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Research Article

Optimizing Price of Credit Default Swaps for Dynamic Project System of Public-Private Partnership

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Most project operations management belongs to the type of public-private partnership (PPP), which is usually dynamic. This paper aims to propose a method for optimizing the price of credit default swaps (CDS) for the dynamic PPP system. This study investigates the credit risk measurement of PPP project financing and the pricing of risk mitigation instruments which are widely used in the case of immature markets in the early stage of China's PPP development. Based on the credit risk measurement theory of the corporate and debt ratings, this paper considers the differences in various credit enhancement methods in the equity-like debt agreement and determines the credit rating of the equity-like debt in PPP projects. Some optimization methods are also proposed to derive the probability of default, so as to determine the price of the credit risk mitigation instrument of CDS which is based on the equity-like debt.

1. Introduction

As a long-term partnership of public sector (government) and private sector (for-profit or nonprofit companies) which is established by signing formal agreements, public-private partnership (PPP) has the advantages of revitalizing existing assets, transforming government functions, and improving the quality of public services in the provision of public goods or services. In recent years, the system established for PPP has been very mature and complete in many developed countries. Compared with developed countries, developing countries have greater needs in the area of large-scale infrastructure construction, and governments are more likely to fall into the embarrassing situation of insufficient funds. Therefore, PPP related policies have been intensively published in China since 2013 and PPP has been widely used in various fields of China's infrastructure and public service construction. The World Bank [1] comprehensively considered asset ownership, operating rights, investment relations, business risks, and contract terms and then classified the PPP contracts into six types: service contract, management contract, Lease, Concession, BOT/BOO, and Divestiture. Due to the long franchise period of the PPP project, the huge amount of investment, the different participation purposes of the project participants, and the complicated financing structure, the use of PPP to build public infrastructure and provide services will face multiple risks. Therefore, it is necessary to correctly identify risks. Only after identifying risks effectively can the risk be reasonably distributed. In the identification and evaluation of PPP project risks and reasonable distribution, a large number of scholars have conducted relevant research.

In the area of PPP risk identification, readers can refer to the relevant literatures given by Hastak and Shaked [2], Grimsey and Lewis [3], Li et al. [4], Shen et al. [5], Zhang [6], Ameyaw and Chan [7], and Ghorbani et al. [8]. Among them, Li et al. [4] divided the risk into macrolevels, medium levels, and microlevels. Zhang [6] has classified PPP risks in more detail: political, economic, and social environment, natural environment, third-party infringement, engineering decision-making and preparation, payment, supervision, completion and handover process, coordination, and relations. This helps participants who are not familiar with the project identify risks quickly. Ameyaw and Chan [7] analyzed six cases of the PPP project and classified 40 risk factors of the PPP project into eight categories, the common risks that practitioners are concerned with include the weak regulatory

and monitoring regime, financing, absence of risk allocation mechanisms, inexperience in PPPs, public opposition, and delayed and nonpayment of bills. Ghorbani et al. [8] divided the risk into the development phase, the operation phase and the entire life cycle. Each phase has different risk categories and factors.

In the PPP risk assessment, representative researches are shown as follows. Gumming and Macintosh [9] created the general theory of venture capital exit and considered that the best exit timing of social capital is that the marginal cost of investment is equal to the marginal return. The investor must determine the equity exit mechanism within the project duration. Khallaf et al. [10] applied game theory to analyze the interactions between parties to understand dynamic outcome of the associated risks in PPP projects. A case study of the railway project in Tanzania was also presented. Kumar et al. [11] applied a standard risk-analysis model of NPV-at-risk tool supported by Monte Carlo Simulation to the real-world PPP based highway infrastructure projects, which is able to identify which source of uncertainty has the most influence on the project's financial returns and what is the actual relationship between the critical influencing parameters and associated NPV. Wu et al. [12] combined the 2-dimension linguistic information of fuzziness and randomness with the cloud model to put forward a risk assessment framework of PPP waste-to-energy incineration projects.

In terms of PPP risk allocation, representative researches are shown as follows. Ewijk [13] identified, evaluated, and allocated risks by establishing a camp model for the operation of a PPP project. Carbonara et al. [14] established a win-win model of fair risk sharing during the franchise period of the PPP project by adopting the NPV method. Nasirzadeh et al. [15] calculated the proportion of risk sharing between the owner and the contractor based on system dynamics, fuzzy logic, and other methods. Almarri and Blackwell [16] studied and proposed two methods to improve the risk sharing and investment evaluation of PPP of large-scale greening projects. The first is to review the contract of the PPP project; the second is to improve the risk simulation method by improving input variables and characteristics.

