Research Article

Feedback Control and Parameter Invasion for a Discrete Competitive Lotka–Volterra System

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Received 1 May 2018; Revised 21 July 2018; Accepted 1 August 2018; Published 16 August 2018

Academic Editor: Mustafa R. S. Kulenovic

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State feedback is used to stabilize the Turing instability at the unstable equilibrium point of a discrete competitive Lotka–Volterra system. In addition, a regularization method is applied to parameter inversion for the given Turing system and numerical simulation can verify the effectiveness of the algorithm. Furthermore, how less or more sample data and dependence on the initial state affect estimation procedure are tested.

1. Introduction

In theoretical ecology, the models governed by difference equations are used to characterize the interactions of species when the size of the population is rarely small [1]. For example, a two-species competitive discrete-time system which reduced from the continuous one with the forward Euler scheme can be obtained as follows [2]:

\[
\begin{align*}
x_{t+1}^i &= r_1 x_t^i \left( 1 - a_{11} x_t^i - a_{12} y_t^i \right) \\
y_{t+1}^i &= r_2 y_t^i \left( 1 - a_{21} x_t^i - a_{22} y_t^i \right)
\end{align*}
\]

(1)

where \( x^i \) and \( y^i \) are the quantities of the two species at \( t \)th generation, \( r_1 > 0 \) and \( r_2 > 0 \) are growth rates of the respective species, \( a_{11} \) and \( a_{12} \) represent the strength of the intraspecific competition, and \( a_{21} \) and \( a_{22} \) are the strength of the interspecific competition.

It is well known that the role of spatial heterogeneity and dispersal in the dynamics of populations has been the subject of much research, both theoretical and experimental, such as the role of dispersal in the maintenance of patchiness, or spatial population variation. It is also a fact that the motion of individuals is random and isotropic, i.e., without any preferred direction, and the individuals are also absolute ones in microscopic sense, and each isolated individual exchanges materials and information by diffusion with its neighbors [3–5]. Then it is reasonable to consider a 2D spatially discrete reaction-diffusion system as follows [6]:

\[
\begin{align*}
x_{i,j}^{t+1} &= r_1 x_{i,j}^t \left( 1 - a_{11} x_{i,j}^t - a_{12} y_{i,j}^t \right) + D_1 \nabla^2 x_{i,j}^t \\
y_{i,j}^{t+1} &= r_2 y_{i,j}^t \left( 1 - a_{21} x_{i,j}^t - a_{22} y_{i,j}^t \right) + D_2 \nabla^2 y_{i,j}^t
\end{align*}
\]

(2)

for \( i, j \in \{1, 2, \cdots, m\} = [1, m] \) and \( t \in \{1, 2, \cdots, n, \cdots\} \), where \( m, n \) are positive integers. Here, \( \nabla^2 \) is discrete Laplace operator

\[
\begin{align*}
\nabla^2 x_{i,j}^t &= x_{i+1,j}^t + x_{i-1,j}^t + x_{i,j+1}^t + x_{i,j-1}^t - 4x_{i,j}^t \\
\nabla^2 y_{i,j}^t &= y_{i+1,j}^t + y_{i-1,j}^t + y_{i,j+1}^t + y_{i,j-1}^t - 4y_{i,j}^t
\end{align*}
\]

(3)

and

\[
\begin{align*}
\nabla^2 x_{i,j}^t &= x_{i+1,j}^t + x_{i-1,j}^t + x_{i,j+1}^t + x_{i,j-1}^t - 4x_{i,j}^t \\
\nabla^2 y_{i,j}^t &= y_{i+1,j}^t + y_{i-1,j}^t + y_{i,j+1}^t + y_{i,j-1}^t - 4y_{i,j}^t
\end{align*}
\]

(4)

This also indicates the coupling or diffusion from the cells to the left \( (i, j-1) \) and right \( (i, j+1) \) and top \( (i+1, j) \) and bottom \( (i-1, j) \) respectively.

