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Research Article

Distributed Consensus of Semi-Markovian Jumping Multiagent Systems with Mode-Dependent Topologies

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This paper investigates the distributed consensus problem of multiagent systems with semi-Markovian jumping dynamics in the mean-square sense. Moreover, the mode-dependent communication topologies and sampled-data consensus protocol over the networks are considered. By semi-Markov jump theory, the consensus problem is first transformed into a mean-square stability problem. Then, sufficient conditions are established with the designed mode-dependent consensus protocol. Finally, a numerical example is provided for verifying the effectiveness of our theoretical results.

1. Introduction

Owing to the rapid development of computer science and network technology, the multiagent systems (MASs) have become a hot research topic with significant applications including mobile robots [1, 2], sensor networks [3, 4], unmanned aerial vehicles (UAVs) [5, 6], and autonomous underwater vehicles (AUVs) [7, 8]. As one key concerned issue, the consensus of MASs has gained great research attention and fruitful results have been reported [9–11]. Generally speaking, certain agreement can be converged with collecting behaviors of the MASs when consensus is reached. Specifically, two significant issues worthy of consideration are the local information exchanges and the agent dynamics in the MASs, respectively. Since it is impractical to apply the continuous communication in the real-world network, various discrete-time communication strategies have been proposed, such as sample-data strategies [12], impulsive strategies [13], and intermittent strategies [14]. In comparison with continuous communication, these discrete-time schemes can obtain effective benefits in saving network resources and energy consumption. However, it is worth mentioning that the hybrid structure of discrete-time communication with continuous-time agent dynamics in MASs would increase the difficulty and complexity in analysis and synthesis. Encouragingly, some remarkable results have been presented in the literature [15–17].

On the other hand, it is noticed that dynamical systems may display mode switching features by abrupt phenomena, which gives rise to researches on switched systems [18, 19]. In particular, some mode switching (or jumping) can be modeled by Markovian jumping systems with finite transition rates. Therefore, it is meaningful and significant to investigate the Markovian jumping MASs. Some recent attempts have been made to tackle these problems [20, 21]. Furthermore, it should be pointed out that the transition rates could be time-varying and the sojourn time cannot be exponentially distributed. As a result, significant efforts have been put into the so-called semi-Markovian jumping systems with some analysis and synthesis methods [22–24]. However, so far, the consensus problem of semi-Markovian jumping MASs still remains open and challenging, which is the first motivation of our study. Another motivation lies in the fact that, for the MASs with switching characteristics, the communication topology and the communication strategy would switch accordingly, such that the consideration of mode-dependent topologies for switched MASs is reasonable. Unfortunately, there have been few results despite its practical importance, let alone those with semi-Markovian jumping MASs.

In response to the above discussions, this paper solves the distributed consensus problem of semi-Markovian jumping MASs with mode-dependent topologies and information exchanges by employing the key idea from semi-Markovian jumping systems. The main contributions of our paper can

be summarized as follows. Firstly, a novel model of semi-Markovian jumping MASs with mode-dependent topologies is proposed for better describing the agent dynamics. Secondly, the distributed mode-dependent consensus protocol with sampled-data information exchanges is designed for guaranteeing the mean-square consensus, which is more applicable for the network environment. Finally, based on model transformation, sufficient consensus criteria are established by applying the mode-dependent Lyapunov-Krasovskii method in the form of linear matrix inequalities (LMIs).

The rest of our paper is outlined as follows. In Section 2, necessary preliminaries on graph theory are introduced and the consensus problem of semi-Markovian jumping MASs is formulated. Section 3 gives the main theoretical results with the developed networked consensus protocol. Section 4 is devoted to numerical simulations for demonstrating the validity of our obtained results. Finally, the concluding remarks are drawn in Section 5.

Notation. The notations are standard throughout this paper. $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote the n dimensional Euclidean space and the space of $m\times n$ real matrices, respectively. $P > 0$ means that P is positive definite. $A \otimes B$ denotes the Kronecker product. $\mathbb{E}\{\cdot\}$ denotes mathematical expectation; $\Pr\{\alpha\}$ represents the probability of an event α. $*$ represents the ellipsis symmetry terms in symmetric block matrices. $\mathrm{diag}\{\cdots\}$ stands for the block-diagonal matrix. Finally, all matrices are compatible for algebraic operations.

