Research Article

Output Feedback Stabilization for Stochastic Nonholonomic Systems under Arbitrary Switching

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The output feedback controllers of stochastic nonholonomic systems under arbitrary switching are discussed. We adopt an observer which can simplify the design process. The designed control laws cause the calculation of the gain parameter to be very convenient since the denominator of virtual controllers does not contain the gain parameter. Finally, an example is given to show the effectiveness of controllers.

1. Introduction

In recent years, switched system’s control, especially under arbitrary switching, has become an active field [1–3]. The global stabilization of switched systems based on arbitrary switching was given [4–6]. The stabilizing controllers of switched systems with arbitrary switching were given [7–10]. The output feedback controllers of nonlinear systems with arbitrary switching were designed [11, 12].

In the past ten years, the problem of stabilization for stochastic nonholonomic systems (SNSs) received much attention. They mainly can be classified into two types. The first is state-feedback control: stabilization [13, 14], adaptive stabilization [15–18], finite-time stabilization [19], stabilization with time-varying delays [20], and stabilization of mobile robots [21, 22]. The second is the output feedback stabilization [23–25].

Since sometimes part of the system states are unmeasurable, output feedback controllers are needed. Zhang et al. discussed the output feedback stabilizing controllers of SNSs whose virtual control $b_i$ contains gain parameter $L$. This will lead to a problem where the calculation of $L$ is very difficult, especially for $n \geq 3$, since the inequalities about $L$ were quintic. In addition, to the authors’ knowledge, there are some results about state-feedback stabilization of SNSs with Markovian switching [13, 14], with few available results for the output stabilization of SNSs under arbitrary switching. Based on the above analysis, there exists a problem, that is, how to choose a proper observer under arbitrary switching where the virtual control $b_i$ in controllers does not contain gain parameter $L$, which causes the calculation of $L$ to be easier.

Notations. $\mathbb{R}$ denotes the set of all real numbers. $\mathbb{R}^n$ denotes the real $n$-dimensional space. For a vector or matrix $X \in \mathbb{R}^{n \times m}$, $X^T$ denotes its transpose, $\|X\|$ denotes the Euclidean norm, $\text{Tr}\{X\}$ is its trace when $X$ is square, and $\mathcal{L}$ is a stochastic differential operator [26].

2. Problem Formulation

The stochastic nonholonomic nonlinear systems are given by

$$\begin{align*}
dx_0 &= u_0 dt + g_{[0,\sigma(t)]}(x_0) d\omega, \\
dx_i &= u_0 x_{i+1} dt + f_{[i,\sigma(t)]}(x_0, x_i) dt \\
&\quad + g_{[i,\sigma(t)]}(x_0, x_i) d\omega, \quad i = 1, \ldots, n-1, \\
dx_n &= u dt + f_{[n,\sigma(t)]}(x_0, x) dt + g_{[i,\sigma(t)]}(x_0, x) d\omega, \\
y &= [x_0, x_1]^T,
\end{align*}$$

(1a)(1b)
where \( u_0 \) and \( u \in \mathbb{R} \) are inputs, \( x_0 \in \mathbb{R} \) and \( (x_1, \ldots, x_p)^T \in \mathbb{R}^p \) are system states, \( \xi = (x_1, \ldots, x_p)^T \), \( f_i \equiv f_i(x_0, \xi) : \mathbb{R}^{i+1} \to \mathbb{R}^m \) are smooth functions named as nonlinear drifts with \( f_i(0, 0) = 0 \), and \( g_i(x_0) : \mathbb{R} \to \mathbb{R}^m \) and \( g : [0, \infty) \to M = \{1, 2, \ldots, m\} \) are smooth functions with \( g_i(0, 0) = 0 \). There are two main differences between systems (1a) and (1b) and those in [24, 25]. The first is the arbitrary switching mentioned in this paper. The second will be illustrated in Remark 7. In addition, Assumptions 1 and 2 are similar to those in [24, 25], but in fact they hold under arbitrary switching; for example, \( \bar{P}_{(i, 0)} = \max\{P_{(i, 1)}, P_{(i, 2)}, P_{(i, 3)}, P_{(i, 4)}\} \) when \( \sigma(t) : [0, \infty) \to M = \{1, 2, 3, 4\} \).

