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Research Article

Study on Single Cycle Production Allocation and Supply Strategy for DCEs Based on the CVaR Criterion

Leiyan Xu,[1,2] **Zhiqing Meng**,[1] **Gengui Zhou**,[1] **Yunzhi Mu**,[1,3] **and Minchao Zheng**[1]

[1] *College of Economics and Management, Zhejiang University of Technology, Liuxia, Hangzhou 210023, China*
[2] *Zhejiang University of Finance and Economics, Xiasha, Hangzhou 310018, China*
[3] *Keyi College of Zhejiang Sci-Tech University, Shaoxing, Zhejiang 312369, China*

Correspondence should be addressed to Zhiqing Meng; mengzhiqing@zjut.edu.cn

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Direct chain enterprises (DCEs) face a decision-making issue as to how to allocate and supply their products to their stores for sales with the minimum losses and maximum profits for the manufacturers. This paper presents a single-cycle optimal allocation model for DCEs under the given total production amount and conditional value at risk loss. The optimal strategy for production allocation and supply is derived. Subsequently, an approximate algorithm for solving the optimal total production amount is presented. The optimal allocation and supply strategy, the minimum total production amount, the minimum allocation strategy, and the discount pricing strategy are obtained for the single cycle. Finally, with the sales data of a food DCE, numerical results corroborate that adopting different production and supply strategies reduces the risk of expected losses and increases the expected return. It is of an important theoretical significance in guiding the production and operation of direct chain enterprises.

1. Introduction

A direct chain enterprise (DCE) is the one whose chain stores are directly operated by the head office of the DCE; that is, the head office implements the unified management of its staff, finances, materials, business, logistics, and information flows for each of its chain stores. Wal-Mart, Carrefour, Starbucks, Uniqlo, Zara, Casamiel, and other companies are typical direct chain enterprises. The production allocation and supply of DCEs has always been a key and difficult problem that must be solved especially for clothing and food enterprises. The phenomenon of oversupply or insufficient production is particularly common. How to provide scientific production allocation and supply strategies has become urgent for the survival of chain enterprises.

Few theoretical researches on the production allocation and supply model for DCEs are available, although there are similar studies on centralized inventory problem =for suppliers and retailers. The idea of the centralized inventory originated in the 1960s. Centralized inventory refers to the fact of a company to provide services from one inventory point to its multiple markets with uncertain demands. Moreover, centralized inventory can effectively reduce the volatility of demands, reduce operating costs, and increase profits. The current research mainly focuses on the affirmation that suppliers provide the centralized supply to several retailers. The relationship between suppliers and retailers is hardly a chain relationship, and the sales and inventory levels among retailers affect each other. The research on the centralized production supply model of DCEs is almost nonexistent. Given that the centralized supply plays an important role in the rapid development of e-commerce or offline direct-operated stores, its impact on enterprises' supply chain management and operation management has earned considerable attention from the academic community. Therefore, studying the production allocation and supply strategy for DCEs' the centralized supply is necessary.

The research methods used in studying the centralized inventory are generally divided into two categories. One is quantitative research methods, such as mathematical