2. Preliminaries and Problem Formulation

Fix a probability space $(\Omega, \mathscr{F}, \mathscr{P})$ and let $\{\sigma(t), t \geq 0\}$ denote a continuous-time discrete-state semi-Markov process on $(\Omega, \mathscr{F}, \mathscr{P})$ taking values in a finite set $\mathscr{S} = \{1, \ldots, s\}$. The transition probability matrices $\Theta := (\pi_{kl}(h))$, $h > 0$, and $\forall k, l \in \mathscr{S}$ are defined by

$$\Pr(\sigma(t+h) = l \mid \sigma(t) = k) = \begin{cases} \pi_{kl}(h)\, h + o(h), & k \neq l, \\ 1 + \pi_{kk}(h)\, h + o(h), & k = l, \end{cases} \tag{1}$$

where

$$\lim \left(\frac{o(h)}{h} \right) = 0. \quad \pi_{kl}(h) \geq 0, \ k \neq l \tag{2}$$

is the transition rate from mode k at time t to mode l at time $t + h$, satisfying

$$\pi_{kk}(h) = -\sum_{l=1, k\neq l}^{s} \pi_{kl}(h), \quad \forall k \in \mathscr{S} \tag{3}$$

2.1. Algebraic Graph Basics. The directed graph $\mathscr{G}(\sigma(t)) = \{\mathscr{V}(\sigma(t)), \mathscr{E}(\sigma(t)), \mathscr{A}(\sigma(t))\}$ is adopted for describing the information exchange topology of the MASs with a fixed mode $\sigma(t)$, where $\mathscr{V}(\sigma(t)) = \{v_1(\sigma(t)), \cdots, v_N(\sigma(t))\}$ is the sets of nodes, $\mathscr{E}(\sigma(t))$ is the sets of edges, and $\mathscr{A}(\sigma(t)) = [a_{ij}(\sigma(t))] \in \mathbb{R}^{N\times N}$ is the weighted adjacency matrix with $a_{ij}(\sigma(t)) > 0$ if $(v_i(\sigma(t)), v_j(\sigma(t))) \in \mathscr{E}$ and $a_{ij}(\sigma(t)) = 0$ otherwise. In addition, define the Laplacian $L(\sigma(t)) = [l_{ij}(\sigma(t))] \in \mathbb{R}^{N\times N}$ as $l_{ii}(\sigma(t)) = \sum_{j=1, j\neq i}^{N} a_{ij}(\sigma(t))$ and $l_{ij}(\sigma(t)) = -a_{ij}(\sigma(t)), i \neq j$. If $\mathscr{G}(\sigma(t))$ has a directed spanning tree, then $L(\sigma(t))$ has a simple zero eigenvalue and all the other eigenvalues are real.

2.2. Semi-Markovian Jumping MASs. Consider the semi-Markovian jumping MASs consisting of N agents, which are described as follows:

$$\dot{x}_i(t) = A(\sigma(t))\, x_i(t) + B(\sigma(t))\, u_i(t), \quad i = 1, 2, \ldots, N, \tag{4}$$

where $x_i(t) \in \mathbb{R}^n$ and $u_i(t) \in \mathbb{R}^p$ denote the system state and the control input of the ith agent, respectively. $A(\sigma(t))$ and $B(\sigma(t))$ are constant matrices for a fixed mode $\sigma(t)$.

Remark 1. It is worth mentioning that the communication topologies could dynamically switch according to the different modes of multiagent systems, which leads to the semi-Markovian jumping mode-dependent topologies. Without loss of generality, it is assumed that all the switching modes can be detected to the agent dynamics and the communication topologies.

The consensus is said to be achieved in the mean-square sense if and only if it holds that

$$\lim_{t\longrightarrow\infty} \mathbb{E}\left\{\left\|x_i - x_j\right\|\right\} = 0, \quad i, j = 1, 2, \ldots, N. \tag{5}$$

The following lemma is given for the subsequent analysis.

Lemma 2 (see [24]). *For matrix $\mathscr{X} \succ 0$, parameters $\overline{\tau} > 0$, $\tau(t)$ satisfying $0 \leq \tau(t) \leq \overline{\tau}$, and $\dot{x}(t) : [-\overline{\tau}, 0] \longrightarrow \mathbb{R}^n$ such that the concerned integrations are well defined; then*

$$-\overline{\tau} \int_{t-\overline{\tau}}^{t} \dot{x}^T(s)\, \mathscr{X}\, \dot{x}(s)\, ds \leq \zeta^T(t)\, \Delta\zeta(t), \tag{6}$$

where

$$\zeta(t) = \left[x^T(t), x^T(t-\tau(t)), x^T(t-\overline{\tau})\right]^T,$$
$$\Delta = \begin{bmatrix} -\mathscr{X} & \mathscr{X} & 0 \\ * & -2\mathscr{X} & \mathscr{X} \\ * & * & -\mathscr{X} \end{bmatrix}. \tag{7}$$

