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Research Article

Exploring General Equilibrium Points for Cournot Model

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In Cournot model, when there are many competitions, the competitive equilibrium becomes chaotic. It is extremely difficult to derive the general equilibrium points. There is no previous research to explore a further problem with the general equilibrium points of n-contenders in Cournot model. In this paper, a general equilibrium Cournot game is proposed based on an inverse demand function. A market spatial structure model is built. Intermediate value theorem, as a realistic method, is introduced to handle a general competitive equilibrium in Cournot model. The number and stability in general equilibrium points are detected by means of celestial motion theory and spatial agglomeration competition model. The existence of general equilibrium points and the stability of Cournot equilibrium points, which are new and future complement of previously known results. Numerical simulations are given to support the research results.

1. Introduction

There is no previous research using mean value theorem approach to handle a further problem with the general equilibrium points of n-contenders in the Cournot model. The case of economists generally divides market structure into four types: monopolies, monopolistic competition, oligopolies, and complete competition. This article studies incomplete market according to oligopoly market. Auguste Cournot (1838) [1] introduced a model to analyze the market structure of oligopoly, which locates between monopoly and perfect competition, and now it has become a very important concept in microeconomics. Later, many scholars study Cournot model from many different aspects. There are two different topics involved: (a) the Couront equilibrium must exist with the competitive equilibrium when the number of firms is finite, and, (b) with the increasing number of firms, the competing equilibrium state must be stable. By now, the second question raised grave doubts. In this paper, we only consider the equilibrium point of oligopoly market structure. When there are more firms in the Cournot model, how to find the general stable competitive equilibrium point and how many general competitive equilibrium points there are, which is stable or unstable point. In [2], the authors consider that there exists a stable equilibrium point in the Cournot duopoly model. Is this the general equilibrium point? That is not defined. They analyze the existence of solutions of the model in the form of nonlinear second-order difference equation. R. D. Theocharis (1960) [3] research shows when there are two sellers in the Cournot model, the Cournot solution is always dynamically stable; if the number is three, the result shows we shall get finite oscillations about the equilibrium district. If the number is greater than three, the state will always be instability. In CLSC competitive system is composed of one manufacturer and one retailer, which will be easier to enter into chaos [4]. In [5], a dynamic duopoly Cournot game is studied from different adjustment mechanisms and expectations. Anming Zhang and Yimin Zhang (1996) [6] derived stability conditions for a multiproduct duopoly. The work [7] analyzes oligopolistic markets, and it proves existence of nontrivial fulfilled-expectations equilibrium. Tönu Puu (1996) [8] simulated the dynamic process with three oligopolists. Though it is unstable, the process found the Cournot equilibrium point of three oligopolists. If the Cournot equilibrium exists, it is stable; Tönu Puu (2008) [9] derived the equilibrium district.

H. N. Agiza (1998) [10] research showed that there exist stable fixed points in the model of three competitors. A dynamic repeated game system composed of two manufacturers and one retailer is investigated [11]. In a dynamic Cournot model, the limiting production level of the game is a random variable [12]. When competitor is four, the state of equilibrium is still stable. A dynamic four-oligopolist game with different expectations is modeled by four-dimensional nonlinear difference equations. The result shows that four-oligopolist game model behaves chaotically with the variation of the parameters (Xiaosong Pu and Junhai Ma (2013) [13]. E. Ahmed and H. N. Agiza (1998) [14] tried to find the fixed point of n-competitors by the pq = 1 model. The dynamical system of n-competitors in a Cournot game is derived, and the stability of its fixed point is studied. The fixed point is relative to c (cost) and n. When the number of competing firms becomes arbitrarily large, prices converge to marginal costs [15]. But, so far, we are unable to derive a general result relating to stability and the number n of competitors. Many researchers have also paid a great attention to the controlling dynamics of Cournot games, such as Agiza and Elsadany [16], Chen and Chen [17], Holyst and Urbanowicz [18], and Elabbasy et al. [19]. This paper presents a new Cournot duopoly game. The main advantage of this game is that the outputs are nonnegative for all times. The chaotic behavior of the Cournot duopoly game has been stabilized on the Nash equilibrium point by using delay feedback control method (H. N. Agiza, A. A. Elsadany, and M. M. El-Dessoky (2013) [20]. Junhai Ma and Fang Wu (2014) [21] explored a discrete triopoly dynamical game from three aspects, and the parameters of bounded rationality can evoke chaos in identical market.