optimization algorithms, Lagrangian relaxation algorithms, stochastic programming algorithms, and etc. As the model becomes more complex, scholars continue to develop new methods with the aid of computer software. The other is empirical and case study methods used to verify whether the companies' inventory performances, inventory turnover, and costs are improved. For examples, Eppen (1979) [1] compared the centralized inventory with the decentralized inventory for the first time, where it is concluded that the expected inventory holding costs in the centralized inventory model are lower than those of the decentralized inventory system when the demand is normally distributed. After that, Chang and Lin (1991) [2] concluded that the inventory and out-of-stock costs of a centralized inventory are lower than those of a decentralized inventory by different methods where demands follow any distribution. Chen and Lin (1989) [3] considered the centralized inventory problem of Poisson distribution. Cai and Du (2009) [4] considered the centralized inventory problem when the demand is of any distribution. Kevin (1999) [5] proved that the operation costs can be reduced by a centralized inventory when demands are uncertain. Cherikh (2000) [6] studied the centralized inventory system for single-period, single-product, multi-inventory points and demonstrated that centralized inventory can increase the expected profits and reduce the risk caused by uncertain demand. Netessin and Rudi (2003) [7] discussed the substitutability of products and the centralized inventory problem in the case of random demands. Benjaafar (2011) [8] studied the problem of the centralized inventory efficiency. Kemahlīogluzīya and Bartholdi (2011) [9] studied the profit distribution in a centralized system by cooperative game theory, proved that the centralized supply chain can reduce the cost, and analyzed the impact of demand changes on retailers' profits. Swiney (2012) [10] studied the purchasing behavior in centralized inventory, where customers usually buy the discounted products at the end of season and give up the purchase at high price such that companies can choose a centralized or a decentralized inventory. Yang et al. (2014) [11] studied joint inventory and pricing strategies in case of high volume orders. Hossinifard and Abbasi (2018) [12] studied the application of the centralized inventory management in hospital blood management, where they validated that centralized inventory is a key factor in the blood supply chain and can improve the sustainability of the blood supply chain. Shivagi Viral Thakker (2018) [13] proved that centralized inventory is better than decentralized inventory in FFCG industry and that centralized inventory can reduce the bullwhip effect and achieve the maximum profits. Christoph Weskamp (2018) [14] proposed a model of two-stage random integer programming and analyzed the centralized production allocation plan to reduce risk. In sum, the above literatures affirmed that centralized inventory management can not only reduce the risk for enterprises or supply chain, but also increase the profits of enterprises. However, research on centralized inventory mainly focuses on non-DCEs; i.e., research on the centralized supply production and allocation strategies of DCEs has not been seen before. Only part of the research considers the interacting allocation problem, and the centralized supply model [1–6] is based on allocation to retailers. Models focus on centralized inventory management, and some model assumptions are not applicable to production allocation and supply strategy for modern DCEs.

The production allocation and supply of the DCE is a risk decision problem. The expected mean loss model is a decision model usually used for risk neutral situation. When the change in demand is relatively large, the expected risk loss decision model cannot measure very well the loss of demand fluctuations. In the supply chain risk ordering decision problem, many people adopt a risk approach, that is, establishing the ordering model of supply chain based on conditional value at risk (CVaR) (2000) [15]. If a fat tail occurs in radon risk distribution, the accumulated losses are extremely high. Hence, the expected mean loss model cannot reflect such distribution, while CVaR can solve this problem. For instance, the oversupply of the market is the main reason for the huge losses of DCE due to insufficient demands, which makes avoiding such losses difficult for DCEs. Therefore, solving the production and supply problems of DCE by CVaR—a risk criteria— is necessary. Alexander et al. (2004) [16] compared VaR with CVaR in portfolio investment. They also verified that CVaR is more effective than VaR and mean model in measuring high risk. CVaR has been widely used in risk decision models since 2007. Many studies have applied conditional value at risk to supply chain ordering research. For instance, Gotoh and Takano (2007) [17] established a single-cycle newsboy CVaR model to determine the best ordering strategy. Zhou and Chen (2008) [18] constructed a single-cycle multiproduct CVaR ordering model. Wu et al. (2013,2014) [19, 20] studied the risk-averse CVaR newsboy model. Xu and Li (2010) [21] established the CVaR newsboy model. Xu and Meng (2015) [22] proposed a risk-averse CVaR loss newsboy model. Xue et al. (2015) [23] established the CVaR model. Xu and Meng (2016) [24] considered the CVaR newsboy ordering model with profit losses. The above literatures mainly considered newsboy models with orders from retailers to suppliers where wholesale prices apply. These models are not suitable for production and supply decision-making problems of DCEs. DCEs must decide the product supply that should be distributed to their chain stores for sales, and it is their goals to achieve an expected sales the same as the production volume with a maximum total profit or a minimum loss, where wholesale prices and recovery price do not apply. Several studies on the previous newsboy models are order decision problems for single retailer, which are rarely in the DCEs. In recent year, market competition has been fierce, and product oversupply has been widespread. Hence, the risk decision-making problem of production allocation and supply for DCEs is urgently needed, which avoids the loss of the companies due to the inconsistency between production and sales.

Therefore, in order to solve the risk loss problem of DCEs' production allocation and supply, we establish a single-cycle optimal production allocation and supply model based on CVaR losses which derive an optimal allocation strategy with the total production volume for DCEs and obtain the minimum allocation strategy under the maximum CVaR risk loss. By the sales data of a food direct chain company, the numerical results corroborate that using the optimal

supply strategy and discount strategy can reduce the expected risk loss and increase the expected profit. This paper is of important theoretical significance on guiding the centralized production and operation of DCEs.

