Research Article

Sequential Fault-Tolerant Fusion Estimation for Multisensor Time-Varying Systems

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In this paper, the impact of the fusion framework on the fault diagnosis process is discussed. The centralized fusion framework makes it difficult to locate and estimate the sensor fault. Based on the fault detection method, the sequential fusion framework could locate and estimate the sensor fault and realize the fault-tolerant estimation of system states. In the sense of minimum mean square error (MMSE), based on the sequential detection of bias fault of sensors, a sequential fault-tolerant fusion estimation approach is presented to estimate the sensor fault and the system state, simultaneously, optimally, in real time. Further, a novel alternate fault-tolerant fusion estimation method is proposed to alternately estimate the sensor fault and the system state, in the frame of sequential fusion. What is more, the equivalency of the two proposed methods is proved. And the feasibility and equivalency of them are also verified by the computer simulation.

1. Introduction

With the improvement of computer science and sensor technology, various kinds of sensors are applied in the fields of military defense, modern industry, intelligent agriculture, navigation guidance, and other fields. Compared with traditional single senor monitoring systems, multisensor systems gather lots of redundancy and complementarity observation data from different time scales and different space scales and then obtain more comprehensive information of target interested through the information fusion methods [1–3].

Due to their working mechanism and application environment, the sensors will inevitably show components aging, drift point, mechanical wear, and such other phenomena. As a result, the bias faults could occur in the sensors in service. Therefore, several effective fault detection method and fault-tolerant fusion estimation methods are proposed for multisensor systems. Based on the federated filter, the literature [4–7] used the χ^2 detection method to detect and isolate the faulty sensor and realized the fault-tolerant performance of the navigation system. A multimode federated Gaussian sum particle fault-tolerant filtering method was proposed

to improve the estimation precision for nonlinear and non-Gaussian systems in [8]. In [9], the χ^2 detection method was utilized to detect the fault of three height sensors for unmanned aerial vehicles (UAV). The faulty height sensor was isolated to achieve fault-tolerant integrated navigation for UAV. In [10], three kinds of sensor faults were considered: the complete failure of the sensor, partial failure, and normal working conditions. And a fault-tolerant H∞ filter was designed to ensure the estimation accuracy. In [11], the sensor bias fault was introduced and modeled as a state-dependent uncertain term, and a passive fault-tolerant filtering method was designed by using the linear matrix inequation (LMI) technique for the failure fault and bias faults of sensors. In [12], the χ^2 detection method was utilized for the multirate multisensor systems to detect the unreliable measurements. The faulty measurements were also isolated and replaced by its predictions. In [13], it pointed out that the method of discarding faulty sensors would result in resource wastes and even lead to the fact that the system state could not be fusion estimated effectively. Based on fault detection of each sensor by using χ^2 detection criteria, a fault estimator was proposed to realize the distributed fault-tolerant estimation

for multisensor sampling systems. However, the proposed fault estimator took the variance of innovation as the variance of the fault, which results in the conservation of the fault-tolerant fusion estimation method.

In this paper, the shortcoming of the traditional centralized Kalman fusion filtering method for the linear time-varying systems with sensor bias fault is analyzed firstly. Then, in the sense of MMSE, a sequential fault-tolerant estimation method is presented to estimate the system state and the sensor bias fault simultaneously, optimally. Further, an alternate fault-tolerant fusion estimation method is proposed to alternately estimate the sensor fault and the system state, if the sensor is detected with bias fault. The two sequential fault-tolerant fusion estimation methods are performance equivalent, which is proved in theory and in simulation.

This paper is organized as follows. In Section 1, the mathematical models of a class of linear time-varying dynamic systems in normal operation and with sensor bias fault are formulated in Section 2. In Section 3, the traditional centralized Kalman fusion estimation method is reviewed. In Sections 4 and 5 two sequential fault-tolerant fusion estimation methods are presented, and the performance equivalence of these two methods is proved. The feasibility, optimality, and equivalence of the two presented methods are illustrated in the simulation in Section 6, and Section 7 draws conclusions.

2. Problem Formulation

Consider the following linear time-varying dynamic system:

$$x(k) = F(k-1)x(k-1) + w(k-1) \qquad (1)$$

where $x(k) \in R^{n \times 1}$ is the system state at time instant k, $F(k) \in R^{n \times n}$ is the corresponding state transition matrix, and the process noise $w(k) \in R^{n \times 1}$ obeys Gaussian distributions with zero mean and the variance of which is $Q(k)$, namely, $w(k) \sim \mathcal{N}(0, Q(k))$.

There are N sensors used to monitor the system; the corresponding measurement equations are given by

$$z_i(k) = H_i(k)x(k) + v_i(k), \quad i = 1, 2, \ldots, N \qquad (2)$$

in which $z_i(k) \in R^{p_i \times 1}$ is the measurement sampled by the sensor i, $H_i(k)$ is associated measurement matrix, and the measurement noise $v_i(k) \in R^{p_i \times 1} \sim \mathcal{N}(\mathbf{0}, R_i(k))$.

Because of the aging of the parts, the drift of the operating point, and the mechanical wear, the sensors may cause faults such as sensor bias, which are usually relative fixed over a period of time. Therefore, the measurement equations with fault sensors can be modeled as follows:

$$z_i(k) = H_i(k)x(k) + v_i(k) + f_i(k) \qquad (3)$$

where $f_i(k)$ is the sensor bias fault of sensor i and $f_i(k+1) = f_i(k)$.

Assumption 1. For the initial state of system (1), $x(0) \sim \mathcal{N}(x_0, P_0)$ holds.

