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Research Article

Research on a Triopoly Dynamic Game with Free Market and Bundling Market in the Chinese Telecom Industry

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Based on the real situation of telecom industry in China, we establish a triopoly game model, which includes two competitive telecom firms and their correlative corporation which produces the complementary product. Both free market and bundling market will be concerned in this dynamic model. Moreover, we consider a one-to-many bundling way instead of two complementary products in terms of the proportion of one to one in a bundling product. By numerical simulation, we find that stable space will decrease and decision chaos appears when the degree of the competition becomes fierce between the two competitive telecom firms. Besides, increasing the amount of bundling services provided by a telecom firm can lead to different impacts on the prices of the three firms investigated. This paper enriches the decision-making for the strategy of bundling pricing and will be valuable for the telecom operators.

1. Introduction

As a form of marketing activity, bundling sales are widespread in operations. Since this kind of behavior may increase sales and profits, many large enterprises even small businesses should consider adopting bundling sales strategy. The form of bundling sell is varied, like bundling pricing, exchanging prizes, presenting the prizes, etc. It is worth noting that the phenomenon of bundling pricing not only occurs between tangible products but may also happen to service products; the telecommunications companies sell smart phone with their own service packages, for instance.

In this paper, we focus on two kinds of bundling products provided by two competitive oligarchies of telecom industry in China and their important partner, Apple Inc. The two telecom companies are China Mobile Communications Corporation (CMCC, for short) and China Union Communication Corporation (CUCC, for short). CMCC constitutes the majority of market share and CUCC, which also has a large market share, firstly collaborates with Apple. Both of them cooperate with Apple by the best-selling iPhone. As a high-end product in the smart phone market, iPhone, compared with other cheaper mobile phones, could be bundled with more communication services provided by the telecom firms. On the other hand, many customers prefer to choose a package including more services instead of a package including fewer services, because the discount would become deeper under the former situation. As Table 4 shows, an extreme case is that when customers choose CUCC's bundling service package 9, which has numerous services, the price of the iPhone seems to reduce to zero. It is no doubt that this is a stratagem for attracting customers. Since each of the three firms has their own channel for marketing, we also consider three free markets.

Anyhow, the essence of bundling pricing is stimulating the demand according to the discount. Therefore, we concentrate on how the discounts provided by the three firms severally or the different degree of competition between the two telecom firms impacts the stability margin in the dynamic model. In our gaming system, a linear demand function and a constant marginal cost are supposed. In order to conform to the actual situation, we investigated all the data of the service packages for the assignment of the parameters.

2. Literature Review

Based on game theory and rank-dependent utility theory, Tan et al. [1] studied an incomplete information Cournot

game in an ambiguous decision environment, and they found players decision-making is affected by their behavioral characteristics and psychological preference. Many researchers have contributed much to the field of dynamic system and dynamic system bifurcation and chaos theory. Li and Chen[2] investigated some new nonlinear equations which have exact explicit parametric representations of breaking loop-solutions. Aziz-Alaoui and Chen [3] researched a new piecewise-linear continuous-time three-dimensional autonomous chaotic system and briefly discussed its complex chaotic dynamics. Zelinka et al. [4] introduced the notion of chaos synthesis and developed a new method for chaotic systems synthesis.

With the consideration of mixed bundling pricing and decision research, Shao and Li [5] studied the bundling and product strategy involving the channel competition, and they found that the retailer preferred the bundling strategy in two channel competitions. Under the consideration of the price bundle offers in the telecommunications industry, Le et al. [6] introduced a Bayesian game model. They investigated the case where operators propose substitutable offers and compete for customers, in order to maximize the operator's revenue. Based on game theory, Ma and Mallik [7] pointed out that, under the manufacturer bundling scenario, the manufacturer would not provide the full product line composed of the basic product, the premium product, and the bundle. Pan and Zhou [8] established a two-layer supply chain with bundling and pricing decisions and found that the manufacturer could earn more if he sells complementary products separately. Mantovani and Vandekerckhove [9] researched the strategies of merging and bundling decisions and showed that bundle strategy could not benefit the consumers because it brings surge in prices in their model. By calculating, R. et al. [10] discussed the optimal bundling and pricing. Chen and Wang [11] researched the pricing policies, subsidy policies, and channel selection policies of different power supply chain structures. In the perspective of the mergers producer, Gaudet and Salant [12] compared the profits of firms under different circumstances. Martin [13] studied the price pooling equilibrium under various conditions that firms produce strategic substitutes or strategic complements. Dilip and John [14] figured out that the price bundling as a usual marketing action could result in the diminution of demand. In the field of decision research, Li and Liu [15] studied the coordination of supply chain between a supplier and a retailer with price and sales effort dependent demand.

With the rise of bifurcation and chaos theory, many scholars have been interested in combining the theories to the game in the domain of economy. Agiza and Elsadany [16] built a nonlinear discrete-time duopoly game assuming the oligarchies have heterogeneous expectations. With the discussion of a 6-dimension dynamical system, Ma and Wu [17] described a triopoly game with two products. They investigated the impacts of the adjustment parameter and the reform of flexible manufacturing could decrease fluctuation risk caused by the instability in multiproduct market. By applying continuation theorem in their research, Wang et al. [18] obtained the sufficient conditions of the existence of positive periodic solutions of a predator-prey system and improved the methods of computation on topological degree. Besides, Yassen and Agiza [19] proposed a delayed bounded rationality model and figured out that the stability region of the model with bounded rationality is reduced when the competitors use different production methods. Bischi et al. [20] showed a dynamic game model and discussed the question of whether the economic hypothesis of the "representative agent" may sometimes impact the results of research. Sarafopoulos [21] conducted the study of a duopoly game which proposes one player has limited rationality and the other uses adaptive expectations. By studying a triopoly price game model, Ma and Wu [22] found the result that the number of time-delay decision makers has no obvious relationship with stability of the system. Based on the Cournot-Bertrand duopoly model, Ma and Pu [23] studied the stability of the system and found only one Nash equilibrium point after calculation. They also showed the bifurcation diagram which could exhibit the behaviors of the system. Elsadany et al. [24] derived a dynamic system with four various firms and indicated that applying a delayed feedback control method can get the stabilization of the chaos. Srivastava and Srivastava [25] confirmed the FACTS devices could regulate the dynamic bifurcations and chaos effectively. Considering a nonlinear cost function, Du et al. [26] investigated the effects of upper and lower limiter on a duopoly game. By establishing a game model which relates to the power market, Tan et al. [27] analyzed the dynamic behaviors in different market parameters and found that the chaos could be controlled to the stable equilibrium point with the time-delayed feedback control method. As to building two different game models, Ma and Guo (2016) investigated the effect of information on the stability. As to building a Cournot model, Ma and Guo [28] investigated a game whose player's decision was based on his estimation. According to the research done by Yue et al. (2016), a point of view that information sharing may not be beneficial to the firms all through is presented. Ma and Ma [29] established a model and found that market competition could lead to bullwhip effect in a supply chain.