However, the development of PPP is not mature in China, due to the monopoly of state-owned capital in the area of infrastructure construction, social capital is relatively unfamiliar to the field and lacks the relevant experience of investment, financing, and operational management. The enthusiasm for participating in PPP projects is limited. Local governments usually disguise their liabilities in the name of PPP to meet the large amount of funds and long-term capital requirements. The phenomenon of "equity-like debt" is ubiquitous in the process of project construction. The equity-like debt in PPP projects refers to the fact that, in a public-private partnership construction project, the investor nominally obtains the equity of the project company, while the financing side provides the investor with a revenue commitment and guarantees the mature repurchase, which is actually an indirect debt financing. Therefore, studying the credit risk of debt financing of PPP projects has become a major issue for each investor in the PPP projects. Therefore, Credit Default Swaps (CDS), which are subject to PPP projects, have become important financial instruments for identifying and hedging credit risk.

The common CDS pricing models are mainly divided into structured models and parsimonious models. The parsimony model considers that there is no clear correlation between the default condition of the reference entity and its value. The default phenomenon of the reference entity is random and the market information is used to directly calculate the risk-neutral probability of default under probability model. Shuai Li et al. [17] presented a general approach to PPP debt financing and CDS pricing in mature markets. In the typical nonmature Chinese market, due to the lack of the secondary market price of the project company's equity, the complete disclosure of operating and financial information, and the related complete financing service system, we combine the corporate credit risk measurement with the measurement of supporting credit of third-party's forward repurchase equity and making up principal and interest shortfalls, based on the bond yield curve of the Chinese market, we use the Jarrow-Turnbul binary tree model to derive its default probability and determine the price of CDS which is based on the debt.

2. Credit Risk Measurement of Equity-Like Debt in PPPs

Since the underlying rights and obligations (such as equity financing and the committed repurchasing of nominal equity, making up of investment principal and interest) have been signed by both parties and are subject to the "Contract Law," the transaction structures can affect the credit rating of the company's debts. Although project financing is in the form of equity, the guarantee terms for investment principal and interest, such as repurchases and making up shortfalls, make the financing risk essentially have the property of debt financing. Therefore, the default risk of equity-like debt needs to be considered from the relevant corporate credit risk and the characteristics of transaction structures; then its corresponding credit risk rating can be determined.

In addition to considering the default risk of debt entities, when determining the credit rating of equity-like debt, we also need to take into account trading structure risks (such as the risk of equity pricing, insufficient capital, project control risk, repurchase risk, and supporting credit loss), participating institution risks, and overall systemic market risks. The debt rating is determined based on a combination of the above factors. Therefore, in order to simplify the quantification and summarize its rating method, we take the support rating of the most critical credit enhancement measures as an example to give a corresponding debt rating. All other factors can be considered similarly; then a comprehensive rating of the debt will be determined.

2.1. Corporate and Debt Rating Methods. We proceed from the general framework of the corporate credit rating and consider the credit rating of the project company based on the default probability model, and the corresponding credit rating will be determined by considering the factor of equity-like debt. For the basic framework of the corporate credit rating in this paper, we quote from the market generally

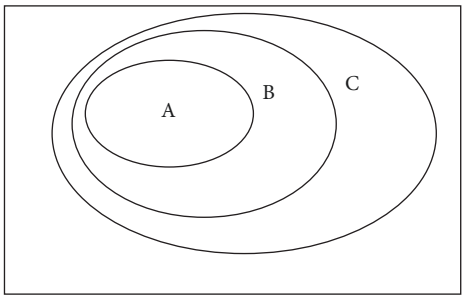

Figure 1: The relationship between macroeconomy, industry, and corporation.

accepted frameworks and variables which have been published by major international rating companies such as S&P, Moody's, and Fitch. The definition of default in the default probability model refers to the basic definition of default in the Basel Capital Accord. Among the existing default probability models, we use the logistic regression model which has been widely used by considering the accuracy, robustness, and performance.

The general framework of the credit rating model mainly refers to which factors have been considered to analyze the corporate credit risk and the role and interaction of these risk factors in the occurrence of default probability. The corporate credit rating of the bond market generally follows the principles of independence, objectivity, consistency, and reasonable prudence. The basic framework of corporate credit analysis includes three aspects: the macroeconomic environment, the industry conditions, and the operating financial status of the issuer. The issuer's financial status is the result of the entity operating in a specific industry under the macroeconomic environment, which can measure the credit status of the issuer to a large extent. However, the financial data has a certain lag, which cannot reflect the macroeconomic environment and industry conditions. When determining the credit rating, it is required that the credits of different industries, the different subject credits in the industry, and the issuer's own time series under the macroindustry are comparable. Therefore, the corporate credit analysis method in this paper is based on the fundamentals of the issuing entity such as operational and financial information. Meanwhile, the adjustment effects of the industry and the macroeconomic environment are also needed to be considered.

