Research Article

Stochastic P-Bifurcation of a Bistable Viscoelastic Beam with Fractional Constitutive Relation under Gaussian White Noise

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In this paper, we study the stochastic P-bifurcation problem for axially moving of a bistable viscoelastic beam with fractional derivatives of high order nonlinear terms under Gaussian white noise excitation. First, using the principle for minimum mean square error, we show that the fractional derivative term is equivalent to a linear combination of the damping force and restoring force, so that the original system can be simplified to an equivalent system. Second, we obtain the stationary Probability Density Function (PDF) of the system’s amplitude by stochastic averaging, and using singularity theory, we find the critical parametric condition for stochastic P-bifurcation of amplitude of the system. Finally, we analyze the types of the stationary PDF curves of the system qualitatively by choosing parameters corresponding to each region within the transition set curve. We verify the theoretical analysis and calculation of the transition set by showing the consistency of the numerical results obtained by Monte Carlo simulation with the analytical results. The method used in this paper directly guides the design of the fractional order viscoelastic material model to adjust the response of the system.

1. Introduction

Fractional calculus is a generalization of integer-order calculus, it extends the order of calculus operation from the traditional integer order to the case of noninteger order, and it has a history of more than 300 years as so far. Due to the limitation of the definition of integer-order derivative, it cannot express the memory property of viscoelastic substances. The definition of fractional derivative contains convolution, which can well express the memory effect and show the cumulative effect over time. Compared with the traditional integer-order calculus, fractional calculus has more advantages and is a suitable mathematical tool for describing the memory characteristics [1–11] and in recent years, it has become the powerful mathematical tool in many disciplines, especially in the study of viscoelastic materials.

The fractional derivative can accurately describe the constitutive relation of viscoelastic materials with fewer parameters, so the studies of fractional differential equations on the typical mechanical properties and the influences of fractional order parameters on the system are very necessary and have important significance. In recent years, many scholars have done a lot of work and achieved fruitful results in this field: Li and Tang studied the nonlinear parametric vibration of an axially moving string made by rubber-like materials, a new nonlinear fractional mathematical model governing transverse motion of the string is derived based on Newton’s second law, the Euler beam theory, and the Lagrangian strain, and the principal parametric resonance is analytically investigated via applying the direct multiscale method [12]. Liu et al. introduced a transfer entropy and surrogate data algorithm to identify the nonlinearity level of the system by using a numerical solution of nonlinear response of beams, the Galerkin method was applied to discretize the dimensionless differential governing equation of the forced vibration, and then the fourth-order Runge-Kutta method was used to obtain the time history response of the lateral displacement [13]. Liu et al. investigated the stochastic stability of coupled viscoelastic system with nonviscously damping driven by white noise through moment Lyapunov exponents and Lyapunov exponents, obtained the coupled Itô stochastic differential equations of the norm of
Due to the complexity of fractional derivative, the analysis method of it becomes more difficult, the study on the vibration characteristics of the parameters can only be qualitatively analyzed, and the critical conditions of the parametric influences cannot be found, which affect the analysis and design of such systems, as well as the stochastic P-bifurcation of bistability for the viscoelastic beam with fractional derivatives of high order nonlinear terms under random noise excitation has not been reported. In view of the above situation, the nonlinear vibration of viscoelastic beam with fractional constitutive relation under Gaussian white noise excitation is taken as an example, the transition set curve of the fractional order system as well as the critical parametric condition for stochastic P-bifurcation of the system is obtained by the singularity theory, and then the types of stationary PDF curves of the system in each region in the parametric plane divided by the transition set are analyzed. By the method of Monte Carlo simulation, the numerical results are compared with the analytical results obtained in this paper, it can be seen that the numerical solutions are in good agreements with the analytical solutions, and thus the correctness of the theoretical analysis in this paper is verified.

2. Equation of Axially Moving Viscoelastic Beam

There are many definitions of fractional derivatives, and the Riemann-Liouville derivative and Caputo derivative are commonly used. The initial conditions corresponding to the Riemann-Liouville derivative have no physical meanings, however, the initial conditions of the systems described by the Caputo derivative have clear physical meanings and their forms are the same as the initial conditions for the differential equations of integer order. So in this paper, the Caputo-type fractional derivative is adopted as follows:

\[ C^\alpha_0 D^\alpha_t \{ x(t) \} = \frac{1}{\Gamma (m - p)} \int^t_0 \frac{x^{(m)}(u)}{(t - u)^{1 + p - m}} du, \]  

(1)

where \( m - 1 < p \leq m, m \in N, t \in [a, b], \Gamma(m) \) is the Euler Gamma function, and \( x^{(m)}(t) \) is the m order derivative of \( x(t) \).