3. Main Results

3.1. Distributed Consensus Protocol Design. In this paper, the sampled-data communication strategy with different modes is adopted. Suppose that the MASs communicate with their local neighbors over the communication network according to a global discrete-time sequence: $0 = t_0 < t_1 < \cdots < t_k < \cdots$ with $t_{k+1} - t_k \leq \overline{\tau}, \overline{\tau} > 0$, such that the transmitted information of the ith agent at instance t_k is $x_i(t_k)$.

The following mode-dependent consensus protocol is designed:

$$u_i(t) = -K(\sigma(t)) \sum_{i=1}^{N} a(\sigma(t))_{ij} \left(x_i(t_k) - x_j(t_k)\right), \quad t_k \le t < t_{k+1}, \tag{8}$$

where $K(\sigma(t))$ is the mode-dependent control gains to be determined.

Remark 3. It is assumed that the semi-Markovian jumping modes can be detected at the sampling instances, such that the system modes can be obtained by the agents and the communication topologies based on sampled-data communication strategy.

Consequently, the closed-loop dynamics of the MASs can be obtained as follows:

$$\begin{aligned} \dot{x}_i(t) &= A(\sigma(t)) x_i(t) - B(\sigma(t)) K(\sigma(t)) \\ &\quad \cdot \sum_{i=1}^{N} a_{ij}(\sigma(t)) \times \left(x_i(t_k) - x_j(t_k)\right), \\ &\qquad t_k \le t < t_{k+1}, \end{aligned} \tag{9}$$

which can be further rewritten as

$$\begin{aligned} \dot{x}(t) &= (I \otimes A(\sigma(t))) x(t) \\ &\quad - (L(\sigma(t)) \otimes B(\sigma(t)) K(\sigma(t))) x(t_k), \\ &\qquad t_k \le t < t_{k+1}, \end{aligned} \tag{10}$$

where $x(t) = [x_1^T(t), x_2^T(t), \ldots, x_N^T(t)]^T$ and $x(s) = x(0), s \in [-\overline{\tau}, 0]$.

It can be verified that when $\mathcal{G}(\sigma(t))$ has a directed spanning tree, it follows that

$$W(\sigma(t))^{-1} L(\sigma(t)) W(\sigma(t)) = \begin{bmatrix} \overline{L}(\sigma(t)) & 0 \\ 0 & 0 \end{bmatrix}, \tag{11}$$

where the last column of $W(\sigma(t))$ is $[1, 1, \ldots, 1]^T$.

By defining $y(t) = (W(\sigma(t))^{-1} \otimes I)x(t) = [\widetilde{x}^T(t), \overline{x}^T(t)]^T$, it can be obtained that

$$\begin{aligned} \dot{y}(t) &= \left(I_N \otimes A(\sigma(t))\right) y(t) \\ &\quad - W(\sigma(t))^{-1} L(\sigma(t)) W(\sigma(t)) \\ &\quad \otimes B(\sigma(t)) K(\sigma(t)) y(t_k) \end{aligned} \tag{12}$$

which yields

$$\begin{aligned} \dot{\widetilde{x}}(t) &= \left(I_{N-1} \otimes A(\sigma(t))\right) \widetilde{x}(t) \\ &\quad - \left(\overline{L}(\sigma(t)) \otimes B(\sigma(t)) K(\sigma(t))\right) \widetilde{x}(t_k), \end{aligned} \tag{13}$$

$$\dot{\overline{x}}(t) = A(\sigma(t)) \overline{x}(t). \tag{14}$$

Thus, it can be obtained that the consensus can be achieved when $\widetilde{x}(t)$ is asymptotically stable in the mean square.

To this end, denote $\sigma(t)$ by l indices for simplicity and use the input-delay approach. Then, (13) can be rewritten by

$$\dot{\widetilde{x}}(t) = \left(I_{N-1} \otimes A_l\right) \widetilde{x}(t) - \left(\overline{L}_l \otimes B_l K_l\right) \widetilde{x}(t - \tau(t)), \tag{15}$$

where $\tau(t) = t - t_k$ denotes the virtual delay satisfying $0 \le \tau(t) < \overline{\tau}$.

3.2. Sufficient Consensus Criteria. Based on the above consensus protocol, the following theorems are derived for the consensus analysis and synthesis of semi-Markovian jumping MASs.

