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Research Article

A Viral Product Diffusion Model to Forecast the Market Performance of Products

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To investigate the diffusion of products in the market, this paper proposes a viral product diffusion model using an epidemiological approach. This model presents the process of product diffusion through the dynamics of human behaviors. Based on the stability theory of Ordinary Differential Equations, we demonstrate the conditions under which a product in the market persists or dies out eventually. Next, we use Google data to validate the model. Fitting results illustrate that the viral product diffusion model not only depicts the steady growth process of products, but also describes the whole diffusion process during which the products increase at the initial stage and then gradually decrease and sometimes even exhibit multiple peaks. This shows that the viral product diffusion model can be used to forecast the developing tendency of products in the market through early behavior of these products. Moreover, our model also provides useful insights on how to design effective marketing strategies via social contagions.

1. Introduction

The promotion and diffusion of products in the market are a lasting and interesting research problem. To promote a new product which has just begun to occupy the market is very difficult, even if the product has more advantages than the existing products. However, a fraction of people who are the first users to accept a new product will take the role of a long fuse that can trigger the diffusion of the corresponding product throughout the market [1]. Therefore, it is of both theoretical and practical significance to forecast the long-term performance of products and explore effective marketing strategy of new products through their early behavior.

The most famous model for the study of product diffusion was the Bass model, which was proposed by Bass in 1969 [2]. The 1969 Bass model paper was one of the most highly cited papers in the marketing literature. This model had spawned the development of a core group of scholars in diffusion theory that was growing in number and influence [3]. In this model, buyers in the decision-making process are divided into innovators and imitators. The Bass model discusses at length the impact of the dynamic changes of innovators and imitators on the product sales. Based on the theoretical framework of the Bass model, a large number of product diffusion models are directly divided buyers, then analyze the changes of behaviors of these buyers in the decision-making process, and forecast product sales via these changes [4–7]. Although these models are effective for forecasting product sales, these models only present the changes of individual behaviors in the decision-making stage and fail to reveal how those buyers are formed. In other words, product diffusion models in the framework of the Bass model ignore the fact that people will experience an awareness stage before making a purchase decision. The motivation of this study is to supplement the discussion of the dynamic changes of individual behavior in the awareness stage, which not only helps to forecast the product sales, but also provides some useful suggestions for the formulation of marketing strategy.

Product diffusion is a process in which people communicate product information in the awareness stage and make a purchase decision during the decision-making stage. Whether in the awareness stage or in the decision-making stage, people's behavior is highly contagious. This makes many similarities between the diffusion of products and the spread of infectious diseases. For this reason, scholars often use the epidemic model to study the diffusion of products [2, 8–10]. The success of the Bass model also shows that the epidemiological approach is very effective in the aspects of studying the product diffusion. The theoretical basis presented this paper still stems from the epidemiological approach. Since our model and the Bass model are derived from the same theory, our model can be considered as an extension of the Bass model.

The epidemic compartmental model was proposed in 1927 [11], and it characterized the spread mechanism of infectious disease. Since then, the epidemiological approaches had been widely applied to study the spread of memes [12], rumors [13], ideas [14, 15], and interest [16]. Using epidemiological approach to investigate the diffusion of products can be summarized in the following steps. First, we divide the crowd into different classes according to the individual perception and behavior on a product. According to the dynamic changes of the number of people in different classes, we propose a viral product diffusion model. Next, we analyze the dynamic properties of the model. Finally, Google Trends Data are used to test the model. As we shall see, our model fits well with real data.

The contribution of this paper is that we propose a new product diffusion model, which forecasts the developing tendency of products in the market and offers some marketing advices. In contrast with classic models, our model adds to the analysis of dynamic behaviors of individuals before they become buyers and discusses the impact of these dynamic behaviors on the performance of products in the market. These analyses and discussions can be helpful to understand how a product occupies the market or disappears from the market and can help us to adjust marketing strategies more purposefully. In our model, the mechanism and process of product diffusion are better described and presented. Moreover, we also analyze the reasons for the formation of buyers. We believe that some of them are out of interest in the product, while others become buyers because of persuasive advertisement, or the influence of peers or others. We find that potential customers have different psychological characteristics and behavior patterns. For this reason, marketing strategies should also reflect the differences even if these strategies are all aimed at potential customers. This finding provides useful insights on designing more effective marketing strategies.

The rest of this paper is organized as follows. Section 2 states some research works related to product diffusion model, viral marketing, and social contagions. In Section 3, we propose a viral product diffusion model and analyze its dynamic properties. The validity analysis and discussion are laid out in Sections 4 and 5, respectively. Finally, Section 6 provides some concluding remarks.

2. Related Work

2.1. Product Diffusion Model. The classical model for the study of new product diffusion was the Bass model (BM), which divided individuals into the innovators and imitators according to the decision behavior of individuals in a social system [2, 3]. The BM provided a useful framework for understanding the diffusion of new products and technologies.

Based on this framework, Li and Jin [17] proposed that the new product diffusion could be divided into two stages, which were the awareness stage and the decision-making stage. They found that new product diffusion was not be affected by persuasive advertisement in the decision-making stage. Chung et al. [5] provided a sales forecast model for short-life-cycle products, such as movies and games. Van Den Bulte and Joshi [18] discussed the role of influentials and imitators in the diffusion of new products. Moreover, scholars expanded and modified the BM due to the inherent limitations of the original model. Niu [19] provided a Stochastic Bass Model (SBM) so that the trajectory of cumulative adoptions in the BM was stochastic trajectory. Ismail and Abu [6] built a robust BM to forecast the sales volume. Kim and Hong [7] proposed a Bass model with integration constant, which effectively dealt with the nonzero initial level.