For a given physical system, due to the fact that the initial moment of the oscillator is \( t = 0 \), the following form of the Caputo derivative is often used:

\[ C^\alpha_0 D^\alpha_t \{ x(t) \} = \frac{1}{\Gamma (m - p)} \int^t_0 \frac{x^{(m)}(u)}{(t - u)^{1 + p - m}} du, \]  

(2)

In this paper, the transverse vibration \( y(x, t) \) of a viscoelastic simply supported beam under lateral excitation \( F(x, t) \) as shown in Figure 1 is considered; applying the d’Alembert principle, the governing equation can be written as \[42]\n
\[ \frac{\partial^2 Q}{\partial x^2} = F(x, t) - \rho A \frac{\partial^2 y}{\partial t^2}, \] \[ \frac{\partial M}{\partial x} = Q - N \frac{\partial y}{\partial x}, \]  

(3)
where \( \rho \) is the mass density of the beam, \( A \) is the area of cross-section, \( M \) is the bending moment, \( Q \) is the lateral force, and \( N \) is the horizontal force. From (3), we have

\[
\frac{\partial^2 M}{\partial x^2} + N \frac{\partial^2 y}{\partial x^2} + \rho A \frac{\partial^2 y}{\partial t^2} - F(x, t) = 0. \tag{4}
\]

Assuming the material of the beam obeys a fractional derivative viscoelastic constitutive relation:

\[
\sigma(x, z, t) = E_0 \varepsilon(x, z, t) + E_1 C_0 D^p \varepsilon(x, z, t) \]

\[
= E_0 \left(1 + \eta C_0 D^p\right) \varepsilon(x, z, t) \equiv S\varepsilon(x, z, t), \tag{5}
\]

where \( p \) is the order of fractional derivative \( C_0 D^p[\varepsilon(x, z, t)] \) as is defined in (2), \( \eta = E_1/E_0 \) is the material modulus ratio, and \( \varepsilon(x, z, t) \) is axial strain component.

When the deformation of the beam is small, the axial strain \( \varepsilon \) and lateral displacement \( y(x, t) \) satisfy the relationship as follows:

\[
\varepsilon(x, z, t) = -z \frac{\partial^2 y(x, t)}{\partial x^2}. \tag{6}
\]

Substituting (6) into (5) yields

\[
\sigma(x, z, t) = S \left[-z \frac{\partial^2 y(x, t)}{\partial x^2}\right]. \tag{7}
\]

The relationship between bending moment \( M(x, t) \) and axial stress \( \sigma(x, z, t) \) of the beam can be expressed as follows:

\[
M(x, t) = \int_{-h/2}^{h/2} z \sigma(x, z, t) \, dz, \tag{8}
\]

where \( h \) is the thickness of the beam.

From (7) and (8), the expression of the bending moment \( M \) can be obtained as follows:

\[
M(x, t) = I \left[ E_0 \left(\frac{\partial^2 y}{\partial x^2}\right) + E_1 \cdot C_0 D^p \left(\frac{\partial^2 y}{\partial x^2}\right) \right]
\]

\[
= IS \left(\frac{\partial^2 y}{\partial x^2}\right), \tag{9}
\]

where \( I = h^3/12 \).

The expression of the horizontal tension is

\[
N = \frac{E_0 A}{2L} \int_0^L \left[ \left(\frac{\partial y}{\partial x}\right)^2 - \left(\frac{\partial y}{\partial x}\right)^4 \right] \, dx
\]

\[
+ \frac{E_1 A}{2L} \int_0^L C_0 D^p \left[ \left(\frac{\partial y}{\partial x}\right)^2 - \left(\frac{\partial y}{\partial x}\right)^4 \right] \, dx \tag{10}
\]

Substituting (9) and (10) into system (4), system (4) can be rewritten as

\[
IS \left(\frac{\partial^2 y}{\partial x^2}\right) + S \frac{\partial^2 y}{\partial t^2} - F(x, t) = 0. \tag{11}
\]