### 3. Output Feedback Stabilization

The controller design in Sections 3.1 and 3.2 is under \( x_0(t_0) \neq 0 \). The other one is discussed in Section 3.3.

#### 3.1. Controller \( u_0 \) Design

For subsystem (1a), one obtains

\[
\mathcal{L}V_0(x_0) \leq -2\lambda V_0(x_0).
\]

with Lyapunov function \( V_0(x_0) = (1/2)x_0^2 \) and controller \( u_0 \) as follows:

\[
u_0 = -\eta_0 x_0, \\
\eta_0 = \lambda + \frac{1}{2} m^2,
\]

where \( \lambda > 0 \) is a design real number.

**Theorem 4.** For system (1a), the closed-loop system with controller (4) is asymptotically stable in probability.

By (4) and (1a), one has

\[
dx_0 = -\eta_0 x_0 dt + g_{(0,0)}^T(x_0) dw.
\]

**Remark 5.** For any \( x_0(t_0) \neq 0 \), one has \( x_0(t) \neq 0 \) in (6) at the time interval \( t \in (t_0, +\infty) \) a.s. with a similar proof of Proposition 1 in [25].

### 3.2. Controller \( u \) Design

In order to design controller \( u \), let

\[
z_i = \frac{x_i}{u_0^i}, \quad 1 \leq i \leq n.
\]

**Remark 6.** For any \( x_0(t_0) \neq 0 \), from Remark 5, we have that (7) is meaningful a.s.

By (1b) and (7), one has

\[
dz_i = z_{i+1} dt + \phi_{i,(0)}(x_0, \xi) dt + \psi_{i,(0)}(x_0, \xi) d\omega,
\]

\[
y_i = z_i, \quad i = 1, \ldots, n.
\]

where \( \xi = (z_1, \ldots, z_n)^T \) and

\[
\phi_{i,(0)}(x_0, \xi) = \frac{z_i - 1}{u_0^i} - \eta_0 (n - i), \quad \psi_{i,(0)}(x_0, \xi) = \frac{z_i}{u_0^i} - \eta_0 (n - i).
\]

We adopt the following observer [27]:

\[
\hat{x}_i = \hat{x}_{i+1} + L^i a_i (y_i - \hat{z}_i), \quad i = 1, \ldots, n - 1,
\]

\[
\hat{x}_n = u + L^n a_n (y_1 - \hat{z}_1),
\]

where \( L \geq 1 \) is a gain parameter and real numbers \( a_i > 0 \), \( i = 1, \ldots, n \), such that \( p(s) = s^2 + a_1 s^{i-1} + \cdots + a_n \) is Hurwitz.

**Remark 7.** The second main difference between this manuscript and [25] is that the observer (10) we choose is the same as that in [27], but different from that in [25]. This observer has two advantages. The first is that it can simplify the process of designing controllers. The second is that \( b_i \) in virtual control \( a_i \) that we design in the following does not contain the gain parameter \( L \).

Denoting

\[
\xi_i = \frac{(z_i - \hat{z}_i)}{L^{i-1}}, \quad i = 1, \ldots, n,
\]

one has the error systems

\[
d\xi_i = LA_\xi dt + \Phi_{\alpha(t)}dt + \Psi_{\alpha(t)} d\omega,
\]

where \( A_\xi = \Phi_{\alpha(t)} - \Psi_{\alpha(t)} \).