Assumption 2. The process noises $w(k-1)$ and the measurement noises $v_i(j)$ are independent; namely, the following cross-variance hold:

$$E\left\{w(k-1)v_i^T(j)\right\} = 0, \quad i = 1, 2, \ldots, N, \ \forall k, j \qquad (4)$$

Assumption 3. $(F(k-1), H_i(k))$, $(i = 1, 2, \ldots, N)$ is uniformly observable.

Assumption 4. In this paper, it is assumed that the sensors cannot exchange some information to each other, and the measurement sampled by them is all transmitted to the fusion center and fused in it.

3. Centralized Fusion Estimation and Fault Detection

In this section, the centralized fusion estimation method and the further sensor fault detection method are reviewed.

Firstly, the following auxiliary variables are denoted, in the centralized fusion frame:

$$Z(k) = \begin{bmatrix} z_1(k) \\ z_2(k) \\ \vdots \\ z_N(k) \end{bmatrix};$$

$$H(k) = \begin{bmatrix} H_1(k) \\ H_2(k) \\ \vdots \\ H_N(k) \end{bmatrix}; \qquad (5)$$

$$V(k) = \begin{bmatrix} v_1(k) \\ v_2(k) \\ \vdots \\ v_N(k) \end{bmatrix}$$

According to the measurement equations (2), a similar augmented measurement equation is obtained as shown in

$$Z(k) = H(k)x(k) + V(k) \qquad (6)$$

where $V(k)$ is the augmented measurement noise which satisfies

$$E\{V(k)\} = \mathbf{0};$$

$$E\left\{V(k)V^T(k)\right\} = \begin{bmatrix} R_1(k) & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & R_2(k) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & R_N(k) \end{bmatrix} \qquad (7)$$

For the case all the sensors works normally, the state of the dynamic system can be estimated by the following centralized fusion estimation method.

Lemma 5. *For the linear time-varying system (1)-(2), assume the estimate of the state at $k-1$ is $\hat{x}(k-1 \mid k-1)$, the corresponding estimation error covariance is $P(k-1 \mid k-1)$, and then the fusion estimate of the state at k is*

$$
\begin{aligned}
\hat{x}(k \mid k) &= \hat{x}(k \mid k-1) \\
&\quad + K(k)(Z(k) - H(k)\hat{x}(k \mid k-1))
\end{aligned} \tag{8}
$$

The corresponding estimation error covariance can be given by

$$
P(k \mid k) = (I - K(k)H(k))P(k \mid k-1) \tag{9}
$$

where

$$
\hat{x}(k \mid k-1) = F(k-1)\hat{x}(k-1 \mid k-1)
$$

$$
\begin{aligned}
P(k \mid k-1) &= F(k-1)P(k-1 \mid k-1)F^T(k-1) \\
&\quad + Q(k-1)
\end{aligned}
$$

$$
\begin{aligned}
K(k) &= P(k \mid k-1)H^T(k) \\
&\quad \cdot \left(H(k)P(k \mid k-1)H^T(k) + R(k)\right)^{-1}
\end{aligned} \tag{10}
$$

Remark 6. The centralized fusion filter in Lemma 5 is optimal in the sense of MMSE, for the case that all the sensors work normally. Actually, the above centralized fusion filter is a standard Kalman filter for the argument system described by (1) and (6), which collects all the normal measurement of the sensors.

However, the sensor bias fault cannot be avoided due to the long-time utility of the sensor and the large-scale change of the environment. In this case, the sensor bias fault should be detected and estimated. On this basis, then, achieve the fault-tolerant estimation of the system state.

According to the estimation criterion of MMSE, for the case that all the sensors works normally, the measurement residual $\widetilde{Z}(k \mid k-1) = Z(k) - H(k)\hat{x}(k \mid k-1)$ should satisfy the zero-mean Gaussian distribution with the covariance $P_{zz}(k \mid k-1) = H(k)P(k \mid k-1)H^T(k) + R(k)$. Otherwise, if a sensor is faulty, the probability distribution of $\widetilde{Z}(k \mid k-1)$ will change. Therefore, the following index can be given to detect the sensor fault:

$$
\gamma(k) = \widetilde{Z}^T(k \mid k-1)\left(P_{zz}(k \mid k-1)\right)^{-1}\widetilde{Z}(k \mid k-1) \tag{11}
$$

Obviously, the detect index above satisfies the χ^2 distribution with the degree $\sum_{i=1}^{N} p_i$. The detect standard can be used as a measure to judge whether the augmented measurement $Z(k)$ is faulty or not. If T_α is set as the one-sided χ^2 distribution value with confidence α, the detect standard can be given by [12]

$$
\begin{aligned}
\gamma(k) &> T_\alpha, \quad fault \\
\gamma(k) &\le T_\alpha, \quad fault\ free
\end{aligned} \tag{12}
$$

Remark 7. In Lemma 5, the augmented measurement $Z(k)$ is established including the measurements sampled by all the sensors at k. The fault detection index above is with a high degree. If a small bias fault happened in a sensor, it is difficult to detect the fault and determine which sensor is failed. Therefore, the sequential fault-tolerant fusion estimation approach will be presented in the following section, which could detect and locate the sensor bias fault for multisensor time-varying systems in real time.

4. Sequential Fault-Tolerant Fusion Estimation I

For linear time-varying systems shown in (1)-(2), the bias fault may appear in each sensor. Therefore, the fault detection should be added in the fusion estimator to ensure its reliability. In order to detect the sensor bias fault and locate the failed sensor in real time, the fault detection method and the fault-tolerant estimation method are presented in the sequential fusion frame, in this section.