In the field of dynamic game, many authors have investigated the products bundled in terms of putting one product A to another product B in a package. However, the reality is often not the ratio of one to one. Therefore, we construct a model in which bundling products accord with ways of one-to-many. Furthermore, our research not only investigates the behavior of bundling pricing but also considers a competition between two monopolies. This initiate consideration enriches the previous studies of bundling pricing and makes the game more realistic.

After the literature review in Section 2, the remaining part of this paper is divided into four sections. Section 3 describes a new-type game model including five subdivided markets, with the consideration of bundling pricing. Besides, we figure out the Nash equilibrium. In Section 4, we do some numerical experiments and analyze the results of the experiments from the perspective of chaos and complexity. Finally, Section 5 is the conclusion of this paper.

3. The Model

3.1. The Assumptions

(a) **Assumption.** We suppose the market demand is divided into five sections, which is shown in Figure 1. Here, markets A, B, and C represent the market of CMCC's product, the market of CUCC's product, and the market of iPhone separately, and the three markets only have their own products without bundling pricing. Besides, markets AC and BC, respectively, mean the market of bundling product, which is a CMCC's product or a CUCC's product bundled a new iPhone 7. Customers would only buy products from the three free markets, like markets A, B, and C, or buy the bundling product in the bundling market, like market AC and market BC.

(b) **Assumption.** As shown in Tables 1 and 2, the details of the service packages already fixed by the two telecom companies are presented. We consider the customers only decide which service package to choose, instead of thinking more about the costs incurred by the service out of the package.

(c) **Assumption.** We assume the period of a service is a month. By the exhibition of Tables 1–4, we define both of the service packages, which are CM 1 and CU 1, as a unit of service, which includes 100 minutes' call and 500 M data plan. Thus, the amount of the other service packages is m_i or n_i (m_i, n_i are constants not less than 1) times as the quantity of the CM 1 or CU 1. We assume the amount of the services offered in market AC (or BC) is m (or n), which is the expectation of m_i (or n_i). In the same manner, the discounts of the two bundling products are μ_1^i and μ_2^i, and their expectations are μ_1 and μ_2.

(d) **Assumption.** The price of the two telecom firms refers to the price of a unit of service in one month. We also assume the price of the iPhone 7 equals the real price divided by 24, because of the research period in contrast of the services. All markets use the price list as Tables 1 and 2 show.

(e) **Assumption.** In order to make the research feasible, we assume that all the discounts of the iPhone 7, which is bundled with one of the service packages in the two bundling markets, are the same since the exact discounts for the telecom firms are trade secret. Because of playing a leading role, we assume the discount of the iPhone 7 is a fixed value, 0.84, according to the price of the phone in Table 4, CU 1.

(f) **Assumption.** Each of the three companies adjusts their price every period under the bounded rationality, in order to maximize their own profits.

3.2. Variable Enactment. A series of specification and variables in this Bertrand game model will be enacted in this section: $p_i(t)$ $(i = 1, 2, 3)$ are the definitions for the three products per unit made by the three companies in the period of t. $q_1^A(t)$, $q_2^B(t)$, and $q_3^C(t)$ are the demands of products provided by CMCC and CUCC and the demands of iPhone 7 in markets A, B, and C severally during the period of t. The linear demand functions are as follows:

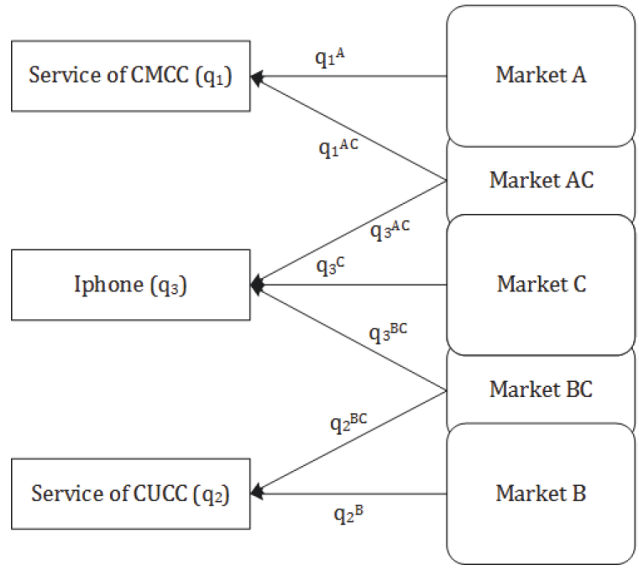


Figure 1: The structure of the model.

$$q_1^A(t) = a_1 - b_1 p_1(t) + d_1 p_2(t). \quad (1)$$

$$q_2^B(t) = a_2 - b_2 p_2(t) + d_2 p_1(t). \quad (2)$$

$$q_3^C(t) = a_3 - b_3 p_3(t) \quad (3)$$

where a_i $(i = 1, 2, 3)$ are the positive parameters of potential demand in markets A, B, and C, on condition that the firms fix their prices to zero. And, then, b_i $(i = 1, 2, 3)$ reflect the self-price sensitive coefficients for the three kinds of products. Besides, d_i $(i = 1, 2, 3)$ are denoted as the mutual product substitution ratios between the two telecom firms.

