Research Article

Group Decision-Making Approach without Weighted Aggregation Operators

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This paper introduces an approach for group decision-making problems (GDMP) without weighted aggregation operators. This approach is more suitable for scenarios with infinite number of individuals. A mathematical model approach is established based on the new concept of s^* -optimal concession equilibrium solution without weighted aggregation operators for group decision-making problems. It is of practical significance for all decision-makers (experts) to find an optimal solution or to sort out all the candidate solutions. We prove that the s^* -optimal concession equilibrium solution is equivalent to solving a single objective optimization problem, and, under certain conditions, the s^* -optimal equilibrium solution always exists. Moreover, it is proven that the s^* -optimal concession equilibrium solution is equivalent to the robust optimal solution of the group weight aggregation and the optimal solution under the worst weighted aggregation operators.

1. Introduction

The group decision-making problem has always been a hot topic in the research of decision theory and an important branch of scientific research. Many applications are in social network, supplier selection, competitive business environment, economic analysis, strategic planning, medical diagnosis, venture capital, and so forth. The mainstream method of group decision-making is studied through a ranking scheme by a selection function, that is, ranking the preference of individuals in groups. For the scenario with limited number of individuals, this method is very effective. But, for that with the infinite number, it becomes difficult or invalid. For example, the retailer and the manufacturer decide the quantity (in unit of kilogram) of production and supply together in the supply chain such as food (Sharafali and Co, 2000) [1]. The number of productions of the product is infinite.

In the last twenty years, almost all approaches studied are either the weighted aggregation operator methods or the weighted utility methods. There are r decision-makers or experts in GDMP, a group of experts (DMs) including $DM_1, DM_2, \dots, DM_r$, with their cost or benefit evaluating function $f_i(\mathbf{x})$ (scored as a candidate), $i = 1, 2, \dots, r$, for DM_i ,

where $\mathbf{x}$ is an individual candidate (called a solution) and a set of all individual candidates $X \subset R^n$. Each DM_i selects a best individual candidate or sorts out a best one from X by evaluating his evaluating function $f_i(\mathbf{x})$. In general, the group decision method is to establish a group utility function:

$$U(\mathbf{x}) := U(f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_r(\mathbf{x})) \\ = U\left(\sum_{i=1}^r \lambda_i f_i(\mathbf{x})\right), \quad (1)$$

where $\lambda_i \in [0, 1]$ is a weight value of DM_i ($i = 1, 2, \dots, r$); $\sum_{i=1}^r \lambda_i = 1$. All DMs select the best individual candidate or sort out one from X by $U(\mathbf{x})$. The different weights of DMs lead to different sorting results. Because usually the DMs are from different fields, the weights of DMs are then different. Therefore, how to take the weights of DMs is very important.

It is found that many researches focus on the weighted aggregation of DMs. For example, Choi (1998) [2] and Kim (1999) [3] et al. presented and developed a mathematical programming model that can establish dominance relations among the preference information about utilities, attribute weights, and group member's weights. Wei (2000) [4] et al. described a minimax principle based procedure of preference

adjustments with a finite number of steps to find the compromise weight. A preemptive goal programming method was proposed for aggregating OWA operator weights (Wang et al., 2007) [5]. Sadi-Nezhad et al. (2008) [6] investigated the generation of a possibilistic model for multidimensional analysis of preference, where the model assesses the fuzzy weights as well as locating the ideal solution with fuzzy decision-making preference on attributes and fuzzy decision matrix. Another good approach aggregating these individual decision matrices into a group decision matrix by using the additive weighted aggregation (AWA) operator was developed by Xu (2009) [7]; then a convergent iterative algorithm to gain a consentaneous group decision matrix is established. And Wu et al. (2009 and 2010) [8, 9] developed some induced continuous ordered weighted geometric (ICOWG) operators and studied some desired properties of the ICOWG operator, where the ordering of the argument values based upon the reliability of the information sources is applied. Yue (2012) [10] determined the weights of decision-makers (DMs), where the weights of decision-makers derived from individual decision are determined with interval numbers.

Some new weight aggregation operator methods have been proposed. For example, Zhou, Chen, and Liu (2012) [11] presented a new aggregation operator called the generalized ordered weighted exponential proportional averaging (GOWEPA) operator, which is based on an optimal model. Liu, Cai, and Martnez (2013) [12] proposed the important weighted continuous generalized ordered weighted averaging (IW-CGOWA) operator and its attitudinal character. Liu, Zhang, and Zhang (2014) [13] studied a group decision-making model based on a generalized ordered weighted geometric average operator with interval preference matrices. Merig, Casanovas, and Yang (2014) [14] introduced the uncertain generalized probabilistic weighted averaging (UGPWA) operator.

Many good methods of multiple attribute group decision-making have been studied and applied to many practical fields. For example, Qi, Liang, and Zhang (2015) [15] focused on the multiple attribute decision-making problems widespread in industry engineering, typically the supplier selection problems, and investigated effective methods utilizing preference information objectively for multiple attribute group decision-making (MAGDM) with unknown attribute weights and expert weights under interval-valued intuitionistic fuzzy environments (IVIFEs). Gao, Li, and Liu (2015) [16] developed a new class of aggregation operator based on utility function, which introduces the risk attitude of decision-makers (DMs) in the aggregation process of investment problem.

In recent years, some new complex methods are put forward for group decision-making in uncertain or fuzzy environment. For example, Yan and Ma (2015) [17] proposed a novel two-stage group decision-making approach to simultaneously address the two types of uncertainties underlying quality function deployment. Dong, Xiao, Zhang, and Wang (2016) [18] put forward a novel consensus framework to manage the consensus and weights (i.e., weights of the experts and attributes) in the iterative multiple attribute group

decision-making (MAGDM) problem. Xu, Chen, Rodriguez, Herrera, and Wang (2016) [19] introduced a new type of fuzzy preference structure, called incomplete HFPRs, to describe hesitant and incomplete evaluation information in the group decision-making (GDM) process. In order to eliminate the limitations of deterministic and fuzzy MAGDM methods, Bayrama and Sahin (2016) [20] presented a probabilistic methodology, which is based on TOPSIS and Monte Carlo simulation of triangular data. Chen and Kuo (2017) [21] proposed a new method for autocratic decision-making using group recommendations based on interval type-2 fuzzy sets, enhanced Karnik-Mendel (EKM) algorithms, and the ordered weighted aggregation (OWA) operator. Banaeian, Mobli, Fahimnia, Nielsen, and Omid (2018) [22] compared the application of three popular multicriteria supplier selection methods in the fuzzy environment.