Assumption 8. Without loss of generality, assume the order in which the measurements reach the fusion center is the same as that of the sensor.

Denote the fusion estimate of the state at $k-1$ as $\hat{x}(k-1 \mid k-1)$ and the corresponding estimation error covariance as $P(k-1 \mid k-1)$. When the measurement sampled by Sensor 1 reaches the fusion center, the prediction of the state at k can be given by

$$
\hat{x}_0(k \mid k) = \hat{x}(k \mid k-1) = F(k-1)\hat{x}(k-1 \mid k-1)
$$

$$
\begin{aligned}
P_0(k \mid k) &= P(k \mid k-1) \\
&= F(k-1)P(k-1 \mid k-1)F^T(k-1) \\
&\quad + Q(k-1)
\end{aligned} \tag{13}
$$

Then, the prediction and innovation of the measurement sampled by Sensor i at k are

$$
\begin{aligned}
\hat{z}_1(k \mid k-1) &= H_1(k)\hat{x}_0(k \mid k) \\
\tilde{z}_1(k \mid k-1) &= z_1(k) - H_1(k)\hat{x}_0(k \mid k)
\end{aligned} \tag{14}
$$

and the covariance of $\tilde{z}_1(k \mid k-1)$ is

$$
P_{zz,1}(k \mid k-1) = H_1(k)P_0(k \mid k)H_1^T(k) + R_1(k) \tag{15}
$$

Define the following fault detection index for Sensor 1:

$$
\begin{aligned}
\gamma_1(k) \\
= \tilde{z}_1^T(k \mid k-1)\left(P_{zz,1}(k \mid k-1)\right)^{-1}\tilde{z}_1(k \mid k-1)
\end{aligned} \tag{16}
$$

It is easy to know that $\gamma_1(k)$ satisfies the χ^2 distribution with the degree p_1. And the detection standard is given by

$$
\begin{aligned}
\gamma_1(k) &> T_1, \quad fault \\
\gamma_1(k) &\le T_1, \quad fault\ free
\end{aligned} \tag{17}
$$

where T_1 is the detection threshold, which can be set by the χ^2 distribution [12].

If Sensor 1 is detected with bias fault, the state space should be described by (1) and (3). And the following augmented state fusion is established:

$$\begin{bmatrix} x(k) \\ f_i(k) \end{bmatrix} = \begin{bmatrix} F(k-1) & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} x(k-1) \\ f_i(k-1) \end{bmatrix} + \begin{bmatrix} w(k-1) \\ 0 \end{bmatrix} \quad (18)$$

Denote $F^*(k-1) = \begin{bmatrix} F(k-1) & 0 \\ 0 & I \end{bmatrix}$, $x^*(k) = \begin{bmatrix} x(k) \\ f_i(k) \end{bmatrix}$, and $w^*(k-1) = \begin{bmatrix} w(k-1) \\ 0 \end{bmatrix}$. And (18) can be rewritten as

$$x^*(k) = F^*(k-1)x^*(k-1) + w^*(k-1) \quad (19)$$

The augmented process noise $w^*(k-1) \sim N(0, Q^*(k-1))$, where $Q^*(k-1) = \begin{bmatrix} Q(k-1) & 0 \\ 0 & 0 \end{bmatrix}$.

The corresponding measurement function is described as

$$z_1(k) = H_1^*(k)x^*(k) + v_1(k) \quad (20)$$

Here, $H_1^*(k) = [H_1(k), I]$.

For the augmented state system (19), (20), the estimate of the augmented state can be given by

$$\begin{aligned} \widehat{x}_1^*(k \mid k) &= \begin{bmatrix} \widehat{x}_1(k \mid k) \\ f_1(k \mid k) \end{bmatrix} \\ &= \widehat{x}_0^*(k \mid k) \\ &\quad + K_1^*(k)(z_1(k) - H_1^*(k)\widehat{x}_0^*(k \mid k)) \end{aligned} \quad (21)$$

and the corresponding covariance of the estimation error is

$$\begin{aligned} P_1^*(k \mid k) &= \begin{bmatrix} P_1(k \mid k) & 0 \\ 0 & P_{1,f}(k \mid k) \end{bmatrix} \\ &= (I - K_1^*(k)H_1^*(k))P_0^*(k \mid k) \end{aligned} \quad (22)$$

In (21)-(22),

$$\begin{aligned} \widehat{x}_0^*(k \mid k) &= \begin{bmatrix} \widehat{x}_0(k \mid k) \\ f_1(k-1 \mid k-1) \end{bmatrix} \\ P_0^*(k \mid k) &= \begin{bmatrix} P_0(k \mid k) & 0 \\ 0 & P_{1,f}(k-1 \mid k-1) \end{bmatrix} \\ &\quad + \begin{bmatrix} Q(k, k-1) & 0 \\ 0 & 0 \end{bmatrix} \end{aligned} \quad (23)$$

$$K_1^*(k) = P_0^*(k \mid k)(H_1^*(k))^T$$
$$\cdot \left(H_1^*(k)P(k \mid k-1)(H_1^*(k))^T + R(k) \right)^{-1}$$

$f_0(k \mid k)$ is the initial estimate of sensor bias fault and $P_{0,f}(k \mid k)$ is the initial estimation error covariance.