In addition to the BM, econometric approach, grey theory, and complex network theory were also used to investigate the product diffusion. Elberse and Eliashberg [20] developed an econometric model to discuss the dynamics of the supply and demand of products in the international market. Wang et al. [21] proposed a time-delayed Verhulst model to describe the delay phenomenon in the diffusion of new products. Since both external and internal factors in the market will influence the new product diffusion, the problem of insufficient data was often encountered in practical research. To solve this problem, Guo et al. [22] gave a grey diffusion model based on grey system theory. Nowadays, the network based on a variety of links is everywhere. Research on product diffusion from the perspective of networks has become inevitable. Wu and Zhang [23] investigated the impact of consumer network structure on the new product diffusion. By simulating the diffusion process in the small world network, they found that the new product will spread faster if there are more weak links.

Although the researches on the product diffusion had been very rich, little attention was paid to the dynamic changes of human behaviors. This study is to fill such a gap. We propose to model the diffusion of products via the dynamics of human behaviors and then investigate the influence of these behavior changes on product performance and marketing strategy.

2.2. Viral Marketing and Social Contagions. Viral marketing was used as a term that started in 1997 [24]. In essence, viral marketing was a special form of electronic word of mouth. It encouraged people to exchange product information or their opinions on products, brands, and companies through social contagions among individuals [25]. Like a virus, viral marketing took advantage of multiplication to spread a message to thousands and even millions [26, 27].

Instead of broadcasting directly the existing advertising to a huge number of users [28], viral marketing targeted a limited number of initial users [29]. Initiating from these seed users, it could activate a "chain-reaction" driven by word of mouth [30]. If the initial users were influential people, then the chain-reaction will be more rapid because these influential people had stronger contagion power in social activities. There were many methods to find influential users or nodes, such as using the characteristics of network members [31] or discovering algorithm based on user trust network [32].

As a marketing strategy, the success of viral marketing is because this strategy gives full play to the role of social contagions in product promotion. If opinion leaders were selected as the seed users, then evidence of contagion was an important driving force to trigger word of mouth chain-reaction [33]. Moreover, social contagions in the product market could arouse repetitive behaviors [34, 35]. These repetitive behaviors have great commercial value because they would bring a lot of potential customers. How to present these repetitive behaviors is one of the highlights of this study.

3. Model and Dynamic Analysis

3.1. Model. Using the epidemiological approach to study a problem requires dividing the population into different classes. To pay respects to the heritage of the epidemic model, we use similar terms to define different classes in our model.

The diffusion of the product in the market is a complex process. It is not only reflected in the visible sales data. In effect, product diffusion should be evaluated with more metrics, such as the transmission of product information and product attention. This not only conforms to the cognitive law, but also can more accurately assess the performance of the product in the market. In such a broad sense of product diffusion, we explain how people are divided into different classes.

When a new product is introduced in the market, some people are very keen on the appearance of this new product. Those individuals who are exposed to and interested in a product at the first time constitute a class, which is called the "susceptible" class S_1. After an individual has a little understanding of a product, he tends to share the information about the product. The "infected" class I is composed of individuals who actively spread the product information in social interactions.

In the Internet age, each individual is not isolated, and the behavior of the individual is likely to be affected by social contagions. When people talk about a product in social interaction, social contagions will lead to the repetitive behaviors that people pay attention to the product. In our model, we also introduce the second type of "susceptible" class. It is denoted by S_2, which consists of individuals who pay repeated attention to the same product. In other words, if an individual has known a product but he still shows great interest in obtaining more product information, then the individual belongs to the "susceptible" class S_2. Note that individuals in S_1 and S_2 have distinct differences, though these individuals are all susceptible individuals. The main differences between S_1 and S_2 are reflected in two aspects. First, the formation of the former is derived from individual psychology, and the latter is the result of social contagions. Second, individuals in S_1 know a little product information. Compared with these individuals, persons in S_2 obtain more information about the product.

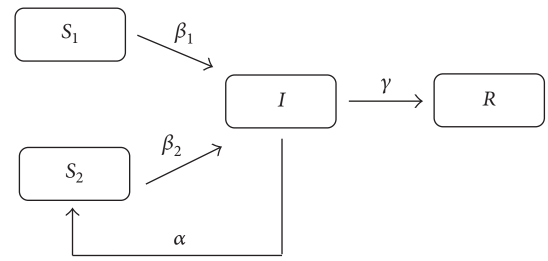


FIGURE 1: Flow chart of individual transfer among different classes.

One product can trigger people's sustained interest, which is what we expect. However, due to the impact of many subjective and objective factors, people lose interest in the product when they know the basic product information. Thus, the last class is called the "recovered" class R, which consists of individuals who have experienced a product and lost interest in the product.

By dividing the population into four classes, the transmission and diffusion of a product in the market can be abstracted as the following dynamic process.

In Figure 1, parameter β_1 is defined as the transmission contact rate, which represents the probability that a susceptible individual becomes an infected individual. Parameter β_2 is the reinfection rate, which shows the rate at which a susceptible individual is repeatedly infected by a product. Parameter α is the proportionality constant of sustained interest, which governs the rate at which infected individuals are affected by social contagions and have sustained interest in a product. Finally, parameter γ is the proportionality constant of loss of interest, which controls the rate at which infected individuals lose interest in the same product and move into the recovered class.

