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Research Article

Dynamic Behaviors of a Nonautonomous Impulsive Competitive System with the Effect of Toxic Substance

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We firstly propose a nonautonomous impulsive Lotka-Volterra competitive system with the effect of toxic substance. Only one of the two species could produce toxic substance. Sufficient condition which guarantees the extinction of one of the species and the global attractivity of the other species is obtained. We also present an example to verify our main results, which show that species still is possibly driven to extinction when only one of the two species produces toxic substances. The results of this paper supplement the existing results.

1. Introduction

As we know, investigating the effect of toxic substances on ecological communities is significant. More and more researchers are engaged in this aspect and have obtained lots of excellent results. Smith [1] firstly introduced the effects of toxic substances into a two-species Lotka-Volterra competitive system under an assumption that each species produces a toxic substance to the other only when the other is present. Chattopadhyay [2] obtained a set of sufficient conditions which ensure the system admits a unique globally stable positive equilibrium by constructing a suitable Lyapunov function. Recently, a developing number of scholars pay attention to the effects of toxic substances on competitive model and many distinguished results have been given(see [3–9] and the references cited therein). For example, Chen et al. [3] and Xie et al. [4] considered the global stability of the interior equilibrium point of the competition model of plankton allelopathy. The authors [5–9] discussed the extinction for the competitive model with the effect of toxic substances.

However, we notice that all the above [1–10] only pay attention to the continuous or discrete system. In fact, the actual real system is easily disturbed by human exploring activities such as planting and harvesting. Different from the continuous or discrete systems, the theory of impulsive differential equation is much richer and has a wider application when analyzing many real-world phenomena [11, 12]. Recently, various kinds of systems with impulse have attracted a lot of attention [13–19], to name just a few. Chen et al. [20] considered the extinction property of the following impulsive competitive system with toxic substance:

$$\begin{aligned}\dot{x}_1(t) &= x_1(t)\big(b_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t) \\ &\quad - d_1(t)x_1(t)x_2(t)\big) \\ \dot{x}_2(t) &= x_2(t)\big(b_2(t) - a_{21}(t)x_1(t) - a_{22}(t)x_2(t) \\ &\quad - d_2(t)x_1(t)x_2(t)\big) \\ &\qquad t \neq \tau_k,\ k \in N, \\ x_1(\tau_k^+) &= (1 + h_{1k})x_1(\tau_k), \\ x_2(\tau_k^+) &= (1 + h_{2k})x_2(\tau_k),\end{aligned} \tag{1}$$

where $x_i(t)$, $i = 1, 2$, denotes the population density of the i-th phytoplankton species at time t for a common pool of resources. The terms $d_1(t)x_1^2(t)x_2(t)$ and $d_2(t)x_1(t)x_2^2(t)$

denote the effects of toxic substances which means that each species produces a toxic substance to the other. And $b_i(t)$ and $a_{ij}(t), i, j = 1, 2$, are all continuous functions which are bounded above and below by positive constants. Here the jump conditions reflect the possibility of impulsive effects on the species x_1 and x_2. $h_{ik}x_i(\tau_k) < 0$ may present the impulsive harvesting amount of the species at $t = \tau_k$, while $h_{ik}x_i(\tau_k) > 0$, the perturbations, may stand for the impulsive stocking amount of the species at $t = \tau_k$. That is, the impulsive harvesting amount (or stocking amount) is proportional to the current density of the species.

Given a function $g(t)$, let g^L and g^M denote $\inf_{-\infty<t<+\infty} g(t)$ and $\sup_{-\infty<t<+\infty} g(t)$, respectively.

Also, throughout this paper, the following conditions are assumed.

(A_1) $b_i(t)$, $a_{ij}(t)(i, j = 1, 2)$, $d_i(t)$ are continuous T−periodic functions such that $b_i^L > 0$, $a_{ij}^L > 0$, $d_i^L > 0$ and $N = \{1, 2, \ldots\}$.

(A_2) $\{\tau_k\}_{k\in N}$ satisfies $0 = \tau_0 < \tau_1 < \tau_2 < \cdots < \tau_k < \cdots$ and $\lim_{k\longrightarrow\infty} \tau_k = +\infty$.