If Sensor 1 is detected as fault free, the system state can be estimated by the traditional Kalman filter; namely,

$$\begin{aligned} \widehat{x}_1(k \mid k) &= \widehat{x}_0(k \mid k) + K_1(k) \\ &\quad \cdot (z_1(k) - H_1(k)\widehat{x}_0(k \mid k)) \\ P_1(k \mid k) &= (I - K_1(k)H_1(k))P_0(k \mid k) \quad (24) \\ K_1(k) &= P_0(k \mid k)H_1^T(k) \\ &\quad \cdot \left(H_1(k)P_0(k \mid k)H_1^T(k) + R_1(k) \right) \end{aligned}$$

When the measurement sampled by Sensor i ($i = 2, 3, \ldots, N$) reaches the fusion center, the sensor bias fault can be detected by the detection standard as in (14)-(17), where the time subscript is i ($i = 2, 3, \ldots, N$) but not 1, on the basis of $\widehat{x}_{i-1}(k \mid k)$, $P_{i-1}(k \mid k)$.

If Sensor i is detected with bias fault, the fault-tolerant estimation is similar to (21)-(23). Denote

$$\begin{aligned} \widehat{x}_i^*(k \mid k) &= \begin{bmatrix} \widehat{x}_i(k \mid k) \\ \widehat{f}_i(k \mid k) \end{bmatrix}, \\ P_i^*(k \mid k) &= \begin{bmatrix} P_i(k \mid k) & 0 \\ 0 & P_{i,f}(k \mid k) \end{bmatrix} \end{aligned} \quad (25)$$

The sensor bias fault and the system state can be estimated simultaneously and optimally in the sense of MMSE.

$$\begin{aligned} \widehat{x}_i^*(k \mid k) &= \widehat{x}_{i-1}^*(k \mid k) + K_i^*(k) \\ &\quad \cdot (z_i(k) - H_i^*(k)\widehat{x}_{i-1}^*(k \mid k)) \\ P_i^*(k \mid k) &= (I - K_i^*(k)H_i^*(k))P_{i-1}^*(k \mid k) \\ K_i^*(k) &= P_{i-1}^*(k \mid k)(H_i^*(k))^T \\ &\quad \cdot \left(H_i^*(k)P_{i-1}^*(k \mid k)(H_i^*(k))^T + R(k) \right)^{-1} \\ \widehat{x}_{i-1}^*(k \mid k) &= \begin{bmatrix} \widehat{x}_{i-1}(k \mid k) \\ \widehat{f}_{i,0}(k \mid k) \end{bmatrix} \\ H_i^*(k) &= [H_i(k), I] \\ P_{i-1}^*(k \mid k) &= \begin{bmatrix} P_{i-1}(k \mid k) & 0 \\ 0 & P_{i,0,f}(k \mid k) \end{bmatrix} \end{aligned} \quad (26)$$

in which $f_{i,0}(k \mid k) = f_i(k-1 \mid k-1)$, $P_{i,0,f}(k \mid k) = P_{i,f}(k-1 \mid k-1)$, if Sensor i is also detected with bias fault at k-1. Otherwise, $f_{i,0}(k \mid k)$, $P_{i,0,f}(k \mid k)$ is the initial estimate of sensor bias fault and the corresponding estimation error covariance.

If Sensor i is detected as fault free, the system state can be estimated by

$$\hat{x}_i(k \mid k) = \hat{x}_{i-1}(k \mid k) + K_i(k)$$

$$\cdot (z_i(k) - H_i(k)\hat{x}_{i-1}(k \mid k))$$

$$P_i(k \mid k) = (I - K_i(k)H_i(k))P_{i-1}(k \mid k) \qquad (27)$$

$$K_i(k) = P_{i-1}(k \mid k)H_i^T(k)$$

$$\cdot \left(H_i(k)P_{i-1}(k \mid k)H_i^T(k) + R_i(k)\right)^{-1}$$

When all the measurements sampled at k reach the fusion center, the global optimal fusion estimate can be given by

$$\hat{x}(k \mid k) = \hat{x}_N(k \mid k),$$

$$P(k \mid k) = P_N(k \mid k) \qquad (28)$$

To sum up, in the sequential fusion frame, a fusion estimation method is given by (27) to estimate the sensor bias fault and the system state simultaneously and optimally, in which $i = 1, 2, \ldots, N$.

Remark 9. It is noted that the fault detection index can also be establish for the centralized fusion estimation method in Lemma 5. However, the centralized fusion estimation method converts all the measurements sampled at k into an augmented measurement. When there are errors in individual sensors, they are may be submerged in the influence of augmented measurement noise on the observation process. Therefore, it is difficult to detect the sensor bias fault quickly. What is more, even if the fault is detected, it is often difficult to determine the number and the locations of the faulty sensors. The sequential fault-tolerant fusion estimation method given in this section not only avoids the high-dimensional matrix operations in the centralized fusion estimation method, but also can detect and estimate the bias fault of each sensor in real time.

5. Sequential Fault-Tolerant Fusion Estimation II

In the last section, a sequential fault-tolerant fusion estimation method is presented to simultaneously estimate the sensor bias fault and the system state, if the sensor is detected with bias fault. In this section, an alternate fault-tolerant fusion estimation method is proposed to alternately estimate the sensor fault and the system state. If Sensor i is detected with bias fault, the bias fault is estimated firstly, then the measurement sampled by Sensor i can be compensated. And the system state can be estimated by using the compensated measurement.