The hesitant fuzzy linguistic term set (HFLTS) and the linguistic distribution are becoming popular tools in modelling linguistic expressions with multiple linguistic terms in decision problems. For example, Wu, Li, Chen, and Dong (2018) [23] proposed a new linguistic group decision model called the maximum support degree model, aiming at maximizing the support degree of the group opinion as well as guarantying the accuracy of the group opinion. Wu, Dai, Chiclana, Fujita, and Herrera-Viedma (2018) [24] presented a minimum adjustment cost feedback mechanism for higher consensus in social network group decision-making under distributed linguistic trust information. Li, Rodriguez, Martnez, Dong, and Herrera (2018) [25] personalized individual semantics in the hesitant GDM with comparative linguistic expressions to show the individual difference in understanding the meaning of words. Furthermore, Wu and Xu (2016) [26] presented a new framework model to address multiple attribute GDM with hesitant fuzzy linguistic information, a good idea where the hesitant fuzzy linguistic weighted average operator and the hesitant fuzzy linguistic ordered weighted average operator are proposed. Wu and Xu (2018) [27] proposed the large-scale group decision-making consensus model in which the clusters are allowed to change and the decision-makers provide preferences using fuzzy preference relations. Wu, Jin, and Xu (2018) [28] developed a new consensus measure that is based on the distances between the individuals on three levels: an alternative pair level, an alternative level, and a preference relation level. Also they designed an algorithm that adopts a local feedback strategy to improve the consensus reaching process.

All the above weighted aggregation methods are more suitable for group decision-making problems with limited number of alternatives. It is not easy to apply the above methods when there is unlimited number of alternatives in product selection of supply chain. Evidently, the cooperation in the supply chain plays an important role in the efficiency of the supply chain (Sharafali and Co, 2000) [1]. Xu, Meng, and Shen (2015) [29] introduced an s^* -optimal equilibrium solution in a model that can decide the minimum concession that the manufacturer and the retailer need to give for their cooperation and proved that the s^* -optimal equilibrium solution can be obtained by solving a goal programming

problem. Based on the idea, this paper introduces a new concept of solution to GDMP: s -concession equilibrium solution and s -optimal concession equilibrium solution. It is characterized by the fact that, for each decision-maker, each objective attribute gives the corresponding concession value s , and the s -optimal concession equilibrium solution is the minimum concession given. We also prove that any solution to GDMP is an s -concession equilibrium solution. It shows that, regardless of the method used to obtain a solution to a group decision, the solution is an s -concession equilibrium solution. Therefore, the s -optimal concession equilibrium solution provides a natural criterion for evaluating the merits of alternatives. Obviously, it is different from other existing methods with weighted aggregation operators. We explain the rationale of the s -optimal concession equilibrium solution. According to the definition of s^* -optimal concession equilibrium solution, the s^* -optimal concession equilibrium solution is obviously not dependent on the decision function of one DM. Therefore, it is not coercive. That is, the final solutions are not determined by some DMs. On the other hand, for each individual, the equilibrium value is the same for each goal for each decision-maker. Therefore, the s -optimal concession equilibrium solution has its individual rationality. The s^* -optimal concession equilibrium solution has the Pareto validity (i.e., the artificial ill solution of all group decision-makers must not be the s^* -optimal concession equilibrium solution).

On the other hand, the linear weighted aggregation method is a very common method used in group decision-making. So, the determination of weights is always the focus of the study in all references. But, its fatal weakness is that different weights lead to different sorting, and it is impossible to prove which linear weighting aggregation method is the best. In this paper, it is proven that the optimal concession equilibrium solution is equivalent to the robust optimal solution of the weighted linear weights of all the decision-makers at the worst weights. When the decision-maker deviates from the worst weight situation, the given weight or the calculated weight is used to compute the optimal solution obtained by the linear weighting method, which is not necessarily the optimal equilibrium solution. Therefore, the motivation and innovation of this paper are that the proposed optimal concession equilibrium solution is a natural law of the consistency of group interests without weighted aggregation operators and when the candidate is infinite, this method avoids the difficulty of calculating the weights so that it is more effective in solving group decision-making problems. In the second section, we discuss the problem of s -optimal concession equilibrium solution with the same concession value. It is proven that the s -optimal concession equilibrium solution is equivalent to solving a single objective optimization problem. In the third section, we discuss the problem of a product ordering and production operation decision between the retailer and the manufacturer using the s -optimal concession equilibrium solution. Numerical results show that the s -optimal concession equilibrium solution increases the quantity of order and production.

2. S^* -Optimal Concession Equilibrium Solution

Suppose that there are r decision-makers, $DM_1, DM_2, \dots, DM_r$, a group of experts and their evaluating (cost or benefit) function $f_i : R^n \rightarrow R^1, i = 1, 2, \dots, r$, for DM_i . Let $\mathbf{x}$ be a candidate scheme (solution) and let $X \subset R^n$ be a set of all candidate schemes. Each decision-maker selects a solution or sorts out one from X by evaluating the function $f_i(\mathbf{x})$. We have the following group decision-making problem:

$$(GDMP) \begin{cases} (P_1) & \min f_1(\mathbf{x}) \\ (P_2) & \min f_2(\mathbf{x}) \\ & \dots \\ (P_r) & \min f_r(\mathbf{x}) \\ \text{s.t.} & \mathbf{x} \in X \end{cases} \quad (2)$$

Let $\boldsymbol{\epsilon} = (\epsilon_1, \epsilon_2, \dots, \epsilon_r)^T \in R^r$.

Definition 1. Let $\mathbf{x}^* \in X$, and $\boldsymbol{\epsilon} \geq 0$. If

$$s = \operatorname{argmin} \{v \mid f_i(\mathbf{x}^*) - v - \epsilon_i \leq f_i(\mathbf{x}), \forall \mathbf{x} \in X, i = 1, 2, \dots, r\}, \quad (3)$$

then $\mathbf{x}^*$ is called an s -concession equilibrium solution to (GDMP) at the value $\boldsymbol{\epsilon}$. ϵ_i is called the target concession value of DM_i ($i = 1, 2, \dots, r$). s is called an equilibrium value of GDMP. The set of all equilibrium values of GDMP is denoted as S . If $\mathbf{x}^*$ is the s^* -concession equilibrium solution to GDMP and s^* is the minimum of the set S , then $\mathbf{x}^*$ is called the s^* -optimal concession equilibrium solution to GDMP at the value $\boldsymbol{\epsilon}$. s^* is called the optimal equilibrium value, and obviously the optimal equilibrium value is unique. Obviously, we have the following properties.

Property 2. Let $\mathbf{x}^*$ be the s -concession equilibrium solution to GDMP at the value $\boldsymbol{\epsilon}$.