And, then, the parameters $q_3^{AC}(t)$ and $q_3^{BC}(t)$ mean the demands of iPhone 7 in market AC and in market BC, respectively, during the period of t. The demand functions in bundling markets are given as

$$q_3^{AC}(t) = a_{31} - b_{31}\mu p_3(t) - b_{31} r_{31} \mu_1 p_1(t). \quad (4)$$

$$q_3^{BC}(t) = a_{32} - b_{32}\mu p_3(t) - b_{32} r_{32} \mu_2 p_2(t) \quad (5)$$

where the positive parameters a_{3i} $(i = 1, 2)$ are the potential demand for the CMCC's bundling product and the CUCC's bundling product from markets AC and BC. And b_{3i} $(i = 1, 2)$, which are greater than zero, are the self-price sensitive coefficients for the two bundling products. The parameters μ, μ_1, and μ_2 denote the discount given by the three firms. The positive constants r_{3i} $(i = 1, 2)$ are the complementarity degree between two products and $b_{3i} r_{3i}$ $(i = 1, 2)$ are the cross-price sensitive coefficients.

Table 1: The price list of CMCC.

service package	call plan (min)	data plan (M)	price of service package (RMB)	m_i
CM 1	100	500	58	1.00
CM 2	220	700	88	1.58
CM 3	500	1000	138	2.67
CM 4	500	2000	158	4.22
CM 5	1000	2000	238	5.35
CM 6	1000	3000	268	6.90
CM 7	2000	3000	338	9.15
CM 8	4000	6000	588	18.29

Table 2: The price list of CUCC.

service package	call plan (min)	data plan (M)	price of service package (RMB)	n_i
CM 1	100	500	56	1.00
CM 2	200	800	76	1.67
CM 3	300	1000	106	2.19
CM 4	500	1000	136	2.56
CM 5	500	2000	166	4.19
CM 6	500	3000	196	5.81
CM 7	1000	4000	296	8.37
CM 8	2000	6000	396	13.5
CM 9	3000	11000	596	23.5

Table 3: Details of the service package offered by CMCC.

service package	CM 1	CM 2	CM 3	CM 4	CM 5	CM 6	CM 7	CM 8
service price in the actual situation	58	88	138	158	238	268	338	588
price of the phone in bundling product	5388	5388	5388	5388	5388	5388	5388	5388
price of the phone after mathematic transaction	4540	4540	4540	4540	4540	4540	4540	4540
gift calls	348	528	828	948	1428	1608	2028	3528
service price after mathematic transaction	78.8	101.3	138.8	153.8	213.8	236.3	288.8	476.3
total cost	6432	6972	7872	8232	9672	10212	11472	15972
price ratio (p_{ri}^M)	1.0	1.29	1.76	1.95	2.71	3.00	3.66	6.04
m_i	1.0	1.6	2.7	4.2	5.3	6.9	9.1	18.3
μ_1^i	1.00	0.81	0.66	0.46	0.51	0.43	0.40	0.33

In bundling markets AC and BC as well, $q_1^{AC}(t)$ and $q_2^{BC}(t)$ denote the demand of CMCC's service and CUCC's service in period t. The demand functions are demonstrated as follows:

$$\begin{aligned} q_1^{AC}(t) &= m q_3^{AC}(t) \\ &= m\left[a_{31} - b_{31}\mu p_3(t) - b_{31} r_{31} \mu_1 p_1(t)\right]. \end{aligned} \tag{6}$$

$$\begin{aligned} q_2^{BC}(t) &= n q_3^{BC}(t) \\ &= n\left[a_{32} - b_{32}\mu p_3(t) - b_{32} r_{32} \mu_2 p_2(t)\right] \end{aligned} \tag{7}$$

where the nonintegral parameters m and n which are not less than 1 are the multiples of the unit of service.

$q_i(t)$ $(i = 1, 2, 3)$, the aggregate demand functions of three products offered by CMCC, CUCC, and Apple in the period t, are showed as follows:

$$\begin{aligned} q_1(t) &= q_1^A(t) + q_1^{AC}(t) \\ &= a_1 - b_1 p_1(t) + d_1 p_2(t) \\ &\quad + m\left[a_{31} - b_{31}\mu p_3(t) - b_{31} r_{31} \mu_1 p_1(t)\right]. \end{aligned} \tag{8}$$

Table 4: Details of the service package offered by CUCC.

service package	CU 1	CU 2	CU 3	CU 4	CU 5	CU 6	CU 7	CU 8	CU 9
service price in the actual situation	56	76	106	136	166	196	296	396	596
price of the phone in bundling product	4549	4359	4069	3779	3489	3199	2239	1279	0
price of the phone after mathematic transaction	4549	4549	4549	4549	4549	4549	4549	4549	4549
gift calls	0	0	0	0	0	0	0	0	0
service price after mathematic transaction	56	68.1	86.0	103.9	121.8	139.8	199.8	259.8	406.5
total cost	5893	6183	6613	7043	7473	7903	9343	10783	14304
price ratio (p_{ri}^{U})	1.00	1.22	1.54	1.86	2.18	2.50	3.57	4.64	7.26
n_i	1.0	1.7	2.2	2.6	4.2	5.8	8.4	13.5	15.1
μ_2^i	1.00	0.73	0.70	0.72	0.52	0.43	0.43	0.34	0.48

$$\begin{aligned} q_2(t) &= q_2^B(t) + q_2^{BC}(t) \\ &= a_2 - b_2 p_2(t) + d_2 p_1(t) \\ &\quad + n\left[a_{32} - b_{32}\mu p_3(t) - b_{32} r_{32}\mu_2 p_2(t)\right]. \end{aligned} \tag{9}$$

$$\begin{aligned} q_3(t) &= q_3^C(t) + q_3^{AC}(t) + q_3^{BC}(t) \\ &= a_3 - b_3 p_3(t) \\ &\quad + \left[a_{31} - b_{31}\mu p_3(t) - b_{31} r_{31}\mu_1 p_1(t)\right] \\ &\quad + \left[a_{32} - b_{32}\mu p_3(t) - b_{32} r_{32}\mu_2 p_2(t)\right]. \end{aligned} \tag{10}$$

The total profit functions of the three firms are defined as $\pi_i(t)$ $(i = 1, 2, 3)$ in the period of t. In addition, we suppose a fixed marginal cost greater than zero for each product. c_0 is the cost per unit for the service of two telecom firms, and c_3 is the cost per unit for the iPhone 7.