(A_3) h_{ik} $(i = 1, 2,\ k \in N)$ are constants and there exists an integer $q > 0$ such that $h_{i(k+q)} = h_{ik}, \tau_{k+q} = \tau_k + T$. A natural constraint in this case is $1 + h_{ik} > 0,\ i = 1, 2,\ k \in N$.

(A_4) $\overline{b_1^L} = b_1^L + (1/T)\sum_{k=1}^{q} \ln(1 + h_{1k}) > 0$ and $\overline{b_2^M} = b_2^M + (1/T)\sum_{k=1}^{q} \ln(1 + h_{2k}) > 0$.

Chen et al.[20] proved that if the coefficients of system (1) satisfy

$$\begin{aligned} \overline{b_1^L} a_{22}^L &> \overline{b_2^M} a_{12}^M, \\ \overline{b_1^L} a_{21}^L &\geq \overline{b_2^M} a_{11}^M, \\ \overline{b_1^L} d_2^L &\geq \overline{b_2^M} d_1^M, \end{aligned} \tag{H_1}$$

then the species x_2 of system (1) will be driven to extinction while the species x_1 will be globally attractive with any positive solution of an impulsive logistic equation.

In [21], the author considered the following Lotka-Volterra type of model for two interesting phytoplankton species:

$$\begin{aligned} \dot{x}_1(t) &= x_1(t)\left(r_1 - a_1 x_1(t) - b_1 x_2(t) - c_1 x_1(t) x_2(t)\right), \\ \dot{x}_2(t) &= x_2(t)\left(r_2 - a_2 x_1(t) - b_2 x_2(t)\right). \end{aligned} \tag{2}$$

Since many biological or environmental parameters are subject to fluctuate with time, a plausible mathematical model may take the following form:

$$\begin{aligned} \dot{x}_1(t) &= x_1(t)\left(r_1(t) - a_1(t) x_1(t) - b_1(t) x_2(t) - c_1(t) x_1(t) x_2(t)\right), \\ \dot{x}_2(t) &= x_2(t)\left(r_2(t) - a_2(t) x_1(t) - b_2(t) x_2(t)\right). \end{aligned} \tag{3}$$

To the best of the authors' knowledge, until this day, there exist no results which focus on the dynamic behaviors of the above system with impulse. All the facts stated above motivate us to investigate the extinction problem for a Lotka-Volterra competitive system with impulse and one toxin producing phytoplankton, i.e., the following system:

$$\begin{aligned} \dot{x}_1(t) &= x_1(t)\left(b_1(t) - a_{11}(t) x_1(t) - a_{12}(t) x_2(t) - d_1(t) x_1(t) x_2(t)\right) \\ \dot{x}_2(t) &= x_2(t)\left(b_2(t) - a_{21}(t) x_1(t) - a_{22}(t) x_2(t)\right) \quad t \neq \tau_k,\ k \in N, \\ x_1(\tau_k^+) &= (1 + h_{1k}) x_1(\tau_k), \\ x_2(\tau_k^+) &= (1 + h_{2k}) x_2(\tau_k). \end{aligned} \tag{4}$$

with initial condition

$$\begin{aligned} x_1(0) &> 0, \\ x_2(0) &> 0. \end{aligned} \tag{5}$$

Here $x_1(t)$ denotes the density of nontoxic phytoplankton. In other words, the first species could not produce toxic substance since $d_2(t) = 0$. And the second species $x_2(t)$ is toxic liberating phytoplankton which could produce toxic substance due to $d_1(t) \neq 0$. And all the coefficients in the above system have the same restriction as that of system (1). Comparing (4) with (1), one could find that (4) is the special case of (1), i.e., $d_2(t) = 0$. Hence, one may conjecture that the results of [20] could be applied directly to system (4) in this paper. However, unfortunately, one can notice that the inequality

$$\overline{b_1^L} d_2^L \geq \overline{b_2^M} d_1^M \tag{6}$$

in (H_1) no longer holds since $d_2(t) = 0$. As a result, for (4), it is very necessary to figure out a new sufficient condition to guarantee the extinction of the species x_2 and the global attractivity of the species x_1. The new sufficient condition which will be presented in the next section is different from (H_1). And the obtained results in the next section will be a good complement of those in [20].