The rest of this paper is organized as follows. Section 2 presents a new model of optimal production allocation and supply for DCEs and the optimal production allocation solution. Section 3 deals with the sensitivity analysis of the optimal production allocation and supply strategy. Section 4 proposes an approximate algorithm of the production allocation and supply strategy based on the above model. Section 5 presents a numerical example, which affirms that the optimal production allocation and supply strategy for the food DCE is efficient. Section 6 concludes the paper.

2. Model of Optimal Production Allocation and Supply

The fact that every chain store faces a supply loss must be taken into consideration by DECs. Given its capacity, the DCE must decide the production volume that should be produced and supplied to all its direct chain stores to reach an optimal allocation and the minimum total loss, which is very difficult.

It is assumed that the number of DCES' chain store is limited and every store sells the identical products. All products are supplied once in a single-cycle, and no supplementary is provided in the single period. A sale cycle is divided into two periods: the normal price period and the discount period. Hence, products are sold at a uniform retail price during the normal price period. After the normal price sale period, each retail store checks the inventory of its remaining products and decides whether the discount price will be used according to the inventory. When the inventory is smaller than the given value (minimum inventory), the remaining products are not discounted for sale. Otherwise, the remaining products are sold at the discounted price, which is determined according to the number of the remaining products.

After our field survey, the above description is consistent with the business model of many DCEs. Hence, assume: a be the retail price of the product; c_i the cost price (including the transportation, operation, and storage fee), where $a > c_i$; x_i the allocation assigned to the i-th store; e_i the unit discount price of the remaining unsold products, where $e_i > 0$ indicates that disposal fee is required and products have no residual value, $e_i < 0$ means that products have the residual value and are sold at discount price during the discount sales period, or $e_i = 0$ indicates that the value of remaining unsold products is zero. Let Q be the total production supply volume; hence, we have $x_1 + x_2 + \cdots + x_m = Q$. Let ξ_i be the random demand from the i-th chain store, with corresponding probability density $p_i(\xi_i)$ and probability distribution function $P_i(\xi_i)$, where $P_i(\xi_i) = 0 (\xi_i \le 0)$, $P_i(+\infty) = 1$. The loss function of the i-th chain store can be expressed as follows:

$$f_i(x_i, \xi_i) = (c_i + e_i)(x_i - \xi_i)^+ + (a - c_i)(\xi_i - x_i)^+, \quad (1)$$

where $(c_i + e_i)(x_i - \xi_i)^+$ is the loss of oversupply and $(a - c_i)(\xi_i - x_i)^+$ is the loss of short supply, $c_i + e_i \ge 0$.

Based on CVaR [15], we establish a single-cycle CVaR loss model of production allocation for the DCEs. Let the probability distribution $\Psi_i(x_i, z_i)$ where the loss function $f_i(x_i, \xi_i)$ of the i-th store is less than the given loss z_i:

$$\Psi_i(x_i, z_i) = P\{f_i(x_i, \xi_i) \le z_i\} = \int_{f_i(x_i,\xi_i)\le z_i} p_i(\xi_i)\, d\xi_i. \quad (2)$$

Let $\alpha_i \in (0, 1)$ be the given confidence level, then we define the VaR loss value of the i-th store:

$$z_i^*(x_i) = \min\{z_i \mid \Psi_i(x_i, z_i) \ge \alpha_i\}. \quad (3)$$

$z_i^*(x_i)$ is maximum possible loss of supply x_i to the i-th store under the confidence level α_i. Hence, the CVaR loss at $z_i^*(x_i)$ is defined by the following:

$$\Phi_i(x_i, z_i^*(x_i)) = \frac{1}{1-\alpha_i} \int_{f_i(x_i,\xi_i)\ge z_i^*(x_i)} f_i(x_i, \xi_i)\, p_i(\xi_i)\, d\xi_i. \quad (4)$$

$\Phi_i(x_i, z_i^*(x_i))$ is the cumulative loss of supply x_i to the i-th store under the confidence level α_i. According to [15], (4) can be solved by the following loss function:

$$F_i(x_i, y_i) = y_i + \frac{1}{1-\alpha_i} \int_0^{+\infty} [f_i(x_i, \xi_i) - y_i]^+ p_i(\xi_i)\, d\xi_i. \quad (5)$$

If there is only one optimal solution to $\min_{y_i \in R^1} F_i(x_i, y_i)$, then $\min_{y_i \in R^1} F_i(x_i, y_i) = \Phi_i(x_i, z_i^*(x_i))$.