Another related body of literature deals with Cournot competition in spatial models. Static spatial Cournot-Nash oligopoly models were formulated on optimization and variational inequality theories (Harker, 1986) [22]. Anderson and Neven (1991) and Hamilton, Thisse, and Weskamp (1989) [23] study spatial discrimination under Cournot oligopoly where locations are strategic choice variables. These papers establish that Cournot competition yields spatial agglomeration. In S. P. Anderson and D. J. Neven work (1991) [24], the research of their article shows the competition between Cournot oligopolists that discriminate over space leads to spatial agglomeration. At the same time, the authors implicate that firms do not necessarily earn supernormal profit by the free-entry equilibrium state. Byung-Wook Wie and Roger L. Tobin (1997) [25] developed a methodology for the study of dynamic oligopolistic competition in serving spatially separated markets. In [26], in spatial markets, locations and quantities are selected sequentially. Many firms are agglomerate equidistantly from each other in a circular city. Numerous firms agglomerate at the center in a linear city. Corrado Benassi (2014) [27] explored the spatial Cournot competition with two firms and shows the existence of dispersion equilibrium, the existing state of which is not unimodal and that asymmetry is not too stronger than normal distribution. The behavior of the tow- and three-group is compared with that of tow-firm and three-firm to shed light on roles of the number of the firms. The local asymptotic behavior of the system is identical with that of the adaptive adjustment in the Cournot model (Akio Matsumoto and Ferenc Szidarovszky (2011) [28]. The work [29] shows symmetric two-player oligopoly game over continuous action spaces can be extended to a population game in which each player interacts with a subset of the entire population.

However, so far, we are not in a position to derive a general result relating stability and instability to the number n of competition in the Cournot model. The aim of our study is how to find the general stable and unstable equilibrium points with a method in the Cournot model. This article studies further Cournot model between the district of the two points of monopoly and complete competition. The district that we define is chaos. Whether or not the chaotic district is stable, we will do our utmost to find at least a fixed equilibrium point. How many points are in the chaotic district: that is the interesting point of this article. Our result shows that we find the general equilibrium points in that chaotic district. Although the research seems slight, it may have remarkable effects on the stability of the system.

2. Model Formulation

In oligopoly competition, we may find inner optional equilibrium production of two firms in the Cournot model. Now, we assume the total production of two firms: $Y = y_1 + y_2$. They produce homogeneous product. On the similar level of product, we define market price: $P(Y) = P(y_1 + y_2)$, which is the inverse demand function. The cost function of any firm is assumed $c_i(y_i)$, i=1, 2 (we omit the factor prices in the cost function because we will assume that they are fixed). Then, the most profit of firm i may be denoted

$$\max \mathbf{\Pi_i}\left(\mathbf{y_1},\mathbf{y_2}\right) = P\left(y_1 + y_2\right) y_i - c_i\left(y_i\right) \tag{1}$$

The condition of inner most optimal profit per firm is satisfied with Cournot model:

$$\begin{aligned}\frac{\partial \pi_i\left(y_1 + y_2\right)}{\partial y_i} &= P\left(y_1 + y_2\right) + P'\left(y_1 + y_2\right) y_i - c_i'\left(y_i\right) \\ &= 0, \quad i = 1, 2\,(2.2)\end{aligned} \tag{2}$$

$$\begin{aligned}\frac{\partial \pi_i\left(y_1 + y_2\right)}{\partial y_i^2} &= 2P'\left(y_1 + y_2\right) + P''\left(y_1 + y_2\right) y_i \\ &\quad - c_i''\left(y_i\right) \ll 0, \quad i = 1, 2\end{aligned} \tag{3}$$