$$\pi_1(t) = q_1^A(t)\, p_1(t) + q_1^{AC}(t)\, \mu_1 p_1(t) - c_0 q_1(t). \tag{11}$$

$$\pi_2(t) = q_2^B(t)\, p_2(t) + q_2^{BC}(t)\, \mu_2 p_2(t) - c_0 q_2(t). \tag{12}$$

$$\begin{aligned} \pi_3(t) &= q_3^C(t)\, p_3(t) + \left[q_3^{AC}(t) + q_3^{BC}(t)\right] \mu p_3(t) \\ &\quad - c_3 q_3(t). \end{aligned} \tag{13}$$

3.3. The Dynamic Model. On the basis of the bounded rationality in Assumption (f), three firms treat their price as the decision variable. Meanwhile, they adjust their price according to the marginal profit $\partial\pi_i(t)/\partial p_i(t)$ in each period. If the marginal profit is positive in period t, increasing the price in period $t + 1$ will be profitable. Conversely, if the marginal profit is negative in period t, decreasing the price in period $t + 1$ will be profitable. The nonlinear functions of the marginal profit are as follows:

$$\begin{aligned} \frac{\partial\pi_1(t)}{\partial p_1(t)} &= a_1 - b_1 p_1(t) + d_1 p_2(t) + b_1\left[c_0 - p_1(t)\right] \\ &\quad - m\mu_1\left[b_{31}\mu p_3(t) - a_{31} + b_{31} r_{31}\mu_1 p_1(t)\right] \\ &\quad + m b_{31} r_{31}\mu_1\left[c_0 - \mu_1 p_1(t)\right]. \end{aligned} \tag{14}$$

$$\begin{aligned} \frac{\partial\pi_2(t)}{\partial p_2(t)} &= a_2 - b_2 p_2(t) + d_2 p_1(t) + b_2\left[c_0 - p_2(t)\right] \\ &\quad - n\mu_2\left[b_{32}\mu p_3(t) - a_{32} + b_{32} r_{32}\mu_2 p_2(t)\right] \\ &\quad + n b_{32} r_{32}\mu_2\left[c_0 - \mu_2 p_2(t)\right]. \end{aligned} \tag{15}$$

$$\begin{aligned} \frac{\partial\pi_3(t)}{\partial p_3(t)} &= a_3 - b_3 p_3(t) - \mu\left[b_{31}\mu p_3(t) - a_{32} - a_{31}\right. \\ &\quad \left. + b_{32}\mu p_3(t) + b_{31} r_{31}\mu_1 p_1(t) + b_{32} r_{32}\mu_2 p_2(t)\right] + \left[c_3\right. \\ &\quad \left. - \mu p_3(t)\right]\left(b_{31} + b_{32}\right)\mu + b_3\left[c_3 - p_3(t)\right]. \end{aligned} \tag{16}$$

Hence, the dynamic equations in this model could be given by following form:

$$\begin{aligned} p_1(t+1) &= p_1(t) + \alpha p_1(t)\frac{\partial\pi_1(t)}{\partial p_1(t)} \\ p_2(t+1) &= p_2(t) + \beta p_2(t)\frac{\partial\pi_2(t)}{\partial p_2(t)} \\ p_3(t+1) &= p_3(t) + \gamma p_3(t)\frac{\partial\pi_3(t)}{\partial p_3(t)} \\ &\qquad \alpha, \beta, \gamma > 0 \end{aligned} \tag{17}$$

where we suppose the three companies adjust their price in the speed of α, β, and γ, which reflect the decision-making of

the three enterprises. After taking (14)–(16) into (17), we can get functions as follows:

$$\begin{aligned}
p_1(t+1) &= p_1(t) + \alpha p_1(t)\{a_1 - b_1 p_1(t) + d_1 p_2(t) \\
&\quad + b_1[c_0 - p_1(t)] - m\mu_1[b_{31}\mu p_3(t) - a_{31} \\
&\quad + b_{31} r_{31}\mu_1 p_1(t)] + m b_{31} r_{31}\mu_1[c_0 - \mu_1 p_1(t)]\} \\
p_2(t+1) &= p_2(t) + \beta p_2(t)\{a_2 - b_2 p_2(t) + d_2 p_1(t) \\
&\quad + b_2[c_0 - p_2(t)] - n\mu_2[b_{32}\mu p_3(t) - a_{32} \\
&\quad + b_{32} r_{32}\mu_2 p_2(t)] + n b_{32} r_{32}\mu_2[c_0 - \mu_2 p_2(t)]\} \\
p_3(t+1) &= p_3(t) + \gamma p_3(t)\{a_3 - b_3 p_3(t) \\
&\quad - \mu[b_{31}\mu p3(t) - a_{32} - a_{31} + b_{32}\mu p_3(t) \\
&\quad + b_{31} r_{31}\mu_1 p_1(t) + b_{32} r_{32}\mu_2 p_2(t)] + [c_3 - \mu p_3(t)] \\
&\quad \cdot (b_{31} + b_{32})\mu + b_3[c_3 - p_3(t)]\}.
\end{aligned} \tag{18}$$

In order to get the positive stable fixed points of the dynamic model, we need the condition of Nash equilibrium state which is $p_i(t+1) = p_i(t)$ $(i = 1, 2, 3)$. After calculation, we obtain only one significant solution in which all the prices are positive; that is,

$$E^* = (p_1^*, p_2^*, p_3^*) \tag{19}$$

Due to the fact that the length of p_i^* $(i = 1, 2, 3)$ is too long to exhibit in this section, we will show the form of them which have taken the fixed parameters in Section 4.

