A Maximum Entropy Multisource Information Fusion Method to Evaluate the MTBF of Low-Voltage Switchgear

Jing-Qin Wang,1 Zhi-Gang Zhang,1 Ching-Hsin Wang,2 and Li Wang1

1School of Electrical Engineering, Hebei University of Technology, Tianjin 300130, China
2Institute of Project Management, Department of Leisure Industry Management, National Chin-Yi University of Technology, Taichung 41170, Taiwan

Correspondence should be addressed to Ching-Hsin Wang; thomas_6701@yahoo.com.tw

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1. Introduction

About 80% of electric energy in our country is provided by low-voltage switchgear; therefore, the reliability of low-voltage switchgear is significant for the safety and efficient operation of electric power system [1, 2]. When analyzing the reliability by Bayesian theory, although it is a solid theory foundation for the evaluation of low-voltage switchgear that rational utilization of experience and prior information, the determination of prior information is an important problem. It has received attention at home and abroad.

Bayesian network is a commonly used tool in probabilistic reasoning of uncertainty in industrial processes [3]. Cai et al. [4–6] proposed a multisource information fusion based fault diagnosis methodology using Bayesian network, which can increase the fault diagnostic accuracy for single fault. Huang et al. [7] developed a practical approach for fatigue lifetime assessment of aircraft engine discs by combining a multisource information fusion method with a Bayesian inference technique and the method quantifies the subjective information, checks different experts’ information, and fuses multiple prior distributions. Rigas et al. [8] presented a research for the use of multisource information fusion, which can currently evaluate specifically for the eye movement biometrics. Wang et al. [9] designed a framework for multisource heterogeneous information fusion in the IoT and used an experimental simulation platform to build an environmental monitoring system to assess the framework. Zhao et al. [10] investigated the empirical entropy method for right censored data, which gives better coverage probability than that of the empirical likelihood method for contaminated and censored lifetime data. Singh et al. [11] used a theoretical method based on maximum Shannon entropy framework to study the finite buffer system, and the advantage of the method is that it has
enabled one to derive the analytical closed form generalized expression of the probability distribution of queue size in finite buffer system.

The collection of prior information is very important, when analyzing the reliability of low-voltage switchgear by Bayesian method. The maximum entropy principle has advantages for the use of prior information effectively [12, 13]. Multisource information fusion can fuse experts experience and historical data effectively [14, 15]. It considers prior information and the optimal prior distribution is selected by the maximum entropy under the boundary conditions [16, 17]. Caticha Ariel used maximum entropy to translate the information contained in the form of the likelihood into a prior distribution for Bayesian inference [18]. Kim demonstrated the method that obtains prior information by entropy [19], which showed the effectiveness of the method. Therefore, in this paper, the maximum entropy multisource information fusion method was proposed to obtain the prior information of low-voltage switchgear and then evaluate the reliability index MTBF. The maximum entropy multisource information fusion method not only considers the credibility of prior information, but also obtains reliable results from prior information completely. Firstly, the credibility of prior information was analyzed by expert experience and historical data. Secondly, the consistency test was completed by Smirnov test to check whether prior information and field test information obey the same distribution. Thirdly, the prior distribution type was known, and hyperparameters of prior distribution function were calculated by bootstrap method. At last, the a posteriori distribution was obtained, and the evaluation of MTBF was completed.

2. The Credibility Analysis and Consistency Test of Prior Information

2.1. The Reliability Evaluation Step of Low-Voltage Switchgear

The flowchart of reliability evaluation step of low-voltage switchgear was shown in Figure 1.

2.2. The Collection of Prior Information

The prior information of low-voltage switchgear is obtained mainly by two aspects.

(1) Historical Information. The information is obtained by historical data, and it is the most reliable and trustworthy way.

(2) Expert Opinion and Engineering Experience. Experts who work in the forefront production or design have a rich engineering experience, and they have a thorough understanding for low-voltage switchgear. The information can be obtained by the communication with them. It is unavoidable that there have some subjective components, but it accords with the project.