(1) Then

$$f_i(\mathbf{x}^*) - s - \epsilon_i \leq f_i(\mathbf{x}), \quad \forall \mathbf{x} \in X, i = 1, 2, \dots, r. \quad (4)$$

(2) If

$$f_i(\mathbf{x}^*) - v - \epsilon_i \leq f_i(\mathbf{x}), \quad \forall \mathbf{x} \in X, i = 1, 2, \dots, r, \quad (5)$$

then $s \leq v$.

(3) Then $\mathbf{x}^*$ is the 0-concession equilibrium solution to GDMP at the value $\boldsymbol{\epsilon}(s)$, where $\boldsymbol{\epsilon}(s) = (s + \epsilon_1, s + \epsilon_2, \dots, s + \epsilon_r)^T$.

(4) If $\epsilon_1 = \epsilon_2 = \dots = \epsilon_r$, then $\mathbf{x}^*$ is the $s + \epsilon_1$ -concession equilibrium solution to GDMP at the value $\mathbf{0}$.

(5) If X has only a finite number of solutions, then the number of s -concession equilibrium solutions is finite, and there must be an s^* -optimal concession equilibrium solution to GDMP.

According to Property 2, the s -concession equilibrium solution is to give the corresponding concession value $s +$

ϵ_i to each of the decision-makers on the decision set or the candidate set X . Each decision-maker wants to identify the minimum concession equilibrium value of $s + \epsilon_i$ as the optimal solution for all the decision-makers. Given their respective concession values ϵ_i , the optimal concession equilibrium solution of the minimum equilibrium value is the best solution they all can accept. Therefore, the s^* -optimal concession equilibrium solution can be seen as a fair solution to GDMP.

What interests us is the minimum equilibrium value s^* and its corresponding s^* -optimal concession equilibrium solution in all equilibrium values. We have an example as follows.

Example 3. Consider the following GDMP:

$$(GDMP) \begin{cases} (P_1) & \min f_1(x, y) = y^2 + (x - 2)^2 \\ (P_2) & \min f_2(x, y) = x^2 + (y - 2)^2 \\ \text{s.t.} & (x, y) \in X = R^1 \times R^1 \end{cases} \quad (6)$$

Obviously, $(2, 0)$ is the optimal solution of decision-maker 1, and $(0, 2)$ is the optimal solution of decision-maker 2.

If $\epsilon = (0, 0)$, one has

$$\begin{aligned} f_1(1, 1) - 2 - 0 &\leq f_1(x, y), \\ f_2(1, 1) - 2 - 0 &\leq f_2(x, y), \\ (x, y) &\in R^1 \times R^1. \end{aligned} \quad (7)$$

$(1, 1)$ is the 2-optimal concession equilibrium solution to the problem at the concession value $(0, 0)$. This solution gives the minimum equilibrium value of each decision-maker's individual objective.

If $\epsilon = (2, 2)$, one has

$$\begin{aligned} f_1(1, 1) - 0 - 2 &\leq f_1(x, y), \\ f_2(1, 1) - 2 &\leq f_2(x, y), \\ (x, y) &\in R^1 \times R^1. \end{aligned} \quad (8)$$

$(1, 1)$ is the 0-optimal concession equilibrium solution to the problem at the concession value $(2, 2)$. This solution gives the minimum equilibrium value of each decision-maker's individual objective.

If $\epsilon = (8, 0)$, one has

$$\begin{aligned} f_1(0, 2) - 0 - 8 &\leq f_1(x, y), \\ f_2(0, 2) - 0 - 0 &\leq f_2(x, y), \\ (x, y) &\in R^1 \times R^1. \end{aligned} \quad (9)$$

$(0, 2)$ is the 0-optimal concession equilibrium solution to the problem at the concession value $(8, 0)$.

The above results show that the concession or compromise values given by the decision-makers are different, and the different optimal concession equilibrium solutions are obtained.

Now, we define the order in the set X .

Definition 4. Let $\mathbf{x}^1, \mathbf{x}^2 \in X$, and $\epsilon \geq 0$. Let $\mathbf{x}^1$ be an s_1 -concession equilibrium solution to GDMP at the value ϵ and let $\mathbf{x}^2$ be an s_2 -concession equilibrium solution to GDMP at the value ϵ . If $s_1 < s_2$, we denote $\mathbf{x}^1 <_s \mathbf{x}^2$ to indicate that $\mathbf{x}^1$ is superior to $\mathbf{x}^2$. If $s_1 = s_2$, we denote $\mathbf{x}^1 \equiv_s \mathbf{x}^2$ to indicate that $\mathbf{x}^1$ is equivalent to $\mathbf{x}^2$.

Obviously, the set X is a serially ordered set about the order $<_s$ or $\equiv_s$.

Property 5. Let $\mathbf{x}^1, \mathbf{x}^2 \in X$, and $\epsilon \geq 0$. Let $\mathbf{x}^1$ be an s_1 -concession equilibrium solution to GDMP at the value ϵ and let $\mathbf{x}^2$ be an s_2 -concession equilibrium solution to GDMP at the value ϵ . If $f_i(\mathbf{x}^1) \leq f_i(\mathbf{x}^2)$, $i = 1, 2, \dots, r$, then $\mathbf{x}^1 <_s \mathbf{x}^2$ or $\mathbf{x}^1 \equiv_s \mathbf{x}^2$.

Proof. According to assumption, we have

$$f_i(\mathbf{x}^1) - s_1 - \epsilon_i \leq f_i(\mathbf{x}), \quad \forall \mathbf{x} \in X, \quad i = 1, 2, \dots, r, \quad (10)$$

$$\begin{aligned} f_i(\mathbf{x}^1) - s_2 - \epsilon_i &\leq f_i(\mathbf{x}^2) - s_2 - \epsilon_i \leq f_i(\mathbf{x}), \\ \forall \mathbf{x} \in X, \quad i &= 1, 2, \dots, r. \end{aligned} \quad (11)$$

By Property 2, we have $s_1 \leq s_2$. $\square$

Property 5 shows that the optimal concession equilibrium solution must be nondominated for all decision-makers.

Example 6. Consider a GDMP, where three decision-makers try to sort out one from three candidate solutions as scoring in Table 1.

Obviously, according to the simple majority rule or the total score rule, the ordering of decisions is the same. Here, by Definition 1, when $\epsilon = (0, 0, 0)$, we have $\mathbf{x}^1 \equiv_s \mathbf{x}^2 \equiv_s \mathbf{x}^3$. When the decision-maker has the same concession value, the ranking of the three solutions is the same. But when $\epsilon = (0, 1, 2)$, we have $\mathbf{x}^1 <_s \mathbf{x}^2 <_s \mathbf{x}^3$. That is, when the decision-maker does not have the same concession value, the ranking of the three solutions is not the same.