Let w_i ($w_i > 0$) be the loss weight of the i-th store. Then the expected loss based on CVaR of the i-th store can be expressed as

$$w_i F_i(x_i, y_i) = w_i \left(y_i + \frac{1}{1-\alpha_i} \int_0^{+\infty} [f_i(x_i, \xi_i) - y_i]^+ p_i(\xi_i)\, d\xi_i \right). \quad (6)$$

Hence, the total loss model based on CVaR of a DCE can be expressed as

$$(DCEP) \quad \min \sum_{i=1}^{m} w_i F_i(x_i, y_i) \quad \text{s.t.} \quad x_1 + x_2 + \cdots + x_m = Q, \quad y \in R^m, \quad (7)$$

where $x = (x_1, x_2, \cdots, x_m)^T$, $y = (y_1, y_2, \cdots, y_m)^T$. We prove the following.

Theorem 1. *If there is a λ^* satisfying: $-w_i(c_i + e_i) \le \lambda^* \le w_i(a - c_i)$ and $\sum_{i=1}^m x_i(\lambda^*) = Q$ for $i = 1, 2, \cdots, m$, then the optimal allocation and supply strategies to (DECP) are*

$$x_i(\lambda^*) = \frac{a - c_i}{a + e_i} P_i^{-1}\left(\frac{(1-\alpha_i)(a - c_i - \lambda^*/w_i)}{a + e_i} + \alpha_i \right) + \frac{c_i + e_i}{a + e_i} P_i^{-1}\left(\frac{(1-\alpha_i)(a - c_i - \lambda^*/w_i)}{a + e_i} \right) \quad (8)$$

Proof. The Lagrangian function of ($DCEP$) can be expressed as

$$L(x, y, \lambda) = \sum_{i=1}^{m} w_i F_i(x_i, y_i) + \lambda(x_1 + x_2 + \cdots + x_m - Q), \tag{9}$$

where λ is Lagrangian parameter. Then, with $\partial L/\partial x_i = 0$ and $\partial L/\partial y_i = 0$, we can get $w_i \partial F_i(x_i, y_i)/\partial x_i + \lambda = 0$, so

$$\begin{aligned} F_i(x_i, y_i) &= y_i + \frac{1}{1-\alpha_i}\int_0^{+\infty}\left[(c_i+e_i)(x_i-\xi_i)^+ + (a-c_i)(\xi_i-x_i)^+ - y_i\right]^+ p_i(\xi_i)\,d\xi_i = y_i \\ &+ \frac{1}{1-\alpha_i}\left(\int_0^{x_i}\left[(c_i+e_i)(x_i-\xi_i)-y_i\right]^+ p_i(\xi_i)\,d\xi_i + \int_{x_i}^{+\infty}\left[(a-c_i)(\xi_i-x_i)-y_i\right]^+ p_i(\xi_i)\,d\xi_i\right). \end{aligned} \tag{10}$$

Three situations are discussed as follows.

(1) When $y_i \le 0$, (10) can be expressed as

$$\begin{aligned} F_i(x_i, y_i) &= y_i + \frac{1}{1-\alpha_i}\left(\int_0^{x_i}\left[(c_i+e_i)(x_i-\xi_i)-y_i\right] p_i(\xi_i)\,d\xi_i + \int_{x_i}^{+\infty}\left[(a-c_i)(\xi_i-x_i)-y_i\right] p_i(\xi_i)\,d\xi_i\right) = y_i \\ &+ \frac{1}{1-\alpha_i}\left[(c_i+e_i)\int_0^{x_i} P_i(\xi_i)\,d\xi_i + (a-c_i)\int_0^{x_i} P_i(\xi_i)\,d\xi_i - (a-c_i)x_i\right] - \frac{y_i}{1-\alpha_i}. \end{aligned} \tag{11}$$