Theorem 4. *For given scalar $\overline{\tau}$, the distributed consensus of semi-Markovian jumping MASs (4) can be achieved with the given mode-dependent consensus protocol (8), if there exist mode-dependent real matrix $P_l > 0$ and matrices $Q > 0$ and $R > 0$, such that $\Pi_{l,\kappa} < 0$ for each $l \in \mathcal{S}$ and $\kappa = 1, 2, \ldots, \mathcal{K}$, where*

$$\begin{aligned} \Pi_{l,\kappa} &:= \begin{bmatrix} \Pi_{1l,\kappa} & \Pi_{2l,\kappa} \\ * & \Pi_{3l,\kappa} \end{bmatrix}, \\ \Pi_{1l,\kappa} &:= \begin{bmatrix} \Pi_{11l,\kappa} & -\left(\overline{L}_l \otimes P_l B_l K_l\right) + \left(I_{N-1} \otimes R\right) \\ * & -2\left(I_{N-1} \otimes R\right) \end{bmatrix}, \\ \Pi_{2l,\kappa} &:= \begin{bmatrix} 0 & \overline{\tau}\left(I_{N-1} \otimes A_l^T R\right) \\ \left(I_{N-1} \otimes R\right) & \overline{\tau}\left(\overline{L}_l^T \otimes K_l^T B_l^T R\right) \end{bmatrix}, \\ \Pi_{3l,\kappa} &:= \begin{bmatrix} -\left(I_{N-1} \otimes Q\right) - \left(I_{N-1} \otimes R\right) & 0 \\ * & -\left(I_{N-1} \otimes R\right) \end{bmatrix}, \\ \Pi_{11l,\kappa} &:= \left(I_{N-1} \otimes P_l A_l\right) + \left(I_{N-1} \otimes A_l^T P_l\right) + \left(I_{N-1} \otimes Q\right) \\ &\quad - \left(I_{N-1} \otimes R\right) + \sum_{k=1}^{N} \pi_{lk,\kappa} \left(I_{N-1} \otimes P_l\right). \end{aligned} \tag{16}$$

Proof. For each mode l, choose the following Lyapunov-Krasovskii functional:

$$V(l,t) = \sum_{p=1}^{3} V_p(l,t), \tag{17}$$

where

$$\begin{aligned} V_1(l,t) &:= \widetilde{x}^T(t) \left(I_{N-1} \otimes P_l\right) \widetilde{x}(t) \\ V_2(l,t) &:= \int_{t-\overline{\tau}}^{t} \widetilde{x}^T(\varphi) \left(I_{N-1} \otimes Q\right) \widetilde{x}(\varphi) \, d\varphi, \\ V_3(l,t) &:= \overline{\tau} \int_{-\overline{\tau}}^{0} \int_{t+\eta}^{t} \dot{\widetilde{x}}^T(\varphi) \left(I_{N-1} \otimes R\right) \dot{\widetilde{x}}(\varphi) \, d\varphi d\eta. \end{aligned} \tag{18}$$

The weak infinitesimal operator $\mathscr{L}$ of $V(l,t)$ is defined by

$$\mathscr{L}V(l,t) \triangleq \lim_{\Delta\longrightarrow 0}\frac{1}{\Delta}\{\mathbb{E}\{V(\sigma(t+\Delta),t+\Delta) \mid \sigma(t)=l\} - V(l,t)\}. \tag{19}$$

Then, it can be derived that

$$\begin{aligned}\mathscr{L}V_1(l,t) &\coloneqq \dot{\tilde{x}}^T(t)(I_{N-1}\otimes P_l)\tilde{x}(t) \\ &\quad + \tilde{x}^T(t)(I_{N-1}\otimes P_l)\dot{\tilde{x}}(t) \\ &\quad + \sum_{k=1}^{N}\pi_{lk}(h)\tilde{x}^T(t)(I_{N-1}\otimes P_l)\tilde{x}(t) \\ &= 2\tilde{x}^T(t)(I_{N-1}\otimes P_l)(I_N\otimes A_l)\tilde{x}(t) \\ &\quad - 2\tilde{x}^T(t)(\overline{L}_l\otimes P_lB_lK_l)\tilde{x}(t-\tau(t)) \\ &\quad + \sum_{k=1}^{N}\pi_{lk}(h)\tilde{x}^T(t)(I_{N-1}\otimes P_l)\tilde{x}(t), \\ \mathscr{L}V_2(l,t) &\coloneqq \tilde{x}^T(t)(I_{N-1}\otimes Q)\tilde{x}(t) \\ &\quad - \tilde{x}^T(t-\overline{\tau})(I_{N-1}\otimes Q)\tilde{x}(t-\overline{\tau}), \\ \mathscr{L}V_3(l,t) &\coloneqq \overline{\tau}^2\dot{\tilde{x}}^T(t)(I_{N-1}\otimes R)\dot{\tilde{x}}(t) \\ &\quad - \overline{\tau}\int_{t-\overline{\tau}}^{t}\tilde{x}^T(\varphi)(I_{N-1}\otimes R)\tilde{x}(\varphi)\,d\varphi.\end{aligned} \tag{20}$$