Manufacturers which dedicated low-voltage switchgear design development can be researched, and the entire user’s feedback information can be collected. It is equivalent to gain the first-hand information from the production line. The historical data of low-voltage switchgear from the year 2007 to 2011 are shown in Figures 2–6.

Because of the difference of external environment and attainment methods, the prior information may not come from the same population distribution. And credibility analysis and consistency test are the key to this question [20]. Only credibility analysis and consistency test can ensure the validity of prior information.

2.3. The Credibility Analysis of Prior Information

Credibility analysis for prior information is important, because only credible prior information can reduce the risk in decision making.

It is denoted that $X = (x_1, x_2, \ldots, x_m)$ is the field test information and $F_m(x)$ is the distribution function, $Y = (y_1, y_2, \ldots, y_n)$ is the prior information and $G_n(y)$ is the distribution function, and the two samples $X$ and $Y$ are independent.

To check whether the two types of information are the same population distribution, the hypotheses were as follows:

$H_0$: $X$ and $Y$ are the same population distribution.
$H_1$: $X$ and $Y$ are not the same population distribution.
To illustrate the concept of prior information credibility, it was denoted that

\( A \) accepts event \( H_0 \),

\( \overline{A} \) refuses event \( H_0 \) and accepts event \( H_1 \).

Therefore, \( P(A \mid H_0) = 1 - \alpha \) and \( P(\overline{A} \mid H_0) = \alpha \), where \( \alpha \) is the probability of rejection event \( A \) when it is true.

Then the prior information credibility of \( Y \) is

\[
P(H_0 \mid A) = \frac{P(\overline{A} \mid H_0) P(H_0)}{P(A \mid H_0) P(H_0) + P(A \mid H_1) (1 - P(H_0))} \tag{1}
\]

where \( \beta \) is the probability of the acceptance of event \( A \) when it is false.

If the prior probability \( P(H_0) \), \( \alpha \), and \( \beta \) are known, the credibility of \( Y \) can be obtained.

The prior probability \( P(H_0) \) of every year was shown in Table 1. It is got according to the expert experience and historical data.

The risk equal principle [21, 22] is used to confirm \( \alpha \) and \( \beta \), \( \alpha = \beta = 0.05 \). Then the credibility of prior information from the year 2007 to 2011 was summarized in Table 2.

From Table 2, it can be observed that the credibility of prior information of low-voltage switchgear is very high. And the higher credibility of prior information shows the higher confidence level of the reliability evaluation.

2.4. The Consistency Test of Prior Information. To use prior information accurately, the consistency between prior and field test information should be taken into account, if the
Table 3: The results of Smirnov test.

<table>
<thead>
<tr>
<th>$X_i$</th>
<th>$Y_i$</th>
<th>$F_m(x) - G_n(y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>$1/12 - 1/15 = 1/60$</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>$2/12 - 2/15 = 2/60$</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>$3/12 - 3/15 = 3/60$</td>
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<tr>
<td>2</td>
<td>2</td>
<td>$4/12 - 3/15 = 8/60$</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>$5/12 - 5/15 = 5/60$</td>
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<tr>
<td>5</td>
<td>6</td>
<td>$6/12 - 5/15 = 10/60$</td>
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<td>6</td>
<td>6</td>
<td>$7/12 - 7/15 = 7/60$</td>
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<tr>
<td>6</td>
<td>6</td>
<td>$7/12 - 8/15 = 3/60$</td>
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<tr>
<td>7</td>
<td>7</td>
<td>$7/12 - 9/15 = -1/60$</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>$8/12 - 8/15 = -4/60$</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>$10/12 - 13/15 = -2/60$</td>
</tr>
<tr>
<td>12</td>
<td>12</td>
<td>$11/12 - 13/15 = 3/60$</td>
</tr>
<tr>
<td>14</td>
<td>14</td>
<td>$11/12 - 1 - 5/60$</td>
</tr>
<tr>
<td>17</td>
<td>1</td>
<td>$1 - 1 = 0$</td>
</tr>
</tbody>
</table>

The Smirnov test statistics of the two samples was shown in Table 3.