Since class S_1 represents individuals who have not yet been exposed to a product and the number of individuals in this class is influenced by the market capacity and the subjective factors, we assume that the number of increased individuals in class S_1 follows the logistic growth. Suppose that infected individuals have a socially attractive effect on the individuals of class S_1 and class S_2. Hence the infection rate is proportional to $S_1 I$, and the reinfection rate is proportional to $S_2 I$. In addition, the recovery rate is proportional to I when infected individuals transform into recovered individuals. At time t, the population densities of these four classes are denoted by $S_1(t)$, $S_2(t)$, $I(t)$, and $R(t)$, respectively. According to the individual dynamics described in Figure 1, our viral product diffusion model is defined as follows:

$$\begin{aligned}
\frac{dS_1}{dt} &= rS_1\left(1-\frac{S_1}{K}\right)-\beta_1 S_1 I,\\
\frac{dS_2}{dt} &= \alpha I-\beta_2 S_2 I,\\
\frac{dI}{dt} &= \beta_1 S_1 I+\beta_2 S_2 I-\alpha I-\gamma I,\\
\frac{dR}{dt} &= \gamma I.
\end{aligned} \tag{1}$$

In model (1), it is assumed that the increment of susceptible individuals in class S_1 is governed by the logistic growth with a market capacity $K > 0$ as well as intrinsic increase rate constant $r > 0$. According to the actual meaning of parameters, it is easy to know that α, β_1, β_2, and γ are all positive constants and the feasible region of model (1) is $R_+^4 = \{(S_1, S_2, I, R) \mid S_1 \geq 0,\ S_2 \geq 0,\ I \geq 0,\ R \geq 0\}$. Moreover, there is a point to note about the formulation of model (1). In this model, we assume that all individuals have equal effect on other individuals. In other words, for the social contagions between susceptible and infected populations, every individual in I has an equal influence on every individual in S_1 or S_2 and vice versa.

3.2. Dynamic Analysis. Because the first three equations of model (1) do not contain the class R, the dynamic behavior of model (1) is equivalent to the following model:

$$\begin{aligned}\frac{dS_1}{dt} &= rS_1\left(1-\frac{S_1}{K}\right)-\beta_1 S_1 I,\\ \frac{dS_2}{dt} &= \alpha I-\beta_2 S_2 I,\\ \frac{dI}{dt} &= \beta_1 S_1 I+\beta_2 S_2 I-\alpha I-\gamma I.\end{aligned}\tag{2}$$

To solve the equilibria of model (2), let the right side of each differential equation be equal to zero; then we have

$$\begin{aligned}rS_1\left(1-\frac{S_1}{K}\right)-\beta_1 S_1 I &= 0,\\ \alpha I-\beta_2 S_2 I &= 0,\\ \beta_1 S_1 I+\beta_2 S_2 I-\alpha I-\gamma I &= 0.\end{aligned}\tag{3}$$

We obtain two equilibria of model (2) by solving equations (3). They are $M_1(K, \overline{S}_2, 0)$ and $M_2(S_1^*, S_2^*, I^*)$, where $0 \leq \overline{S}_2 < S_2(0)$, $S_1^* = \gamma/\beta_1$, $S_2^* = \alpha/\beta_2$, and $I^* = (rK\beta_1 - r\gamma)/K\beta_1^2$. Here $S_2(0)$ represents the initial value of class S_2. Let $R_0 = K\beta_1/\gamma$, if $R_0 < 1$; then model (2) has a unique equilibrium $M_1(K, \overline{S}_2, 0)$; if $R_0 > 1$, then model (2) has a boundary equilibrium $M_1(K, \overline{S}_2, 0)$ and a unique positive equilibrium $M_2(S_1^*, S_2^*, I^*)$.

Based on the Ordinary Differential Equations (ODE) theory, we analyze the dynamic properties of model (2).

Theorem 1. *Consider model (2) with any given initial condition $S_1(0) > 0$, $S_2(0) > 0$, and $I(0) > 0$. Whether the viral product persists or dies out eventually depends on the value of $R_0 = K\beta_1/\gamma$. If $R_0 < 1$, and $\beta_2\overline{S}_2 - \alpha < 0$, then the solution $(S_1(t), S_2(t), I(t))$ of model (2) asymptotically converges to $(K, \overline{S}_2, 0)$, where $\overline{S}_2 \in [0, S_2(0))$ is initial condition dependent, and the viral product dies out. If $R_0 > 1$, then $\lim_{t\to\infty}(S_1(t), S_2(t), I(t)) = (\gamma/\beta_1, \alpha/\beta_2, (rK\beta_1 - r\gamma)/K\beta_1^2)$, and the viral product persists.*

Proof. Let

$$\begin{aligned}rS_1\left(1-\frac{S_1}{K}\right)-\beta_1 S_1 I &\triangleq P(S_1,S_2,I),\\ \alpha I-\beta_2 S_2 I &\triangleq Q(S_1,S_2,I),\\ \beta_1 S_1 I+\beta_2 S_2 I-\alpha I-\gamma I &\triangleq R(S_1,S_2,I),\end{aligned}\tag{4}$$

that is, $P(\cdot)$, $Q(\cdot)$, and $R(\cdot)$ are functions of variables S_1, S_2, and I. Then the Jacobian matrix J of model (2) associated with equilibrium M_1 is

$$\begin{aligned}J_{M_1} &= \left.\begin{pmatrix}\dfrac{\partial P}{\partial S_1} & \dfrac{\partial P}{\partial S_2} & \dfrac{\partial P}{\partial I}\\ \dfrac{\partial Q}{\partial S_1} & \dfrac{\partial Q}{\partial S_2} & \dfrac{\partial Q}{\partial I}\\ \dfrac{\partial R}{\partial S_1} & \dfrac{\partial R}{\partial S_2} & \dfrac{\partial R}{\partial I}\end{pmatrix}\right|_{M_1}\\ &= \left.\begin{pmatrix} r-\dfrac{2r}{K}S_1 & 0 & -\beta_1 S_1\\ 0 & -\beta_2 I & \alpha-\beta_2 S_2\\ \beta_1 I & \beta_2 I & \beta_1 S_1+\beta_2 S_2-\alpha-\gamma\end{pmatrix}\right|_{M_1}\\ &= \begin{pmatrix} -r & 0 & -\beta_1 K\\ 0 & 0 & \alpha-\beta_2\overline{S}_2\\ 0 & 0 & \beta_1 K+\beta_2\overline{S}_2-\alpha-\gamma\end{pmatrix}.\end{aligned}\tag{5}$$