By Lemma 2, it holds that

$$\begin{aligned}&-\overline{\tau}\int_{t-\overline{\tau}}^{t}\dot{\tilde{x}}^T(\varphi)(I_{N-1}\otimes R)\dot{\tilde{x}}(\varphi)\,d\varphi \\ &\le \begin{bmatrix}\tilde{x}(t)\\ \tilde{x}(t-\tau(t))\\ \tilde{x}(t-\overline{\tau})\end{bmatrix}^T \\ &\times\begin{bmatrix}-(I_{N-1}\otimes R) & (I_{N-1}\otimes R) & 0\\ * & -2(I_{N-1}\otimes R) & (I_{N-1}\otimes R)\\ * & * & -(I_{N-1}\otimes R)\end{bmatrix} \\ &\times\begin{bmatrix}\tilde{x}(t)\\ \tilde{x}(t-\tau(t))\\ \tilde{x}(t-\overline{\tau})\end{bmatrix}.\end{aligned} \tag{21}$$

Thus, one has

$$\mathscr{L}V(l,t) \le \eta^T(t)\widetilde{\Pi}_l\eta(t) + \overline{\tau}^2\dot{\tilde{x}}^T(t)(I_{N-1}\otimes R)\dot{\tilde{x}}(t), \tag{22}$$

where

$$\begin{aligned}\widetilde{\Pi}_l &\coloneqq \begin{bmatrix}\widetilde{\Pi}_{1l} & \widetilde{\Pi}_{2l}\\ * & \widetilde{\Pi}_{3l}\end{bmatrix}, \\ \widetilde{\Pi}_{1l} &\coloneqq \begin{bmatrix}\widetilde{\Pi}_{11l} & -(\overline{L}_l\otimes P_lB_lK_l)+(I_{N-1}\otimes R)\\ * & -2(I_{N-1}\otimes R)\end{bmatrix}, \\ \widetilde{\Pi}_{2l} &\coloneqq \begin{bmatrix}0\\ (I_{N-1}\otimes R)\end{bmatrix}, \\ \widetilde{\Pi}_{3l} &\coloneqq -(I_{N-1}\otimes Q)-(I_{N-1}\otimes R), \\ \widetilde{\Pi}_{11l} &\coloneqq 2(I_{N-1}\otimes P_lA_l)+(I_{N-1}\otimes Q)-(I_{N-1}\otimes R) \\ &\quad + \sum_{k=1}^{N}\pi_{lk}(h)(I_{N-1}\otimes P_l)\end{aligned} \tag{23}$$

and $\eta(t) = [\tilde{x}^T(t), \tilde{x}^T(t-\tau(t)), \tilde{x}^T(t-\overline{\tau})]^T$.

It follows by Schur complement that $\mathscr{L}V(l,t) < 0$ holds if $\overline{\Pi}_l < 0$, such that (15) is asymptotically stable in the mean square, where

$$\begin{aligned}\overline{\Pi}_l &\coloneqq \begin{bmatrix}\overline{\Pi}_{1l} & \overline{\Pi}_{2l}\\ * & \overline{\Pi}_{3l}\end{bmatrix}, \\ \overline{\Pi}_{1l} &\coloneqq \begin{bmatrix}\overline{\Pi}_{11l} & -(\overline{L}_l\otimes P_lB_lK_l)+(I_{N-1}\otimes R)\\ * & -2(I_{N-1}\otimes R)\end{bmatrix}, \\ \overline{\Pi}_{2l} &\coloneqq \begin{bmatrix}0 & \overline{\tau}(I_{N-1}\otimes A_l)^T\\ (I_{N-1}\otimes R) & \overline{\tau}(\overline{L}_l\otimes B_lK_l)^T\end{bmatrix}, \\ \overline{\Pi}_{3l} &\coloneqq \begin{bmatrix}-(I_{N-1}\otimes Q)-(I_{N-1}\otimes R) & 0\\ * & -(I_{N-1}\otimes R)^{-1}\end{bmatrix}, \\ \overline{\Pi}_{11l} &\coloneqq (I_{N-1}\otimes P_lA_l)+(I_{N-1}\otimes A_l^TP_l)+(I_{N-1}\otimes Q) \\ &\quad -(I_{N-1}\otimes R)+\sum_{k=1}^{N}\pi_{lk}(h)(I_{N-1}\otimes P_l).\end{aligned} \tag{24}$$