For each $i = 1, 2, \dots, r$, we have the following single objective optimization problem:

$$\begin{aligned} (P_i) \quad & \min f_i(\mathbf{x}) \\ \text{s.t.} \quad & \mathbf{x} \in X. \end{aligned} \quad (12)$$

Let f_i^* be the optimal target value of (P_i) . When $f_i^* = -\infty$, a special scenario, there is no optimal concession equilibrium solution to GDMP. In the following, it is assumed that $f_i^* > -\infty$, $i = 1, 2, \dots$.

Lemma 7. Assume that there is an optimal solution to (P_i) ($i = 1, 2, \dots, r$). Then, for any given $\bar{\mathbf{x}} \in X$, $\bar{\mathbf{x}}$ is an $\bar{s}$ -concession equilibrium solution to GDMP, where $\bar{s} = \max\{f_i(\bar{\mathbf{x}}) - f_i^* - \epsilon_i \mid i = 1, 2, \dots, r\}$.

Proof. For any $\mathbf{x} \in X$, we have

$$f_i^* \leq f_i(\mathbf{x}), \quad i = 1, 2, \dots, r. \quad (13)$$

TABLE 1: The score of three decisions and the equilibrium value under different concession values.

Solution	Score of DM1	Score of DM2	Score of DM3	$\epsilon = (0, 0, 0)$	$\epsilon = (0, 1, 2)$
X	$f_1(x)$	$f_2(x)$	$f_1(x)$	s	s
x^1	1	2	3	2	0
x^2	2	3	1	2	1
x^3	3	1	2	2	2

So, we have

$$f_i(\bar{\mathbf{x}}) - [f_i(\bar{\mathbf{x}}) - f_i^* - \epsilon_i] - \epsilon_i \leq f_i(\mathbf{x}), \quad i = 1, 2, \dots, r, \quad (14)$$

and

$$f_i(\bar{\mathbf{x}}) - \bar{s} - \epsilon_i \leq f_i(\mathbf{x}), \quad i = 1, 2, \dots, r, \quad (15)$$

where $\bar{s} = \max\{f_i(\bar{\mathbf{x}}) - f_i^* - \epsilon_i \mid i = 1, 2, \dots, r\}$. There is $i = k$ such that $\bar{s} = f_k(\bar{\mathbf{x}}) - f_k^* - \epsilon_k$. And there is some s' so that $s' < \bar{s}$ and

$$f_i(\bar{\mathbf{x}}) - s' - \epsilon_i \leq f_i(\mathbf{x}), \quad i = 1, 2, \dots, r. \quad (16)$$

By the above inequations, let $i = k$ and let $\mathbf{x}_k^* \in X$ be an optimal solution to (P_k) . That is,

$$f_k(\bar{\mathbf{x}}) - s' - \epsilon_k \leq f_k(\mathbf{x}_k^*) = f_k^*. \quad (17)$$

We have $\bar{s} = f_k(\bar{\mathbf{x}}) - f_k^* - \epsilon_k \leq s'$. This is a contradiction. Therefore, by Definition 1, the conclusion of the theorem is true. $\square$

From Lemma 7, it is known that, for any $\mathbf{x} \in X$, there exists an equilibrium value ϵ of GDMP at the concession value s , which makes the point $\mathbf{x}$ be an s -concession equilibrium solution to GDMP at the concession value ϵ . Lemma 7, on the other hand, also illustrates such an important fact that, whether we adopt other existing group decision methods to choose a solution $\mathbf{x}'$ in X , there is a value s that makes the solution $\mathbf{x}'$ be an s -concession equilibrium solution to GDMP at the concession value ϵ . Obviously, the s^* -optimal concession equilibrium solution is the best in all s -concession equilibrium solutions. Unless you change the concession value ϵ , the s^* -optimal concession equilibrium solution will not change.

Define the following optimization problem:

$$\begin{aligned} (S) \quad & \min \quad s \\ \text{s.t.} \quad & f_i(\mathbf{x}) - s - \epsilon_i \leq f_i^*, \quad i = 1, 2, \dots, r \\ & s \in R^1, \mathbf{x} \in X \end{aligned} \quad (18)$$

Theorem 8. Assume that there is an optimal solution to (P_i) ($i = 1, 2, \dots, r$). Then $\mathbf{x}^*$ is s^* -optimal concession equilibrium solution to GDMP at the value ϵ if and only if $(\mathbf{x}^*, s^*)$ is an optimal solution to (S).

Proof. First, assume that if $(\mathbf{x}^*, s^*)$ is an optimal solution to (S), then, for any $\mathbf{x} \in X$, we have

$$f_i(\mathbf{x}^*) - s^* - \epsilon_i \leq f_i^* \leq f_i(\mathbf{x}), \quad i = 1, 2, \dots, r. \quad (19)$$

So, by Definition 1, we have that $\mathbf{x}^*$ is an s^* -concession equilibrium solution to GDMP at the value ϵ . Let $\bar{\mathbf{x}}$ be an $\bar{s}$ -optimal concession equilibrium solution to GDMP at the value ϵ . By Property 2, we have $s^* \geq \bar{s}$ and

$$f_i(\bar{\mathbf{x}}) - \bar{s} - \epsilon_i \leq f_i(\mathbf{x}), \quad \forall \mathbf{x} \in X, i = 1, 2, \dots, r. \quad (20)$$

Therefore, $(\bar{\mathbf{x}}, \bar{s})$ is a feasible solution to (S). So, $\bar{s} \geq s^*$. That is, $\bar{s} = s^*$ and $\mathbf{x}^*$ is the s^* -optimal concession equilibrium solution to GDMP at the value ϵ .

Now, assume that $\mathbf{x}^*$ is the s^* -optimal concession equilibrium solution to GDMP at the value ϵ ; then, by Definition 1, we know that $(\mathbf{x}^*, s^*)$ is a feasible solution to (S). Let $(\bar{\mathbf{x}}, \bar{s})$ be an optimal solution to (S). By the above proof, $\bar{\mathbf{x}}$ is an $\bar{s}$ -optimal concession equilibrium solution to GDMP at the value ϵ . Therefore, $\bar{s} = s^*$. So, $\mathbf{x}^* \equiv_s \bar{\mathbf{x}}$ and $(\mathbf{x}^*, s^*)$ is an optimal solution to (S). $\square$

If the optimal solution exists for (P_i) , then the s^* -optimal concession equilibrium solution also exists. In fact, just assume that X is a compact set and f_i ($i = 1, 2, \dots, r$) is a continuous function on X ; then, according to Lemma 7, it is known that the s^* -optimal concession equilibrium solution to GDMP exists.

Theorem 9. Assume that X is a compact set and f_i ($i = 1, 2, \dots, r$) is a continuous function on X . Then the s^* -optimal concession equilibrium solution to GDMP exists.