By the basic theories of impulsive differential equations in [22, 23], system (4) has a unique solution $x(t) = x(t, t_0) \in PC([0, +\infty), R^2)$ and $PC([0, +\infty), R^2) = \{\phi : [0, +\infty) \longrightarrow R^2, \phi$ is continuous for $t \neq \tau_k$. Also $\phi(\tau_k^-)$ and $\phi(\tau_k^+)$ exist, and $\phi(\tau_k^-) = \phi(\tau_k^+),\ k \in N\}$ for each initial value $x(0) = x_0 \in R^{2+}$.

The organization of this paper is as follows. In Section 2, necessary preliminaries are presented. In Section 3, we give the sufficient conditions ensuring the extinction and global attractivity of the system. In Section 4, numerical simulations are presented to illustrate the feasibility of our main results. In the last section, we give a brief discussion.

2. Lemmas

In this section, we state the following lemmas [20] which will be useful in the proof of our main results.

Lemma 1. *Assume that $m \in PC[R_+, R]$ with points of discontinuity at $t = \tau_k$ and is left continuous at $t = \tau_k$, $k \in N$, and that*

$$D^+ m(t) \le g(t, m(t)), \quad t \ne \tau_k, \qquad m(\tau_k^+) \le \phi_k(m(\tau_k)), \tag{7}$$

where $g \in C[R_+ \times R_+, R]$, $\phi_k \in C[R, R]$ and $\phi_k(u)$ is nondecreasing in u for each $k \in N$. Let $r(t)$ be the maximal solution of the scalar impulsive differential equation:

$$\dot{u}(t) = g(t, u(t)), \quad t \ne \tau_k, \qquad u(\tau_k^+) = \phi_k(u(\tau_k)) \ge 0, \quad \tau_k > t_0, \ k \in N, \qquad u(t_0^+) = u_0, \tag{8}$$

existing on $[t_0, \infty)$, then $m(t_0^+) \le u_0$ implies $m(t) \le r(t)$, $t \ge t_0$.

Lemma 2. *Consider the following impulsive system:*

$$\dot{x}(t) = x(t)(a(t) - b(t)x(t)), \quad t \ne \tau_k, \qquad x(\tau_k^+) = (1 + c_k)x(\tau_k), \quad k \in N, \tag{9}$$

where $a(t)$, $b(t)$ are continuous T-periodic functions and $b(t) > 0$, and there exists an integer $q > 0$ such that $c_{k+q} = c_k$, $t_{k+q} = t_k + T$ and $c_k > -1$ for $k \in N$. Then (9) has a unique positive T-periodic solution $\theta_{[a,b]}$ if and only if

$$\prod_{k=1}^{q}(1 + c_k)\exp\left\{\int_0^T a(t)\,dt\right\} > 1, \tag{10}$$

which is equivalent to the inequality $\int_0^T a(t)dt + \sum_{k=1}^{q}\ln(1 + c_k) > 0$. Moreover, the unique positive T-periodic solution $\theta_{[a,b]}$ is globally asymptotically stable.

Lemma 3. *Let $X(t) = (x_1(t), x_2(t))^T$ be any solution of system (4) such that $x_i(0^+) > 0$, then $x_i(t) > 0$ for all $t \ge 0$.*

Lemma 4. *Let $X(t) = (x_1(t), x_2(t))^T$ be any solution of system (4) such that $x_i(0^+) > 0$, $i = 1, 2$, then we have*

$$\limsup_{t \to +\infty} x_i(t) \le \theta^M_{[b_i, a_{ii}]}, \quad i = 1, 2, \tag{11}$$

where $\theta_{[b_i, a_{ii}]}$ is the unique positive solution of the following Logistic equation with impulse

$$x'(t) = x(t)(b_i(t) - a_{ii}(t)x(t)), \quad t \ne \tau_k, \ k \in N, \qquad x(\tau_k^+) = (1 + h_{ik})x(\tau_k), \tag{12}$$

3. Main Results

In this section, we present our main results for system (4). In the following, we firstly present the extinction of the species x_2.