The characteristic equation associated with the linearization of model (2) at M_1 is

$$\begin{aligned}\left|\lambda E - J_{M_1}\right| &= \begin{vmatrix}\lambda+r & 0 & \beta_1 K\\ 0 & \lambda & -\alpha+\beta_2\overline{S}_2\\ 0 & 0 & \lambda-\left(\beta_1 K+\beta_2\overline{S}_2-\alpha-\gamma\right)\end{vmatrix}\\ &= (\lambda+r)\cdot\begin{vmatrix}\lambda & -\alpha+\beta_2\overline{S}_2\\ 0 & \lambda-\left(\beta_1 K+\beta_2\overline{S}_2-\alpha-\gamma\right)\end{vmatrix}\\ &= (\lambda+r)\cdot\lambda\cdot\left[\lambda-\left(\beta_1 K+\beta_2\overline{S}_2-\alpha-\gamma\right)\right]\\ &= 0,\end{aligned}\tag{6}$$

where E is a unit matrix. The three eigenvalues of the Jacobian matrix at equilibrium M_1 are $\lambda_1 = 0$, $\lambda_2 = -r$, and $\lambda_3 = \beta_1 K + \beta_2\overline{S}_2 - \alpha - \gamma$, respectively. Clearly if $R_0 = K\beta_1/\gamma < 1$, model (2) has a unique positive equilibrium M_1. Moreover, if $\beta_2\overline{S}_2 - \alpha < 0$, then $\lambda_1 = 0$, $\lambda_2 = -r < 0$, and $\lambda_3 = \beta_1 K + \beta_2\overline{S}_2 - \alpha - \gamma < 0$. According to the ODE stability theory [36], equilibrium M_1 is locally asymptotically stable in feasible region $R_+^3 = \{(S_1, S_2, I) \mid S_1 \geq 0,\ S_2 \geq 0,\ I \geq 0\}$. That is, the solution $(S_1(t), S_2(t), I(t))$ of model (2) asymptotically converges to $(K, \overline{S}_2, 0)$. The proof is complete.

Next, consider Jacobian matrix J at equilibrium M_2; we obtain

$$J_{M_2}=\left.\begin{pmatrix}\frac{\partial P}{\partial S_1} & \frac{\partial P}{\partial S_2} & \frac{\partial P}{\partial I}\\ \frac{\partial Q}{\partial S_1} & \frac{\partial Q}{\partial S_2} & \frac{\partial Q}{\partial I}\\ \frac{\partial R}{\partial S_1} & \frac{\partial R}{\partial S_2} & \frac{\partial R}{\partial I}\end{pmatrix}\right|_{M_2}$$

$$=\left.\begin{pmatrix}r-\frac{2r}{K}S_1 & 0 & -\beta_1 S_1\\ 0 & -\beta_2 I & \alpha-\beta_2 S_2\\ \beta_1 I & \beta_2 I & \beta_1 S_1+\beta_2 S_2-\alpha-\gamma\end{pmatrix}\right|_{M_2} \quad (7)$$

$$=\begin{pmatrix}r-\frac{2r}{K}S_1^*-\beta_1 I^* & 0 & -\beta_1 S_1^*\\ 0 & -\beta_2 I^* & \alpha-\beta_2 S_2^*\\ \beta_1 I^* & \beta_2 I^* & \beta_1 S_1^*+\beta_2 S_2^*-\alpha-\gamma\end{pmatrix}$$

$$=\begin{pmatrix}r-\frac{2r}{K}S_1^*-\beta_1 I^* & 0 & -\gamma\\ 0 & -\beta_2 I^* & 0\\ \beta_1 I^* & \beta_2 I^* & 0\end{pmatrix}.$$

Then the characteristic equation associated with the linearization of model (2) at M_2 is

$$\left|\lambda E-J_{M_2}\right|=\begin{vmatrix}\lambda-\left(r-\frac{2r}{K}S_1^*-\beta_1 I^*\right) & 0 & \gamma\\ 0 & \lambda+\beta_2 I^* & 0\\ -\beta_1 I^* & -\beta_2 I^* & \lambda\end{vmatrix} = 0, \quad (8)$$

where E is a unit matrix. This implies that

$$\begin{aligned}&\left[\lambda-\left(r-\frac{2r}{K}S_1^*-\beta_1 I^*\right)\right]\cdot\begin{vmatrix}\lambda+\beta_2 I^* & 0\\ -\beta_2 I^* & \lambda\end{vmatrix}+\left(-\beta_1 I^*\right)\\ &\cdot\begin{vmatrix}0 & \gamma\\ \lambda+\beta_2 I^* & 0\end{vmatrix}=\left[\lambda-\left(r-\frac{2r}{K}S_1^*-\beta_1 I^*\right)\right]\cdot\lambda\\ &\cdot\left(\lambda+\beta_2 I^*\right)-\beta_1 I^*\cdot(-\gamma)\cdot\left(\lambda+\beta_2 I^*\right)\\ &=\left(\lambda+\beta_2 I^*\right)\\ &\cdot\left[\lambda^2-\lambda\left(r-\frac{2r}{K}S_1^*-\beta_1 I^*\right)+\beta_1\gamma I^*\right]=0.\end{aligned} \quad (9)$$

Clearly $\lambda_1'=-\beta_2 I^*<0$; let $b=r-(2r/K)S_1^*-\beta_1 I^*$, $c=\beta_1\gamma I^*$; then $\lambda_{2,3}'=(b\pm\sqrt{b^2-4c})/2$.