So, $\partial F_i/\partial x_i = (1/(1-\alpha_i))[(a+e_i)P_i(x_i)-(a-c_i)]$ and $\partial F_i/\partial y_i = -\alpha_i/(1-\alpha_i) < 0$. We know that $F_i(x_i, y_i)$ is monotonic decrease with y_i, and $\partial^2 F_i/\partial x_i^2 = (1/(1-\alpha_i))[(a+e_i)p_i(x_i)] \ge 0$. So, we have

$$\begin{vmatrix} \dfrac{\partial^2 F_i}{\partial x_i^2} & \dfrac{\partial^2 F_i}{\partial x_i \partial y_i} \\ \dfrac{\partial^2 F_i}{\partial y_i \partial x_i} & \dfrac{\partial^2 F_i}{\partial y_i^2} \end{vmatrix} = 0. \tag{12}$$

(2) When $0 < y_i \le (c_i + e_i)x_i$, (10) can be expressed as

$$\begin{aligned} F_i(x_i, y_i) &= y_i + \frac{1}{1-\alpha_i}\left(\int_0^{x_i}\left[(c_i+e_i)(x_i-\xi_i)-y_i\right] \cdot p_i(\xi_i)\,d\xi_i + \int_{x_i}^{+\infty}\left[(a-c_i)(\xi_i-x_i)-y_i\right] \cdot p_i(\xi_i)\,d\xi_i\right) = y_i \\ &+ \frac{1}{1-\alpha_i}\left(\int_0^{x_i-y_i/(c_i+e_i)}\left[(c_i+e_i)(x_i-\xi_i)-y_i\right] \cdot p_i(\xi_i)\,d\xi_i + \int_{x_i+y_i/(a-c_i)}^{+\infty}\left[(a-c_i)(\xi_i-x_i)-y_i\right] \cdot p_i(\xi_i)\,d\xi_i\right) = y_i \\ &+ \frac{1}{1-\alpha_i}\left(\int_0^{x_i-y_i/(c_i+e_i)}(c_i+e_i)P_i(\xi_i)\,d\xi_i + \int_0^{+\infty}\left[(a-c_i)(\xi_i-x_i)-y_i\right] p_i(\xi_i)\,d\xi_i + \int_0^{x_i+y_i/(a-c_i)}(a-c_i)P_i(\xi_i)\,d\xi_i\right). \end{aligned} \tag{13}$$

Solve the first derivative of $F_i(x_i, y_i)$:

$$\frac{\partial F_i}{\partial x_i} = \frac{1}{1-\alpha_i}\left[(c_i+e_i)P_i\left(x_i - \frac{y_i}{c_i+e_i}\right) + (a-c_i)P_i\left(x_i + \frac{y_i}{a-c_i}\right) - (a-c_i)\right], \tag{14}$$

$$\frac{\partial F_i}{\partial y_i} = 1 + \frac{1}{1-\alpha_i}\left[-P_i\left(x_i - \frac{y_i}{c_i+e_i}\right) + P_i\left(x_i + \frac{y_i}{a-c_i}\right) - 1\right]. \tag{15}$$

From $\partial L/\partial x_i = 0$ and $\partial L/\partial y_i = 0$, we have

$$x_i(\lambda) = \frac{a-c_i}{a+e_i}P_i^{-1}\left(\frac{(1-\alpha_i)(a-c_i-\lambda/w_i)}{a+e_i} + \alpha_i\right) + \frac{c_i+e_i}{a+e_i}P_i^{-1}\left(\frac{(1-\alpha_i)(a-c_i-\lambda/w_i)}{a+e_i}\right), \tag{16}$$

$$y_i(\lambda) = \frac{a+e_i}{(c_i+e_i)(a-c_i)}\left[P_i^{-1}\left(\frac{(1-\alpha_i)(a-c_i-\lambda/w_i)}{a+e_i} + \alpha_i\right) - P_i^{-1}\left(\frac{(1-\alpha_i)(a-c_i-\lambda/w_i)}{a+e_i}\right)\right]. \tag{17}$$

Solve the second derivative of $F_i(x_i, y_i)$:

$$\begin{aligned}\frac{\partial^2 F_i}{\partial x_i^2} &= \frac{1}{1-\alpha_i}\left[(c_i+e_i)\,p_i\left(x_i-\frac{y_i}{c_i+e_i}\right)\right.\\ &\quad\left.+(a-c_i)\,p_i\left(x_i+\frac{y_i}{a-c_i}\right)\right]>0,\\ \frac{\partial^2 F_i}{\partial x_i\partial y_i} &= \frac{1}{1-\alpha_i}\left[-p_i\left(x_i-\frac{y_i}{c_i+e_i}\right)\right.\\ &\quad\left.+p_i\left(x_i+\frac{y_i}{a-c_i}\right)\right],\\ \frac{\partial^2 F_i}{\partial y_i^2} &= \frac{1}{1-\alpha_i}\left[(c_i+e_i)^{-1}\,p_i\left(x_i-\frac{y_i}{c_i+e_i}\right)\right.\\ &\quad\left.+(a-c_i)^{-1}\,p_i\left(x_i+\frac{y_i}{a-c_i}\right)\right]>0,\end{aligned} \tag{18}$$