The deterioration of the macroeconomic environment is denoted by A, the deterioration of the industry is denoted by B, and the occurrence of corporate default is denoted by C. The relationship of the macroeconomy, industry, and corporation is shown in Figure 1.

So we obtained $P(B \mid A) = 1,\ P(C \mid B) = 1$.

Based on the full probability formula, $P(C)$ can be calculated by the following formula:

$$\begin{aligned} P(C) &= P(C \mid B)\, P(B) + P\left(C \mid \overline{B}\right) P\left(\overline{B}\right) \\ &= P(B) + P\left(C \mid \overline{B}\right) P\left(\overline{B}\right) \\ &= P(B \mid A)\, P(A) + P\left(B \mid \overline{A}\right) P\left(\overline{A}\right) \\ &\quad + P\left(C \mid \overline{B}\right) P\left(\overline{B}\right) \\ &= P(A) + P\left(B \mid \overline{A}\right) P\left(\overline{A}\right) + P\left(C \mid \overline{B}\right) P\left(\overline{B}\right) \\ &= P(A) + P\left(B \mid \overline{A}\right)(1 - P(A)) \\ &\quad + P\left(C \mid \overline{B}\right)(1 - P(B)) \end{aligned} \tag{1}$$

In the default probability model, we assume that if there is a problem with the macroeconomy, the corporate entity must have credit risks. Therefore, for the corporate entity and the industry, the probability of default risk arising from the macroeconomic environment and the probability of default risk when the macroeconomy is not at risk have additivity, so the probability of default risk arising from the macroeconomy will be used as an adjustment factor to affect the upper limit of the industry and corporate's rating.

The assumption applies to the relationship between the industry and the corporate. The probability of default risk arising from the industry will be used as an adjustment factor to affect the upper limit of the corporate's rating.

Measuring the probability of the corporate credit default risk $P(C \mid \overline{B})$ with the entity's operational and financial information will be the key to the corporate rating. For the aspect of the operational information, the basic conditions of the business entity (such as capital injection size, owner nature, and social influence), the level of managers, the ability to control the company's operations and resources, and the decentralized operations are generally considered when determining the credit risk. For the aspect of financial information, some factors are often considered, such as the profitability of the entity, the existing debt structure, especially the financial leverage, the financial status of the company, and the protection of the debt. In the following section, we will set the indicators of these influencing factors to be the explanatory variable X of the generalized linear model, which affect the value of the corporate default probability $= P(C \mid \overline{B})$.

Before estimating the default probability model, we need to introduce the definition of breach of contract. According to the Basel Accord, when borrowers are not willing to fulfill their credit obligations in full for over 90 days, they can be categorized into breach of contract. In addition, Standard & Poor's generally define that if borrowers have not paid the bond within a grace period of 10 to 30 days, it will be considered as a default event. European credit institutions generally considers that if borrowers fail to pay the bond overdue 120 days, it will be considered as a default event. Moody's defines a default event if it is overdue for more than one day. In summary, this paper defines that if the project company is unwilling or unable to perform its debt payment obligations (including principal, interest, and fees stipulated in related agreements) in full more than 90 overdue days and is still unable to perform its debt payment obligations in full

Table 1: Credit ratings with the corresponding default probability of Moody, S&P, and domestic commercial banks.

Rating Symbols	Default probability interval of Moody		Default probability interval of S&P		Default probability interval of domestic commercial banks	
AAA	0.00%	0.02%	0.00%	0.02%	0.00%	0.01%
AA	0.02%	0.05%	0.02%	0.04%	0.01%	0.5%
A	0.05%	0.15%	0.04%	0.11%	0.5%	1%
BBB	0.15%	0.50%	0.11%	0.27%	1%	2%
BB	0.50%	2.00%	0.27%	0.88%	2%	5%
B	2.00%	7.00%	0.88%	4.86%	5%	10%
CCC	7.00%	15.00%	4.86%	15.94%	10%	20%
CC	15.00%	20.00%	15.94%	17.61%	20%	100%
C	20.00%	100.00%	17.61%	19.10%	100%	100%
D	100.00%		19.10%	20%	100%	100%

within the grace period (t days), it will be considered as PPP's breach of contract.