Due to $S_1^*=\gamma/\beta_1$, $I^*=(rK\beta_1-r\gamma)/K\beta_1^2$, it is easy to know that

$$\begin{aligned}b&=r-\frac{2r}{K}S_1^*-\beta_1 I^*=r-\frac{2r}{K}\cdot\frac{\gamma}{\beta_1}-\beta_1\cdot\frac{rK\beta_1-r\gamma}{K\beta_1^2}\\ &=\frac{-r\gamma}{K\beta_1}<0.\end{aligned} \quad (10)$$

Moreover, $c=\beta_1\gamma I^*>0$ when $R_0=K\beta_1/\gamma>1$; it is clear that λ_2' and λ_3' have negative real parts. Consequently, all roots of the characteristic equation (8) have negative real parts when $R_0=K\beta_1/\gamma>1$. According to the ODE stability theory, the unique positive equilibrium M_2 is locally asymptotically stable in feasible region R_+^3. This means that $(S_1(t),S_2(t),I(t))\to(\gamma/\beta_1,\alpha/\beta_2,(rK\beta_1-r\gamma)/K\beta_1^2)$ as $t\to\infty$. □

4. Model Validation

4.1. Data. The rapid development of Internet technology makes it possible to obtain the historical search data of a product. These historical data record the search behavior of a product and reflect the people's attention to the product. To test our model, we collect Google search engine data, which are obtained from the Google Trends tool. Google search data began in 2004 and spaced at weekly intervals. These data report the relative number of search volume for a given product.

As the data provided by Google Trends are very rich, the selection of sample data is very flexible. Generally speaking, we only need to follow some major rules to determine the search strings. First, search strings are not confined to specific physical products. They can also be new applications in social networks or popular brands. Diverse choices can ensure the applicability of the model. Second, the selection of search strings should be relatively new because Google Trends provides search data since 2004. Finally, popular products are the best choice. For a given search string, we obtain the search data which are normalized by Google Trends. In a sense, if a product is more popular, then Google Trends can provide more effective product data. Based on the above criteria, we choose three search strings as sample data. They are Instagram, Nokia Lumia 920, and The North Face.

4.2. Parameter Estimation. In order to test the validity of the model, we first need to estimate the unknown parameters in the model. In this section, we introduce the theory of parameter estimation and how to estimate those unknown parameters according to the sample data. Considering system (2) with initial conditions, which can be expressed in the following form:

$$\begin{aligned}\dot{z}&=f(t,z,\theta),\\ z(t_0)&=z_0,\end{aligned} \quad (11)$$

where $z=(S_1,S_2,I)\in R^3$ is the vector of state variables, $\theta=(r,K,\beta_1,\beta_2,\alpha,\gamma)\in R^6$ is the vector of unknown parameters, and $t\in R$ is a time variable.

Suppose that the observed values of system (11) are denoted by z_j; then $z_j=z(t_j,\theta_0)+\varepsilon_j$, $j=1,2,\ldots,n$. Here $z(t_j,\theta_0)$ are the real values of system (11) when parameters $\theta=\theta_0$ and ε_j are random deviations. And these random deviations satisfy the following conditions.

(1) ε_j $(j=1,2,\ldots,n)$ are i.i.d. random variables.

(2) $\forall j$, then $E(\varepsilon_j)=0$.

(3) $\forall j$, then $\mathrm{var}(\varepsilon_j)=\sigma^2<\infty$.

Table 1: Estimated parameter values.

Product	Parameter					
	r (10^{-2})	K (10^{2})	β_1 (10^{-4})	β_2 (10^{-4})	α (10^{-3})	γ (10^{-3})
Instagram	5.0	3.5	0.03	0.9	0.3	0.5
Nokia Lumia 920	0.1	5.0	6.5	81.0	35.0	3.3
The North Face	1.5	8.0	4.7	5.0	0.042	0.027

According to the Ordinary Least Squares (OLS), we can solve an estimator $\widehat{\theta}$ which satisfies:

$$\widehat{\theta}_{\mathrm{OLS}} = \min_{\theta\in\Theta} f(\theta) = \min_{\theta\in\Theta} \sum_j \left[z_j - z\left(t_j;\theta\right)\right]^2, \quad (12)$$

where Θ is the feasible region for parameter θ.

To solve a best estimator, we can perform the following steps. First, we compute the numerical solutions of system (11), which can be solved using MATLAB ode45. Note that it is very important to give a reasonable initial value. This is because the choice of the initial value directly affects the speed and the accuracy of numerical computation. Second, according to (12), that is, the OLS theory, we can obtain a new set of parameters by making numerical solutions of system (11) better fit the observed data. This step can also be achieved via MATLAB. MATLAB's optimization toolbox provides many optimization algorithm functions, such as function "fminbnd," "fminsearch," and "fminunc." Here we use the function "fminsearch" to optimize the objective function $f(\theta)$. Third, using the new parameters obtained by the second step, we compute new numerical solutions of system (11), and then these solutions are compared with the observed data again. The iteration process between numerical solutions of system (11) and parameter updating will stop when the system residual meets the demands. Through the above three steps, we solve an estimator $\widehat{\theta}$ that meets the convergence criteria of a given optimization algorithm. This $\widehat{\theta}$ is an optimal estimation of the unknown parameters in the feasible region Θ.

4.3. Results. According to the above calculation steps, the results of the parameter estimation for the three strings are shown in Table 1.

The first search string is "Instagram," which is a very popular social networking service. Google Trends Data showed that the search volume of "Instagram" gradually increased since 2011. Up to now, the number of people who are concerned about Instagram is still in a stable growth state. Figure 2 presents the fitted curve $I(t)$ plotted alongside the Google Trends Data.

The second example is a digital product. The exact search string is "Nokia Lumia 920" to obtain sample data. The search volume of this product reached its peak in 2013 and then began to decline gradually. Google Trends Data showed that the search trend of "Nokia Lumia 920" presented an obvious peak. The fitted curve of the product can be seen in Figure 3.

The last search string is "The North Face," which is a fashion brand. Unlike the above two examples, this brand existed prior to 2004 so that its search records began in 2004.