In Table 3, it can be known that $m = 12$, $n = 15$, $D_{m,n} = 15/60 = 1/4$. Also, it can be observed that $P(D_{m,n} < 23/60) = 0.80$ from the attached list 10 in literature [23], and $P(D_{m,n} > 1/4) > 0.2$. Therefore, $H_3$ is accepted, and the field test information and the prior information are the same entire sample.

The historical data of 2011 is the field test information, and the historical data of 2007, 2008, 2009, and 2010 is the prior information, respectively. According to Smirnov test, the historical data of 2011 is the same entire sample with the year of 2007, 2008, 2009, and 2010.

3. The Prior Distribution Type of Low-Voltage Switchgear

The distribution type of prior information has a direct impact on reliability evaluation of low-voltage switchgear. In general, the reliability distributions of electrical products contain the following types, exponential distribution, normal distribution, and Weibull distribution [24, 25]. Because of having three parameters, Weibull distribution can fit data precisely and contains exponential distribution and normal distribution. Therefore, it is assumed that the failure time of low-voltage switchgear obeys Weibull distribution.

The probability density distribution function of Weibull distribution [26] with three parameters is

$$f(t; \eta, m, \gamma) = \frac{m}{\eta} \left( \frac{t-\gamma}{\eta} \right)^{m-1} \exp \left( - \left( \frac{t-\gamma}{\eta} \right)^m \right), \quad t > 0,$$

where $\eta$ is the scale parameter, $m$ is the shape parameter, and $\gamma$ is the time extension function. In this paper, there is no time extension; that is to say, $\gamma$ is 0.

Therefore, (3) is reduced to

$$f(t; \eta, m) = \frac{m}{\eta} \left( \frac{t-\gamma}{\eta} \right)^{m-1} \exp \left( - \left( \frac{t}{\eta} \right)^m \right), \quad t > 0.$$  

The historical data was ordered from small to large $t_1 \leq t_2 \leq \cdots \leq t_r$, and $X_i = \ln t_i$. Then the test statistics $W$ is

$$W = \frac{\sum_{i=1}^{r-1} (k_i / (r - r_i - 1))}{\sum_{i=1}^{r-1} (t_i / r_i)},$$

where

$$r_1 = \left\{ \begin{array}{ll} \frac{r}{2} & \text{when } r \text{ is a even number} \\ \frac{(r - 1)}{2} & \text{when } r \text{ is an odd number} \end{array} \right.$$  

$$k_i = \frac{x_{i+1} - x_i}{\ln \left[ \ln \left( (4 (r - i - 1) + 3) / (4r + 1) \right) / \ln \left( (4 (r - i) + 3) / (4r + 1) \right) \right]}.$$
When the significance level is $\alpha$, if the prior information of low-voltage switchgear obeys Weibull distribution, the $W$ should obey
\begin{equation}
F_{\alpha/2}(2(r - r_1 - 1), 2r_1) \leq W
\end{equation}
\begin{equation}
\leq F_{1-\alpha/2}(2(r - r_1 - 1), 2r_1).
\end{equation}
According to data in Figure 6, it can be seen that $r = 12$, $r_1 = 6$. If $\alpha = 0.05$, it can be calculated that $W = 0.3502$, $F_{0.975}(10, 12) = 2.91$, $F_{0.025}(10, 12) = 0.3436$, and $F_{0.025}(10, 12) < W < F_{0.975}(10, 12)$. Therefore, the prior information in 2011 obeys Weibull distribution. 

4. The Maximum Entropy Method for the Prior Distribution

4.1. Prior Distribution of Low-Voltage Switchgear. Formula (8) is the expression of the joint prior distribution $\pi(\eta, m)$, and formula (9) to formula (11) are the constraint conditions.

\begin{equation}
\max H(\eta, m) = -\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \pi(\eta, m) \ln \pi(\eta, m) d\eta dm
\end{equation}
\begin{equation}
\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \pi(\eta, m) d\eta dm = 1
\end{equation}
\begin{equation}
\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \eta^i \pi(\eta, m) d\eta dm = E(\eta^i) = \eta_i,
\end{equation}
\begin{equation}
\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} m^j \pi(\eta, m) d\eta dm = E(m^j) = m_j,
\end{equation}
where $\eta_i$, $m_j$ are the origin moments of $i$ order and $j$ order of two-parameter $\eta$ and $m$, respectively; $k$ and $n$ are the highest order of two-parameter origin moment, respectively.