The partial difference systems (space-time discrete systems) for biological patterns resulted from diffusion-driven instability (Turing instability) in plants and animals and the set of equilibrium patterned solutions have been studied in great detail over the last several years; for example, see [6–11]. Models such as discrete competition reaction-diffusion
models (2) have also been proposed to explain a wide variety of biological patterning processes and various patterns such as spiral wave, trigger wave, stripes, and chaotic Turing structure can be exhibited in the Turing instability region [6].

In some situations, one may wish to recover its stability by means of some ways, in order to move the trajectory towards the desired orbit. Feedback control is an effective one, which is of significance in the control procedure of ecology balance. If one may wish to alter the positions of positive equilibrium and to obtain its stability, to achieve the aim, one of the techniques used is to alter system structurally by introducing “indirect control” variables. Though there are many works on the single species or multispecies competition systems with feedback controls [12–16]. To the best of the authors’ knowledge, there are still no scholars who are investigating the stability property of the 2D spatially discrete reaction-diffusion competitive system with feedback controls; this motivates us to propose such a model as follows:

\[
\begin{aligned}
x_{i,j}^{t+1} &= r_i x_{i,j}^t \left(1 - a_{11} x_{i,j}^t - a_{12} y_{i,j}^t - b_1 u_{i,j}^t\right) + D V^2 x_{i,j}^t \\
y_{i,j}^{t+1} &= r_j y_{i,j}^t \left(1 - a_{21} x_{i,j}^t - a_{22} y_{i,j}^t - b_2 v_{i,j}^t\right) + D V^2 y_{i,j}^t \\
u_{i,j}^{t+1} &= (1 - c_1) u_{i,j}^t(t) + d_1 x_{i,j}^t \\
v_{i,j}^{t+1} &= (1 - c_2) v_{i,j}^t(t) + d_2 y_{i,j}^t
\end{aligned}
\]  

(5)

where \(u\) and \(v\) are feedback control variables and the parameters \(c_1, c_2, d_1, d_2, e_1\), and \(e_2\) are positive constants.

It may be a fact that stability analysis, in mathematics, belongs to direct problem which pays attention on the dynamical behavior of the system state. Otherwise, one may be much more concerned to know what reason or what system environment to result into the current state. In mathematics, it can belong to inverse problems of identification and determination of parameters. On the other hand, an important and often difficult step from the viewpoint of testing models against experimental observations is the determination of model parameters from limited data when details of the mechanistic steps involved are not known. Parameter identification is the foundation of state estimation, controller design, diagnosis and fault detection, etc. Therefore, much research on such parameters inversion problems based on equation or reaction-diffusion systems has emerged; for example, see [17–20]. And, there is a few work on parameters identification or estimation for Turing systems [21–23]. Parameter identification for the classic Gierer-Meinhardt reaction-diffusion system is considered in [21], which can result in diffusion-driven instability, and the parameters are extended in time and space and used as distributed control variables. In [22], it is shown how using different combinations of spatial and temporal data can improve parameter estimation in a postulated model and how postprocessing with sensitivity analysis can be used to address the complexity issue. The authors present a Bayesian inference approach to solve both the parameter and the state estimation problem for stochastic reaction-diffusion systems in [23].

In [24], a parameter estimation method called regularization method is applied to estimate the discrete Lotka–Volterra cooperative system and comparison experiments are also done using the regularization method and least square method to confirm the algorithm’s effectiveness. Similarly, a revised parameter estimation method can be used for the 2D spatially discrete reaction-diffusion competitive system (2) and the case to be discussed in this work will hold linear feature.

So the paper is organized as follows. After a brief presentation of the model with diffusion for a completely symmetric case of the system (2), local instability conditions can be deduced combining linearization method and inner product technique for a the symmetric system with feedback control in Section 2. A parameter estimation method called regularization method is applied to estimate the discrete Lotka–Volterra competitive system in Section 3, and numerical examples will also support this inference. The final section is the conclusion.