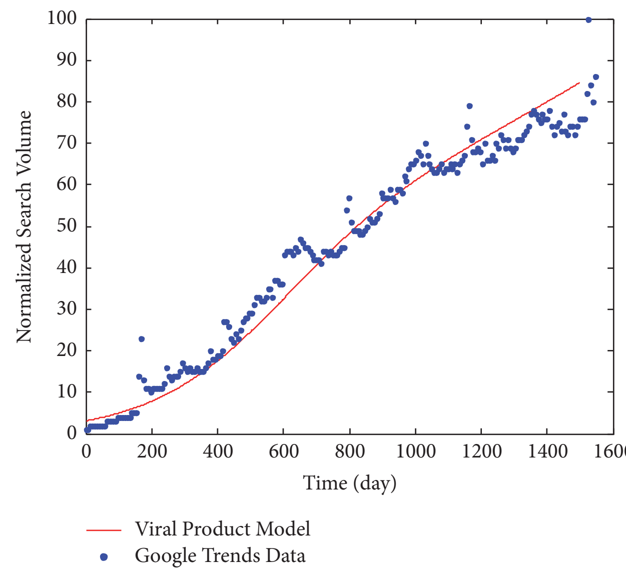


Figure 2: Model fitting on "Instagram" from October 2011 to January 2016.

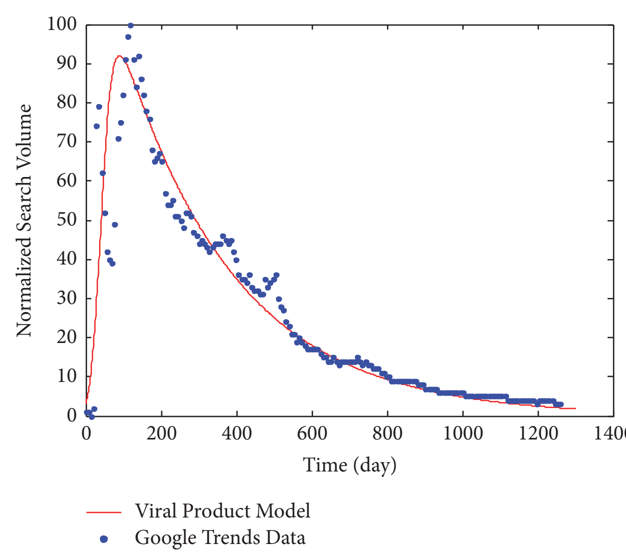


Figure 3: Model fitting on "Nokia Lumia 920" from August 2012 to January 2016.

The search volume of this brand showed multiple peaks in 2004–2016. That is to say, the performance of the brand in the market presents fluctuating changes. Figure 4 shows the fitted curve of Google Trends Data.

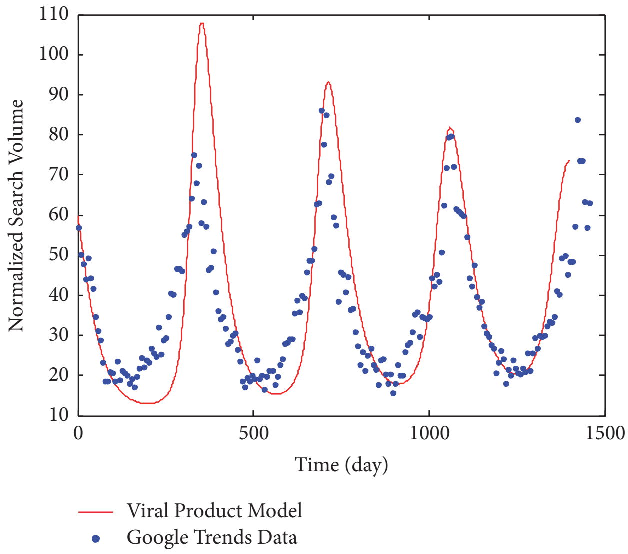


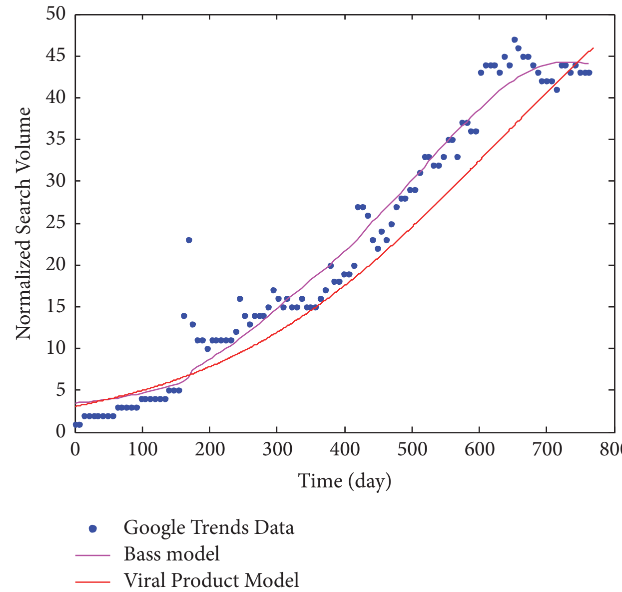
FIGURE 4: Model fitting on “The North Face” from January 2012 to January 2016.

The above fitting results illustrate that our model has shown good adaptability for the various performances of the product. The model not only depicts the steady growth process of products, but also describes the whole process that the products increase in the initial stage and then gradually decrease and even can handle multiple peaks of products. Hence, our model can be used to forecast the performance of products in the market.