Theorem 5. *If there exists $\rho \in [0, 1]$ such that the following condition*

$$\overline{b_1^L} a_{22}^L > \overline{b_2^M}\left(a_{12}^M + \rho d_1^M \theta^M_{[b_1, a_{11}]}\right), \qquad \overline{b_1^L} a_{21}^L > \overline{b_2^M}\left(a_{11}^M + (1 - \rho) d_1^M \theta^M_{[b_2, a_{22}]}\right) \tag{H_2}$$

holds, then any positive solution $(x_1(t), x_2(t))^T$ of system (4) satisfies

$$\lim_{t \to +\infty} x_2(t) = 0. \tag{13}$$

Proof. By (H_2), we can choose positive constants γ and η such that

$$\frac{\overline{b_1^L}}{\overline{b_2^M}} > \frac{\eta}{\gamma} > \frac{a_{12}^M + \rho d_1^M \theta^M_{[b_1, a_{11}]}}{a_{22}^L}, \qquad \frac{\overline{b_1^L}}{\overline{b_2^M}} > \frac{\eta}{\gamma} > \frac{a_{11}^M + (1 - \rho) d_1^M \theta^M_{[b_2, a_{22}]}}{a_{21}^L}. \tag{14}$$

So from Lemma 4, there exist positive constants ε, T_0 such that $x_i(t) \le \theta_{[b_i, a_{ii}]} + \varepsilon$ and

$$\eta a_{22}(t) - \gamma\left(a_{12}(t) + \rho d_1(t)\left(\theta_{[b_1, a_{11}]} + \varepsilon\right)\right) > 0, \qquad \eta a_{21}(t) - \gamma\left(a_{11}(t) + (1 - \rho) d_1(t)\left(\theta_{[b_2, a_{22}]} + \varepsilon\right)\right) > 0 \tag{15}$$

when $t \ge T_0$. Also there exists $\delta > 0$ such that

$$\eta\left(b_2(t) + \frac{1}{T}\sum_{k=1}^{q}\ln(1 + h_{2k})\right) - \gamma\left(b_1(t) + \frac{1}{T}\sum_{k=1}^{q}\ln(1 + h_{1k})\right) < -\frac{\delta}{T} < 0. \tag{16}$$

From system (4) and (15), we have

$$\begin{aligned}\frac{d}{dt}\left[\ln\frac{(x_2(t))^\eta}{(x_1(t))^\gamma}\right] &= (\eta b_2(t) - \gamma b_1(t)) - (\eta a_{22}(t) \\ &\quad - \gamma a_{12}(t))x_2(t) - (\eta a_{21}(t) - \gamma a_{11}(t))x_1(t) \\ &\quad + \gamma d_1(t)x_1(t)x_2(t) \le (\eta b_2(t) - \gamma b_1(t)) \\ &\quad - \left(\eta a_{22}(t) - \gamma a_{12}(t) - \gamma\rho d_1(t)\left(\theta_{[b_1, a_{11}]} + \varepsilon\right)\right) \\ &\quad \cdot x_2(t) - \left(\eta a_{21}(t) - \gamma a_{11}(t)\right. \\ &\quad \left. - \gamma(1 - \rho)d_1(t)\left(\theta_{[b_2, a_{22}]} + \varepsilon\right)\right)x_1(t) \le \eta b_2(t) \\ &\quad - \gamma b_1(t), \quad t \ne \tau_k, \ t > T_0\end{aligned} \tag{17}$$

and

$$\ln\left(\frac{(x_2(\tau_k^+))^\eta}{(x_1(\tau_k^+))^\gamma}\right)=\ln\left(\frac{(1+h_{2k})^\eta}{(1+h_{1k})^\gamma}\right)+\ln\left(\frac{(x_2(\tau_k))^\eta}{(x_1(\tau_k))^\gamma}\right). \quad (18)$$

Integrating both sides of (17) over internals $[T_0, T_0+\tau_1)$, $[T_0+\tau_1, T_0+\tau_2)$, $[T_0+\tau_2, T_0+\tau_3)$, $[T_0+\tau_{\sigma-1}, T_0+\tau_\sigma)$ and $[T_0+\tau_\sigma, t)$, respectively, and adding the σ inequalities, for $t \in [T_0+\tau_\sigma, T_0+\tau_{\sigma+1})$ and $T_0+\tau_\sigma \in [m_1 T, (m_1+1)T)$, $m_1 \in N$, we obtain