The boundary conditions are

\[
\frac{\partial^2 y}{\partial x^2} = 0, \quad \text{when } x = 0, \ x = L. \tag{12}
\]

According to the boundary conditions (12), the solution of system (11) can be expressed as the Fourier series:

\[
y(x, t) = \sum_{n=1}^{\infty} u_n(t) \sin \left(\frac{n\pi x}{L}\right). \tag{13}
\]
Assume that the initial transverse vibration of the system is $y_0(x) = 0$ and the transverse load of the system satisfies the following form:

$$F (x, t) = \rho A \cdot \sin \left( \frac{\pi x}{L} \right) \xi (t),$$  \hspace{1cm} (14)

where $\xi(t)$ is the Gaussian white noise, which satisfies $E[\xi(t)] = 0, E[\xi(t)\xi(t-\tau)] = 2D\delta(\tau)$, $D$ represents the noise intensity, and $\delta(\tau)$ is the Dirac function.

By the discrete format based on Galerkin method, system (11) can be reduced to the fractional differential equation as follows:

$$\ddot{u} + k_1 u + \eta k_1 C^0 D^p u - k_3 u^3 - \eta k_3 u^0 C^0 D^p(u^2) + k_5 u^5$$

$$+ \eta k_5 u^0 C^0 D^p (u^4) = \xi (t),$$  \hspace{1cm} (15)

where $C^0 D^p u$ is the $p$ $(0 < p < 1)$ order Caputo derivative of $u(t)$ as defined in (2), and

$$k_1 = \left( \frac{\pi}{L} \right)^4 \frac{1E_0}{\rho A},$$

$$k_3 = \left( \frac{\pi}{L} \right)^4 \frac{E_0}{4\rho},$$

$$k_5 = \left( \frac{\pi}{L} \right)^6 \frac{3E_0}{16\rho}.$$  \hspace{1cm} (16)

For convenience, system (15) can be represented as follows:

$$\ddot{u} + \omega^2 u + \eta \omega^2 C^0 D^p u - k_3 u^3 - \eta k_3 u^0 C^0 D^p (u^2)$$

$$+ k_5 u^5 + \eta k_5 u^0 C^0 D^p (u^4) = \xi (t),$$  \hspace{1cm} (17)

where $\omega$ is the natural frequency of system (17).

Based on (22), we can obtain

$$\ddot{u} (t) = -a(t) \sin \phi(t).$$  \hspace{1cm} (23)

Substituting (19) into (20) yields

$$E \left[ c (a) \dot{u}^2 + w^2 (a) u \dot{u} - \dot{u}^2 C^0 D^p u \right]$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_0^T \left( c (a) \dot{u}^2 + w^2 (a) u \dot{u} - \dot{u}^2 C^0 D^p u \right) dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_0^T \left( c (a) \dot{u}^2 + w^2 (a) u \dot{u} - \dot{u}^2 C^0 D^p u \right) dt$$

$$= 0.$$  \hspace{1cm} (21)

Assume that the solution of system (18) has the following form:

$$u(t) = a(t) \cos \phi(t)$$

$$\phi(t) = \omega t + \theta,$$  \hspace{1cm} (22)

where $\omega$ is the natural frequency of system (17).

Substituting (22) and (23) into (21) yields

$$\lim_{T \to \infty} \frac{1}{T} \int_0^T \left( c (a) \dot{u}^2 + w^2 (a) u \dot{u} - \dot{u}^2 C^0 D^p u \right) dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_0^T \left( c (a) \dot{u}^2 + w^2 (a) u \dot{u} - \dot{u}^2 C^0 D^p u \right) dt$$

$$= 0.$$  \hspace{1cm} (24)

$$\int_0^T \dot{x} (t-\tau) d\tau = \frac{c (a) a^2 \omega^2}{2}$$

$$- \frac{1}{\Gamma (1-p)} \lim_{T \to \infty} \frac{1}{T} \int_0^T \left( a \omega \sin \phi \frac{\sin \phi \cos (\omega \tau) - \cos \phi \sin (\omega \tau)}{\tau^a} \right) dt = 0$$

$$\lim_{T \to \infty} \frac{1}{T} \int_0^T \left( c (a) \dot{uu} + w^2 (a) uu - u \dot{u} C^0 D^p u \right) dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_0^T \left( c (a) \dot{uu} + w^2 (a) uu - u \dot{u} C^0 D^p u \right) dt$$

