficient compared with the same scale random networks can be represented as

\[
L \leq L_{\text{random}},
\]

\[
C \gg C_{\text{random}},
\]

\[
L_{\text{random}} = \frac{\ln N}{\ln K},
\]

\[
C_{\text{random}} = \frac{K}{N^2},
\]

where \( L_{\text{random}} \) is the average shortest path length of random network, \( C_{\text{random}} \) is the clustering coefficient of random network, \( K \) is the average degree of network that can be expressed as \( 2Q/N \), \( Q \) is the number of edges in the network, and \( N \) is the network size, which is the total number of nodes in the network.

Compared with the same scale random network parameters, the average path length and clustering coefficient of cross-shareholdings network are presented in Table 2. We can see that the average path length of cross-shareholdings network is less than the same scale random network, and the clustering coefficient is greater than the same scale random network. Therefore, cross-shareholdings network is the typical small world network. The shorter average path length indicates that any two companies can be easily connected in network. If a company’s shareholding relationship changes, the other companies are easy to be affected. The larger clustering coefficient indicates that the average condensation between companies is higher. If the companies’ shareholding changes, it could spread faster in neighboring group, and the extent of incidence is large.

3.3. Scale-Free Properties. Degree distribution is a main metric that depicts scale-free properties. The node degrees obeying power law distribution means that the network is a scale-free network. That is to say, most of the network nodes degree is small and only a handful of nodes degrees is large, that is, the key nodes of network. Degree distribution can reflect the randomness of the network structure, network structure of important indicators. When the network is random and the degree of each node in the network is
Table 2: Cross-shareholding network and same scale random network parameter.

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of nodes</th>
<th>Number of links</th>
<th>Average degree</th>
<th>Average path length ($L_{\text{random}}$)</th>
<th>Clustering coefficient ($C_{\text{random}}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>172</td>
<td>256</td>
<td>2.977</td>
<td>1.696 (4.7189)</td>
<td>0.029 (0.017)</td>
</tr>
<tr>
<td>2008</td>
<td>160</td>
<td>169</td>
<td>2.113</td>
<td>1.938 (6.786)</td>
<td>0.014 (0.013)</td>
</tr>
<tr>
<td>2009</td>
<td>138</td>
<td>193</td>
<td>2.797</td>
<td>1.516 (4.790)</td>
<td>0.012 (0.011)</td>
</tr>
<tr>
<td>2010</td>
<td>203</td>
<td>229</td>
<td>2.256</td>
<td>1.805 (6.530)</td>
<td>0.013 (0.012)</td>
</tr>
<tr>
<td>2011</td>
<td>176</td>
<td>196</td>
<td>1.979</td>
<td>1.675 (6.457)</td>
<td>0.04 (0.011)</td>
</tr>
<tr>
<td>2012</td>
<td>195</td>
<td>211</td>
<td>2.164</td>
<td>1.656 (6.830)</td>
<td>0.038 (0.011)</td>
</tr>
<tr>
<td>2013</td>
<td>189</td>
<td>187</td>
<td>1.979</td>
<td>1.409 (7.680)</td>
<td>0.038 (0.011)</td>
</tr>
</tbody>
</table>

Table 3: The coefficients of linear fit at company-level in scale-free index.

(a) Degree distribution

<table>
<thead>
<tr>
<th>Year</th>
<th>$\gamma$</th>
<th>$R^2$ square</th>
<th>Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>2.311</td>
<td>0.8168</td>
<td>0.2767</td>
</tr>
<tr>
<td>2008</td>
<td>2.465</td>
<td>0.8213</td>
<td>0.2961</td>
</tr>
<tr>
<td>2009</td>
<td>2.558</td>
<td>0.873</td>
<td>0.4485</td>
</tr>
<tr>
<td>2010</td>
<td>2.625</td>
<td>0.9207</td>
<td>0.441</td>
</tr>
<tr>
<td>2011</td>
<td>2.721</td>
<td>0.9537</td>
<td>0.5297</td>
</tr>
<tr>
<td>2012</td>
<td>2.722</td>
<td>0.8237</td>
<td>0.4334</td>
</tr>
<tr>
<td>2013</td>
<td>2.724</td>
<td>0.806</td>
<td>0.4419</td>
</tr>
</tbody>
</table>