We use a logistic regression model to describe the default probability $Y = P(C \mid \overline{B})$ of the corporate entity in the absence of macroeconomic and industry risk events. The parameters of the model are estimated and $P(C \mid \overline{B})$ is obtained by substituting the sample data and applying the maximum likelihood estimation method.

Similarly, we can use the same method to describe the probability $Z = P(B \mid \overline{A})$ of general default of the industry in the absence of a macroeconomic risk event and obtained the probability of industry risk $P(B \mid \overline{A})$. Then we substitute $P(B \mid \overline{A})$ into (1) and calculate the corporate default probability $P(C)$.

After obtaining the probability of default, to obtain the corresponding credit rating of the corporate, it is necessary to correspond with the main scale of the credit rating. At present, the main scale of the rating agencies in the financial market is based on a large amount of historical data and years of experience in rating, which has both an explanatory and statistically significant economic meaning. The corporate credit rating is determined by referring to the main scale of rating agencies.

We compared the credit ratings of Moody's and Standard & Poor's and domestic commercial banks' internal rating methods with their corresponding default probability, which is shown in Table 1.

Considering that the credit risk management system of domestic commercial banks is becoming increasingly standard, and it can reflect the actual situation of domestic companies' breach of contract accurately, we will refer to the main scale of the domestic commercial banks to determine the credit rating symbol which corresponds to the default probability of the PPP project company.

2.2. Credit Rating after the Credit Enhancement under Various Financing Structures of Equity-Like Debt. In some PPP projects, the major shareholders of the project company and the local government often support the project company's operations and financing in many ways, such as jointing guarantees and making up shortfalls of principal and interest. Therefore, for this type of project, besides the corporation's individual rating, the support rating is also needed to be considered.

Support rating refers to the promotion of individual ratings by considering supporting factors. To determine the degree of individual rating improvement, it is necessary to evaluate the status and strategy of individuals or local governments and the possible manifestations of support for the individual default (such as a predefault support and a postdefault bailout). For state-owned enterprise groups, the status and role of the group in the local state-controlled enterprise system are also needed to be considered when assessing the degree of support given by the local government and determining the government support rating.

Due to the nonstandardized characteristics of the financial instruments of equity-like debt, on the basis of identifying the corporate credit risk, it is necessary to determine the support rating from the following three aspects. The first is the comprehensiveness and rigorousness of the repurchase arrangement or the shortfall replenishment measure on the debt agreement, which is used to ensure that the contract that contains the payment obligation becomes a debt of the supporter. The second is to compare the repurchase or difference between complemented debts of the agreement with the ordinary debts of the supporter and set that whether the right holders of repurchase or difference complement can claim the creditor's rights declaration to be the reference for the priority or inferior layer of the debts. The third is to inspect the financial quality, economic value, and social value of the project company and pay attention to the value of relief in extremely unfavorable situations. It is used to assess the financial risk, the sustainability of its operations, and the strength of obtaining external support of the project.

The above three aspects are the key aspects of measuring the support rating of equity-like debt. In the following section, we will analyze the effectiveness of the credit enhancement measures by combining several major credit enhancement methods. Since the process of generating the credit rating symbol ultimately needs to be combined with the experience of rating analysts, the descriptions of several major credit enhancement methods are given in Table 2.

Table 2: The description of four main credit enhancement methods.