Noticing the fact that $\pi_{lk}(h) = \sum_{\kappa=1}^{\mathscr{K}}\varkappa_\kappa\pi_{lk,\kappa}$, $\sum_{\kappa=1}^{\mathscr{K}}\varkappa_\kappa = 1$, and $\varkappa_\kappa \ge 0$ for time-varying dwell time, it can be obtained that $\Pi_{l,\kappa} < 0$, which completes the proof. □

Theorem 5. *For given scalar $\overline{\tau}$, the distributed consensus of semi-Markovian jumping MASs (4) can be achieved, if there exist mode-dependent real matrix $\widetilde{P}_l > 0$ and matrices $\widetilde{Q} > 0$ and $\widetilde{R} > 0$, such that $\Xi_{l,\kappa} < 0$ for each $l \in \mathscr{S}$ and $\kappa = 1,2,\ldots,\mathscr{K}$, where*

$$\begin{aligned}\Xi_{l,\kappa} &\coloneqq \begin{bmatrix}\Xi_{1l,\kappa} & \Xi_{2l,\kappa}\\ * & \Xi_{3l,\kappa}\end{bmatrix}, \\ \Xi_{1l,\kappa} &\coloneqq \begin{bmatrix}\Xi_{11l,\kappa} & \Xi_{12l,\kappa}\\ * & \Xi_{13l,\kappa}\end{bmatrix},\end{aligned}$$

$$\Xi_{11l,\kappa} := \begin{bmatrix} \Xi_{111l,\kappa} & -\left(\overline{L}_l \otimes B_l G_l\right) + \left(I_{N-1} \otimes \widetilde{R}\right) \\ * & -2\left(I_{N-1} \otimes \widetilde{R}\right) \end{bmatrix},$$

$$\Xi_{12l,\kappa} := \begin{bmatrix} 0 & \overline{\tau}\left(I_{N-1} \otimes A_l^T \widetilde{P}_l\right) \\ \left(I_{N-1} \otimes \widetilde{R}\right) & \overline{\tau}\left(\overline{L}_l^T \otimes G_l^T B_l^T\right) \end{bmatrix},$$

$$\Xi_{13l,\kappa} := \operatorname{diag}\left\{-\left(I_{N-1} \otimes \widetilde{Q}\right) - \left(I_{N-1} \otimes \widetilde{R}\right), \left(I_{N-1} \otimes \widetilde{R}\right) - 2\left(I_{N-1} \otimes \widetilde{P}_l\right)\right\},$$

$$\Xi_{111l,\kappa} := \left(I_{N-1} \otimes A_l \widetilde{P}_l\right) + \left(I_{N-1} \otimes \widetilde{P}_l A_l^T\right) + \left(I_{N-1} \otimes \widetilde{Q}\right) - \left(I_{N-1} \otimes \widetilde{R}\right) + \pi_{ll,\kappa}\left(I_{N-1} \otimes \widetilde{P}_l\right),$$

$$\Xi_{2l,\kappa} := \left[\Xi_{21l,\kappa}, \Xi_{22l,\kappa}, \Xi_{23l,\kappa}\right],$$

$$\Xi_{21l,\kappa} := \begin{bmatrix} \sqrt{\pi_{l1,\kappa}}\left(I_{N-1} \otimes \widetilde{P}_l\right) & \cdots \\ 0 & \cdots \\ 0 & \cdots \\ 0 & \cdots \end{bmatrix},$$

$$\Xi_{22l,\kappa} := \begin{bmatrix} \sqrt{\pi_{l(l-1),\kappa}}\left(I_{N-1} \otimes \widetilde{P}_l\right) & \sqrt{\pi_{l(l+1),\kappa}}\left(I_{N-1} \otimes \widetilde{P}_l\right) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\Xi_{23l,\kappa} := \begin{bmatrix} \cdots & \sqrt{\pi_{lN,\kappa}}\left(I_{N-1} \otimes \widetilde{P}_l\right) \\ \cdots & 0 \\ \cdots & 0 \\ \cdots & 0 \end{bmatrix},$$