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where
\[
A = \begin{bmatrix}
-\alpha_1 \\
-\alpha_2 & I_{n-1} \\
\vdots \\
-\alpha_n & 0
\end{bmatrix},
\]
\[
\Phi_{\sigma(t)} = \begin{bmatrix}
\phi_{[1,\sigma(t)]} \\
\phi_{[2,\sigma(t)]} \\
\vdots \\
\phi_{[n-1,\sigma(t)]} \\
\frac{\phi_{[n,\sigma(t)]}}{L^{n-2}} \\
\frac{\phi_{[n+1,\sigma(t)]}}{L^{n-1}}
\end{bmatrix},
\]
\[
\Psi_{\sigma(t)}^T = \begin{bmatrix}
\psi_{[1,\sigma(t)]}^T \\
\psi_{[2,\sigma(t)]}^T \\
\vdots \\
\psi_{[n-1,\sigma(t)]}^T \\
\frac{\psi_{[n,\sigma(t)]}^T}{L^{n-2}} \\
\frac{\psi_{[n+1,\sigma(t)]}^T}{L^{n-1}}
\end{bmatrix}.
\]

Therefore, for positive definite matrix \(P\), \(A\) is a Hurwitz matrix; that is, \(A^T P + PA = -I\). Now, one has
\[
d\xi = LA\xi dt + \Phi_{\sigma(t)} dt + \Psi_{\sigma(t)}^T \omega dt,
\]
\[
dy = z_2 dt + \phi_{[1,\sigma(t)]} dt + \psi_{[1,\sigma(t)]}^T \omega dt,
\]
\[
\dot{z}_i = \dot{z}_{i+1} + L t_i (y_1 - \dot{z}_1), \quad i = 1, \ldots, n - 1,
\]
\[
\dot{z}_n = u + L^T a_n (y_1 - \dot{z}_1).
\]

**Proposition 8.** By Assumptions 1 and 2, there exist constants \(\tau_1 > 0\) and \(\tau_2 > 0\) a.s., such that
\[
\|\Phi_{\sigma(t)}(x_0, z)\|^2 \leq \left( |z_1| + |z_2| + \cdots + |z_i| \right)^2 \tau_1,
\]
\[
\psi_{\sigma(t)}^T(x_0, z)\|^2 \leq \left( |z_1| + \frac{|z_2|}{L} + \cdots + \frac{|z_n|}{L^{n-1}} \right)^2 \tau_2^2,
\]
\[
where \(z = [z_1, \ldots, z_n]^T\).

**Remark 9.** The proof of the above proposition is similar to that of Proposition 2 in [25]. In fact, we only need to let \(\overline{\tau}_i = \eta_0 + \max_{s \in [0, T]} |\overline{\tau}_i|\).

One can define variables \(\epsilon_i\) as follows in order to utilize the backstepping method:
\[
\epsilon_1 = z_1,
\]
\[
\epsilon_i (z_1, z_2, \ldots, z_i) = \dot{z}_i - \alpha_{i-1} (z_1, z_2, \ldots, z_{i-1}),
\]
with virtual smooth controllers \(\alpha_{i-1}\) (\(i = 2, \ldots, n\)).

**Step 1.** For positive parameter \(\delta\), letting \(V_0 = \delta \xi^T P \xi\), with a similar method in [24], one has
\[
\mathcal{L} V_0 \leq - \left[ \delta \left( L - 2 \|P\|^2 \right) - 2nC_c \right] \|\xi\|^2
\]
\[
+ 2nC_c \left( |z_1|^2 + \frac{|z_2|^2}{L^2} + \cdots + \frac{|z_n|^2}{L^{2n-2}} \right),
\]
where \(C_c = n\delta r_1^2 + n\delta \lambda_{\text{max}}(P) r_2^2\). Defining \(V_1 = V_0 + (1/2) \epsilon_1^2\), we have
\[
\mathcal{L} V_1 \leq - \left[ \delta \left( L - 2 \|P\|^2 \right) - 2nC_c \right] \|\xi\|^2
\]
\[
+ 2nC_c \left( |z_1|^2 + \frac{|z_2|^2}{L^2} + \cdots + \frac{|z_n|^2}{L^{2n-2}} \right)
\]
\[
+ \epsilon_1 \left[ \epsilon_2 + \alpha_1 + L \epsilon_2 + \phi_{[1,\sigma(t)]} \right]
\]
\[
+ \frac{1}{2} \text{Tr} \left[ \psi_{[1,\sigma(t)]} \psi_{[1,\sigma(t)]}^T \right].
\]