5.1. Bias Fault Estimation. The bias fault of Sensor i at k can be estimated by

$$\hat{f}_{i,k}(k \mid k) = \hat{f}_{i,k-1}(k-1 \mid k-1) + K_{i,f}(k)\big(z_i(k)$$

$$- H_i(k)\hat{x}_{i-1}(k \mid k) - \hat{f}_{i,k-1}(k-1 \mid k-1)\big) \qquad (29)$$

in which $\hat{f}_{i,k-1}(k-1 \mid k-1)$ is the bias fault estimate of Sensor i at $k-1$, if Sensor i is also detected with bias fault at k-1. Otherwise, $\hat{f}_{i,k-1}(k-1 \mid k-1)$ is the initial estimate of the bias fault of Sensor i.

The corresponding estimation gain and estimate error covariance, respectively, are

$$K_{i,f}(k) = P_{i,k-1,f}(k-1 \mid k-1)\big(R(k)$$

$$+ P_{i,k-1,f}(k-1 \mid k-1)$$

$$+ H_i(k)P_{i-1}(k \mid k)H_i^T(k)\big)^{-1} \qquad (30)$$

$$P_{i,k,f}(k \mid k) = \left(I - K_{i,f}(k)\right)P_{i,k-1,f}(k-1 \mid k-1)$$

5.2. Faulty Measurement Compensation. The measurement sampled by Sensor i can be compensated by utilizing the estimate shown in (29).

$$z_i^{\bullet}(k) = z_i(k) - \hat{f}_{i,k}(k \mid k) \qquad (31)$$

5.3. Fault-Tolerant Estimation. The fault-tolerant estimate of the system state can be given by

$$\hat{x}_i^{\bullet}(k \mid k)$$

$$= \hat{x}_{i-1}^{\bullet}(k \mid k) + K_i^{\bullet}(k)\left(z_i^{\bullet}(k) - H_i(k)\hat{x}_{i-1}^{\bullet}(k \mid k)\right)$$

$$M(k)$$

$$= H_i(k)P_{i-1}^{\bullet}(k \mid k)H_i^T(k) + P_{i,k-1,f}(k \mid k)$$

$$+ R(k) \qquad (32)$$

$$P_i^{\bullet}(k \mid k)$$

$$= P_{i-1}^{\bullet}(k \mid k)$$

$$- P_{i-1}^{\bullet}(k \mid k)H_i^T(k)M^{-1}(k)H_i(k)P_{i-1}^{\bullet}(k \mid k)$$

$$K_i^{\bullet}(k) = P_{i-1}^{\bullet}(k \mid k)H_i^T(k)M^{-1}(k)\left(I - K_{i,f}(k)\right)^{-1}$$

Theorem 10. *The sequential fusion fault-tolerant estimation algorithm based on augmented state (sequential fault-tolerant estimation fusion algorithm I) has the same estimation accuracy as the alternate fault-tolerant fusion estimation algorithm (sequential fault-tolerant estimation fusion algorithm II).*

Proof. The main difference between the two sequential fusion fault-tolerant estimation algorithms is mainly reflected in the different processing methods for the measurement sampled by the sensor with bias fault. Therefore, the proof of the above theorem is focused on the equivalence of (26) and (32).

In (28)

$$K_i^*(k) = P_{i-1}^*(k \mid k)(H_i^*(k))^T$$

$$\cdot \left(R(k) + H_i^*(k) P_{i-1}^*(k \mid k)(H_i^*(k))^T \right)^{-1}$$

$$= \begin{bmatrix} P_{i-1}(k \mid k) H_i^T(k) M^{-1} \\ P_{i,0,f}(k \mid k) M^{-1} \end{bmatrix} \tag{33}$$

Then the estimate of the system state at k and the corresponding estimation error covariance can be described by

$$\hat{x}_i(k \mid k) = \hat{x}_{i-1}(k \mid k) + P_{i-1}(k \mid k) H_i^T(k)$$

$$\cdot M^{-1} \left(z_i(k) - H_i(k) \hat{x}_{i-1}(k \mid k) - \hat{f}_{i,0}(k \mid k) \right)$$

$$P_i(k \mid k) = P_{i-1}(k \mid k) - P_{i-1}(k \mid k) H_i^T(k) M^{-1} H_i(k)$$

$$\cdot P_{i-1}(k \mid k) \tag{34}$$

The innovation of the measurement compensated by (33) is given by

$$\tilde{z}_i^{\bullet}(k) = z_i(k) - \hat{f}_{i,k}(k \mid k) - H_i(k) \hat{x}_{i-1}^{\bullet}(k \mid k)$$

$$= H_i(k) \tilde{x}_{i-1}^{\bullet}(k \mid k) + \tilde{f}_{i,k}(k \mid k) + v_i(k) = \left(I \right.$$

$$\left. - K_{i,f}(k) \right) \left(H_i(k) \tilde{x}_{i-1}^{\bullet}(k \mid k) + v_i(k) \right.$$

$$\left. + \tilde{f}_{i,k-1}(k-1 \mid k-1) \right) \tag{35}$$

Substituting it into (32),

$$\hat{x}_i^{\bullet}(k \mid k) = \hat{x}_{i-1}^{\bullet}(k \mid k) + K_i^{\bullet}(k) \left(z_i^{\bullet}(k) \right.$$

$$\left. - H_i(k) \hat{x}_{i-1}^{\bullet}(k \mid k) \right) = \hat{x}_{i-1}^{\bullet}(k \mid k) + K_i^{\bullet}(k) \left(I \right.$$

$$\left. - K_{i,f}(k) \right) \left(v_i(k) + H_i(k) \tilde{x}_{i-1}^{\bullet}(k \mid k) \right.$$