If there are n firms in the market, formula (2) is also denoted:

$$\mathrm{P}\left(\mathrm{Y}\right)\left[1 + \frac{d_p}{d_y}\frac{y_i}{p}\right] = c_i'\left(y_i\right), \quad i = 1, 2 \cdots n \tag{4}$$

where

$m_i = y_i / y$ and m_i is market share of i firm's production in whole output, so

$$\mathrm{P}\left(\mathrm{Y}\right)\left[1 + \frac{d_p}{d_y}\frac{y}{p}m_i\right] = c_i'\left(y_i\right) \tag{5}$$

where

$$\frac{1}{\varepsilon} = \frac{d_p}{d_y}\frac{y}{p} \tag{6}$$

where $1/\varepsilon$ is the derivative of elasticity of market demand.

$$\mathrm{P}(\mathrm{Y})\left[1+\frac{m_i}{\varepsilon_i}\right]=c_i'(y_i) \tag{7}$$

Proposition 1. *When $m_i \longrightarrow 0$, $\mathrm{P(Y)} = C_i'(y_i)$.*

In this situation, the profit of market does not exist. Every firm in the market is an infinitesimal share of the market, which becomes a completely competitive market. The Cournot equilibrium is close to a completely competitive equilibrium.

Proposition 2. *When $m_i = 1$, we have exactly the monopoly market condition. There is only one company on the market. The question is raised. When the number of firms is little, we might find the equilibrium production of competition in the Cournot model. In reality, some firms locate between monopoly and complete competition structure. Whether or not we can find the optimal quantity between them is what we will study.*

Definition 3. Any firm i has common cost

$$\mathrm{C}(q_j)=\mathrm{c}q^j \tag{8}$$

Definition 4. The inverse demand function is expressed as

$$\mathrm{P}(q^j)=\mathrm{a}-\mathrm{b}\sum_{j=1}^{n}q^j \tag{9}$$

and *parameter* is

$$\begin{aligned} &a>0,\\ &b>0,\\ &a>c. \end{aligned} \tag{10}$$

According to (8) and (9), the profit of firm j is

$$\mathrm{f}(\mathrm{Q})=\Pi^j(q^1,\cdots q^n)=\left(a-b\sum_{k=1}^{n}q^k\right)q^j-cq^j \tag{11}$$

(11) is in accordance with Lagrange's mean value theorem.

Lemma 5. *When $j = k = 1$, it is only a firm in the market. It belongs to a monopoly firm.*

$$\mathrm{f}(\mathrm{Q})=\mathrm{f}(\mathrm{q})=-\mathrm{b}q^2+(a-c)q \tag{12}$$

Now, we will find the most value by taking partial derivative of $f(\mathrm{Q})$:

$$\mathrm{f}'(\mathrm{Q})=\mathrm{f}'(\mathrm{q})=0 \Rightarrow q=\frac{a-c}{2b} \tag{13}$$

Lemma 6. *$j = k = n$, and there are more firms in the market, which is completely competitive market.*

Formula (11) is taken partial derivative:

$$\mathrm{a}-2\mathrm{b}q^j-b\sum_{k\neq j}^{n}q^k-c=0 \Longrightarrow \mathrm{b}q^j=a-c-b\sum_{k=1}^{n}q^k \tag{14}$$

Definition 7. $\overline{q}$ is average production. Equation (14) also is denoted:

$$\mathrm{b}\overline{q}=a-c-nb\overline{q} \Rightarrow \overline{q}=\frac{a-c}{b(n+1)} \tag{15}$$

Proposition 8. *When $n \longrightarrow \infty$*

$$\lim_{n\longrightarrow\infty}q=\lim_{n\longrightarrow\infty}n\overline{q}=\lim_{n\longrightarrow\infty}\frac{n(a-c)}{b(n+1)}=\frac{a-c}{b} \tag{16}$$

3. A General Equilibrium Point

Based on Lagrange's mean value theorem, consider the following.