(b) Degree of cumulative probability distribution

<table>
<thead>
<tr>
<th>Year</th>
<th>$\lambda$</th>
<th>$R^2$ square</th>
<th>Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>0.16</td>
<td>0.9103</td>
<td>0.4215</td>
</tr>
<tr>
<td>2008</td>
<td>0.141</td>
<td>0.6851</td>
<td>0.2153</td>
</tr>
<tr>
<td>2009</td>
<td>0.297</td>
<td>0.9869</td>
<td>0.9198</td>
</tr>
<tr>
<td>2010</td>
<td>0.405</td>
<td>0.9712</td>
<td>0.955</td>
</tr>
<tr>
<td>2011</td>
<td>0.436</td>
<td>0.9633</td>
<td>1.1972</td>
</tr>
<tr>
<td>2012</td>
<td>0.437</td>
<td>0.9223</td>
<td>1.1458</td>
</tr>
<tr>
<td>2013</td>
<td>0.394</td>
<td>0.9682</td>
<td>0.8403</td>
</tr>
</tbody>
</table>

approximately same, this network is disorderly. When the network is scale-free, there are some key nodes that possess higher degree and a large number of nodes possess lower degree; there is a significant difference among the degrees of nodes; this network is ordered.

Cross-shareholdings network is the nonuniform network, the degree of the vast majority of listed companies is relatively low, but there are a small number of listed companies that own the higher degree; they are core firms of the cross-shareholdings network. The degree distribution scatter plot of network is presented in Figure 2(a). The cross-shareholdings network degree distribution is unbalanced. With capital, technology, and policy advantages, some high quality companies hold other companies’ share. Therefore, the high quality companies are key nodes in the network, and their degrees are higher than other listed companies.

The cross-shareholdings network scale-free index and its evolution law shown as $\gamma$ in Table 3(a) are from 2007 to 2013. Table 3(a) shows that fitted degree distribution of network through the power law distribution and goodness of fit were greater than 0.8 and close to 1. The range of goodness of fit is $[0, 1]$, the more it is close to 1, the better the power law distribution curve fitting effect is. Therefore, cross-shareholdings network node degree distribution possesses power law distribution properties (as shown in Figure 2(c), $P_{2013}(k) = 0.4419k^{-2.724}$, $R^2 = 0.806$, the power law index is $\gamma = 2.724$ between 2 and 3) and has high confidence level; the overall structure of the network is scale-free networks. As evident from Figure 2(b), the network power law index showed a trend of increase, near the peak until 2013. The results show that the cross-shareholdings network node degrees obey the power law distribution; then $P(k)$ approximate the power function $P(k) \sim k^{-\gamma}$. It shows that the degree distribution of network is uneven distribution, a part of the nodes degrees are large, referred to as the Hub node of the network. In scale-free network, the Hub node has strong influence; their change impacts on the stability of the entire network.

In general, we can reach the following conclusions. First, the goodness of fit of degree distribution is close to 1 and shows a trend of gradual increase. Therefore, the network structure presents the development trend of scale-free networks. That is to say, cross-shareholdings network connection is following preferential connection "Matthew effect." Second, there are some Hub nodes which high degree in cross-shareholdings network. There is a part of listed firms that have a strong influence in the stock market, and they usually have influence whether or not the network can be in normal operation.

The cumulative probability distribution scatter diagram of network is presented in Figure 2(d). The cross-shareholdings network cumulative probability distribution index and its evolution law are shown as $\lambda$ in Table 3(b) from 2007 to 2013. The degree of cumulative probability distribution of cross-shareholdings network obeys exponential distribution (as shown in Figure 2(f), $P_{2013}(k) = 0.8403e^{-0.394k}$, $R^2 = 0.9682$, and the cumulative probability distribution index is $\lambda = 0.394$) and better fitting. It suggests that the connection of nodes (degrees) is not evenly distributed. Most of the nodes connected only a small number of listed companies and a small number of nodes connected more listed companies. Therefore the greater the node degree, the more likely the contact with more other nodes and the stronger the company shareholding capacity; this further verifies that the cross-shareholdings network possesses scale-free feature. The fitting index of network showed gradual increase first and then decrease from 2007 to 2013 (as shown $\lambda$ in Figure 2(e)).
Figure 2: Degree distribution evolution of the cross-shareholding network.
and the topology of the network is changing constantly with the time. In addition, most goodness of fit is above 0.9, close to 1; this indicates that the fitting is getting better and better.

4. Conclusions

In this paper we have studied the cross-shareholdings network evolution law as a no weighted graph of listed firms. The network clearly displays small world properties and scale-free properties. The characteristics of network are also computed, wherein these companies are identified as potential shareholding points. Only a small number of crucial companies connect more listed companies; if their shareholding relationships change, they can transmit faster to the others and strongly affect other companies’ ownership in stock market.

The beauty of complex network theory is that it is a powerful tool with limitless application possibilities. This study contributes a complex network analysis of the physical state of the cross-shareholdings network. Such a study would not only reveal the topological aspects but also provide a detailed insight into the network dynamics by identifying the stations with more shareholding relationship and help the policy makers to further enhance the securities market regulation to achieve efficient flow.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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