$$\left. + \tilde{f}_{i,k-1}(k-1 \mid k-1) \right) = \hat{x}_{i-1}^{\bullet}(k \mid k) + P_{i-1}^{\bullet}(k \mid k)$$

$$\cdot H_i^T(k) M^{-1}(k) \left(z_i(k) - H_i(k) \hat{x}_{i-1}(k \mid k) \right.$$

$$\left. - \hat{f}_{i,k-1}(k-1 \mid k-1) \right) \tag{36}$$

If $\tilde{x}_{i-1}^{\bullet}(k \mid k) = \hat{x}_{i-1}(k \mid k), P_{i-1}^{\bullet}(k \mid k) = P_{i-1}(k \mid k), \hat{f}_{i,k-1}(k-1 \mid k-1) = \hat{f}_{i,0}(k \mid k)$, then

$$\hat{x}_i^{\bullet}(k \mid k) = \hat{x}_i(k \mid k) \tag{37}$$

□

Remark 11. Although the sequential fault-tolerant fusion estimation method I given in Section 4 could estimate that bias fault and obtain the fault-tolerant fusion estimate of the system state, it is noted that the estimation result of this method is an estimate of the augmented state matrix which is constituted of the bias fault and the system state, when the bias fault is detected. It is implied that the fusion

center needs the augmented matrix computation ability. While the sequential fault-tolerant fusion estimation method II proposed in this section estimates the bias fault and the system state alternately, which need not to deal with the augmented matrix. What is more, the alternate fault-tolerant fusion estimation method is more accessible. The same estimate accuracy of the two sequential fault-tolerant fusion estimation methods proves the effectiveness of the alternate fault-tolerant fusion estimation method. Namely, there is no accuracy-loss in the two alternate estimation processes.

6. Simulation

In this section, two simulation examples are utilized to prove the effectiveness and feasibility of the two sequential fault-tolerant fusion estimation methods proposed in this paper. The first one is for the CV motion model, and the second one is for the hydrological data assimilation

6.1. Simulation I. Consider the following CV motion model:

$$x(k) = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} x(k-1) + w(k, k-1) \tag{38}$$

where $T = 1s$ is the fusion period and the covariance of $w(k-1)$ is

$$Q(k-1) = \begin{bmatrix} \dfrac{T^3}{3} & \dfrac{T^2}{2} \\ \dfrac{T^2}{2} & T \end{bmatrix} \cdot q \tag{39}$$

and $q = 0.15$.

There are two sensors utilized to observe the target. The measurement functions are given by

$$z_i(k) = H_i(k) x(k) + v_i(k), \quad i = 1, 2 \tag{40}$$

in which the covariance of $\mathbf{v}_i(k_i)$, $i = 1, 2$ are $R_1(k_1) = 1$ and $R_2(k_2) = 0.81$. And the measurement matric are $H_1(k_1) = [1, 0.5]$ and $H_2(k_2) = [0.5, 1]$. The initial state estimate is $x_0 = [1, 1]$, and the corresponding estimation error covariance is $P_0 = \text{diag}\{1, 1\}$.

Assume Sensor 1 appear bias fault at 50, and the bias fault is

$$f(k) = \begin{cases} 10 & k \geq 50 \\ 0 & k < 50 \end{cases} \tag{41}$$

The simulation results are shown as in Figures 1–4.

As illustrated in Figures 1–4, the sequential fault-tolerant fusion estimation methods presented in this paper could detect the bias faults of Sensor 1 in real time and fault-tolerant could estimate the system state. The centralized fusion estimation method cannot determine the order of the sensor with bias fault, so its estimates are obtained by taking all the measurements as normal ones. Therefore, the tracking accuracy of the centralized fusion estimation method turns
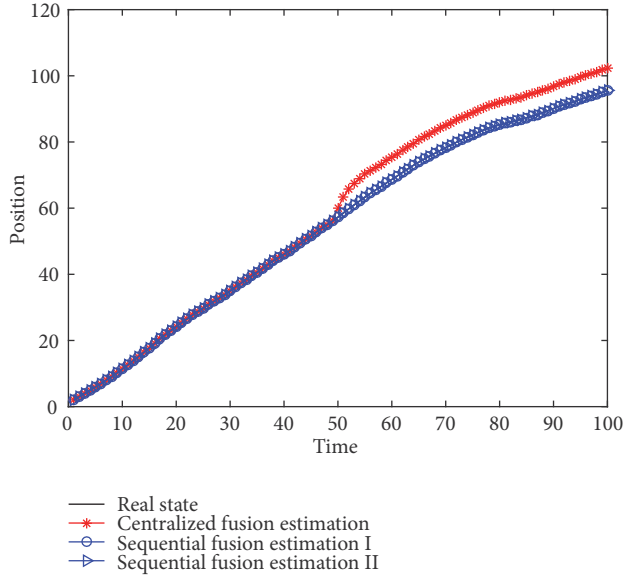
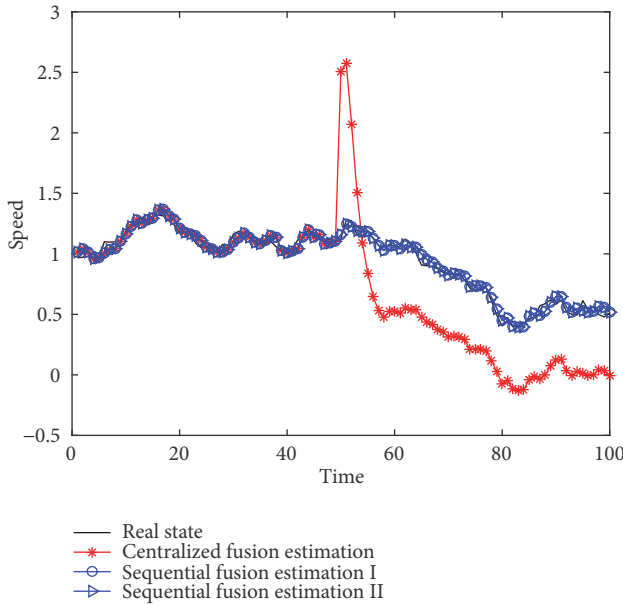FIGURE 1: The real state and estimate curves of position.