Besides, the Jacobian matrix of dynamic equation (18) is that

$$J = \begin{pmatrix} J_{11} & \alpha d_1 p_1 & -\alpha b_{31} m p_1 \mu\mu_1 \\ \beta d_2 p_2 & J_{22} & -\beta b_{32} n p_2 \mu\mu_2 \\ -\gamma b_{31} p_3 r_{31}\mu\mu_1 & -\gamma b_{32} p_3 r_{32}\mu\mu_2 & J_{33} \end{pmatrix} \tag{20}$$

where

$$\begin{aligned}
J_{11} &= \alpha(a_1 - b_1 p_1 + d_1 p_2 + b_1(c_0 - p_1) \\
&\quad - m\mu_1(b_{31} p_3\mu - a_{31} + b_{31} p_1 r_{31}\mu_1) + b_{31} m r_{31}\mu_1(c_0 \\
&\quad - p_1\mu_1)) - \alpha p_1(2b_{31} m r_{31}\mu_1^2 + 2b_1) + 1. \\
J_{22} &= \beta(a_2 - b_2 p_2 + d_2 p_1 + b_2(c_0 - p_2) - n\mu_2(b_{32} p_3\mu \\
&\quad - a_{32} + b_{32} p_2 r_3 2\mu_2) + b_{32} n r_{32}\mu_2(c_0 - p_2\mu_2)) \\
&\quad - \beta p_2(2b_{32} n r_{32}\mu_2^2 + 2b_2) + 1. \\
J_{33} &= \gamma(a_3 - b_3 p_3 - \mu(b_{31} p_3\mu - a_{32} - a_{31} + b_{32} p_3\mu \\
&\quad + b_{31} p_1 r_{31}\mu_1 + b_{32} p_2 r_{32}\mu_2) + (c_3 - p_3\mu)(b_{31}\mu \\
&\quad + b_{32}\mu) + b_3(c_3 - p_3)) - \gamma p_3(2b_3 + 2\mu(b_{31}\mu \\
&\quad + b_{32}\mu)) + 1.
\end{aligned} \tag{21}$$

The characteristic polynomial of the matrix of (20) is as the function equation (22) shows. Consider

$$P(\lambda) = \lambda^3 + A'\lambda^2 + B'\lambda + C' \tag{22}$$

where A'; B'; and C' will be discussed in Section 4. In addition, we can also figure out the stable region of parameters α, β, and γ by the condition of Jury criterion as follows:

$$\begin{aligned}
\Omega_{E^*}(\alpha, \beta, \gamma) = \{(\alpha, \beta, \gamma) : \\
1 + A' + B' + C' > 0 \\
1 - A' + B' - C' > 0 \\
1 - C'^2 > 0 \\
\left(1 - C'^2\right) - \left(B' - A'C'\right)^2 > 0\}.
\end{aligned} \tag{23}$$

The stable region is the range where E^* will be stable.

4. The Numerical Modeling and Analysis

4.1. The Parameter Setting. It is worth mentioning that we investigate the service package in practice, in order to determine the values of parameters, for instance, m, n, μ_1, μ_2, and μ. As (A.1) and (A.2) show, the tariffs include different service packages which are posted on their official website, respectively. Based on the analysis for each of the first 8 packages, we figure out and suppose that the utility of 1.45 M data is equal to the utility of 1-minute call in CMCC, and the utility of 1.15 M data is equal to the utility of 1-minute call in CUCC. And then we could obtain all the values of the parameters m and n, which are the amount of the service. Each of p_{ri}^M and p_{ri}^U refers to each of their service package prices divided by their price of unit service, separately. We can calculate the parameters μ_1^i and μ_2^i, which are equal to p_{ri}^M/m and p_{ri}^U/n, respectively.

Furthermore, compared with the group clients who will be excluded in our research, most individual customers who choose an iPhone 7 in bundling markets would be more likely to buy several specific service packages. The parameters m, μ_1, n, and μ_2 are assigned the value of 5.5, 0.47, 6.1, and 0.46, respectively. The calculation of the values of them will be presented in Figure 9. In our research, so as to do in-depth study, we assign values for the rest of the parameters which are as follows:

$$\begin{aligned}
a_1 &= 3.8, \\
a_2 &= 3.8, \\
a_3 &= 4.5, \\
b_1 &= 0.55, \\
b_2 &= 0.55, \\
b_3 &= 0.6, \\
d_1 &= 0.2, \\
d_2 &= 0.2,
\end{aligned}$$

$$\begin{aligned} c_0 &= 0.0006, \\ c_3 &= 0.95, \\ a_{31} &= 1.5, \\ a_{32} &= 1.5, \\ b_{31} &= 1.15, \\ b_{32} &= 1.15, \\ r_{31} &= 0.5, \\ r_{32} &= 0.5, \\ \mu &= 0.84. \end{aligned} \tag{24}$$

Moreover, $\alpha = 0.16$, $\beta = 0.25$, and $\gamma = 0.15$ when the adjustment parameter is not changed in the experiments.

4.2. The Stable Region. We will do some numerical modeling experiments to investigate how the parameters impact the stable region of Nash equilibrium.

Experiment 1. By the parameters assigned into (14)–(16) and (18)–(22), the Nash equilibrium, the Jacobian matrix, and the characteristic polynomial of the matrix are as follows, respectively.

With valuing all (14); (15); and (16) equal to 0, we can get

$$\begin{aligned} 0.2p_2 - 2.5p_1 - 2.5p_3 + 7.68 &= 0 \\ 0.2p_1 - 2.58p_2 - 2.71p_3 + 8.01 &= 0 \\ 9.43 - 0.222p_2 - 4.45p_3 - 0.227p_1 &= 0. \end{aligned} \tag{25}$$

Solving (25), we can get

$$E^* = (1.181, 1.272, 1.996). \tag{26}$$

Equation (25) is the Nash equilibrium.

And, then,

$$J = \begin{pmatrix} 1 - 2.95\alpha & 0.236\alpha & -2.95\alpha \\ 0.254\beta & 1 - 3.74\beta & -3.45\beta \\ -0.453\gamma & -0.444\gamma & 1 - 8.87\gamma \end{pmatrix}. \tag{27}$$

Equation (26) is the Jacobian matrix.