$$\begin{aligned}
&\ln\left(\frac{(x_2(t))^\eta}{(x_1(t))^\gamma}\right)-\ln\left(\frac{(x_2(T_0))^\eta}{(x_1(T_0))^\gamma}\right)\\
&\quad<\int_{T_0}^t(\eta b_2(t)-\gamma b_1(t))\,dt+\ln\frac{\prod_{T_0<\tau_k<t}(1+h_{2k})^\eta}{\prod_{T_0<\tau_k<t}(1+h_{1k})^\gamma}\\
&\quad=m_1\int_0^T(\eta b_2(t)-\gamma b_1(t))\,dt\\
&\qquad+m_1\eta\sum_{k=1}^q\ln(1+h_{2k})-m_1\gamma\sum_{k=1}^q\ln(1+h_{1k})\\
&\qquad+\int_{m_1T}^t(\eta b_2(t)-\gamma b_1(t))\,dt\\
&\qquad+\ln\frac{\prod_{m_1T<\tau_k<t}(1+h_{2k})^\eta}{\prod_{m_1T<\tau_k<t}(1+h_{1k})^\gamma}\\
&\quad\le m_1\eta\left(Tb_2^M+\sum_{k=1}^q\ln(1+h_{2k})\right)\\
&\qquad-m_1\gamma\left(Tb_1^L+\sum_{k=1}^q\ln(1+h_{1k})\right)+B\\
&\quad\le -m_1\delta+B,
\end{aligned} \quad (19)$$

where $B=\max_{0\le s\le T}(\int_0^s(\eta b_2(t)-\gamma b_1(t))dt+\ln(\prod_{0\le\tau_k<s}(1+h_{2k})^\eta)/\prod_{0\le\tau_k<s}(1+h_{1k})^\gamma))$. This shows that

$$(x_2(t))^\eta\le(x_1(t))^\gamma\exp(-m_1\delta+B)\frac{(x_2(T_0))^\eta}{(x_1(T_0))^\gamma}. \quad (20)$$

According to Lemma 4, we notice that $x_1(t)$ is ultimately upper bounded; hence, we obtain $\lim_{t\longrightarrow+\infty}x_2(t)=0$. This completes the proof of Theorem 5. □

In the above, we have provided the sufficient condition to ensure that species x_2 is extinct. Similar to the proof of Theorems 3.2 and 3.3 in [20], we can also obtain the following result about the global attractivity of the species x_1.

Theorem 6. *Assume that (H_2) holds, then the species x_1 is globally attractive; i.e., for any positive solution $(x_1(t), x_2(t))^T$ of system (4) and any positive solution $x(t)$ of the impulsive logistic equation,*

$$\begin{aligned}
&\dot{x}(t)=x(t)(b_1(t)-a_{11}(t)x(t)), \quad t\ne\tau_k,\ k\in N,\\
&x(\tau_k^+)=(1+h_{1k})x(\tau_k),
\end{aligned} \quad (21)$$

one has

$$\lim_{t\longrightarrow+\infty}(x_1(t)-x(t))=0. \quad (22)$$

Remark 7. For system (4), condition (H_1) is not satisfied due to $d_2(t)=0$. As a result, we cannot easily deduce the extinction of the species x_2 and the globally attractivity of the species x_1 from Theorem 3.1-3.3 in [20]. However, in Theorems 5–6 of this paper, we have obtained the sufficient condition (H_2) under which the corresponding dynamic behaviors for (4) are presented. Also we can find that conditions (H_1) and (H_2) cannot be derived from each other. In other words, the above two conditions are complementary. Thus, the main results in this paper are indeed a good complement of those in [20].

Remark 8. According to the main results in [20], for system (4), one can easily induce the extinction of the species x_1 and the global attractivity of the species x_2 under the condition $\overline{b_2^L}a_{12}^L > \overline{b_1^M}a_{22}^M$, $\overline{b_2^L}a_{11}^L \ge \overline{b_1^M}a_{21}^M$. This paper pays attention to figure out a new sufficient condition to guarantee the extinction of the species x_2 and the global attractivity of the species x_1. The obtained results show that the species x_2 can still be extinct even though the species x_1 do not produce toxic substance.