Proof. By the assumption, there is an optimal solution to each (P_i) . By Lemma 7, we have $S \neq \emptyset$. We prove that S is close. Assume that a sequence $\{s_k\} \subset S$ converges to s^* . For $k = 1, 2, \dots$, let $\mathbf{x}_k^* \in X$ be an s_k -concession equilibrium solution to GDMP. Because X is compact, the sequence $\{\mathbf{x}_k^*\}$ has a convergent subsequence. Without loss of generality, let $\mathbf{x}_k^* \rightarrow \mathbf{x}^* \in X$. By Definition 1, we have

$$\begin{aligned} f_i(\mathbf{x}_k^*) - s_k - \epsilon_i &\leq f_i(\mathbf{x}), \\ \forall \mathbf{x} \in X, i &= 1, 2, \dots, r, k = 1, 2, \dots \end{aligned} \quad (21)$$

Let $k \rightarrow +\infty$; then we have

$$f_i(\mathbf{x}^*) - s^* - \epsilon_i \leq f_i(\mathbf{x}), \quad \forall \mathbf{x} \in X, i = 1, 2, \dots, r. \quad (22)$$

So $\mathbf{x}^*$ is the s^* -concession equilibrium solution. Therefore S is close. By Lemma 7, we know that S has a minimum s^* . Given a sufficiently large $A > 0$, define the problem

$$\begin{aligned} (S)_A \quad & \min \quad s \\ \text{s.t.} \quad & f_i(\mathbf{x}) - s - \epsilon_i \leq f_i^*, \quad i = 1, 2, \dots, r \\ & A \geq s \geq s^*, \quad \mathbf{x} \in X. \end{aligned} \quad (23)$$

It is obvious that the problem $(S)_A$ is equivalent to the problem (S) , and the feasible set of the problem $(S)_A$ is compact too. Therefore, there exists the optimal solution $(\mathbf{x}_A^*, s_A^*)$ to $(S)_A$; then $(\mathbf{x}_A^*, s_A^*)$ is also the optimal solution to the problem (S) . By Theorem 8, the conclusion is true. $\square$

According to Theorem 8, we obtain a method to find out an s^* -optimal concession equilibrium solution to GDMP. First, the optimal solution to the problem (P_i) is solved, respectively. Then the programming problem (S) is solved. If the optimal solution $(\mathbf{x}^*, s^*)$ to (S) is obtained, then we obtain that $\mathbf{x}^*$ is an s^* -optimal concession equilibrium solution to GDMP. Obviously, the above method is easy for the linear GDMP. But, for the nonlinear GDMP, many nonlinear problems need to be solved, which is clearly difficult and brings a lot of inconvenience in the calculation. Then, how can we find a suitable alternative problem for solving the problem? We consider the following single objective programming problem:

$$\begin{aligned} (\bar{S}) \quad & \min \quad s + \sum_{i=1}^r f_i(\mathbf{x}^i) \\ \text{s.t.} \quad & f_i(\mathbf{x}) - s - \epsilon_i \leq f_i(\mathbf{x}^i), \quad i = 1, 2, \dots, r \\ & \mathbf{x}, \mathbf{x}^1, \dots, \mathbf{x}^r \in X \end{aligned} \quad (24)$$

where $s, \mathbf{x}, \mathbf{x}^1, \mathbf{x}^2, \dots, \mathbf{x}^r$ are variables of $(\bar{S})$. We have the better results below.

Theorem 10. Suppose that $\bar{\mathbf{x}}^{*i}$ is an optimal solution to (P_i) ($i = 1, 2, \dots, r$). If $[(\mathbf{x}^*, \bar{s}), (\mathbf{x}^1, \mathbf{x}^2, \dots, \mathbf{x}^r)]$ is an optimal solution to $(\bar{S})$, then $\mathbf{x}^*$ is an s^* -optimal concession equilibrium solution to GDMP at the value ϵ , where $s^* = \bar{s} + \hat{s}$.

$$\hat{s} = \max \{f_i(\mathbf{x}^{*i}) - f_i^* \mid i = 1, 2, \dots, r\} \quad (25)$$

Proof. Let $\bar{\mathbf{x}}^*$ be an $\bar{s}^*$ -optimal concession equilibrium solution to GDMP. By Theorem 8, we have that $(\bar{\mathbf{x}}^*, \bar{s}^*)$ is an optimal solution to (S) . It is clear that $[(\bar{\mathbf{x}}^*, \bar{s}^*), (\bar{\mathbf{x}}^1, \dots, \bar{\mathbf{x}}^r)]$ is a feasible solution to $(\bar{S})$. Therefore, we have

$$\bar{s} + \sum_{i=1}^r f_i(\mathbf{x}^{*i}) \leq \bar{s}^* + \sum_{i=1}^r f_i(\bar{\mathbf{x}}^{*i}). \quad (26)$$

By (25) and (26) and $f_i(\bar{\mathbf{x}}^{*i}) = f_i^*$, we have $s^* \leq \bar{s}^*$. On the other hand, according to the theorem hypothesis, we have

$$f_i(\mathbf{x}^*) - \bar{s} - \epsilon_i \leq f_i(\mathbf{x}^{*i}), \quad i = 1, 2, \dots, r. \quad (27)$$

So,

$$f_i(\mathbf{x}^*) - \bar{s} - \epsilon_i \leq f_i(\mathbf{x}^{*i}) - f_i^* + f_i^*, \quad i = 1, 2, \dots, r. \quad (28)$$

Hence,

$$f_i(\mathbf{x}^*) - \epsilon_i \leq s^* + f_i^*, \quad i = 1, 2, \dots, r. \quad (29)$$

We have that $(\mathbf{x}^*, s^*)$ is a feasible solution to (S) . Therefore, $s^* \geq \bar{s}^*$ and $\mathbf{x}^*$ is an s^* -optimal concession equilibrium solution to GDMP at the value ϵ with $s^* = \bar{s}^*$. $\square$

Remark 11. According to Theorem 10, we can find out an optimal solution to the problem $(\bar{S})$, and the s^* -optimal concession equilibrium solution to GDMP can be obtained. However, the optimal equilibrium value of s^* is not obtained unless the optimal solutions to all $(P_1), (P_2), \dots, (P_r)$ are found for all the problems.

If X is a convex set and f_i ($i = 1, 2, \dots, R$) is convex function, then it is easy to prove that the set of all s^* -optimal concession equilibrium solutions is a convex set. Furthermore, if f_i ($i = 1, 2, \dots, R$) is a strictly convex function, then s^* -optimal concession equilibrium solution is unique. We have the following corollary.