then

$$\begin{aligned}\begin{vmatrix}\dfrac{\partial^2 F_i}{\partial x_i^2} & \dfrac{\partial^2 F_i}{\partial x_i\partial y_i}\\ \dfrac{\partial^2 F_i}{\partial y_i\partial x_i} & \dfrac{\partial^2 F_i}{\partial y_i^2}\end{vmatrix} &= \frac{1}{(1-\alpha_i)^2}\left[\frac{(a+e_i)^2}{(c_i+e_i)(a-c_i)}\right.\\ &\quad\left.\cdot\, p_i\left(x_i-\frac{y_i}{c_i+e_i}\right)p_i\left(x_i+\frac{y_i}{a-c_i}\right)\right]>0.\end{aligned} \tag{19}$$

Therefore, $\nabla^2 L(x, y, \lambda)$ is positive definite.

(3) When $y_i \geq (c_i + e_i)x_i$, (10) can be expressed as

$$\begin{aligned}F_i(x_i, y_i) &= y_i + \frac{1}{1-\alpha_i}\left(\int_{x_i}^{+\infty}\left[(a-c_i)(\xi_i-x_i)-y_i\right]\right.\\ &\quad\left.\cdot\, p_i(\xi_i)\,d\xi_i\right) = y_i\\ &\quad+\frac{1}{1-\alpha_i}\left(\int_{x_i+y_i/(a-c_i)}^{+\infty}\left[(a-c_i)(\xi_i-x_i)-y_i\right]\right.\\ &\quad\left.\cdot\, p_i(\xi_i)\,d\xi_i\right) = y_i\\ &\quad+\frac{1}{1-\alpha_i}\left(\int_0^{+\infty}\left[(a-c_i)(\xi_i-x_i)-y_i\right]p_i(\xi_i)\,d\xi_i\right.\\ &\quad\left.+\int_0^{x_i+y_i/(a-c_i)}(a-c_i)\,P_i(\xi_i)\,d\xi_i\right).\end{aligned} \tag{20}$$

Solve the first derivative of $F_i(x_i, y_i)$:

$$\begin{aligned}\frac{\partial F_i}{\partial x_i} &= \frac{a-c_i}{1-\alpha_i}\left[P_i\left(x_i+\frac{y_i}{a-c_i}\right)-1\right]<0,\\ \frac{\partial F_i}{\partial y_i} &= 1+\frac{1}{1-\alpha_i}\left[P_i\left(x_i+\frac{y_i}{a-c_i}\right)-1\right].\end{aligned} \tag{21}$$

$F_i(x_i, y_i)$ is monotonic decrease by y_i, and $\begin{vmatrix}\partial^2 F_i/\partial x_i^2 & \partial^2 F_i/\partial x_i\partial y_i\\ \partial^2 F_i/\partial y_i\partial x_i & \partial^2 F_i/\partial y_i^2\end{vmatrix} = 0$.

According to above, $L(x, y, \lambda)$ is a convex on (x, y). Evidently with (16) and (17), for the fixed λ, we have a unique optimal solution to $\min_{(x,y)} L(x, y, \lambda)$. So

$$P_i\left(x_i(\lambda)-\frac{y_i(\lambda)}{c_i+e_i}\right) = \frac{(1-\alpha_i)(a-c_i-\lambda/w_i)}{a+e_i},\quad i=1,2,\cdots,m, \tag{22}$$

$$P_i\left(x_i(\lambda)+\frac{y_i(\lambda)}{a-c_i}\right) = \frac{(1-\alpha_i)(a-c_i-\lambda/w_i)}{a+e_i}+\alpha_i,\quad i=1,2,\cdots,m, \tag{23}$$

thus $0 \leq P_i(x_i(\lambda)-y_i(\lambda)/(c_i+e_i)) \leq 1$, $0 \leq P_i(x_i(\lambda)+y_i(\lambda)/(a-c_i)) \leq 1$, $i = 1, 2, \cdots, m$, where λ must satisfy the following inequalities:

$$-w_i(\alpha_i(a-c_i)+c_i+e_i) \leq \lambda \leq w_i(a-c_i),\quad i=1,2,\cdots,m, \tag{24}$$

$$-w_i(c_i+e_i) \leq \lambda \leq w_i(a-c_i)+\frac{\alpha_i w_i(a+e_i)}{1-\alpha_i},\quad i=1,2,\cdots,m. \tag{25}$$