Besides,

$$\begin{aligned} P(\lambda) = {} & \lambda^3 + (2.95\alpha + 3.74\beta + 8.87\gamma - 3)\lambda^2 + (11\alpha\beta \\ & - 7.48\beta - 17.7\gamma - 5.9\alpha + 24.8\alpha\gamma + 31.7\beta\gamma + 3)\lambda \\ & + 2.95\alpha + 3.74\beta + 8.87\gamma - 11\alpha\beta - 24.8\alpha\gamma \\ & - 31.7\beta\gamma + 87.2\alpha\beta\gamma - 1 \end{aligned} \tag{28}$$

where

$$\begin{aligned} A' &= 2.95\alpha + 3.74\beta + 8.87\gamma - 3 \\ B' &= 11\alpha\beta - 7.48\beta - 17.7\gamma - 5.9\alpha + 24.8\alpha\gamma \\ &\quad + 31.7\beta\gamma + 3 \\ C' &= 3.74\beta + 8.87\gamma - 11.0\alpha\beta - 24.8\alpha\gamma - 31.7\beta\gamma \\ &\quad + 87.2\alpha\beta\gamma - 1 \end{aligned} \tag{29}$$

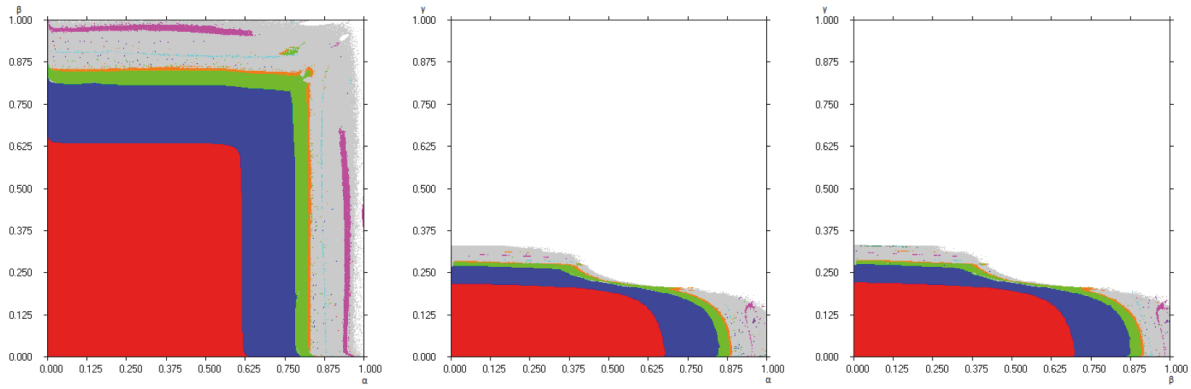
Equation (27) is the characteristic polynomial. Moreover, putting (29) into Jury criterion equation (23), we can obtain the system of inequalities which is the stable region. In addition, we acquire the stable region shown in Figure 2(a). As it shows, we set γ equal to 0.15 and calculate the stable region by Jury criterion with both α and β changing from zero to one. The figure illustrates that the red region is the stable region, and the deep blue, green, orange, light blue, and pink region refer to the 2, 4, 8, 6, and 5 cycles' regions, severally. The rest of the regions, gray region and white region, are the variable overflow area which means the region of chaos. From this figure, we can know that the prices of the firms will have only one Nash equilibrium solution under the fixed adjusted speed in red region. Also, Figure 2(b) shows the stable region and the other regions using the same colors as α and γ are changed from 0 to 1, on the condition that $\beta = 0.25$ is fixed. Similarly, with α assigned the value of 0.16, Figure 2(c) indicates the stable region in the same way as β and γ are changed from 0 to 1.

Experiment 2. In order to investigate the impacts to the stable region on conditions of different degree of competition between the two telecom firms, we did the second experiment and exhibited the result by Figure 3. Obviously, we can find that the stable region decreased when the degree of competition became fierce between the CMCC and CUCC from the picture. Under the circumstance of $\gamma = 0.25$, the green, the blue, and the red regions reflect the stability of $d_1 = d_2 = 0.1$, $d_1 = d_2 = 0.4$, and $d_1 = d_2 = 0.9$, severally.

Experiment 3. By comparing the content plotted in Figure 4, the authors observed that the stable region increased when the degree of the complementarity was enhanced between the telecommunication products to the smart phones. The three regions in Figures 4(a), 4(b), and 4(c) refer to the stabilities under the circumstances of $r_{31} = r_{32} = 0.1$, $r_{31} = r_{32} = 0.5$, and $r_{31} = r_{32} = 0.9$, separately.

In addition, the authors plotted the system's largest Lyapunov exponent (LLE), which was illustrated as the variation of system state, as presented in Figure 5. Note that the system was stable when the LLE was smaller than 0. If the LLE is greater than zero, the system enters into chaos. Without changing others parameters, we obtain the stability scope of α, β, and γ in Figures 5(a), 5(b), and 5(c) separately.

4.3. The Impact of Price Adjustment Parameters. For the sake of investigating the impact of price adjustment parameters, we plot the bifurcation diagrams by changing the adjustment



(a) (b) (c)

Figure 2: (a) The stability with α and β is changed and $\gamma = 0.15$. (b) The stability with α and γ is changed and $\beta = 0.25$. (c) The stability with β and γ is changed and $\alpha = 0.16$.

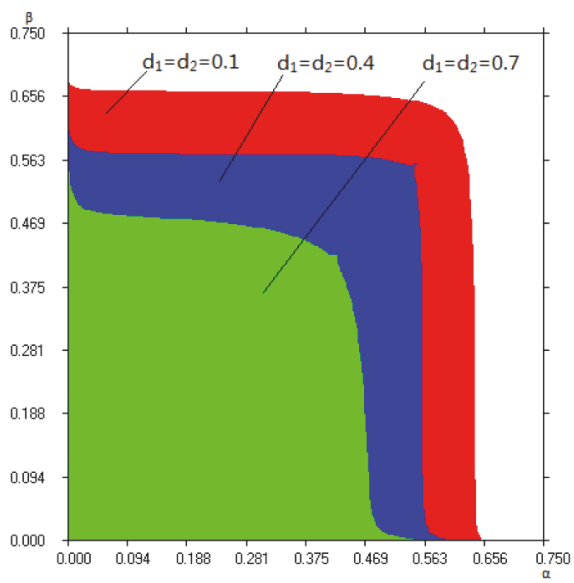


Figure 3: The stability of three values of d_1, d_2, with α, and β is changed, and $\gamma = 0.25$.