Corollary 12. Let $X = R^n$ and let $f_i(\mathbf{x})$ ($i = 1, 2, \dots, r$) on R^n be a continuous differentiable convex function. Suppose that $\bar{\mathbf{x}}^{*i}$ is an optimal solution to (P_i) ($i = 1, 2, \dots, r$). Then $\mathbf{x}^*$ is s^* -optimal concession equilibrium solution to GDMP at the value ϵ if and only if there is an incomplete zero of $(\lambda_1, \lambda_2, \dots, \lambda_r)$ to satisfy the following KKT-condition:

$$\begin{aligned} \lambda_1 + \lambda_2 + \dots + \lambda_r &= 1, \\ \sum_{i=1}^r \lambda_i \nabla f_i(\mathbf{x}^*) &= 0, \end{aligned} \quad (30)$$

$$\lambda_i (f_i(\mathbf{x}^*) - s^* - \epsilon_i - f_i^*) = 0, \quad i = 1, 2, \dots, r$$

$$f_i(\mathbf{x}^*) - s^* - \epsilon_i \leq f_i^*, \quad i = 1, 2, \dots, r$$

$$\lambda_i \geq 0, \quad i = 1, 2, \dots, r$$

According to Corollary 12, the group weight aggregation method is the most commonly used in group decision-making. After the weights are given to the decision-makers, the weights are weighted linearly to obtain the ranking of the alternatives. In the following, we prove that the s^* -optimal concession equilibrium solution is essentially a robust optimal solution for all group decision weights.

Let DM_i 's score be $f_i(\mathbf{x}) - \epsilon_i - f_i^*$ for $\mathbf{x}$. By using weights $1 \geq \lambda_i \geq 0$, DM_i 's score becomes $\lambda_i (f_i(\mathbf{x}) - \epsilon_i - f_i^*)$ for $\mathbf{x}$. With the linear weighting method, the score of all decision-makers for $\mathbf{x}$ is defined as

$$F(\mathbf{x}, \boldsymbol{\lambda}) := \sum_{i=1}^r \lambda_i (f_i(\mathbf{x}) - \epsilon_i - f_i^*), \quad (31)$$

where $\lambda_1 + \lambda_2 + \dots + \lambda_r = 1$, $\lambda_i \geq 0$, $i = 1, 2, \dots, r$. Let group weight set be

$$\Lambda = \left\{ \lambda = (\lambda_1, \lambda_2, \dots, \lambda_r) \mid \sum_{i=1}^r \lambda_i = 1, \lambda_i \geq 0, i = 1, 2, \dots, r \right\}. \quad (32)$$

The worst score of the solution $\mathbf{x}$ solves $\max_{\lambda \in \Lambda} F(\mathbf{x}, \lambda)$ for each $\mathbf{x} \in X$. Let $\lambda(\mathbf{x}) = \operatorname{argmax}\{F(\mathbf{x}, \lambda) \mid \lambda \in \Lambda\}$. Then, we are to find a minimum score from these worst scores $\min_{\mathbf{x} \in X} F(\mathbf{x}, \lambda(\mathbf{x}))$; that is,

$$(\tilde{S}) \min_{\mathbf{x} \in X} \max_{\lambda \in \Lambda} F(\mathbf{x}, \lambda). \quad (33)$$

We prove the following conclusion.

Theorem 13. Suppose that the optimal solution to (P_i) ($i = 1, 2, \dots, r$) exists; then the problem (S) is equivalent to the problem $(\tilde{S})$.

Proof. For a fixed $\mathbf{x}$, the problem $\max_{\lambda \in \Lambda} F(\mathbf{x}, \lambda)$ is a linear programming:

$$\begin{aligned} \Lambda(\mathbf{x}) \max \quad & F(\mathbf{x}, \lambda) = \sum_{i=1}^r \lambda_i (f_i(\mathbf{x}) - \epsilon_i - f_i^*) \\ \text{s.t.} \quad & \lambda_1 + \lambda_2 + \dots + \lambda_r = 1, \\ & \lambda_i \geq 0, i = 1, 2, \dots, r. \end{aligned} \quad (34)$$

The dual problem of $\Lambda(\mathbf{x})$ is

$$\begin{aligned} S(\mathbf{x}) \min \quad & s \\ \text{s.t.} \quad & f_i(\mathbf{x}) - s - \epsilon_i \leq f_i^*, i = 1, 2, \dots, r \\ & s \in R^1. \end{aligned} \quad (35)$$

According to the strong duality theorem, there exists the optimal solution to the problem $\Lambda(\mathbf{x})$ and the problem $S(\mathbf{x})$, and the optimal objective values are equal at their optimal solutions. Let λ^* be an optimal solution to $\Lambda(\mathbf{x})$ and let s^* be an optimal solution to $S(\mathbf{x})$; then $s^* = F(\mathbf{x}, \lambda^*) = \max_{\lambda \in \Lambda} F(\mathbf{x}, \lambda)$. Therefore, the conclusion of theorem is true. $\square$

The evaluation function $f_i(\mathbf{x})$ of all decision-makers should be consistent as far as possible. $(f_1^*, f_2^*, \dots, f_r^*)$ is ideal goal. Obviously, the closer to the ideal goal the better the solution. A deviation function is defined by

$$\begin{aligned} \Delta(\mathbf{x}) &= \frac{1}{r} \\ &\cdot \sqrt{(f_1(\mathbf{x}) - f_1^*)^2 + (f_2(\mathbf{x}) - f_2^*)^2 + \dots + (f_r(\mathbf{x}) - f_r^*)^2}, \end{aligned} \quad (36)$$

$\mathbf{x} \in X$.

According to Theorem 8, if $\mathbf{x}$ is s -concession equilibrium solution, then we have

$$0 \leq f_i(\mathbf{x}) - f_i^* \leq s + \epsilon_i, i = 1, 2, \dots, r, \quad (37)$$

so that $\Delta(\mathbf{x}) \leq s + \max\{\epsilon_i \mid i = 1, 2, \dots, r\}$ for $\mathbf{x} \in X$. So, we have the following conclusion.

Corollary 14. Assume that there is an optimal solution to (P_i) ($i = 1, 2, \dots, r$). If $\mathbf{x}^*$ is s^* -optimal concession equilibrium solution to GDMP at the value ϵ , then $\min_{\mathbf{x} \in X} \Delta(\mathbf{x}) \leq \Delta(\mathbf{x}^*) \leq s^* + \max\{\epsilon_i \mid i = 1, 2, \dots, r\}$ for $\mathbf{x} \in X$.

Corollary 14 means that an optimal concession equilibrium solution $\mathbf{x}^*$ has a minimal deviation bound $s^* + \max\{\epsilon_i \mid i = 1, 2, \dots, r\}$. It is clear that the evaluation function of the optimal concession equilibrium solution is the closest ideal goal.