Credit enhancement methods	Description
Project company's full premiums repurchase + third-party's joint guarantees	The project company repurchase the equity (beneficial right or fund share) held by the financial instruments of equity-like debt (including trust plans, asset management plans, fund shares, etc.) according to the principal and income premium agreed in the investment contract, and the third-party agency (such as sponsoring group companies or local governments) undertakes joint guarantees on the repurchase obligations of the project company. The project company assumes the obligation to pay the debt, and the third-party institution which acts as the trustworthy entity will be the target of the rating. This is a guarantee security method, and the credit enhancement effect is based on the credit quality of the project company and the third-party organization. Any party with a high credit rating is sufficient to support the high level of rating of financing instruments. Therefore, the final credit rating is determined based on the higher rating of the two parties.
Project company's full premiums repurchase + third-party's making up principal and interest shortfalls	In this case, the guarantee effect of the third-party institution's commitment letter must be first determined. If the commitment letter of making up principal and interest shortfalls has strict terms, and the triggering mechanism is conducive to implementation, it can be defined that the supplemental commitments have substantial safeguard effect, but the nature of the liability is the supplementary responsibility instead of the repayment liability, so the replenishment obligor has the counterplead right of telling first and cannot constitute joint guarantees. The "Enterprise Bankruptcy Law" stipulates that the debtor's unexpired guarantee liability is deemed to have been expired and needs to be paid off in advance. However, in the case of the bankruptcy of the third party, the guarantor who bears the supplemental responsibilities still has the problem of handling the counterplead right of telling first. In order to protect the fairness of transactions and achieve the original intention of setting rights, if the guarantor's counterplead right is cancelled, the creditor may first get the repayment from the guarantor. Judging from the legal practice, as the supplement number of debts is difficult to define, the creditor's claims can be denied by the bankruptcy administrator. Therefore, under this situation, if there is a substantial guarantee effect, and the terms in the agreement substantially cancel the guarantor's counterplead right, the higher credit rating of the third party can support the debt rating to the corresponding level. If there is only a substantial guarantee effect and the credit rating of the third party institution is higher than that of the project company, it can support the debt which is higher than the project company level but not higher than the third party institution's credit rating. In other cases, third-party agencies are not considered, and the credit rating of the debt is determined only based on credit rating of the project company.
Project company's principal repurchase + third-party's making up principal and interest shortfalls	Project company repurchase the equity, income right and fund share of the financing instrument, The third party agency has the liability for the repurchase shortfalls of the project company and the interest of the financing instrument. However, as the project company acts as the debtor and its repurchase obligation cannot cover the principal and interest of the financing instrument, its corporate credit has little impact on the credit rating of the financing instrument. The main consideration is the substance of the third-party's replenishment commitment shortfalls. If the credit rating of the third-party company is high, the debt rating can be appropriately close to its credit rating.
Project company's principal repurchase + profit supplement or project company's full premiums repurchase of principal and interest	The project company acts as the debtor and assumes all payment obligations. Therefore, the credit rating of the debt is largely determined by the rating of the project company.

For ease of use, it is assumed that the rating of the project company is R_M, and the rating of the third-party credit agency is R_N,$M, N = 1, 2, \cdots 10, M \geq N$, and the corresponding credit rating symbol of R_M and R_N is shown in Table 3.

The support ratings of the above four typical credit enhancement methods are shown in Table 4.

After obtaining the credit rating of the project company and the third party through the method described in Section 2.1, the final credit rating (denoted by R_k) of the debt can basically be obtained by referring to Table 4.

3. Analysis of the Price of Credit Risk Mitigation Instrument of CDS

After obtaining the value of credit rating, we consider the actual situation of domestic debt default, based on the yield curve of the debenture bond, and we derive the default probability of different maturities by applying the Jarrow-Turnbull model.

The Jarrow-Turnbull model is also known as the Jarrow-Turnbull binary tree model. The model considers the impact of interest rate fluctuation processes and default processes on

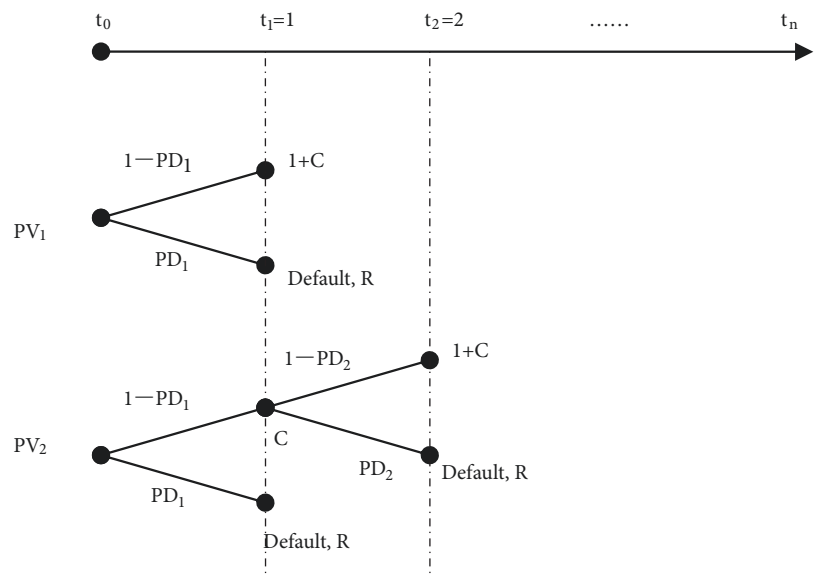

Figure 2: Binary tree-solution of default probability.

Table 3: List of credit rating symbols.