The following inequalities hold with Lemma 2.1 in [28]:
\[
2nC_c \frac{|z_1|^2}{L^2} \leq 4nC_c \frac{\epsilon_1^2}{L} + 4nC_c \frac{\alpha_1^2}{L^2},
\]
\[
\epsilon_1 \epsilon_2 \leq \frac{1}{4} L \epsilon_1^2 + L \epsilon_2^2,
\]
\[
L \epsilon_2 \epsilon_1 \leq \frac{L}{4} \epsilon_1^2 + 2 \epsilon_2^2,
\]
\[
\epsilon_1 \phi_{[1,\sigma(t)]} \leq L \epsilon_1^2 \epsilon_2^2,
\]
\[
\frac{1}{2} \text{Tr} \left[ \psi_{[1,\sigma(t)]} \psi_{[1,\sigma(t)]}^T \right] \leq \frac{1}{2} L \tau_2^2 \epsilon_1^2,
\]
\[
2nC_c |z_1|^2 \leq 2nC_c L \epsilon_2^2,
\]
where, together with (18), \(|z_i/L^{i-1}| \leq \|\xi\| |z_i/L^{i-1}|\). Choosing \(\alpha_1 = -Lb_1 \epsilon_1\),
\[
b_1 = 2n + 1 + 2nC_c + \frac{5}{4} + \tau_1 + \frac{\tau_2^2}{2},
\]
one has
\[
\mathcal{L} V_1 \leq - \left[ \delta \left( L - 2 \|P\|^2 \right) - 2nC_c - L \right] \|\xi\|^2
\]
\[
+ 2nC_c \left[ \frac{|z_1|^2}{L^2} + \cdots + \frac{|z_n|^2}{L^{2n-2}} \right]
\]
\[
- \left[ (2n + 1) L - 4nC_c b_1^2 \right] \epsilon_1 + \frac{1}{4L} \epsilon_2^2 + 4nC_c \frac{\epsilon_2^2}{L}.
\]
Step $i$ ($2 \leq i \leq n - 1$). From (16), we have

$$
\varepsilon_i = \tilde{z}_i + Lb_{i-1}\tilde{z}_{i-1} + L^2b_{i-1}b_{i-2}\tilde{z}_{i-2} + \cdots + L^{i-1}b_{i-1}b_{i-2}\cdots b_1\varepsilon_1,
$$

$$
de_i = \left[ \tilde{z}_{i+1} + L^2\tilde{d} \tilde{z}_{i} + L^2d_i\varepsilon_1 + \cdots + L^2d_{i-1}\varepsilon_{i-1} + Ld_i\varepsilon_i \right] \, dt
+L^{i-1}b_{i-1}b_{i-2}\cdots b_1 \left[ L\delta \chi \, dt + \phi_{|\sigma(0)|} \, dt + \psi_{|\sigma(0)|}^{\top} \, d\omega \right],
$$

where

$$
d_{i,1} = -b_{i-1}\cdots b_1b_1,
$$

$$
d_{i,j} = b_{i-1}\cdots b_jb_j - b_{i-1}\cdots b_jb_{j+1}, \quad 2 \leq j \leq i - 1,
$$

$$
d_{i,i} = b_{i-1},
$$

$$
\tilde{d}_i = a_i + b_{i-1}b_{i-2}\cdots b_2a_2 + b_{i-1}\cdots b_3a_3 + \cdots + b_{i-1}a_{i-1}.
$$