Example 15. Consider the following GDMP:

$$(GDMP) \begin{cases} (P_1) & \min f_1(\mathbf{x}) = x \\ (P_2) & \min f_2(\mathbf{x}) = -x \\ \text{s.t.} & \mathbf{x} \in X = [-1, 1] \end{cases} \quad (38)$$

We know that $f_1^* = f_2^* = -1$. When $\epsilon = (0, 0)$, 0 is the 1-optimal concession equilibrium solution of the problem about the concession value $(0, 0)$. This solution gives the minimum equilibrium value of each decision-maker's individual objective. Define a weighted function by

$$\begin{aligned} F(\mathbf{x}, \lambda) &= \lambda f_1(\mathbf{x}) + (1 - \lambda) f_2(\mathbf{x}) \\ &= \lambda x + (1 - \lambda)(-x) = (2\lambda - 1)x. \end{aligned} \quad (39)$$

By Theorem 13, the optimal solution to $\min_{\mathbf{x} \in X} \max_{\lambda \in [0, 1]} F(\mathbf{x}, \lambda)$ is $\mathbf{x}^* = 0$. As a comparison, we are to use the linear weighted method to solve this problem, a very famous method (Kim and Han (1999)), where weighted value $\lambda \in [0, 1]$. When given $\lambda > 0.5$, an optimal solution to $\min_{\mathbf{x} \in X} F(\mathbf{x}, \lambda)$ is $\mathbf{x}^* = -1$. When $\lambda < 0.5$, an optimal solution to $\min_{\mathbf{x} \in X} F(\mathbf{x}, \lambda)$ is $\mathbf{x}^* = 1$. But when $\lambda = 0.5$, any optimal solution to $\min_{\mathbf{x} \in X} F(\mathbf{x}, \lambda)$ does not exist. On other hand, the deviation function $\Delta(\mathbf{x})$ is minimum at $\mathbf{x}^* = 0$ but maximum at $\mathbf{x}^* = 1$ or $\mathbf{x}^* = -1$. It means that the linear weighted method is invalid or bad in this example. Therefore, no matter what weight you get from a weight method, the linear weighting method may be invalid.

Remark 16. The problem $(\tilde{S})$ is essentially a robust optimization problem for all weights Λ . Its optimal solution (i.e., the optimal solution $(\mathbf{x}^*, \lambda^*)$) is weighted linearly and the worst weight in Λ^* is selected from the scheme set X . In other words, if the given weight value λ deviates from the optimal weight λ^* of the problem $\Lambda(\mathbf{x})$, the obtained score $F(\mathbf{x}, \lambda)$ becomes greater than $F(\mathbf{x}^*, \lambda^*)$. Theorem 13 shows that the s^* -optimal concession equilibrium solution $\mathbf{x}^*$ to GDMP at the concession value ϵ is also the optimal solution to the problem $(\tilde{S})$, that is, an optimal concession equilibrium solution based on the weight of all group decision-makers. From the viewpoint of robustness, the s^* -optimal concession equilibrium solution is the robust solution for the decision-makers under the worst weights in Λ .

3. A GDMP Example: Cooperative Operation of Products

In the cooperative ordering and production problems, a manufacturer produces products and sells them to retailers. The manufacturers' traditional way of operation is to plan their production based on retailers' previous orders. The quantity of the manufacturer's production is entirely determined by the retailer's orders and is produced thereafter. Nowadays, to encourage retailers to place more orders, manufacturers buy back unsold goods from retailers, sharing the risk of excess supply with retailers. Then, manufacturers and retailers jointly determine orders and production, which is becoming more popular.

To simplify the problem, we consider one retailer's order to one manufacturer or supplier. As a result, the retailer and the manufacturer determine the amount of order and production to minimize their risk losses and ensure that the amount of order determined by the retailer receives agreement from the manufacturer. Let order quantity and production quantity be x (infinite plan). Let the demand for the product be a random variable ξ , let the corresponding probability distribution density function be $p(\cdot)$, and let the distribution function be $P(\cdot)$.

Suppose the following parameters: a is the unit retail price of the retailer; e is the cost price per unit product of the manufacturer; b is the unit wholesale price of the manufacturer, where $a > b > e$; c is the repurchase price of the manufacturer for every unit product, and if the retailer does not sell what is ordered, then the manufacturer repurchases the remaining products: $b > c$.

In a sales cycle, the retailer and the manufacturer consider two kinds of losses, where one is the oversupply loss and the other is the loss in short supply. The retailer's oversupply losses and short supply losses are $(b-c)(x-\xi)^+$ and $(a-b)(\xi-x)^+$, respectively. Let the retailer's overall expected loss be

$$Ef_1(x, \xi) = E\{(b-c)(x-\xi)^+ + (a-b)(\xi-x)^+\} \quad (40)$$

If the retailer does not cooperate with the manufacturer in ordering and production, the retailer's objective is to minimize the expected loss:

$$\begin{aligned} \min \quad & Ef_1(x, \xi) \\ \text{s.t.} \quad & x \in R^1. \end{aligned} \quad (41)$$

By solving the above problem, its optimal ordering policy is $x_1^* = P^{-1}((a-b)/(a-c))$, and the expected minimum loss is $f_1^* = Ef_1(x_1^*, \xi)$.

The loss of excess supply and loss in short supply of the manufacturer are $c(x-\xi)^+$ and $b(\xi-x)^+$, respectively. Let the manufacturer's overall expected loss be

$$Ef_2(x, \xi) = E\{c(x-\xi)^+ + (b-e)(\xi-x)^+\} \quad (42)$$

Similarly, if the manufacturer does not cooperate with the retailer, the manufacturer's expected target loss is

$$\begin{aligned} \min \quad & Ef_2(x, \xi) \\ \text{s.t.} \quad & x \in R^1. \end{aligned} \quad (43)$$

By solving the above problem, the manufacturer's optimal production strategy is $x_2^* = P^{-1}((b-e)/(b+c-e))$, and the expected minimum loss is $f_2^* = Ef_2(x_2^*, \xi)$. We have the following conclusion.