Ranking	Rating Symbols
1	AAA
2	AA
3	A
4	BBB
5	BB
6	B
7	CCC
8	CC
9	C
10	D

bond prices. The model assumes that when the breach event occurs, the seller only pays compensation on the agreed fixed trading day.

The bond pricing formula in the Jarrow-Turnbull model [18] is shown as follows:

$$V_{JT}(0,t_n) = \sum_{i=1}^{n} P(0,t_i)\, Q(0,t_i)\, C\Delta t + P(0,t_n)\, Q(0,t_n) + R \cdot P(0,t_n)\left[1 - Q(0,t_n)\right] \tag{2}$$

In formula (2), $V_{JT}(0,t_n)$ represents the present value of the bond at time t_n; $P(0,t_i)$ represents the discount rate at time t_i, $P(t_i,t_i) = 1$; $Q(0,t_i)$ denotes the survival rate of bond at time t_i; C denotes the nominal interest rate of bond; Δt denotes the interval between adjacent payment days; R represents the rate that the bond can be recovered when a credit event occurs.

The idea of using this model to price the CDS is to derivate the default probability curve through the credit spreads of the government bond and the debenture bond of the same rating, and then the CDS price is calculated by using the back-off default probability.

3.1. Calculating Probability of Default. For the sake of simplicity, the face value of the bond is set to be 1; the coupon rate of the bond is set to be C (annual interest payment); the rate that the bond can be recovered when a credit event occurs is denoted by R; the relationship between R and the default loss rate (LGD) is $R = 1 - LGD$.

We mainly calculate default probabilities of debenture bonds with different maturities through variables such as present value of bonds PV, default recovery rate R, and risk-free interest rate r. We use the Chinese government bond yield curve as a risk-free rate. The binary tree method is used to analyze the future cash flow of credit bonds, as shown in Figure 2.

In Figure 2, PD_1 indicates the probability of a default occurring before time t_1; PD_2 indicates the probability of no default occurring before time t_1 and a default occurring between times t_1 and t_2. If the current price of the debenture bond is set as the present value of the future cash flow income, the price of the 1-year debenture bond can be calculated as

$$\mathrm{PV}_1 = \frac{(1+C)(1-PD_1) + R \cdot PD_1}{1+r_1} \tag{3}$$

We can calculate the value of PD_1 by solving the equation

$$PD_1 = \frac{(1+C) - \mathrm{PV}_1(1+r_1)}{1+C-R} \tag{4}$$

Table 4: Credit rating after the credit enhancement of various financing structures of equity-like debt.

Credit Enhancement	Conditions	Support Rating
Project company's full premiums repurchase + third-party's joint guarantees	-	R_N
Project company's full premiums repurchase + third-party's making up principal and interest shortfalls	(a) substantial guarantee effect, and the terms in the agreement substantially cancel the guarantor's counterplead right	R_N
	(b) only a substantial guarantee effect	$R_k, N \le k \le M$
	(c) no guarantee effect	R_M
Project company's principal repurchase + third-party's making up principal and interest shortfalls	(a) only a substantial guarantee effect	$R_k, N \le k$
	(b) no guarantee effect	-
Project company's principal repurchase +profit supplement, or project company's full premiums repurchase of principal and interest	-	R_M

Similarly, the price of a 2-year debenture bond can be calculated as follows:

$$PV_2 = \frac{C\left(1 - PD_1\right) + R \cdot PD_1}{1 + r_1} + \frac{\left[(1 + C)\left(1 - PD_2\right) + R \cdot PD_2\right]\left(1 - PD_1\right)}{\left(1 + r_2\right)^2} \quad (5)$$

The value of PD_1 is obtained by solving the (4), after getting the value of PD_1, and the value of PD_2 is obtained by solving the (5). By analogy, we can use the 3-year debenture bond price to calculate the value of PD_3. Similarly, the value of PD_4, PD_5, and PD_n $(n > 5)$ can be calculated by using the same method. In the above derivation process, r_1 is the 1-year risk-free interest rate, r_2 is the 2-year risk-free interest rate, and r_n is the n-year risk-free interest rate.

Based on the above process, we can obtain the default probability curve of the same credit rating bond.

Since the trading volume of bonds in the secondary market of China bonds (including interbank and exchange-traded markets) is limited, all transaction data are concentrated within a period of 0.5 to 5 years from the expiration date of the bonds, and the transaction data for more than 5 years are extremely limited. The present value of $\mathrm{PV}_1, \mathrm{PV}_2, \cdots, \mathrm{PV}_n$ corresponding to the R_k rating is often not exactly reached at the condition of an integer year of maturity period. Even if there is exactly a trading price for the whole year, due to the small trading volume, there are often a large number of liquidity premiums that cause prices to deviate, so the present value of a bond that is taken from the market price tends to deviate greatly. Therefore, we use the China Bond Yield Curve to estimate the present value of the bond price.