Now, we have finished the design step $i - 1$, and $\alpha_{i-1}$ is chosen as follows:

$$
\alpha_{i-1} = -Lb_{i-1}\varepsilon_{i-1},
$$

$$
b_{i-1} = n + 4nC_e + \frac{5}{4} + \frac{d^2_{i-1,1}}{4} + \frac{1}{4}d^2_{i-1,1} + \cdots + \frac{1}{4}d^2_{i-1,i-1} + \frac{1}{4}(b_{i-2}\cdots b_1)^2 \varepsilon_{i-1} + \tilde{d}_{i-1,i-1}.
$$

Let $V_{i-1} = V_{i-2} + (1/2L^2)^i\varepsilon_{i-1}^2$, such that

$$
\mathcal{L}V_{i-1} \leq -\left[ 8L - 2 \delta \| P \| \right] - 2nC_e - (i - 1) \left[ \| \varepsilon_i \| \right]^2
+ 2nC_e \left( \frac{\| \tilde{z}_{i+1} \|}{L^2} + \cdots + \frac{\| \tilde{z}_n \|}{L^n-2} \right)
- \left\{ 2n - 2(i - 1) + 3 \right\} L
- \sum_{j=1}^{i-2} \left( \frac{b_j \cdots b_1}{2} \right) \varepsilon_j^2 - \sum_{j=2}^{i-1} \frac{1}{L^{i-2}}
\right. \left[ (n + j - i + 1) L - 4nC_e b_j^2 \varepsilon_j^2 + 4nC_e \frac{\varepsilon_j^2}{L^{j-2}} \right]
+ \frac{\varepsilon_i^2}{4L^{2i-3}}.
$$

Defining $V_i = V_{i-1} + (1/2L^2)^i\varepsilon_i^2$, we have

$$
\mathcal{L}V_i \leq -\left[ 8L - 2 \delta \| P \| \right] - 2nC_e - (i - 1) \left[ \| \varepsilon_i \| \right]^2
+ 2nC_e \left( \frac{\| \tilde{z}_{i+1} \|}{L^2} + \cdots + \frac{\| \tilde{z}_n \|}{L^n-2} \right)
- \left\{ 2n - 2(i - 1) + 3 \right\} L
- \sum_{j=1}^{i-1} \frac{1}{L^{i-2}} \left[ (n + j - i + 1) L - 4nC_e b_j^2 \varepsilon_j^2 + 4nC_e \frac{\varepsilon_j^2}{L^{j-2}} \right]
+ \frac{\varepsilon_i^2}{4L^{2i-3}}.
$$

By Lemma 2.1 in [28], one gets

$$
2nC_e \frac{\| \tilde{z}_{i+1} \|^2}{L^2} \leq 4nC_e \frac{\varepsilon_{i+1}^2}{L^2} + 4nC_e \varepsilon_i^2,
$$

$$
\frac{1}{L^{2i-2}} \varepsilon_{i+1}^2 \leq \frac{1}{4L^{2i-2}} \varepsilon_{i+1}^2 + \frac{1}{L^{2i-3}} \varepsilon_i^2.
$$

$$
\frac{d_{i,1}}{L^2} \varepsilon_{i+1}^2 \leq \frac{d_{i,1}^2}{4L^{2i-2}} \varepsilon_{i+1}^2 + L\varepsilon_{i+1}^2,
$$

$$
\frac{d_{i,i}}{L^2} \varepsilon_{i+1}^2 \leq \frac{d_{i,i}^2}{4L^{2i-2}} \varepsilon_{i+1}^2 + L\varepsilon_{i+1}^2,
$$