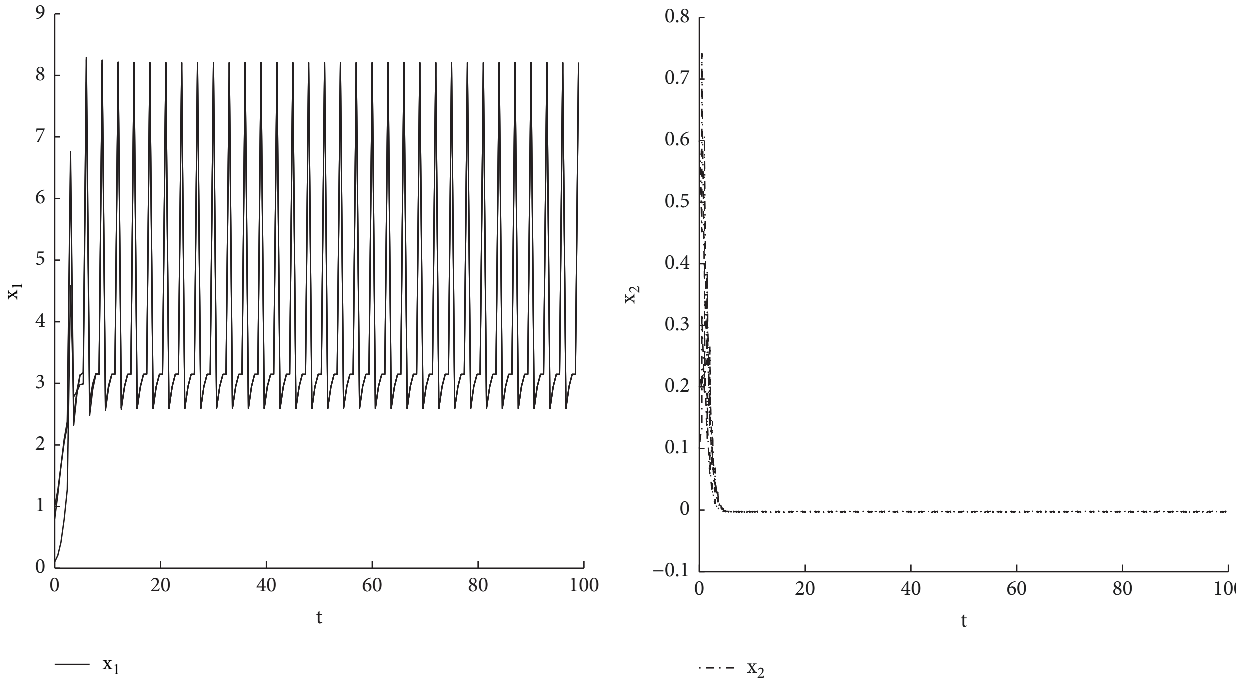
4. Example

In this section, we give numerical simulation to illustrate the main theoretical results above.

Example 1. In system (4), let $b_1(t)=1.4+0.2\sin(4\pi t)$, $a_{11}(t)=0.5$, $a_{12}(t)=0.2$, $d_1(t)=0.4$, $h_{1k}=\exp(1)-1$, $b_2(t)=0.7+0.4\sin(4\pi t)$, $a_{21}(t)=0.5$, $a_{22}(t)=1.3$, $h_{2k}=\exp(1/2)-1$. By computing, we derive $\theta^M_{[b_1,a_{11}]}\le 12$, $\theta^M_{[b_2,a_{22}]}\le 1.8$ and, choosing $\rho=0.4$, we have $\overline{b_1^L}a_{22}^L=4.42$, $\overline{b_2^M}(a_{12}^M+\rho d_1^M\theta^M_{[b_1,a_{11}]})\le 3.604$, $\overline{b_2^M}(a_{11}^M+(1-\rho)d_1^M\theta^M_{[b_2,a_{22}]})\le 1.5844$. So the condition (H_2) holds. From Theorems 5–6, it follows that species x_2 is extinct while species x_1 is globally stable (see Figure 1).

5. Conclusion

In this paper, a nonautonomous impulsive Lotka-Volterra competitive system with the effect of toxic substance has been studied. We have obtained the sufficient condition which guarantees one of the species' extinction and the other species' global attractivity. The results in this paper supplement those in [20]. We also have presented an example to verify our main results, which show that the species can still be driven to extinction when only one of the two species produces toxic substance.



(a) Species x_1 is permanent

(b) Species x_2 is extinct

Figure 1: Dynamic behaviors of system (4) with the initial conditions $(0.1, 0.1)^T$, $(1, 0.6)^T$, and $(0.8, 0.2)^T$.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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