To solve $\pi(\eta, m)$, the following auxiliary function was constructed:
\begin{equation}
J[\pi(\eta, m)] = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left(-\pi(\eta, m) \ln \pi(\eta, m)ight) d\eta dm + \sum_{i=0}^{k} \lambda_i \eta^i \pi(\eta, m) + \sum_{j=0}^{n} c_j m^j \pi(\eta, m),
\end{equation}
where $\lambda_i$ and $c_j$ were the Lagrange Multiplier. 

When $\partial J[\pi(\eta, m)] / \partial \pi(\eta, m) = 0$, it can be solved that
\begin{equation}
\pi(\eta, m) = \exp \left(\lambda_0 - 1 + \sum_{i=1}^{k} \lambda_i \eta^i + \sum_{j=1}^{n} c_j m^j\right).
\end{equation}
According to the expert experience, $k = n = 2$,
\begin{equation}
\pi(\eta, m) = \exp \left(\lambda_0 - 1 - \frac{\lambda_1}{4} \eta^2 - \frac{c_1}{4} m^2 + \lambda_2 \left(\eta + \frac{\lambda_1}{2\lambda_2}\right)^2 + c_2 \left(m + \frac{c_1}{2c_2}\right)^2\right).
\end{equation}

On the constraints, by joint density function normalization, $\pi(\eta, m)$ obeys bivariate normal distribution as
\begin{equation}
\pi(\eta, m) = \frac{1}{2\pi \sigma_{\eta} \sigma_{m}} \exp \left(-\frac{(\eta - \eta_\text{avg})^2}{2\sigma_{\eta}^2} - \frac{(m - m_\text{avg})^2}{2\sigma_{m}^2}\right),
\end{equation}
where $\eta_\text{avg}$, $m_\text{avg}$, $\sigma_{\eta}^2$, and $\sigma_{m}^2$ were the mathematical expectation and the variance of $\eta$ and $m$.

From (15), it can be known that the prior distribution not only contains the known prior information but tries to avoid the introduction of other assumption information.

4.2. The Determination of Parameters. In (15), it contains two parameters $\eta$ and $m$. And to obtain $\eta_\text{avg}$, $m_\text{avg}$, $\sigma_{\eta}^2$, and $\sigma_{m}^2$, the bootstrap method [27–30] was used.

$T = (t_1, t_2, \ldots, t_n)$ denotes a set of obtained samples with total failure. The distribution parameter samples can be obtained by the following steps.
(1) The self-help sample $T^* = (t_1^*, t_2^*, \ldots, t_n^*)$ can be obtained by carrying out sampling with replacement for $T = (t_1, t_2, \ldots, t_n)$.
(2) $\eta_\text{avg}$ and $m_\text{avg}$ can be got from the maximum likelihood estimation with the self-help sample $T^* = (t_1^*, t_2^*, \ldots, t_n^*)$.
(3) Repeat the previous two steps for $N$ times; then the estimated parametric sample can be got.
\begin{equation}
\{(\hat{\eta}_1, \hat{m}_1), (\hat{\eta}_2, \hat{m}_2), \ldots, (\hat{\eta}_N, \hat{m}_N)\}.
\end{equation}

(4) The expectation and variance of the unknown parameter $m$ and $\eta$ can be got by the estimated parametric samples.
\begin{equation}
\eta_\text{avg} = \frac{1}{n} \sum_{i=1}^{n} \eta_i,
\end{equation}
\begin{equation}
\sigma_{\eta}^2 = \frac{1}{n-1} \sum_{i=1}^{n} (\eta_i - \eta_\text{avg})^2
\end{equation}
\begin{equation}
m_\text{avg} = \frac{1}{n} \sum_{j=1}^{n} m_j,
\end{equation}
\begin{equation}
\sigma_{m}^2 = \frac{1}{n-1} \sum_{j=1}^{n} (m_j - m_\text{avg})^2.
\end{equation}