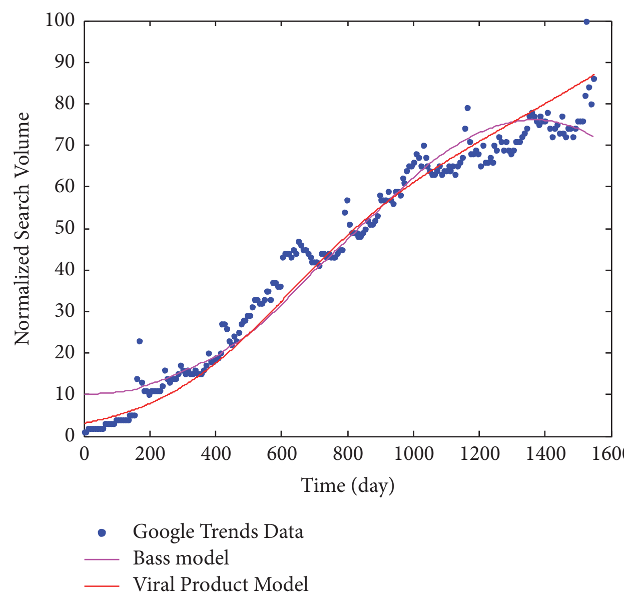
4.4. Comparison Analysis. From Figures 2, 3, and 4, we can observe that the viral product diffusion model has, in some sense, been tested and verified. In order to illustrate the forecasting effect of the model, we compare the model with the Bass model based on the above sample.

The basic form of the Bass model is $S_t = a + bY_{t-1} + cY_{t-1}^2$, $t = 2, 3, \ldots$, where S_t presents sales at t, and Y_{t-1} is cumulative sales through period $t-1$. The specific meaning of parameters a, b, and c can be found in this literature [2]. Based on this model, we first estimate the parameters in the model via “Instagram” and “Nokia Lumia 920” data. Here parameter estimation is computed by regression analysis. Table 2 shows the regression results. On the basis of this, Figures 5 and 6 illustrate the predicted curves and actual curves for the two samples.

Figures 5 and 6 show that the Bass model and our model all provide a good description of the general trend of the actual data. But as shown in Figures 5 and 6, we find that the Bass model provides a more efficient prediction in the vicinity of the peak. The result of forecasting is particularly good for short-term trend before the peak, as shown in Figure 5(a). Contrary to this, our model predicts long-term trend better than the Bass model. In addition, the Bass model can only simulate the inverted U curve. For this reason, we did not use “The North Face” data to test the Bass model. The Bass model, $S_t = a + bY_{t-1} + cY_{t-1}^2$, present a parabola, which is a single peak curve. However, our model can simulate the oscillation of the product curve. It handles multiple peaks of products.



(a) Oct 23, 2011–Nov 30, 2013



(b) Oct 23, 2011–Jan 23, 2016

FIGURE 5: Actual data and curves predicted by two models for “Instagram.”

5. Discussion

To illustrate the general applicability of the model, we use three cases to validate our model. It is interesting to note from Figure 4 that our model fits well with the transient oscillations. Actually, Theorem 1 demonstrates that eventually the solution of model (2) should approach the equilibrium point, which is illustrated by the decreasing amplitudes of transient oscillation in Figure 7. This means that our model is

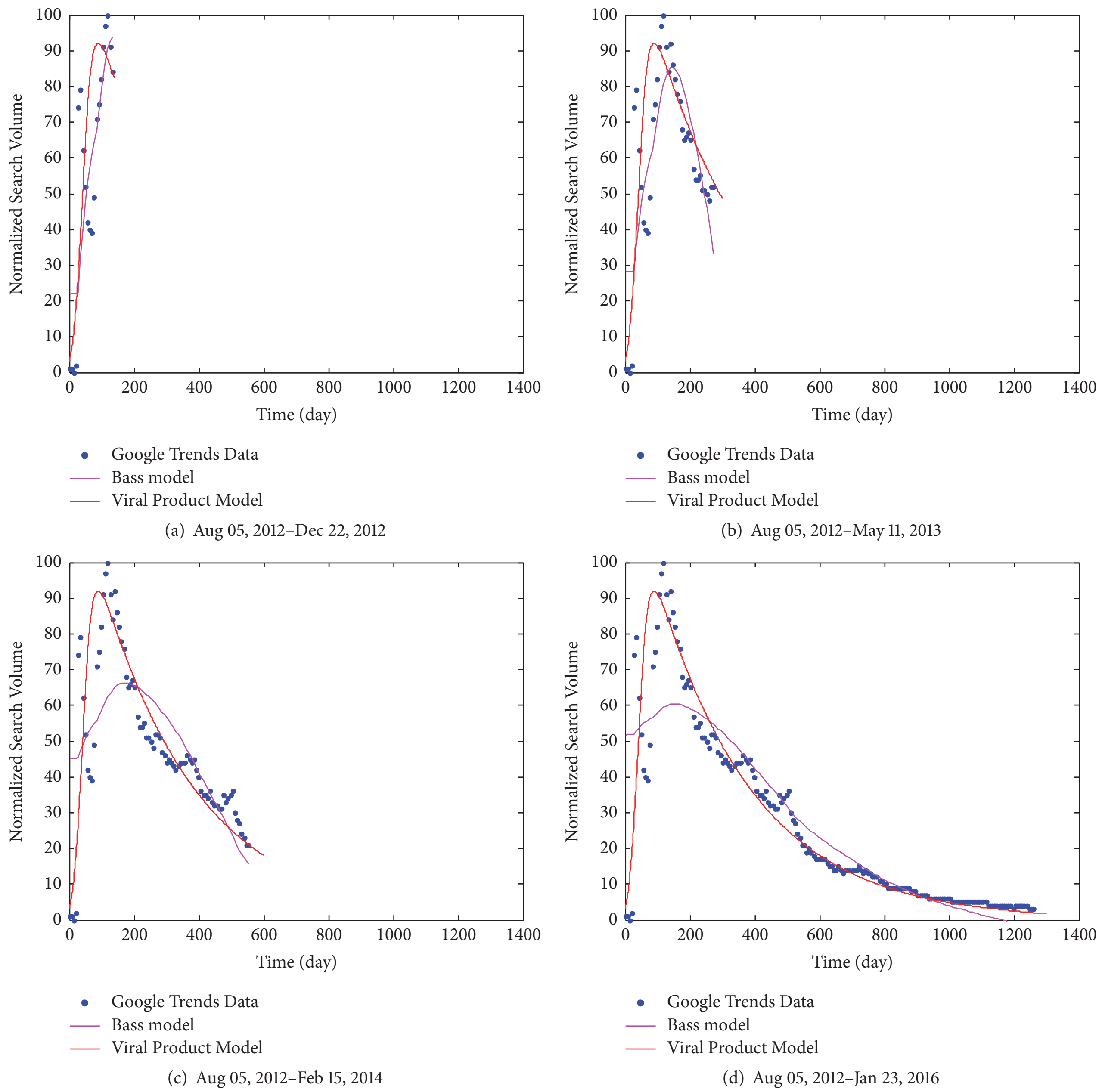


Figure 6: Actual data and curves predicted by two models for "Nokia Lumia 920."