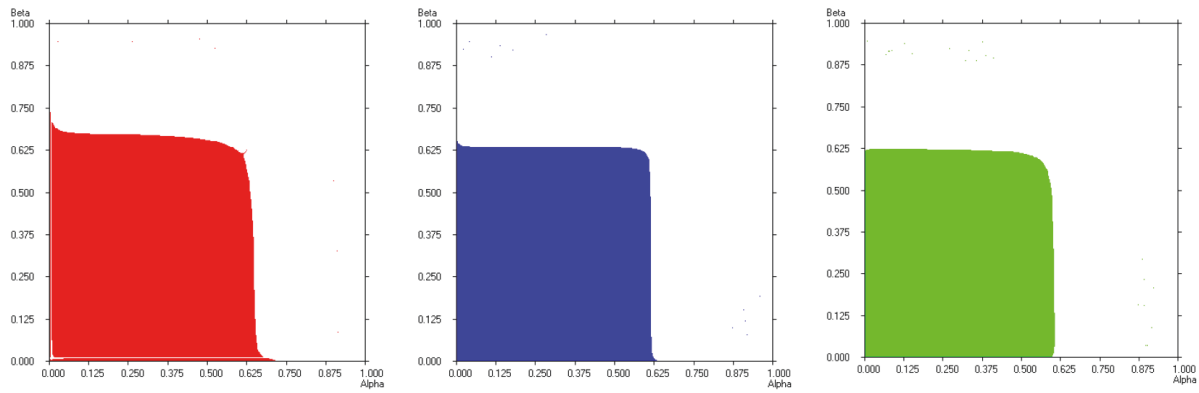
parameters, α, β, and γ in Figures 6(a), 6(b), and 6(c). For the three subfigures, the blue line marked p_1 reflects one unit price of service package offered by CMCC, and the red line refers to one unit price of service package offered by CUCC, and finally the green line represents the price of the iPhone 7. After a period of stability, the system begins bifurcation as the parameter α goes. The system goes into period four and then period eight, and afterwards it enters into chaos. When the chaos appears, it would be hard for the managers to make decision.

4.4. The Impact of Bundling Amount. The impact of parameter m will be investigated in this section according to Figure 7. We can see that both p_1 and p_2 decrease with the value of m increases in Figures 7(a) and 7(b). Conversely, p_3 grows up with the increasing of m in Figure 7(c).

4.5. The Chaos Attractor. Figure 8 shows the chaos attractor. In Figure 8(a), the horizontal axis and vertical axis refer to the prices of the service packages of CMCC and CUCC, and then we set α equal to 0.93. Similarly, in Figure 8(b), the horizontal axis and vertical axis refer to the prices of the service package of CMCC and iPhone 7, and then the authors set α equal to 0.95. The chaos attractor means a little variation in the initial condition could cause a diverse and unpredictable output. Hence, the firms may suffer enormous risks with competition in the market of chaos. The fluctuations in prices, which are the decision variables made by firms every periods, exert the negative effects.

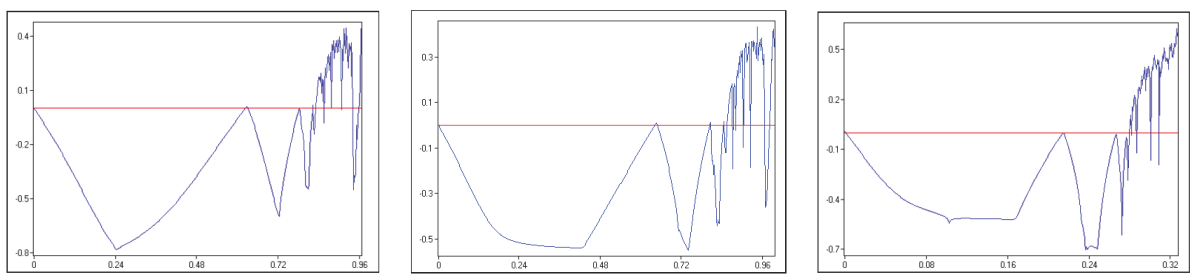
5. Conclusion

In this research, we build a dynamic triopoly model which includes two competitive telecom firms in China and their correlative corporation, which produces the complementary product. Both their free markets and the markets of bundling pricing products are considered in order to make the model more realistic. We do some numerical experiments to research the Nash equilibrium solution, the stable regions, bifurcation of prices, the LLE, and the chaos attractor. The valuable conclusions by our research are as follows: Firstly, we can find that the stable region will decrease when the degree of the competition becomes fierce between the two competitive firms. Secondly, the stable region will increase if the degree of the complementarity enhances between the telecommunication products to the smart phones. Thirdly, moderately increasing the amount of bundling services not only can lead to the decrease of the prices of two telecom firms when the system is stable but also can increase the price of iPhone 7. Hence, for CMCC or CUCC, moderately decreasing their services in packages without the variation of other conditions can increase their service prices. And,



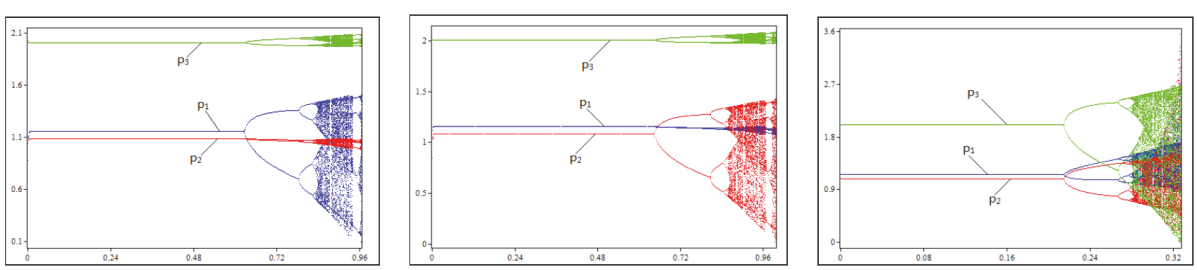
(a) (b) (c)

Figure 4: (a) The stability with α and β is changed, and $\gamma = 0.25$; $r_{31} = r_{32} = 0.1$. (b) The stability with α and β is changed, and $\gamma = 0.25$ and $r_{31} = r_{32} = 0.5$. (c) The stability with α and β is changed, and $\gamma = 0.25$ and $r_{31} = r_{32} = 0.9$.