Theorem 17. *If $b^2 = ac + e(b-c)$, then $x_1^* = x_2^*$; that is, the retailer and the manufacturer have the same optimal order quantity and production quantity and $f_1^* = ((a-c)/(b+c-e))f_2^* = ((b-c)/c)f_2^*$. If $b^2 \neq ac + e(b-c)$, then $x_1^* \neq x_2^*$.*

If the retailer and the manufacturer make the decision to cooperate (group decision-making), they decide on the order and production together. According to the model in the previous section (S), the decision model is

$$\begin{aligned} (S2) \quad & \min \quad s \\ \text{s.t.} \quad & Ef_1(x, \xi) - s - \epsilon_1 \leq f_1^*, \\ & Ef_2(x, \xi) - s - \epsilon_2 \leq f_2^*, \\ & s, x \in R^1. \end{aligned} \quad (44)$$

It is easy to know that $Ef_1(x, \xi)$ and $Ef_2(x, \xi)$ are convex functions at x . The problem (S2) is an obviously convex programming, and the Slater constraint qualification is obviously established. By Corollary 12, problem (S2) has the optimal solution (x^*, s^*) and satisfies the KKT condition; then we get the following conclusion.

Theorem 18. *If (x^*, s^*) is an optimal solution to (S2), then (x^*, s^*) satisfies $x^* = P^{-1}((a-b)/(a-c))$, $s^* = -\epsilon_1$, or $x^* = P^{-1}((b-e)/(b+c-e))$, $s^* = -\epsilon_2$, or*

$$(a-b-2c+e) \int_0^{x^*} P(t) dt - (a-2b+e)x^* \quad (45)$$

$$+ (a-2b+e)E\xi + \epsilon_2 + f_2^* - \epsilon_1 - f_1^* = 0,$$

$$\begin{aligned} s^* = Ef_1(x^*, \xi) - (\epsilon_1 + f_1^*) = Ef_2(x^*, \xi) \\ - (\epsilon_2 + f_2^*), \end{aligned} \quad (46)$$

and (1) for $b^2 > ac + e(b-c)$, $x_1^* \leq x^* \leq x_2^*$, (2) for $b^2 < ac + e(b-c)$, $x_1^* \geq x^* \geq x_2^*$, and (3) for $b^2 = ac + e(b-c)$, $x_1^* = x^* = x_2^*$.

By Theorems 17 and 18, we obtain an algorithm for the optimal ordering and production strategy.

Step 1. Given a, b, c, e , and the distribution function $P(t)$, calculate $x_1^* = P^{-1}((a-b)/(a-c))$, $f_1^* = Ef_1(x_1^*, \xi)$, $x_2^* = P^{-1}((b-e)/(b+c-e))$, and $f_2^* = Ef_2(x_2^*, \xi)$, respectively. If $b^2 = ac + e(b-c)$, then the optimal ordering and production strategy is $x_1^* = x_2^*$; stop. Otherwise, if $b^2 \neq ac + e(b-c)$, then go to Step 2.

Step 2. The concessions values (ϵ_1, ϵ_2) are given for the retailer and the manufacturer.

TABLE 2: Ordering quantity, production, and losses of retailers and manufacturers individually.

Buy-back price c	Retailer's order x_1^*	Manufacturer's output x_2^*	Retailer's loss f_1^*	Manufacturer's loss f_2^*
7	967	1044	1810.56	1323.09
10	1000	1000	1595.77	1595.77
13	1044	967	1323.09	1810.56

TABLE 3: Optimal concession equilibrium solution, concession value, and risk loss value of retailer and manufacturer under different repurchase prices.

Buy-back price c	Optimal concession equilibrium solution (x^*, s^*)	Concession value (ϵ_1, ϵ_2)	(Retailer loss, manufacturer loss) $(Ef_1(x^*, \xi), Ef_2(x^*, \xi))$
7	(1002, 29.11)	(0.00, 0.00)	(1839.67, 1352.20)
7	(1033, 1.88)	(100.00, 0.00)	(1912.44, 1324.97)
13	(1002, 29.11)	(0.00, 0.00)	(1352.20, 1839.67)
13	(1033, 1.88)	(0.00, 100.00)	(1324.97, 1912.44)

Step 3. Solve formula (45); the solution x^* is obtained and then s^* by (46). Calculate $Ef_1(x^*, \xi)$ and $Ef_2(x^*, \xi)$ of the expected loss.

Example 19. Take a numerical example of a food production. Let the unit retail price $a = 30$ yuan (RMB) per box, the unit wholesale price $b = 20$ yuan, and the unit cost price $e = 10$ yuan, while the buy-back price is to be determined between 1 and 19. From the historical sales data, it is understood in our case that the demand data approximately satisfies the normal distribution $N(\mu, \sigma)$, the mean $\mu = 1000$, and variance $\sigma = 200$.

The data in Table 2 is the value of the optimal decision by the retailer and the manufacturer individually.

As shown in Table 2, as the repurchase price increases, the retailer's order quantity increases with the risk loss decreasing, and the manufacturer's production decreases with the risk loss increasing. When the repurchase price is $c = 10$, the optimal order quantity of the retailer is equal to that of the manufacturer, and their risk loss is the same. As shown in Table 2, if the retailer and the manufacturer make decisions individually at the repurchase price $c = 7$, the manufacturer produces 1044 box, but the retailer orders 967 boxes; then the manufacturer will produce 77 more. At the repurchase price of $c = 13$, the manufacturer produces 967 boxes, but the retailer wants to order 1044 boxes, so there is a short supply. Although repurchase policy can increase the quantity of retailers' orders, if the two sides do not cooperate in order and production, there is either an over or a short production.

If the retailer and the manufacturer make decisions to cooperate in the quantity of the order, the s^* -optimal concession equilibrium solution is used to make the order and the production decision. In Table 3, the first column is the repurchase price, the second column is the s^* -optimal concession equilibrium solution (concessed ordering and production capacity and the optimal equilibrium value),

the third column is their concession values, and the fourth column is retailer's and manufacturer's risk loss. As shown in Table 3, when the repurchase price is $c = 7$ or 13, when both parties give the same concession value, their optimal concession equilibrium solution is (1002, 29.11). That is, the result of optimal order and production moves close to that when the buy-back price = 10. When $c = 7$, the retailer's concession value increases by 100 and the order quantity by 33 or 34; or the manufacturer's concession value increases by 100 and the production by 33 or 34.

Therefore, in this case, for the manufacturer, the increase in the repurchase price can stimulate the retailers to order more goods. When the two sides increase the concession value, it can reduce the risk of the retailer. The manufacturer shares the loss risk with retailers, which plays a positive role in stabilizing the retailer's order level. This example means that the retailer and the manufacturer make a decision and cooperate (group decision-making) with each other to reach a better result than they do separately.

4. Conclusion

In this paper, we study the s^* -optimal concession equilibrium solution to multiple attribute group decision-making problem. We prove that it can be obtained by solving an equivalent single objective programming problem, and the s^* -optimal equilibrium solution is equivalent to the optimal solution of the worst weight of the linear weighted objective method. The s^* -optimal concession equilibrium solution to some complex group decision-making problems, such as supply chain order and production problems, avoids constructing preference functions, which makes the problem solving simpler.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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