$$= 0.$$  \hspace{1cm} (25)

$$\int_0^T \dot{\phi} (t-\tau) d\tau = \frac{c (a) a^2 \omega^2}{2}$$

$$- \frac{1}{\Gamma (1-p)} \lim_{T \to \infty} \frac{1}{T} \int_0^T \left( a \omega \sin \phi \frac{\sin \phi \cos (\omega \tau) - \cos \phi \sin (\omega \tau)}{\tau^a} \right) dt = 0$$
Next, we consider the stochastic P-bifurcation of system (28) which comprises the fractional derivatives of high order nonlinear terms and analyze the influence of parametric variation on the system response.

3. The Stationary PDF of Amplitude

For the system (28), the material modulus ratio is given as $\eta = 0.5$, coefficients of nonlinear terms are given as $k_3 = 7.8$, $k_5 = 5.9$, respectively, and nature frequency is given as $\omega = 1$. For the convenience to discuss the parametric influence, the bifurcation diagram of amplitude of the limit cycle along with variation of the fractional order $p$ is shown in Figure 2 when $D = 0$.

As can be seen from Figure 2, the solution corresponding to the solid line is almost completely coincided with the numerical solution, which proves the correctness and accuracy of the approximate analytical result of the deterministic system; at the same time, it shows that the solution corresponding to the solid line is stable and the solution corresponding to the dotted line is unstable. And it also can be seen that there is 1 attractor in the system when $p$ changes in the interval $[0, 0.6817]$; equilibrium, as shown in Figure 3(a); there are 2 attractors when $p$ changes in the interval $[0.6818, 1]$: equilibrium and limit cycle, as shown in Figure 3(b).

In order to obtain the stationary PDF of the amplitude of system (28), the following transformation is introduced:

$$u(t) = a(t) \cos \Phi(t)$$

$$\dot{u}(t) = -a(t) \omega_0 \sin \Phi(t)$$

$$\Phi(t) = \omega_0 t + \theta(t)$$

(29)

where $\omega_0$ is the natural frequency of the equivalent system (28), $a(t)$ and $\theta(t)$ represent the amplitude process and the phase process of the system response, respectively, and they are all random processes.

Substituting (29) into (28), we can obtain

$$\frac{da}{dt} = F_{11}(a, \theta) + G_{11}(a, \theta) \xi(t)$$
\[
\frac{d\theta}{dt} = F_{21}(a, \theta) + G_{21}(a, \theta) \xi(t),
\]

in which
\[
\begin{align*}
F_{11} &= -\frac{\Delta \sin \Phi}{w_0}, \\
F_{21} &= -\frac{\Delta \cos \Phi}{aw_0}, \\
G_{11} &= -\frac{\sin \Phi}{w_0}, \\
G_{21} &= -\frac{\cos \Phi}{aw_0},
\end{align*}
\]

and
\[
\Delta = \eta_{w^p+1}a w_0 \sin \left(\frac{p\pi}{2}\right) \sin \Phi - k_3 a^3 \cos^3 \Phi - \eta k_3
\]
\[
\cdot \frac{a^3}{2} (2w_0)^p \cos \Phi \cos \left(2\Phi + \frac{p\pi}{2}\right) - k_5 a^5 \cos^5 \Phi
\]
\[
- \eta k_3 a^5 \cos \Phi \left[ \frac{1}{8} (4w_0)^p \cos \left(4\Phi + \frac{p\pi}{2}\right) + \frac{p\pi}{2} \right].
\]

Equation (30) can be regarded as a Stratonovich stochastic differential equation, by adding the corresponding Wong-Zakai correction term: it can be transformed into the following Itô stochastic differential equation:
\[
\begin{align*}
d\theta &= \left[ F_{21}(a, \theta) + F_{22}(a, \theta) \right] dt \\
&\quad + \sqrt{2D} G_{21}(a, \theta) dB(t),
\end{align*}
\]

where \(B(t)\) is a standard Wiener process and

\[
\begin{align*}
F_{12} &= D \frac{\partial G_{11}}{\partial a} G_{11} + D \frac{\partial G_{21}}{\partial \theta} G_{21}, \\
F_{22} &= D \frac{\partial G_{21}}{\partial a} G_{11} + D \frac{\partial G_{21}}{\partial \theta} G_{21}.
\end{align*}
\]