FIGURE 2: The real state and estimate curves of speed.

FIGURE 3: The absolute estimation error curves of position.

worse when the bias fault appears in Sensor 1, as shown in Figures 1 and 2. Note that the centralized fusion estimation method is optimal for normal measurements. Combined with the above simulation results, the optimality and feasibility of the presented sequential fault-tolerance fusion estimation methods are proved before the fault occurs. It is illustrated in the time interval $(0, 49]$ in Figures 3 and 4. After Sensor 1 is detected with the bias fault, the alternate fault-tolerant fusion estimation method can estimate the system states on the basis of the effective estimation of sensor bias fault, which obtains the same accuracy with the sequential fault-tolerant fusion estimation method based on augmented state, which also verifies the theorem in Section 5.
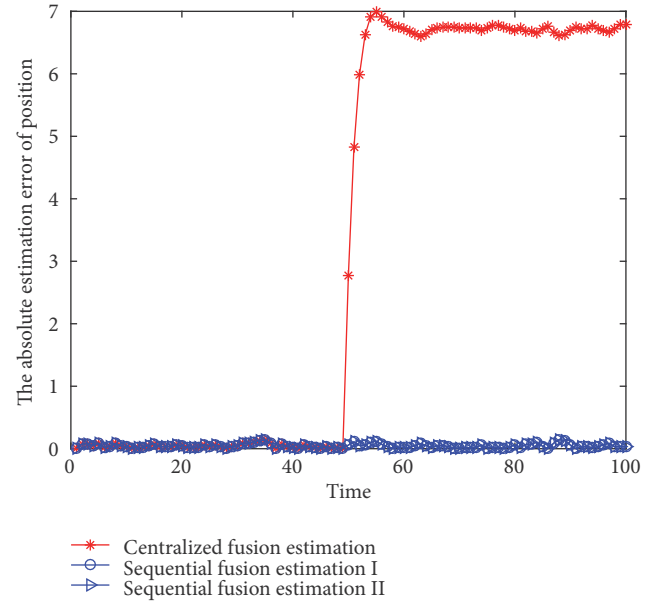
6.2. Simulation II. The Muskingum method is one of the most important kinds of hydrological data assimilation Technologies [14, 15]. The Muskingum method is a river flow calculation model, in which the river reach is divided into m sections, and the flow rate of the Section i at k is denoted as $Q_i(k)$. According to the Muskingum method,

$$
\begin{bmatrix} a_1 & 0 & \cdots & 0 \\ b_2 & a_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & b_m & a_m \end{bmatrix} \begin{bmatrix} Q_1(k) \\ Q_2(k) \\ \vdots \\ Q_m(k) \end{bmatrix}
$$

$$
= \begin{bmatrix} c_1 & 0 & \cdots & 0 \\ d_2 & c_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & d_m & c_m \end{bmatrix} \begin{bmatrix} Q_1(k-1) \\ Q_2(k-1) \\ \vdots \\ Q_m(k-1) \end{bmatrix} \tag{42}
$$

$$
+ \begin{bmatrix} u_1(k-1) \\ u_2(k-1) \\ \vdots \\ u_m(k-1) \end{bmatrix}
$$

where $a_i = K_i(1 - x_i)/2 + \Delta t/2$, $b_i = K_i x_i - \Delta t$, $c_i = K_i(1 - x_j) - \Delta t/2$, $d_i = K_i x_j + \Delta t/2$, and K_i and x_i are the Muskingum parameters. Δt is the sampled period, and $u_i(k - 1)$, $i = 1, 2, \ldots, m$, denotes the local inflow.