Assumption 9. $f(x)$ is a function defined on a closed interval $[\mathrm{m},\mathrm{n}]$ $(m < n)$. Then, there exists z in the open interval (m,n). The derivative of $f(x)$ at z equals the difference quotient $\triangle f(m,n)$.

More explicitly,

$$\triangle f(m,n)=\mathrm{f}'(\mathrm{z})=\frac{f(n)-f(m)}{n-m} \tag{17}$$

In this article, we construct a market space structure model. The point m is defined as a monopoly market structure. The point n is on behalf of a perfect competitive market structure. Cournot competition locates between m and n. Some studies show there are many equilibrium points in the district. In this model, at least a general equilibrium point is derived. The model can provide acquiring optimal production for firms in incomplete competition market, such as Figure 1.

Definition 10. q_m is the middle value of Cournot model, in which we can find q_m on Lagrange's mean value theorem.

There is $\mathrm{m} = (a-c)/2b$; $\mathrm{n} = (a-c)/b$. On the closed interval $[(a-c)/2b,(a-c)/b]$

$$\mathrm{f}'(z)=\mathrm{f}'(q_m)=\frac{f(n)-f(m)}{n-m}=-\frac{a-c}{2} \tag{18}$$

Then, we derive a general equilibrium point of Cournot q_m:

$$-2\mathrm{b}q_m+(a-c)=-\frac{a-c}{2} \Rightarrow q_m=\frac{3(a-c)}{4b} \tag{19}$$

In the space where the oligarch enterprises gather with a three-dimensional dynamic system, the stable state of q_m is a complex surface. The surface is attractor, such as Figure 2.

It is interesting after deduction that we find a general equilibrium point q_m between monopoly and complete market structure in the Cournot model.

Equilibrium price is

$$p_m=\frac{a+3c}{4} \tag{20}$$

Equilibrium profit is

$$\Pi_m=\frac{3(a-c)^2}{16b} \tag{21}$$

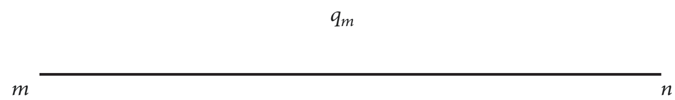

Figure 1: Market space structure model.

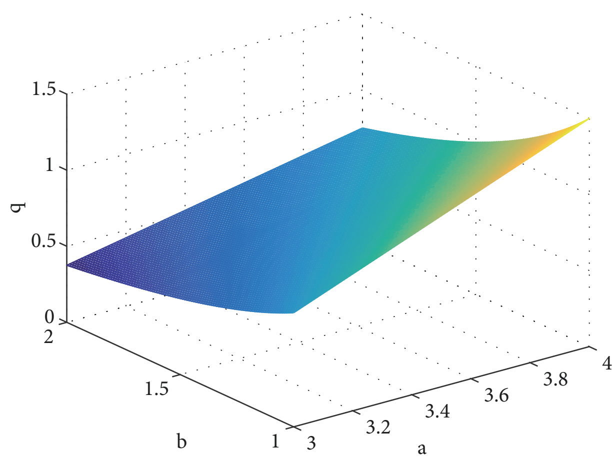

Figure 2: q_m distribution state.

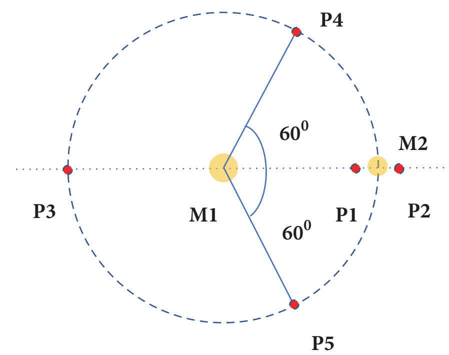

Figure 3: Dynamic spatial agglomeration competition model.