Table 2: Bass model regression results.

Product	Period	a	b	c (10^{-6})	R^2
Instagram	Oct 23, 2011–Nov 30, 2013	3.4952	0.0367	−8.2387	0.9644
Instagram	Oct 23, 2011–Jan 23, 2016	10.0414	0.0169	−1.0801	0.9570
Nokia Lumia 920	Aug 05, 2012–Dec 22, 2012	21.9117	0.1306	−59.1643	0.6064
Nokia Lumia 920	Aug 05, 2012–May 11, 2013	28.1587	0.0952	−39.6549	0.5729
Nokia Lumia 920	Aug 05, 2012–Feb 15, 2014	45.2149	0.0278	−9.2015	0.4596
Nokia Lumia 920	Aug 05, 2012–Jan 23, 2016	51.7469	0.0134	−5.1592	0.7815

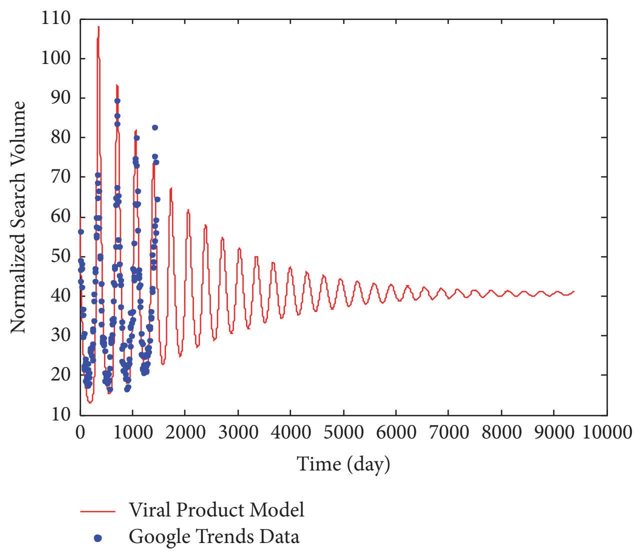


Figure 7: The solution of viral product diffusion model will tend to the equilibrium point.

also applicable for the situation where the peaks of a product are gradually weakened.

By analyzing the dynamic properties of model (2) and fitting the actual data, we find that the diffusion process of the product indeed is reflected in the dynamic changes of the number of individuals in different classes. For this reason, we can regulate the market performance of the product by altering parameters α, β_1, β_2, and γ in model (2).

The ideas to change these parameters are as follows. In view of the differences mentioned in the previous between S_1 and S_2, marketing strategies for adjusting parameters β_1 and β_2 are different. Since individuals in class S_1 are more curious to a new product and they experience the product only once, the strategy of changing parameter β_1 must emphasize the novelty and interest of the product so as to make these individuals move into class I. However, such marketing ideas are not suitable for class S_2. Individuals in class S_2 intend to gain more product information so that they can get a better understanding of a product. Accordingly, the strategy of adjusting parameter β_2 should highlight the function of the product and promote the user feedback. Individuals move from class I to class S_2 due to social contagions. Thus, increasing parameter α needs to draw support from social contagions. For instance, we can devise a method to drive chain-reaction of word of mouth or utilize the infectivity of individual behaviors. Such strategies are easy to arouse the herd effect, which are effective methods to increase the number of individuals in class S_2. As for parameter γ, the key of adjusting this parameter is to focus on how to maintain the individual's interest in one product. As long as we follow this principle, the strategies to change parameter γ are very flexible. In short, we need to fully consider the characteristics of individuals in different classes when we design marketing strategies. Targeted marketing is more effective.

We have a brief explanation on parameters K and r in model (2). Note that parameter K represents the constant population size, which has a convoluted meaning because Google Trends Data are normalized. Parameter r is influenced by many factors so that this value is generally different for different products.

6. Conclusions

In this paper we propose a viral product diffusion model via the epidemiological approach. Dynamic analysis of the model reveals the conditions for a product to persist or die out. The model has been verified according to Google Trends Data. Validation results show that our model describes the steady growth or a single peak of a product very well. Also, this model can reflect the fluctuation of a product. It is remarkable that the diffusion process of the product in the market can be modelled by such a simple viral model and the real data fit a theoretical curve so closely.

Our research provides a new approach to forecast the long-term performance of products through their early behaviors. Depending on accurate prediction of product performance, the company managers may adjust the production project and operation plan and update the products. The new approach provides a theoretical guarantee for the management to make the right decision. Furthermore, this study is also instructive to marketers. In order to achieve the expected sales, the design of marketing strategies should grasp the characteristics of customers, even if these customers are all potential customers. Specifically, marketing methods for individuals in class S_1 should emphasize the novelty and interest of a product, which can stimulate them to purchase or spread the product. While individuals in class S_2 are the marketing targets, marketing strategies should include more information on product functions and user experiences in order to make these potential customers believe and buy a product. These suggestions can help marketers to develop more effective marketing strategies.

There are still some limitations in this study. Parameters needed to be estimated in the three examples are assumed to be time-independent constants while they are likely to be time-varying. We will explore the product diffusion model with time-varying parameters in the future.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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