The China Bond Yield Curve is a yield curve published by the Central Government Securities Depository Trust & Clearing Corporation, the curve is formed based on bond transactions, quotations, and institutional valuation data in the Chinese market, it is used to characterize the various maturity structures of Chinese bonds. The data includes the interbank bond market settlement data, transaction data for exchange bonds, bilateral quotation at the counter, bilateral quotes for the interbank bond market and the profitability valuation data of some of the core members. In the current Chinese bond market, the price discovery mechanism is still incomplete (especially when bilateral interbank quotation does not exert its price discovery function properly), the Chinese bond yield curve mainly refers to the profitability valuation data of some of the core members. Therefore, whether in the maturity stage of better liquidity or in the maturity period of poor liquidity, the China Bond Yield Curve mostly truly reflect the general value of bonds and market conditions, and it has become an important basis for the basic yield curve of the Chinese bond market.

We use the debt credit rating R_k of the PPP fake equity real debt which is obtained in Section 2.2 to corresponds to the spot rate yield curve of corporate bonds rated R_k in the China Bond Yield Curve and get yield rate $y_{k,i}$ in which the maturity period i is an integer $(i = 1, 2 \cdots, n)$, and then PV_i is calculated as follows:

$$PV_i = \frac{1}{\left(1 + y_{k,i}\right)^i} + \sum_{j=1}^{i} \frac{C}{\left(1 + y_{k,j}\right)^j} \quad \left(1 \le j \le i\right) \quad (6)$$

The corresponding default probability sequence PD_i can be obtained by substituting the present value of the bond obtained by (6) into (4) and (5).

3.2. CDS Price in the Case of One-Time Payment of Premiums. In this case, we assume that the CDS buyer pays the CDS premium once at the beginning of the transaction, and the prepaid premium rate at time t_0 is denoted by S. Once a credit event occurs, the delivery will be made in cash on the next future premium payment date of the credit event, and the CDS will be terminated. We simulate the future cash flow during the CDS duration by using the binary tree method, which is shown in Figure 3.

On the other hand, at the beginning of the transaction, the current CDS value is zero for the buyer and the seller, and the CDS rate S_1 is determined by

$$S_1 = \frac{\mathrm{PD}_1\left(1 - \mathrm{R}\right)}{1 + r_1} \quad (7)$$

So S_1 is the quote for 1-year CDS based on the above bond.

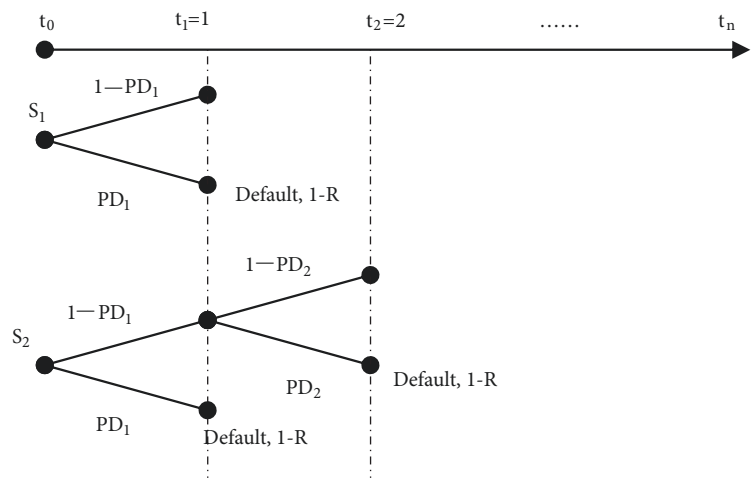

Figure 3: Binary tree-CDS pricing solution (one-time payment of premiums).