If it has only a firm in the market (monopoly firm), its production is q.

$$\mathrm{q} = \frac{a-c}{2b} \tag{22}$$

Its price is denoted

$$\mathrm{p} = \mathrm{a} - \mathrm{bq} \tag{23}$$

Its profit is also denoted

$$\Pi_{oligopoly} = \frac{(a-c)^2}{4b} \tag{24}$$

Obviously $\Pi_{oligopoly} > \Pi_m$, $q_m > q$, which shows that the profit of the monopoly market is the greatest than others. Also, the production is relatively small. When a firm enters into a relative market, the output of enterprises can be referred to the general equilibrium production q_m.

4. Equilibrium Points and Stability

In imperfect competition markets, firms hope to find the stable competitive scenario, which may maximize the utility of the company. The stable competitive case is also called Nash equilibrium. In [30], partial privatization and cross-ownership were introduced to stabilize multiproduct mixed duopoly market. In this paper, we have found that there exists at least a general equilibrium point in the Cournot model. How many stable or unstable equilibrium points in Cournot model, which is handled by Lagrangian points on theory of space mechanics. We construct a dynamic spatial agglomeration competition model, such as Figure 3. In this model, all firms with identical products compete with each other in the space, like celestial bodies motion. The result of competition converges at several points, and there are only a few companies left in the market at last. Detecting stable or unstable equilibrium points, it can provide firms for competitive optimal production strategy. In addition, the limited resources of an enterprise are utilized more effectively. Finally, market competition achieves a win-win situation. Effective competition promotes economic development and enriches people's lives.

Assumption 11. There is an invisible force that brings together many of the similar companies in the market. This phenomenon is like the motion of the celestial body. We may use Lagrange points to find the general stable and unstable equilibrium points of Cournot model in the incomplete market.

Assumption 12. There are two market structure, such as M1 and M2. M1 represents the monopoly market. M2 is the perfect competition market.

It shows five Lagrangian points in Figure 3. They are P1, P2, P3, P4, and P5. M1, M2, P1, P2, and P3 are on the same line. The P1 point lies on the line defined by the two market structures M1 and M2 and between them, the one where the market attraction of M2 partially cancels M1's market attraction. The P2 point lies on the line through the two large markets, beyond the smaller of the two. Here, the market forces of the two large markets balance the centrifugal effect on a body at P2. The P3 point lies on the line defined by the two large markets, beyond the larger of the two. The P4 and P5 points lie at the third corners of the two equilateral triangles in the plane of orbit whose common base is the line between the centers of the two markets, such that the point lies behind (P5) or ahead (P4) of the smaller market with regard to its orbit around the larger market. Based on Lagrangian points, P1, P2, and P3 are stable points. P4 and P5 are unstable points.

P1 is the solution to the following equation, balancing market force:

$$\frac{M1}{(R-r)^2} = \frac{M2}{r^2} + \frac{M1}{R^2} - \frac{r(M1+M2)}{R^3} \tag{25}$$

where r is the correlation coefficient of P1 with the smaller market. R is structure coefficient between the two main markets, and M1 and M2 are the masses of the large and small market, respectively. From (25), given by

$$r \approx R\sqrt[3]{\frac{M2}{3M1}}, \tag{26}$$

P2 is the solution to the following equation, balancing market force:

$$\frac{M1}{(R+r)^2} + \frac{M2}{r^2} = \frac{M1}{R^2} + \frac{r(M1+M2)}{R^3} \tag{27}$$

with parameters defined as for the P1 case.