$$
\frac{d_{i,j}}{L^{2i-2}} \varepsilon_{i+1}^2 \leq \frac{d_{i,j}^2}{4L^{2i-2}} \varepsilon_{i+1}^2 + \frac{1}{L^{2i-3}} \varepsilon_{i+1}^2,
$$

$$
\frac{1}{L^2} \varepsilon_{i+1}^2 \leq \frac{1}{4L^2} \varepsilon_{i+1}^2 + L\varepsilon_{i+1}^2.
$$

$$
\frac{1}{L^2} \varepsilon_{i+1}^2 \leq \frac{1}{4L^2} \varepsilon_{i+1}^2 + L\varepsilon_{i+1}^2,
$$

$$
\frac{1}{L^2} \varepsilon_{i+1}^2 \leq \frac{1}{4L^2} \varepsilon_{i+1}^2 + L\varepsilon_{i+1}^2.
$$
\[
\frac{1}{L-1} b_{i-1} b_{i-2} \cdots b_1 e_i \phi_p \leq \frac{(b_{i-1} b_{i-2} \cdots b_i)^2}{4L^{2-3}} \epsilon_i^2 + L \epsilon_i^1,
\]
\[
\frac{1}{2L^{2-2}} \left( L^{-1} b_{i-1} b_{i-2} \cdots b_1 \right)^2 \text{Tr} \left\{ \psi_{i(L \epsilon_i^{(L)})}^{T} \psi_{i(L \epsilon_i^{(L)})} \right\} \leq \frac{(b_{i-1} b_{i-2} \cdots b_1)^2}{2} \epsilon_i^2, \tag{27}
\]
Choosing \( \alpha_i \) as
\[
\alpha_i = -L b_i e_i,
\]
\[
b_i = n + 4nC_c + \frac{5}{4} + \frac{d_{i,1}^2}{4} + \frac{1}{4} d_{i,1}^2 + \cdots + \frac{1}{4} d_{i,j-1}^2
\]
\[
+ \frac{1}{4} (b_{i-1} \cdots b_1)^2 + \frac{1}{4} (b_{i-1} \cdots b_1)^2 \epsilon_i^1 + d_{i,j}, \tag{28}
\]
one has
\[
\mathcal{L} V \leq - \left[ \delta L - 2 \delta \| P \|^2 - 2nC_c - iL \right] \| \xi \|^2 + 2nC_c \left( \frac{\| \zeta_n \|^2}{L^{2n-2}} + \cdots + \frac{\| \zeta_n \|^2}{L^{2n-2}} \right)
\]
\[
- \left\{ 2n - 2i + 3 \right\} L - \sum_{j=1}^{i} \left( \frac{b_{j-1} \cdots b_1}{4} \right)^2 \frac{\epsilon_i^1}{2} - 4nC_c b_i^2 \right\}
\]
\[
\cdot \epsilon_i^2 - \sum_{j=2}^{i} \frac{1}{L^{2j-3}} \left[ (n + j - i) L - 4nC_c b_j^2 \right] \epsilon_j^2 + 4nC_c \epsilon_i^2 + \frac{\epsilon_{i+1}^2}{L^{2i-1}} + \frac{1}{4L^{2i-1}} \epsilon_{i+1}^2. \tag{29}
\]
**Step n.** Letting \( V_n = V_{n-1} + (1/2L^{2n-2}) \epsilon_n^2 \) and choosing
\[
u = -L b_n e_n,
\]
\[
b_n = n + 4nC_c + \frac{5}{4} + \frac{d_{n,1}^2}{4} + \cdots + \frac{d_{n,n-1}^2}{4}
\]
\[
+ \left( \frac{b_{n-1} \cdots b_1}{4} \right)^2 + \left( \frac{b_{n-1} \cdots b_1}{4} \right)^2 \epsilon_i^1 + d_{n,n}, \tag{30}
\]
one gets
\[
\mathcal{L} V_n \leq -h_0 \| \xi \|^2 - h_1 \epsilon_i^2 - \sum_{j=2}^{n} h_j \frac{1}{L^{2j-3}} \epsilon_j^2 - h_n \frac{1}{L^{2n-3}} \epsilon_n^2 \tag{31}
\]
\[
\leq -\tilde{h} V_n,
\]
where \( \tilde{h} = \min\{h_0/\delta, 2h_1, \ldots, 2h_n\} \) and
\[
h_0 = \delta L - 2 \delta \| P \|^2 - 2nC_c - nL \geq 0,
\]
\[
h_1 = 3L - \sum_{j=1}^{n-1} \left( \frac{b_{j-1} \cdots b_1}{2} \right)^2 \frac{\epsilon_i^1}{2} - 4nC_c b_i^2 \geq 0, \tag{32}
\]
\[
h_j = jL - 4nC_c b_j^2 \geq 0, \quad j = 2, \ldots, n-1,
\]
\[
h_n = n \geq 0, \quad \delta > n.
\]
**Remark 10.** From (20), one has that \( b_i \) do not contain \( L \) under arbitrary switching. From (28), one has that all \( b_i \) do not contain \( L \) under the designed controllers.