### 2. Feedback Control and Its Stability

In this section, a completely symmetric discrete Lotka–Volterra competitive system can be given as follows:

\[
\begin{aligned}
x_{i,j}^{t+1} &= r_i x_{i,j}^t \left(1 - x_{i,j}^t - a y_{i,j}^t\right) + D V^2 x_{i,j}^t \\
y_{i,j}^{t+1} &= r_j y_{i,j}^t \left(1 - a x_{i,j}^t - y_{i,j}^t\right) + D V^2 y_{i,j}^t
\end{aligned}
\]  

(6)

with periodic boundary conditions

\[
\begin{aligned}
x_{i,0}^t &= x_{i,m}^t, \\
x_{i,1}^t &= x_{i,m+1}^t, \\
x_{0,j}^t &= x_{m,j}^t, \\
x_{1,j}^t &= x_{m+1,j}^t
\end{aligned}
\]  

(7)

and

\[
\begin{aligned}
y_{i,0}^t &= y_{i,m}^t, \\
y_{i,1}^t &= y_{i,m+1}^t, \\
y_{0,j}^t &= y_{m,j}^t, \\
y_{1,j}^t &= y_{m+1,j}^t
\end{aligned}
\]  

(8)

for \(i, j \in \{1, 2, \cdots, m\} = \{1, m\}\) and \(t \in \{1, 2, \cdots, n, \cdots\}\), where \(m, n\) are positive integers.

From [6], the above system is a diffusion-driven unstable one at the nontrivial coexistence point ((\(r − 1\))/\(r(\alpha + 1)\), (\(r − 1\))/\(r(\alpha + 1)\)) when the condition \(D k_{ls}^2 > 3 - r\) holds for some positive number \(D\) and \(k_{ls}^2\), where

\[
k_{ls}^2 = 4 \left(\sin^2 \left(\frac{(l-1)\pi}{m}\right) + \sin^2 \left(\frac{(s-1)\pi}{m}\right)\right)
\]  

(9)

for \(l, s \in \{1, m\}\).
In order to stabilize the orbit at an unstable equilibrium point of system (2), we use the state feedback control method and indirect control variables are added; then we can get the point of system (2), we use the state feedback control method for four fixed points, where

\[ \begin{align*}
E_0(0,0,0,0), \\
E_1 \left(0, \frac{a_2 (r-1)}{(a_2 + b_2 c_2) r}, 0, \frac{a_2 c_2 (r-1)}{(a_2 + b_2 c_2) a r} \right), \\
E_2 \left(\frac{a_1 (r-1)}{(a_1 + b_1 c_1) r}, 0, \frac{a_1 c_1 (r-1)}{(a_1 + b_1 c_1) a r} \right), \\
E_3(x^*, y^*, u^*, v^*),
\end{align*} \]

where

\[ \begin{align*}
x^* &= \frac{(r-1)a_1 (a_2 + b_2 c_2 - a_2 a)}{r ((a_1 + b_1 c_1) (a_2 + b_2 c_2) - a_1 a_2 a^2)}, \\
y^* &= \frac{(r-1)a_2 (a_1 + b_1 c_1 - a_1 a)}{r ((a_1 + b_1 c_1) (a_2 + b_2 c_2) - a_1 a_2 a^2)}, \\
u^* &= \frac{c_1}{a_1} x^*, \\
v^* &= \frac{c_2}{a_2} y^*.
\end{align*} \]

There exists positive fixed point if and only if \( r > 1 \) and

\[ \frac{(a_1 + b_1 c_1) (a_2 + b_2 c_2)}{a_1 a_2 a^2} > 1, \]

and

\[ a < \min \left\{ 1 + \frac{b_1 c_1}{a_1} (1 + \frac{b_2 c_2}{a_2}) \right\}. \]