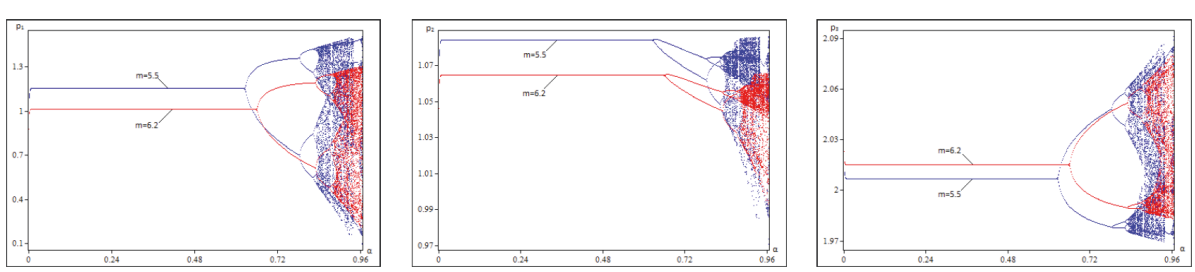
(a) (b) (c)

Figure 5: (a) The LLE as α changes. (b) The LLE as β changes. (c) The LLE as γ changes.



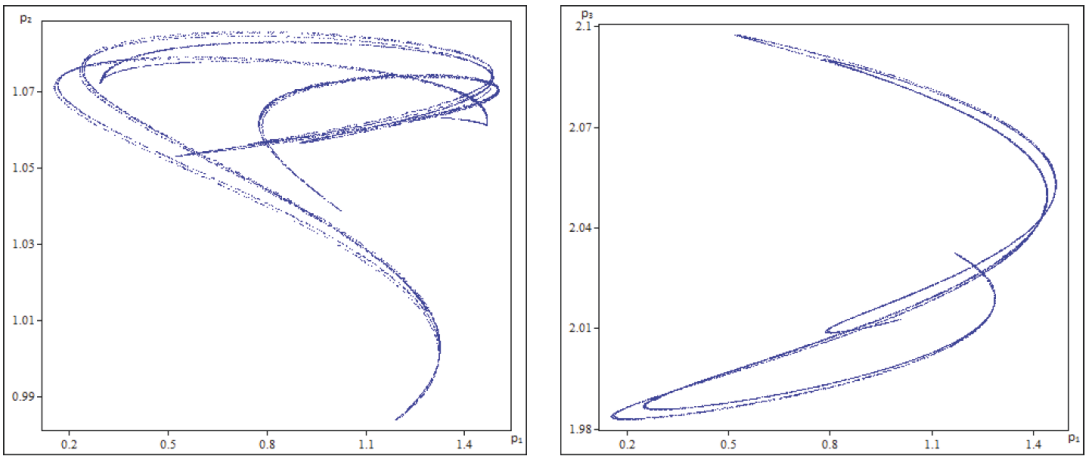
(a) (b) (c)

Figure 6: (a) Bifurcations of p_i as α goes when $\beta = 0.25$ and $\gamma = 0.15$, (b) bifurcations of p_i as β goes when $\alpha = 0.16$ and $\gamma = 0.15$, and (c) bifurcations of p_i as γ goes when $\alpha = 0.16$ and $\beta = 0.25$.



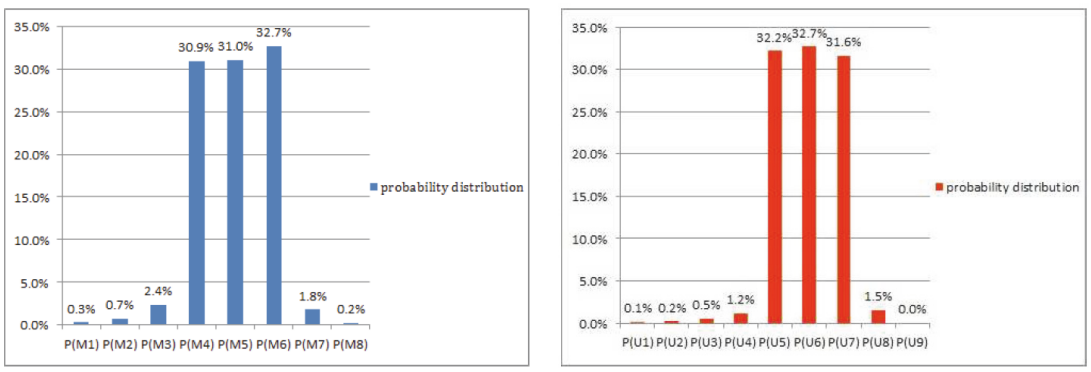
(a) (b) (c)

Figure 7: (a) Bifurcations of p_1 when $m = 5.5$ and 6.2, (b) bifurcations of p_2 when $m = 5.5$ and 6.2, and (c) bifurcations of p_3 when $m = 5.5$ and 6.2.



(a) (b)

Figure 8: (a) The chaos attractor when α is 0.93, $\beta = 0.25$, and γ is 0.15. (b) The chaos attractor when α is 0.95, $\beta = 0.25$, and γ is 0.15.



(a) (b)

Figure 9: (a) The probability distribution of m_i. (b) The probability distribution of n_i.

finally, we give the regions of chaos, which should be avoided, from the perspective of economics.

Our research enriches the existing papers about bundling pricing. This research has managerial significance of not only the three firms discussed, but also the enterprises simulated to the three firms. The result of this model may help the managers adjust their decision-making with their experiences. Certainly, there are more studies that should be done in the future, such as considering the firm, which produces the complements, which has different discounts for the two firms, or the innovation of the products in the background of electronic products, etc.

Appendix

See Tables 1, 2, 3, and 4 and Figure 9.

Based on our research, as shown in Figure 9, we investigate $P(Mi)$, which means the probability of the customers choosing the service package of CM i (i=1,2,. . .,8), and also investigate the $P(Ui)$, which refers to the probability of the customers choosing the service package of CU i (i=1,2,. . .,9). So, the expectations of m, μ_1, n, and μ_2 are as follows:

$$m = \sum_{i=1}^{8} m_i P(Mi) \tag{A.1}$$

$$\mu_1 = \sum_{i=1}^{8} \mu_1^i P(Mi) \tag{A.2}$$

$$n = \sum_{i=1}^{9} n_i P(Ui) \tag{A.3}$$

$$\mu_2 = \sum_{i=1}^{9} \mu_2^i P(Ui) \tag{A.4}$$

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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