$$\Xi_{3l,\kappa} := \operatorname{diag}\left\{-\left(I_{N-1} \otimes \widetilde{P}_1\right), \ldots, -\left(I_{N-1} \otimes \widetilde{P}_{l-1}\right), -\left(I_{N-1} \otimes \widetilde{P}_{l+1}\right), \ldots, -\left(I_{N-1} \otimes \widetilde{P}_N\right)\right\}, \tag{25}$$

and the mode-dependent consensus protocol gain K_l can be obtained by $K_l = G_l \widetilde{P}_l^{-1}$.

Proof. Based on Theorem 4, it can be verified that if $\overline{\Xi}_{l,\kappa} < 0$ then $\Pi_{l,\kappa} < 0$, where

$$\overline{\Xi}_{l,\kappa} := \begin{bmatrix} \overline{\Xi}_{1l,\kappa} & \overline{\Xi}_{2l,\kappa} \\ * & \overline{\Xi}_{3l,\kappa} \end{bmatrix},$$

$$\overline{\Xi}_{1l,\kappa} := \begin{bmatrix} \overline{\Xi}_{11l,\kappa} & \overline{\Xi}_{12l,\kappa} \\ * & \overline{\Xi}_{13l,\kappa} \end{bmatrix},$$

$$\overline{\Xi}_{11l,\kappa} := \begin{bmatrix} \overline{\Xi}_{111l,\kappa} & -\left(\overline{L}_l \otimes P_l B_l K_l\right) + \left(I_{N-1} \otimes R\right) \\ * & -2\left(I_{N-1} \otimes R\right) \end{bmatrix},$$

$$\overline{\Xi}_{12l,\kappa} := \begin{bmatrix} 0 & \overline{\tau}\left(I_{N-1} \otimes A_l\right)^T \\ \left(I_{N-1} \otimes R\right) & \overline{\tau}\left(\overline{L}_l^T \otimes B_l K_l\right)^T \end{bmatrix},$$

$$\overline{\Xi}_{13l,\kappa} := \operatorname{diag}\left\{-\left(I_{N-1} \otimes Q\right) - \left(I_{N-1} \otimes R\right), -\left(I_{N-1} \otimes R\right)^{-1}\right\},$$

$$\overline{\Xi}_{111l,\kappa} := \left(I_{N-1} \otimes P_l A_l\right) + \left(I_{N-1} \otimes A_l^T P_l\right) + \left(I_{N-1} \otimes Q\right) - \left(I_{N-1} \otimes R\right) + \pi_{ll,\kappa}\left(I_{N-1} \otimes P_l\right),$$

$$\overline{\Xi}_{2l,\kappa} := \left[\overline{\Xi}_{21l,\kappa}, \overline{\Xi}_{22l,\kappa}\right],$$

$$\overline{\Xi}_{21l,\kappa} := \begin{bmatrix} \sqrt{\pi_{l1,\kappa}} & \cdots & \sqrt{\pi_{l(l-1),\kappa}} \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix},$$

$$\overline{\Xi}_{22l,\kappa} := \begin{bmatrix} \sqrt{\pi_{l(l+1),\kappa}} & \cdots & \sqrt{\pi_{lN,\kappa}} \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix},$$

$$\overline{\Xi}_{3l,\kappa} := \begin{bmatrix} \overline{\Xi}_{31l,\kappa} & \overline{\Xi}_{32l,\kappa} \\ * & \overline{\Xi}_{33l,\kappa} \end{bmatrix},$$

$$\overline{\Xi}_{31l,\kappa} := \operatorname{diag}\left\{-\left(I_{N-1} \otimes P_1\right)^{-1}, \cdots, -\left(I_{N-1} \otimes P_{l-1}\right)^{-1}\right\},$$

$$\overline{\Xi}_{32l,\kappa} := \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix},$$

$$\overline{\Xi}_{33l,\kappa} := \operatorname{diag}\left\{-\left(I_{N-1} \otimes P_{l+1}\right)^{-1}, \cdots, -\left(I_{N-1} \otimes P_N\right)^{-1}\right\}. \tag{26}$$

Letting $\widetilde{P}_l = P_l^{-1}$, $\widetilde{Q} = P_l^{-1} Q P_l^{-1}$, $\widetilde{R} = P_l^{-1} R P_l^{-1}$, and $K_l \widetilde{P}_l = G_l$ and performing congruent transformation to $\overline{\Xi}_{l,\kappa} < 0$, the results can follow directly from the proof of Theorem 4. □

Remark 6. It can be found that the dimensions of LMIs are related to the l and k, which means that when the numbers of system modes and agents are increased, the dimension of LMIs will be increased accordingly. Although this would increase the computational complexity to some extent, the established LMIs are strict LMIs which can be easily solved by MATLAB.