For the reaction-diffusion system (10), we linearize about the steady state, to get

\[ w_{ij}^{t+1} = Aw_{ij}^t + DV^2 w_{ij}^t, \]

with the periodic boundary conditions

\[ \begin{align*}
w_{ij,0} (t) &= w_{im,0} (t), \\
w_{ij,1} (t) &= w_{im+1,1} (t), \\
w_{ij,0} (t) &= w_{im,0} (t), \\
w_{ij,1} (t) &= w_{im+1,1} (t),
\end{align*} \]

where

\[ A = \begin{bmatrix} f_x \\ f_y \\ f_u \\ f_v \\ g_x \\ g_y \\ g_u \\ g_v \\ d_1 \\ 0 \\ 1 - e_1 \\ 0 \\ 0 \\ d_2 \\ 0 \\ 1 - e_2 \end{bmatrix}, \]

\[ w_{ij} (t) = \begin{bmatrix} x_{ij}^t - x^* \\ y_{ij}^t - y^* \\ u_{ij}^t - u^* \\ v_{ij}^t - v^* \end{bmatrix}, \]

and

\[ D = \begin{bmatrix} D & 0 \\ 0 & D \end{bmatrix}. \]

In order to study instability of (15), we firstly consider eigenvalues

\[ \nabla^2 X_{ij} + \mu X_{ij} = 0, \]

with the periodic boundary conditions

\[ \begin{align*}
X_{i,0} &= X_{im}, \\
X_{i,1} &= X_{im+1}, \\
X_{0,j} &= X_{m,j}, \\
X_{1,j} &= X_{m+1,j}.
\end{align*} \]

In view of [11], the eigenvalue problems (19)-(20) have the eigenvalues

\[ \mu_{ls} = 4 \left( \frac{\sin^2 \left( \frac{(l-1) \pi}{m} \right)}{m} + \frac{\sin^2 \left( \frac{(s-1) \pi}{m} \right)}{m} \right) = k_{ls}^2 \]

for \( l, s \in [1, m] \).
Then taking the inner product of (15), respectively, with the corresponding eigenfunction $X_{ij}^t$ of the eigenvalue $\mu_{ij}$, we see that

$$\sum_{i,j=1}^{m} X_{ij}^{t+1} = f_x \sum_{i,j=1}^{m} X_{ij}^t \dot{X}_{ij} + f_y \sum_{i,j=1}^{m} X_{ij}^t \dot{Y}_{ij} + D \sum_{i,j=1}^{m} X_{ij}^t \dot{V}_{ij} \quad (22)$$

and

$$\sum_{i,j=1}^{m} X_{ij}^{t+1} = g_x \sum_{i,j=1}^{m} X_{ij}^t \dot{X}_{ij} + g_y \sum_{i,j=1}^{m} X_{ij}^t \dot{Y}_{ij} + D \sum_{i,j=1}^{m} X_{ij}^t \dot{V}_{ij} \quad (23)$$

Let $X^t = \sum_{i,j=1}^{m} X_{ij}^t u_{ij}$, $Y^t = \sum_{i,j=1}^{m} X_{ij}^t v_{ij}$, and $V^t = \sum_{i,j=1}^{m} X_{ij}^t w_{ij}$ and use the periodic boundary conditions (20); then we have

$$X^{t+1} = f_x X^t + f_y Y^t + f_u U^t + f_v V^t - Dk_{ls}^2 U^t$$

$$Y^{t+1} = g_x X^t + g_y Y^t + g_u U^t + g_v V^t - Dk_{ls}^2 V^t$$

$$U^{t+1} = (1 - e_1) U^t + d_1 X^t$$

$$V^{t+1} = (1 - e_2) V^t + d_1 Y^t$$

or

$$X^{t+1} = (f_x - DC) X^t + f_y Y^t + f_u U^t + f_v V^t$$

$$Y^{t+1} = g_x X^t + (g_y - Dk_{ls}^2) Y^t + g_u U^t + g_v V^t$$

$$U^{t+1} = (1 - e_1) U^t + d_1 X^t$$

$$V^{t+1} = (1 - e_2) V^t + d_1 Y^t$$

which has the eigenvalue equation

$$\lambda^4 + R_1 \lambda^3 + R_2 \lambda^2 + R_3 \lambda + R_4 = 0 \quad (24)$$

where

$$R_1 = 2Dk_{ls}^2 - f_x - g_y + e_1 + e_2 - 2,$$

$$R_2 = (Dk_{ls}^2 - f_x) (Dk_{ls}^2 - g_y) + (e_1 + e_2 - 2) (2Dk_{ls}^2 - f_x - g_y)$$