**Remark 11.** For example, with \( n = 3 \), by (41) in [25] and (32), we have
\[
h_0 = \delta \left( L - 2 \| P \|^2 - \| B \|^2 \right) - 6C_c - 3L \geq 0,
\]
\[
h_1 = 6L - 4L - \frac{\eta_0^2}{L} \left( \frac{b_{j-1} \cdots b_1}{2} \right)^2 \frac{\epsilon_i^1}{2} - 12C_c b_i^2 \geq 0,
\]
\[
h_2 = 2L - \eta_0^2 \left( \frac{b_{i-1} \cdots b_1}{2} \right)^2 \frac{\epsilon_i^1}{2} - 12C_c b_i^2 \geq 0,
\]
\[
h_3 = 3 \geq 0, \quad \delta > 3,
\]
\[
h_0 = \delta \left( L - 2 \| P \|^2 \right) - 6C_c - 3L \geq 0,
\]
\[
h_1 = 7L - 4L - \sum_{j=1}^{2} \left( \frac{b_{j-1} \cdots b_1}{2} \right)^2 \frac{\epsilon_i^1}{2} - 12C_c b_i^2 \geq 0,
\]
\[
h_2 = 3L - 12C_c b_i^2 \geq 0,
\]
\[
h_3 = 3 \geq 0, \quad \delta > 3,
\]
respectively. It is easy to see that \( h_1 \) and \( h_2 \) in (33) are all quintic functions about \( L \), but they are linear functions about \( L \) in (34). So, the calculation of \( \delta \) and \( L \) will be more simple with the method in this manuscript.

Choose \( V = V_0 + V_n \), which together with (5) and (31) result in
\[
\mathcal{L} V \leq -h V, \tag{35}
\]
where \( h = \min\{h_0/\delta, 2h_1, \ldots, 2h_n\} \).

**Theorem 12.** For system (1b), the closed-loop system with controller (30) is asymptotically stable in probability.

3.3. **Switching Control.** In the above two subsections, we give the controllers \( u_0 \) and \( u \) with \( x_0(t_0) \neq 0 \) as (4) and (30). Now, we turn to the case of \( x_0(t_0) = 0 \). If \( x_0(t_0) = 0 \), we firstly choose constant control \( u_0 = u_0^* \neq 0 \). Secondly, there will exist a time \( t_*^* > 0 \) such that \( x_0(t_*^*) \neq 0 \). After that, controllers \( u_0 \) and \( u \) as (4) and (30) can be applied.

**Theorem 13.** If we apply the above switching procedure, systems (1a) and (1b) will be asymptotically stabilized in probability.