By calculating the transcendental moment, the parameter expressions of the prior distribution can be got,
\begin{equation}
\bar{m} = \bar{m}_\text{avg},
\end{equation}
\begin{equation}
\sigma_{m}^2 = \sigma_{m}^2.
\end{equation}
\[ \eta = \bar{\eta}, \]
\[ \sigma^2_\eta = S^2_\eta. \]  

(18)

Input the data in Figures 2–6 to the MATLAB; the self-help sample can be got by the bootstrap method. And the maximum likelihood estimate can be got by the \texttt{wblfit()} command in MATLAB. Finally, the mathematical expectation and the variance of the two parameters are

\[ \bar{\eta} = 6145.5, \]
\[ m = 1.4000 \]
\[ \sigma^2_\eta = 9832.0, \]
\[ \sigma^2_m = 0.0016. \]  

(19)

The prior density function of low-voltage switchgear is

\[ \pi(\eta, m) = \frac{1}{7.9325\pi} \cdot \exp\left( -\frac{(\eta - 6145.5)^2}{19.664} - \frac{(m - 1.4000)^2}{0.0032} \right). \]  

(20)

5. The MTBF Evaluation of Low-Voltage Switchgear

Based on Bayesian principle, the field life data of low-voltage switchgear can be assumed as \( T = (t_1, t_2, \ldots, t_n) \), and then its likelihood function can be expressed as

\[ p(t_1, t_2, \ldots, t_n | \eta, m) = \left( \frac{m}{\eta} \right)^n \prod_{i=1}^{n} \left( \frac{t_i}{\eta} \right)^{m-1} \exp\left( -\sum_{i=1}^{n} \left( \frac{t_i}{\eta} \right)^m \right). \]  

(21)

Then the joint posterior distribution of parameter \( \eta, m \) can be expressed as

\[ h(\eta, m | t_1, t_2, \ldots, t_n) = \frac{p(t_1, t_2, \ldots, t_n | \eta, m) \pi(\eta, m)}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(t_1, t_2, \ldots, t_n | \eta, m) \pi(\eta, m) d\eta dm}. \]  

(22)

By the numerical calculation, the point estimate of the posterior distribution can be obtained,

\[ \bar{m} = E(m | t_1, t_2, \ldots, t_n) = 1.1640 \]
\[ \bar{\eta} = E(\eta | t_1, t_2, \ldots, t_n) = 7021.3. \]  

(23)

Finally, the estimation value of MTBF of low-voltage switchgear is

\[ \text{MTBF} = \bar{\eta} \left( 1 + \frac{1}{\bar{m}} \right) = 6659.06 \text{ h}. \]  

(24)

6. Conclusion

A maximum entropy multisource information fusion method was proposed to obtain the prior information of low-voltage switchgear and then evaluate the reliability. The proposed method fused the expert experience, engineering experience, and historical data. And the method was illustrated by historical data of the year from 2007 to 2011 and experience of low-voltage switchgear.

The credibility analysis and the compatibility test of the prior information were presented by the Smirnov test method. The higher credibility of prior information shows the higher confidence level of historical data and experience. Also, it is a solid basic for the reliability evaluation of low-voltage switchgear

The type of the prior information was determined and the parameter was solved by maximum entropy. It obeys bivariate normal prior distribution. The estimation values of the two parameters are 6145.5 and 1.4000. According to Bayesian theory, the point estimations of the posterior distribution of the two parameters are 7021.3 and 1.1640. Finally, the estimation value of posterior MTBF of low-voltage switchgear is 6659.03 h, which is about 0.76 years. It is a waste to humans in effort, time, and finance. The evaluation result reduces the experimental period and test cost, which is an improvement for the reliability evaluation and management of low-voltage switchgear and also an improvement for other systems with simple sample data.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References