$$R_3 = (Dk_{ls}^2 - f_x) (Dk_{ls}^2 - g_y) (e_1 + e_2 - 2)$$

$$R_4 = (Dk_{ls}^2 - f_x) (Dk_{ls}^2 - g_y) (e_1 - 1)(e_2 - 1)$$

$$+ d_2 f_v (e_1 - 1) f_v g_v (e_1 - 1)(e_2 - 1)$$

$$- d_1 (e_1 - 1) f_y g_u$$

$$- d_1 f_u (Dk_{ls}^2 - g_y) (e_2 - 1)$$

$$+ d_1 d_2 (f_u g_v - f_v g_u).$$

and $f = rx(1 - x - ay - b_1 u), g = ry(1 - ax - y - b_2 v), f_x = f_r(x^*, y^*, u^*, v^*), f_y = f_r(x^*, y^*, u^*, v^*)$, $f_u = f_r(x^*, y^*, u^*, v^*), f_v = f_r(x^*, y^*, u^*, v^*)$, $g_x = g_r(x^*, y^*, u^*, v^*), g_y = g_r(x^*, y^*, u^*, v^*)$, $g_u = g_r(x^*, y^*, u^*, v^*)$, and $g_v = g_r(x^*, y^*, u^*, v^*)$.

According to the Routh–Hurwitz criterion, we can draw the following conclusion.

**Theorem 1.** The positive homogeneous steady state $E$ is stable if the following conditions are satisfied:

$$R_1 > 0,$$

$$R_1R_2 - R_3 > 0,$$

$$R_3 (R_1R_2 - R_3 - R_4^2) > 0,$$

$$R_4 > 0$$

When conditions (27) are not satisfied, the positive steady state is unstable and bifurcations may occur.

### 3. Parameter Inversion

From above section, it is clear that if all the system parameters are given, we can solve the concentration distribution with time and space, which is possible to generate patterns of species distribution with the system evolution. It can be called direct problem. However, some parameters cannot be determined in advance or measured directly. Thus, we need to estimate the parameters via mathematical algorithms by means of data which can be measured. It can be so-called inverse problem, namely, parameter identification. The purpose of the section is to determine the parameters which best fit the simulations to the measurements.
We only consider the above system (6), which can be denoted as follows:

\[
\begin{pmatrix}
    x_{i,j}^{t+1} \\
y_{i,j}^{t+1}
\end{pmatrix} = \begin{pmatrix}
    (x_{i,j}^t)^2 - x_{i,j}^t y_{i,j}^t & \nabla^2 x_{i,j}^t \\
    (y_{i,j}^t)^2 & \nabla^2 y_{i,j}^t
\end{pmatrix} \begin{pmatrix}
r \\
ar
\end{pmatrix} + \begin{pmatrix}
r \\
ar
\end{pmatrix} \begin{pmatrix}
D \\
D
\end{pmatrix}
\]

(28)

or

\[
\begin{pmatrix}
x_{i,j}^{t+1} \\
y_{i,j}^{t+1}
\end{pmatrix} = \begin{pmatrix}
    (x_{i,j}^t)^2 - x_{i,j}^t y_{i,j}^t & \nabla^2 x_{i,j}^t \\
    (y_{i,j}^t)^2 & \nabla^2 y_{i,j}^t
\end{pmatrix} \begin{pmatrix}
r \\
A
\end{pmatrix} + \begin{pmatrix}
D \\
D
\end{pmatrix}
\]

(29)

where \( A = ar \).