**4. A Simulation Example**

Consider systems (1a) and (1b) with \( \sigma : [0, +\infty) \rightarrow \{1, 2\} \) and \( g_{[0,1]} = c_1 x_0 \cos(x_0), g_{[1,2]} = c_1 x_0 \sin(x_0), f_{[1,1]} = c_1 x_1 \cos^3(x_1), f_{[1,2]} = c_1 x_1 \cos(x_1), g_{[1,1]} = c_1 x_1 \sin^3(x_1), g_{[1,2]} = c_1 x_1 \sin(x_1), f_{[2,1]} = f_{[2,2]} = 0, g_{[2,1]} = c_2 x_2 \sin^2(x_2), g_{[2,2]} = c_2 x_2 \sin(x_2), \) and \( c_1 = 0.001 \).

From (8), we have \( \phi_{[1,1]} = \eta_0 z_1 + c_1 z_1 \cos^3(x_1) + c_1^2 z_1 \cos(x_1) - c_1^2 z_1 \sin^2(x_1) \cos(x_2), \phi_{[1,2]} = 0, \psi_{[1,1]} = \).
\textbf{Figure 1:} The responses of states $x_0$, $x_1$, and $x_2$.

\[ c_1 z_1 \sin^2(x_1) - c_1 z_1 \cos(x_0), \psi_{[2,1]} = c_1 z_2 \sin^2(z_2), \phi_{[1,2]} = \eta_0 z_1 + c_1 z_1 \cos^2(x_1) + c_1^2 z_1 \sin^2(x_0) - c_1^2 z_1 \sin(x_1) \sin(x_0), \phi_{[1,2]} = 0, \psi_{[1,2]} = c_1 z_1 \sin(x_1) - c_1 z_1 \sin(x_0), \] and \[ \phi_{[2,2]} = c_1 z_2 \sin(z_2). \]

Letting $\eta_0 = 0.3$, by Proposition 8, we have $\tau_1 = 0.3023$ and $\tau_2 = 0.001$. Choosing $a_1 = 0.75$ and $a_2 = 1.25$, then $\lambda_{\text{max}}(P) = 2$ and $\|P\| = 2$.

From (20) and (30), one gets

\[ b_1 = 5 + 4C_c + 1 + \frac{1}{4} + \tau_1 + \frac{1}{2} \tau_2^2, \]

\[ b_2 = 2 + 8C_c + 1 + \frac{1}{4} + \left( a_0 + b_1 a_1 \right)^2 + \frac{b_1^2}{4} + b_1 + \frac{b_1^2}{4}, \]

\[ u = -Lb_2 \tilde{z}_2 - L^2 b_1 \tilde{z}_1, \]

where $C_c = 0.4131\delta$. From (32) and (35), we have

\[ h_0 = (\delta - 2)L - 2\delta \|P\|^2 - 4C_c \geq 0, \]

\[ h_1 = 2L - \frac{b_1^2 \tau_1^2}{2} - 8C_c b_1^2 \geq 0, \]

\[ h_2 = 2 \geq 0, \quad \delta > 2. \]

Solving the above inequalities, one has $\delta = 2.26$ and $L = 76$, which means $h_0 = 0.0277$, $h_1 = 5.5526$, and $\mathcal{L}V \leq 0$.

If we choose initial values $x_0(0) = 0.1$, $x_1(0) = 0.0003$, $x_2(0) = 0.2$, $\tilde{z}_1(0) = -0.001$, and $\tilde{z}_2(0) = 7.5$, responses of systems are as in Figures 1, 2, and 3.
Remark 14. From the above example, the observer we adopted is the same as that in [27], but it can simplify the calculation of $\delta$ and $L$ compared with the observer in [25].

5. Conclusions

The output feedback stabilization for SNSs under arbitrary switching is discussed. We proposed an observer which is different from that in [25]. $\beta$ of the designed output feedback stabilizing control laws do not contain the gain parameter. We will give some new results, for example, how to design an adaptive controller with the method LMI based on results in [29, 30].

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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