$$r \approx R\sqrt[3]{\frac{M2}{3M1}} \tag{28}$$

P3 is the solution to the following equation, balancing market force:

$$\begin{aligned} &\frac{M1}{(R-r)^2} + \frac{M2}{(2R-r)^2} \\ &= \left(\frac{M2}{M1+M2}R + R - r\right)\frac{(M1+M2)}{R^3} \end{aligned} \tag{29}$$

with parameters defined as for the P1 and P2 cases. r now indicates how much closer P3 is to the more massive market than the smaller market.

$$r \approx R\frac{7M2}{12M1} \tag{30}$$

The reason five points are in balance is that, at P4 and P5, the locations to the two markets are symmetric. Now, we may derive the value of five points, respectively, such as Figure 3.

$$\begin{aligned} P1 &= \left(R\cdot\left(1-\sqrt[3]{\frac{M2}{3M1}}\right), 0\right) \\ P2 &= \left(R\cdot\left(1+\sqrt[3]{\frac{M2}{3M1}}\right), 0\right) \\ P3 &= \left(-R\cdot\left(1+\frac{5M2}{12M1}\right), 0\right) \\ P4 &= \left(\frac{R}{2}\cdot\left(\frac{M1-M2}{M1+M2}\right), \frac{\sqrt{3}}{2}R\right) \\ P5 &= \left(\frac{R}{2}\cdot\left(\frac{M1-M2}{M1+M2}\right), -\frac{\sqrt{3}}{2}R\right) \end{aligned} \tag{31}$$

Assumption 13. We may make M1=n and M2=m.

Based on Lagrangian points, consider the following.

Substituting M1=n and M2=m back into formula, finally we may derive Cournot general equilibrium stable and unstable points. P1, P2, and P3 are general unstable equilibrium points. P4 and P5 are general stable points.

$$\begin{aligned} P1 &= \left(R\cdot\left(1-\sqrt[3]{\frac{1}{6}}\right), 0\right) \\ P2 &= \left(R\cdot\left(1+\sqrt[3]{\frac{1}{6}}\right), 0\right) \\ P3 &= \left(-R\cdot\left(\frac{29}{24}\right), 0\right) \\ P4 &= \left(\frac{R}{6}, \frac{\sqrt{3}}{2}R\right) \\ P5 &= \left(\frac{R}{6}, -\frac{\sqrt{3}}{2}R\right) \end{aligned} \tag{32}$$

From the above, the stable and unstable points are derived. P1, P2, and P3 are a monopoly equilibrium separately. The monopoly points are unstable points. P4 and P5 are imperfect competition equilibrium, which are stable points. The result shows that all firms compete with each other in identical market. The equilibrium points will converge to the point of P4 or P5 finally. Under the surroundings, the competition is stable. We can derive the equilibrium output, the equilibrium price, and the equilibrium profit further.

5. Concluding Remarks

In addition, these findings provide additional information about competitive equilibrium in incomplete competition market. Many articles imply much vertex points in chaotic district of the Cournot model. In this paper, a fixed equilibrium point is as least derived in the Cournot model by Lagrange's mean value theorem. At the same time, general stable and unstable equilibrium points are found in the Cournot model in incomplete competition market. Three points are unstable points. Two points are stable points. As long as we know the unit cost and the product function of a competitive firm, we can calculate the general equilibrium price and the general equilibrium output. When a firm will be about to enter into a market, this article provides a theoretical foundation how to make itself obtain the optimal profit.

We establish a market structure concept model to explain the chaotic district of Cournot model. Analyzing the uniform linear inverse demand of Cournot model, we, respectively, calculate the production of monopoly and the production of perfect competition. By profit function and Lagrange's mean value theorem, at least an equilibrium point exists. According to the celestial motion theory, we find the general stable and unstable equilibrium points in the Cournot model in incomplete competition market by Lagrange's point. When we substitute the equilibrium price into the profit function, at last, we find that the profit value is the biggest in monopoly structure, which is according to the real market.

Data Availability

The data used to support the findings of this study are included within the article.

Disclosure

The authors claim that none of the material in the paper has been published. The authors are in charge of any mistake that may occur in this paper.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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