Let

\[
U_{1,n} = (x_{i,n})_{m,m},
\]

\[
U_{2,n} = (y_{i,n})_{m,m},
\]

\[
H_1(U_{1,n}, U_{2,n}) = h_1(x_{i,j}, y_{i,j})_{m,m}
\]

\[
= \begin{pmatrix}
    (x_{i,j}^t)^2 - x_{i,j}^t y_{i,j}^t & \nabla^2 x_{i,j}^t \\
    (y_{i,j}^t)^2 & \nabla^2 y_{i,j}^t
\end{pmatrix}
\]

(30)

\[
H_2(U_{1,n}, U_{2,n}) = h_2(x_{i,j}, y_{i,j})_{m,m}
\]

\[
= \begin{pmatrix}
    (y_{i,j}^t)^2 & \nabla^2 y_{i,j}^t
\end{pmatrix}
\]

\[
C = (r, A, D)^T,
\]

where \( C = (c_{i,j}) = (r, A, D)^T \).

System (29) can be represented as the following form:

\[
U_{n+1} = \begin{pmatrix}
    U_{1,n+1} \\
    U_{2,n+1}
\end{pmatrix} = \begin{pmatrix}
    H_1(U_{1,n}, U_{2,n}) \\
    H_2(U_{1,n}, U_{2,n})
\end{pmatrix} C = H(U_n) C.
\]

(31)

Although we obtain the above linear form, \( x_{i,j}^t - (x_{i,j}^t)^2, -x_{i,j}^t y_{i,j}^t \), and \( \nabla^2 x_{i,j}^t \) (or \( y_{i,j}^t - (y_{i,j}^t)^2, -x_{i,j}^t y_{i,j}^t \), and \( \nabla^2 y_{i,j}^t \)) will possess serious collinearity, which will result in the fact that many traditional parameter identification methods, proposed in the past, just like least square method, maximum likelihood method, etc., are not effective. To deal with the problem, some revised parameter identification methods, such as regularization method, have been put forward [24–26]. Here, the regularization method will be used to deal with the parameter identification.

Suppose that \( ||U_{n+1} - U_{n+1}^*|| \leq \delta, \delta > 0, \delta \leq H(U_n) - H^T(U_n) \leq H(U_{n+1}) - H^T(U_{n+1}) \) are real value. The least-squares problem associated with (31) can be written as

\[
\min_{C \in \mathbb{R}^3} \| H(U_n) C - U_{n+1} \|
\]

(32)

which is generally ill-posed based on Hadamard [25].

To overcome the ill-posedness, we can employ the regularization approach to get a best fitted solution. By using the Tikhonov regularization, we can convert problem (32) into the following form:

\[
\min_{C \in \mathbb{R}^3} \left\{ \| H(U_n) C - U_{n+1} \|^2 + \theta \| C \|^2 \right\}
\]

(33)

where \( \theta > 0 \) is regularization parameter. The solution of (33) is given by

\[
C_\theta = (H^T(U_n) H(U_n) + \theta I)^{-1} H^T(U_n) U_{n+1}.
\]

(34)

Table 1: The estimation result when the sample values are generated from rand initial value from \( t = 1 \) to \( t = 100000 \).

<table>
<thead>
<tr>
<th></th>
<th>r</th>
<th>a</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>RV</td>
<td>2.98</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>LS</td>
<td>2.9839</td>
<td>0.5211</td>
<td>0.0947</td>
</tr>
<tr>
<td>RM</td>
<td>2.9758</td>
<td>0.5051</td>
<td>0.0963</td>
</tr>
</tbody>
</table>

Tikhonov regularization, we can convert problem (32) into the following form:

\[
\min_{C \in \mathbb{R}^3} \left\{ \| H(U_n) C - U_{n+1} \|^2 + \theta \| C \|^2 \right\}
\]

(33)

where \( \theta > 0 \) is regularization parameter. The solution of (33) is given by

\[
C_\theta = (H^T(U_n) H(U_n) + \theta I)^{-1} H^T(U_n) U_{n+1}.
\]

(34)

The convergence and feasibility of the Tikhonov regularization have been studied well. Therefore, we no longer discuss the details. Next we will make some numerical experiments to test the effectiveness for the parameter estimation method, namely, generating data computationally for a model with known parameters and then testing our recovery of parameters from the data which is available as panel data.