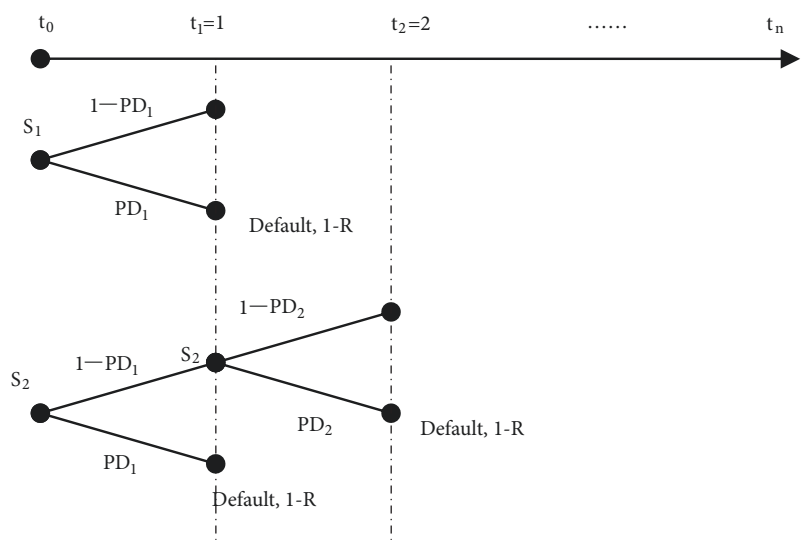

Figure 4: Binary tree-CDS pricing solution (premium payment in installments).

Similarly, the net future cash flow for a 2-year CDS can be described as follows:

$$S_2 - \left[\frac{\mathrm{PD}_1(1-\mathrm{R})}{1+r_1} + \frac{(1-PD_1)\,PD_2\,(1-R)}{(1+r_2)^2}\right] \tag{8}$$

S_2 can be calculated by the

$$S_2 = \frac{\mathrm{PD}_1(1-\mathrm{R})}{1+r_1} + \frac{(1-PD_1)\,PD_2\,(1-R)}{(1+r_2)^2} \tag{9}$$

By analogy, we can obtain the prices of 3-year, 4-year, and n-year CDS and calculate the value of S_3 , S_4, S_n.

3.3. CDS Price in Case of Premium Payment in Installments. Assuming that CDS premiums are paid in installments, they are paid at the beginning of each payment interval, and the payment interval is equal to the bond interest payment interval. Here we assume that the interest payment interval for the bond is one year, so for a CDS with a contract term of n years, the buyer pays premiums at time $t_0 = 0$, $t_1 = 1, \cdots, t_{n-1} = n-1$ and the interval of the adjacent payment time is $\Delta t = t_{i+1} - t_i = 1$, and the premium rate is denoted by S, which means that the CDS buyer needs to pay S times the face value of the bond of the CDS seller's assets each year. The future cash flow during the CDS duration is shown in Figure 4.

Considering the 1-year CDS, the net present value of the future cash flow of the CDS can be described as follows:

$$S_1 - \frac{\mathrm{PD}_1(1-\mathrm{R})}{1+r_1} \tag{10}$$

S_1 can be calculated by

$$S_1 = \frac{\mathrm{PD}_1(1-\mathrm{R})}{1+r_1} \tag{11}$$

Similarly, the net future cash flow for a 2-year CDS can be determined as follows:

$$\left[S_2 + \frac{S_2\left(1 - PD_1\right)}{\left(1 + r_2\right)^2}\right] - \left[\frac{\mathrm{PD}_1\left(1 - \mathrm{R}\right)}{1 + r_1} + \frac{\left(1 - PD_1\right) PD_2 \left(1 - R\right)}{\left(1 + r_2\right)^2}\right] \quad (12)$$

S_2 can be calculated by the formula

$$S_2 = \frac{\left[\mathrm{PD}_1\left(1 - \mathrm{R}\right) / \left(1 + r_1\right) + \left(1 - PD_1\right) PD_2 \left(1 - R\right) / \left(1 + r_2\right)^2\right]}{\left[1 + \left(1 - PD_1\right) / \left(1 + r_2\right)^2\right]} \quad (13)$$

By analogy, we can obtain the prices of 3-year, 4-year, and n-year CDS and calculate the value of S_3, S_4, S_n.

4. Conclusions

For the widely adopted PPP mode in the project operations management, how to design some mechanism to improve the operation efficiency is one of the important decision problems for project managers. Since the PPP project is a dynamic system, the decision is usually very complex. This paper proposes a method for optimizing the price of credit default swaps (CDS) for the dynamic PPP system. This study investigates the credit risk measurement of PPP project financing and the pricing of risk mitigation instruments which are widely used in the case of immature markets in the early stage of China's PPP development. Based on the credit risk measurement theory of the corporate and debt ratings and the differences in various credit enhancement methods in the equity-like debt agreement, this paper proposes the credit rating of the equity-like debt in PPP project. Some optimization methods are also proposed to derive the probability of default, so as to determine the price of the credit risk mitigation instrument of CDS which is based on the equity-like debt.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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