4. Illustrative Example

In this section, an illustrative example with simulation results is provided for showing our proposed consensus design.

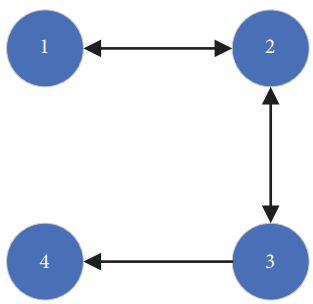

Figure 1: The communication topology with mode 1.

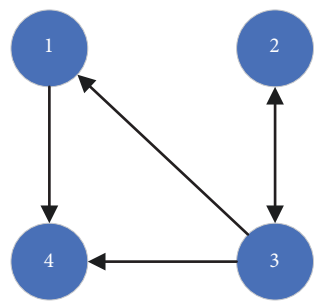

Figure 2: The communication topology with mode 2.

Consider the semi-Markovian jumping MASs with four agents and two jumping modes.

The transition rates are assumed to be $\pi_{11}(h) \in [-2.2, -1.6]$ and $\pi_{22}(h) \in [-2, -1.5]$, which implies that $\pi_{11,1} = -1.6$, $\pi_{11,2} = -2.2$, $\pi_{22,1} = -1.5$, and $\pi_{22,2} = -2$ with $\mathscr{K} = 2$. The sampled-data communication period of the control inputs is given by 0.2s, such that one has $\overline{\tau} = 0.2$.

The agent dynamics are described by the following parameters:

$$A_1 = \begin{bmatrix} -1.5 & 0.8 \\ 0 & -2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.5 \\ 0.3 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1 & 0.4 \\ 0.2 & -2.3 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.4 \\ 0.2 \end{bmatrix}, \tag{27}$$

The communication topologies with directed spanning trees are depicted in Figures 1 and 2, respectively. As a result, the Laplacian matrices of the communication topologies can be obtained as follows:

$$L_1 = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & -1 & 2 \end{bmatrix}. \tag{28}$$

With the above parameters, the mode-dependent consensus control input gains of the MASs can be calculated by solving the LMIs in Theorem 5 in the MATLAB, which are given by

$$K_1 = \begin{bmatrix} 0.4381 & 0.2344 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 0.6535 & 0.4209 \end{bmatrix}. \tag{29}$$

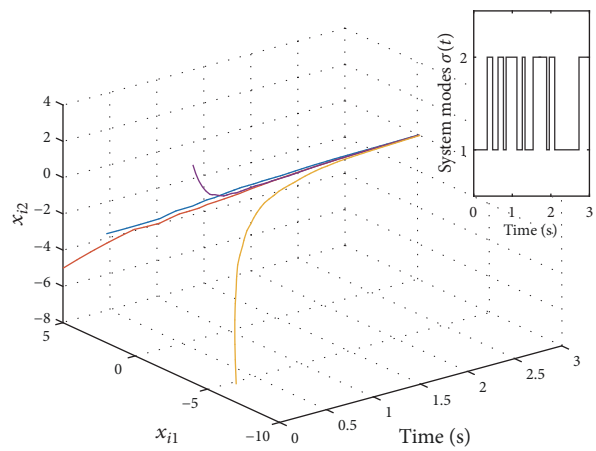

Figure 3: The state responses of the semi-Markovian jumping MASs.

As a result, the consensus state responses of the semi-Markovian jumping MASs by our designed consensus control input gains can be seen in Figure 3. It can be seen that all the states of semi-Markovian jumping MASs would reach the agreement, which confirms the effectiveness of our theoretical results.

5. Conclusions

This paper focuses on the distributed consensus of semi-Markovian jumping MASs with mode-dependent communication topologies. The mode-dependent consensus protocol with sampled-data information is proposed. On the basis of model transformation and Lyapunov-Krasovskii method, sufficient consensus criteria are derived in the mean-square sense and the mode-dependent consensus gains are designed with LMIs. In the end, a simulation example is given for validating the effectiveness of our theoretical results. Our future work would encompass investigating the consensus of semi-Markovian jumping MASs under constrained network environments.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The author declares that he has no conflicts of interest.

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