A flood process is considered in this simulation, the Muskingum parameters are given by $K_i = 25$, $x_i = 0.3$. There are 3 sections all sampled by 2 kinds of flow meters every 5
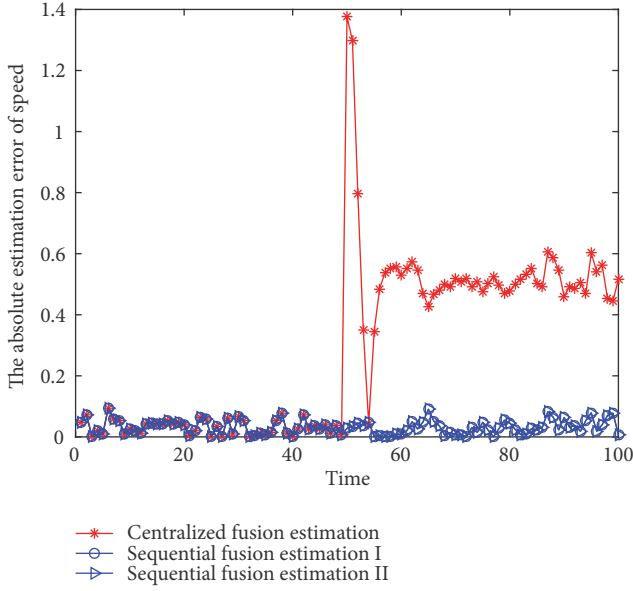
FIGURE 4: The absolute estimation error curves of position.
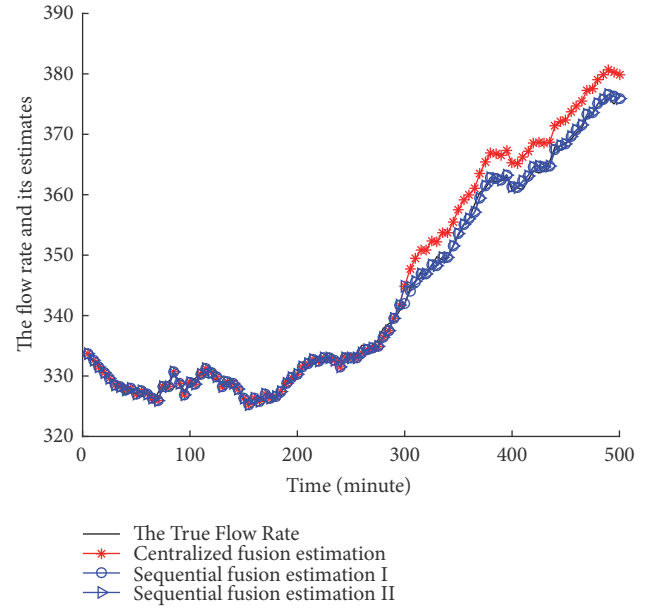

FIGURE 5: The flow rate of its estimates of three methods.

minutes. In this river reach, local inflows have little effect. Therefore, the measurement functions can be given by

$$
\begin{bmatrix} a_1 & 0 & 0 \\ b_2 & a_2 & 0 \\ 0 & b_3 & a_3 \end{bmatrix} \begin{bmatrix} Q_1(k) \\ Q_2(k) \\ Q_3(k) \end{bmatrix}
$$

$$
= \begin{bmatrix} c_1 & 0 & 0 \\ d_2 & c_2 & 0 \\ 0 & d_3 & c_3 \end{bmatrix} \begin{bmatrix} Q_1(k-1) \\ Q_2(k-1) \\ Q_3(k-1) \end{bmatrix} + \begin{bmatrix} w_1(k-1) \\ w_2(k-1) \\ w_3(k-1) \end{bmatrix} \quad (43)
$$

$$
y_{i,j}(k) = Q_i(k) + v_{i,j}(k), \quad i = 1,2,3; \ j = 1,2
$$

500 minutes of the flood process is shown in the simulation results, in which the first flow meter is assumed to be bias fault after 300 minute. $f_{i,1}(k) = 8, i = 1,2,3; k > 300$. In this simulation, the centralized fusion estimation method and the two sequential fault-tolerant fusion estimation methods are compared for the hydrological data assimilation of the 500 minutes of the flood process. The simulation results are given by Figures 5 and 6

In Figures 5 and 6, it is noted that the centralized fusion estimation method and the two sequential fusion estimation methods proposed in this paper could obtain accurate estimates of the flow rate before 300 minutes. Then, the first flow meter is faulty; the centralized fusion estimation method cannot determine which flow meter is fault. The two sequential fusion estimation methods proposed in this paper could detect and determine the faulty flow meter. Then, the bias fault is estimated by different methods. On this basis, the same fault-tolerant fusion estimates and flow rate correction errors illustrate the theorem in Section 5.
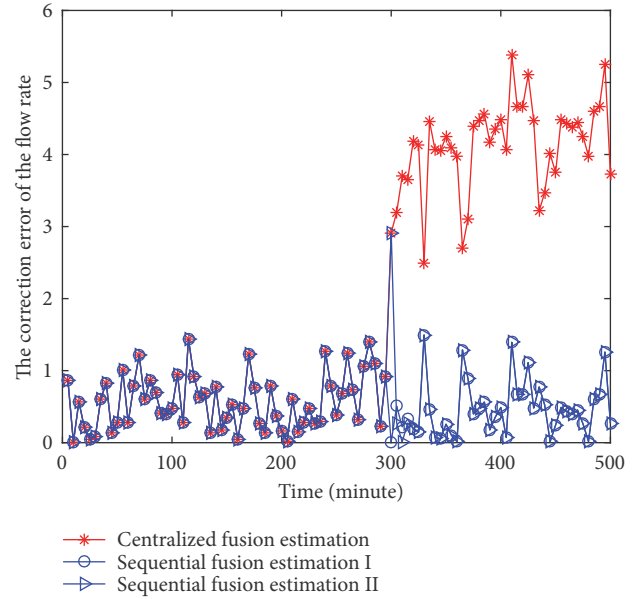

FIGURE 6: The correction errors of the flow rate.

7. Conclusion

The centralized fusion estimation method is usually difficult to locate the faulty sensor when the sensor fault is detected, which makes it impossible to further estimate the fault. In order to simultaneously detect, locate, and estimate sensor bias faults, two sequential fault-tolerant fusion estimation methods are presented in this paper. The system state and the sensor bias faults are simultaneously estimated in the first presented method, while they are alternatively estimated in the second method. The performance equivalence of the

two presented sequential fault-tolerant fusion estimation methods is both proved in theory and in simulation.

Note that the bias faults in this paper are considered as constant faults of sensors. For the time-varying bias faults [16, 17], the two proposed sequential fault-tolerant fusion estimation methods are not suitable. And this problem is an open question for our further research.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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