The test data can be generated by means of iteration computation when \( r = 2.98, a = 0.5, D = 0.1 \), and the small amplitude random perturbation is 1% around the steady state. Firstly, we select the 100000 interaction values of \( x, y \). To confirm the above algorithm’s effectiveness, comparison experiments are done using the regularization method (RM) and least square method (LS) and the numerical result corresponding to the real value (RV) can be found in Table 1.
with the regularization parameter $\theta^* = 1.0875 \times 10^{-6}$. Furthermore, to intuitively reflect the fit goodness, patterns generated from the system whose parameters are obtained by using the regularization method and least square method can be shown in Figure 1.

It may be true that patterns formation can depend on not only fluctuations of system parameters but also variation of the initial conditions. Initial distribution of an immobile reactive species can affect pattern formation. Then we make some numerical experiments to check out the estimation effective when the system parameters are fixed but the initial values are different. We still choose $r = 2.98$, $a = 0.5$, $D = 0.1$, and special initial value is given by

\begin{equation}
    x^0_{i,j} = \begin{cases} 
    \text{rand} \times 0.01 + \frac{(r-1)}{r(a+1)} & \text{if } i = j = \left\lfloor \frac{m}{2} \right\rfloor \\
    0 & \text{other,}
    \end{cases}
\end{equation}

and

\begin{equation}
    y^0_{i,j} = \begin{cases} 
    \text{rand} \times 0.01 + \frac{(r-1)}{r(a+1)} & \text{if } i = j = \left\lfloor \frac{m}{2} \right\rfloor \\
    0 & \text{other,}
    \end{cases}
\end{equation}

Then comparison experiments are also done using the regularization method (RM) and least square method (LS);

<table>
<thead>
<tr>
<th></th>
<th>$r$</th>
<th>$a$</th>
<th>$D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RV</td>
<td>2.9892</td>
<td>0.6735</td>
<td>0.0906</td>
</tr>
<tr>
<td>LS</td>
<td>2.9710</td>
<td>0.5201</td>
<td>0.0924</td>
</tr>
</tbody>
</table>

The estimation result can be found in Table 2. Furthermore, patterns generated from the system whose parameters are obtained by using the regularization method and least square method can be shown in Figure 2. Although there exist interesting spiral structures in Figure 2(a), the symmetry breaking around the fixed point does not emerge in Figure 2(b) and a stable pattern of rhombi shapes can be observed from Figure 2(c). Comparing the results of two experiments, it may be a fact that factors that might affect estimation procedure depend on the initial state which results in different time evolution process and less sample data.

4. Conclusion

We give several concluding remarks in this section.
(1) In this work, the local stability of a positive interior equilibrium for a discrete competitive Lotka–Volterra system model with feedback controls is investigated by means of inner product technique and eigenvalue analysis. Otherwise, the global stability still is not been obtained and the method of global Lyapunov functions may be applied in our further work.

(2) The numerical results obtained demonstrate the feasibility and potential advantages of applying a regularization method to parameter estimation in discrete competitive Lotka–Volterra systems. Some facts can also show that more sample data can increase estimation accuracy relieve colinearity, minute estimation error can result in significant effect, and different patterns can emerge, which may prove the fact that the nonlinear system remarkably depends on the system parameters and initial state.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work was financially supported by the National Natural Science Foundation of China under Grant no. 11371277 and the Cultivation Program for Excellent Youth Teacher in University with Grant no. 507-125RCPY0314, in Tianjin.

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