By the stochastic averaging method, averaging (33) regarding \(\Phi\), the following averaged Itô equation can be obtained as follows:
\[
\begin{align*}
da &= m_1(a) dt + \sigma_1(a) dB(t) \\
d\theta &= m_2(a) dt + \sigma_2(a) dB(t),
\end{align*}
\]

where
\[
\begin{align*}
m_1(a) &= -\frac{\eta k_2 2^p w_0^{p-1} a^5 \sin (p\pi/2)}{8} \\
&\quad - \frac{\eta k_3 2^p w_0^{p-1} a^5 \sin (p\pi/2)}{8} \\
&\quad - \frac{\eta w^p a^8 \cos (p\pi/2) + 5}{16w_0} a^4, \\
m_2(a) &= \frac{k_5}{w_0}, \\
\sigma_1^2(a) &= \frac{D}{w_0^2} \\
\sigma_2^2(a) &= \frac{k_5}{w_0} \left( 2\eta \cdot 2^p w_0^p \cos (p\pi/2) + 5 \right) a^4.
\end{align*}
\]
Equations (35) and (36) show that the averaged Itô equation for \( a(t) \) is independent of \( \theta(t) \) and the random process \( a(t) \) is a one-dimensional diffusion process. The corresponding FPK equation associated with \( a(t) \) can be written as

\[
\frac{\partial p}{\partial t} = -\frac{\partial}{\partial a} \left[ m_1(a) p \right] + \frac{1}{2} \frac{\partial^2}{\partial a^2} \left[ \sigma_1^2(a) p \right].
\]  

(37)

The boundary conditions are

\[
p = c, \ c \in (-\infty, +\infty) \quad \text{when} \quad a = 0
\]

\[\sigma_1^2(a) = \frac{D}{a^2 w_0^2}.\]

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\]

\[\sigma_1^2(a) = \frac{D}{a^2 w_0^2}.\]

(36)
for convenience, each region in Figure 4 is marked with a number.

Taking a given point \((p, D)\) in each of the two subregions of Figure 4, the characteristics of stationary PDF curves are analyzed, and the corresponding results are shown in Figure 5.

As can be seen from Figure 4, the parametric region where the PDF curve appears bimodal is surrounded by an approximately triangular region. When the parameter \((p, D)\) is taken in the region 1, the PDF curve has only a distinct peak, as shown in Figure 5(a); in region 2, the PDF curve has a distinct peak near the origin, but the probability is obviously not zero far away from the origin; there are both the equilibrium and limit cycle in the system simultaneously, as shown in Figure 5(b).

The analysis results above show that the stationary PDF curves of the system amplitude in any two adjacent regions in Figure 4 are qualitatively different. No matter the values of the unfolding parameters cross any line in the figure, the system will occur stochastic P-bifurcation behaviors, so the transition set curve is just the critical parametric condition for the stochastic P-bifurcation of the system, and the analytic results in Figure 5 are in good agreement with the numerical results by Monte Carlo simulation, which further verify the correctness of the theoretical analysis.

5. Conclusion

In this paper, we studied the stochastic P-bifurcation for axially moving of a bistable viscoelastic beam model with fractional derivatives of high order nonlinear terms under Gaussian white noise excitation. According to the minimum mean square error principle, we transformed the original system into an equivalent and simplified system and obtained the stationary PDF of the system amplitude using stochastic averaging. In addition, we obtained the critical parametric condition for stochastic P-bifurcation of the system using singularity theory; based on this, the system response can be maintained at the monostability or small amplitude near the equilibrium by selecting the appropriate unfolding parameters, providing theoretical guidance for system design in practical engineering, and avoiding the instability and damage caused by the large amplitude vibration or nonlinear jump phenomenon of the system. Finally, the numerical results by Monte Carlo simulation of the original system also verify the theoretical results obtained in this paper. We conclude that the order \(p\) of fractional derivative and the noise intensity \(D\) can both cause stochastic P-bifurcation of the system and the number of peaks in the stationary PDF curve of system amplitude can change from 2 to 1 by selecting the appropriate unfolding parameters \((p, D)\); it also shows that the method used in this paper is feasible to analyze
the stochastic P-bifurcation behaviors of viscoelastic material system with fractional constitutive relation.

Data Availability

Data that support the findings of this study are included within the article (and its additional files). All other data are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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