Let $\underline{\lambda} = \max\{-w_i(c_i + e_i) \mid i = 1, 2, \cdots, m\}$, $\overline{\lambda} = \min\{w_i(a - c_i) \mid i = 1, 2, \cdots, m\}$, and $\lambda \in [\underline{\lambda}, \overline{\lambda}]$, and (22) and (23) have solutions. Then the theorem conclusion is established. □

Corollary 2. *When $\lambda = 0$, DCEs are of decentralized supply, and the optimal allocation amount is expressed as*

$$\begin{aligned}x_i(0) &= \frac{a-c_i}{a+e_i}P_i^{-1}\left(\frac{(1-\alpha_i)(a-c_i)}{a+e_i}+\alpha_i\right)\\ &\quad+\frac{c_i+e_i}{a+e_i}P_i^{-1}\left(\frac{(1-\alpha_i)(a-c_i)}{a+e_i}\right),\end{aligned} \tag{26}$$

where total supply $Q = \sum_{i=1}^m x_i(0)$.

According to Theorem 1, set $\lambda = 0$ in (16), the optimal supply for a chain store is (26). If $\alpha_i = 0$ in (16), we get Theorem 3.

Theorem 3. *If there is a λ^* satisfying $-w_i(c_i + e_i) \leq \lambda^* \leq w_i(a-c_i)$, $i = 1, 2, \cdots, m$, and $\sum_{i=1}^m x_i(\lambda^*) = Q$, and set $\alpha_i = 0$ in (DECP), then the optimal allocation strategy based on the excepted loss is*

$$x_i^0(\lambda^*) = P_i^{-1}\left(\frac{a-c_i-\lambda^*/w_i}{a+e_i}\right),\quad i=1,2,\cdots,m. \tag{27}$$

Equation (8) is the optimal solution of the production allocation of the DCE based on CVaR. Equation (26) is the optimal solution of the production allocation under decentralized supply. Equation (27) is the optimal solution of production allocation based on the excepted loss.

3. Sensitivity Analysis of the Optimal Strategy

This section gives sensitivity analysis of the optimal strategy (8), i.e., when any of the parameters $a, e_i, \lambda^*, w_i, c_i, \alpha_i$ changes, how the optimal allocation strategy (8) changes accordingly. According to (8), let

$$x_i^1(\lambda) = P_i^{-1}\left(\frac{(1-\alpha_i)(a-c_i-\lambda/w_i)}{a+e_i}+\alpha_i\right), \tag{28}$$

$$x_i^2(\lambda) = P_i^{-1}\left(\frac{(1-\alpha_i)(a-c_i-\lambda/w_i)}{a+e_i}\right). \tag{29}$$

Then the optimal allocation and supply strategy (8) is as follows:

$$x_i(\lambda) = \frac{a-c_i}{a+e_i}x_i^1(\lambda)+\frac{c_i+e_i}{a+e_i}x_i^2(\lambda). \tag{30}$$

Theorem 4. *Under the optimal allocation strategy, if a, e_i, λ^*, w_i, and c_i are fixed, then the optimal allocation amount $x_i^1(\lambda^*)$ increases with the confidence level α_i, and the optimal allocation amount $x_i^2(\lambda^*)$ decreases when the confidence level α_i increases.*

Proof. According to (28) and (29)

$$\begin{aligned}\frac{\partial x_i^1(\lambda^*)}{\partial \alpha_i} &= \left(\frac{-(a-c_i-\lambda^*/w_i)}{a+e_i}+1\right)p_i\left(x_i^1(\lambda^*)\right)^{-1}\\ &\geq 0,\\ \frac{\partial x_i^2(\lambda^*)}{\partial \alpha_i} &= \left(\frac{-(a-c_i-\lambda^*/w_i)}{a+e_i}\right)p_i\left(x_i^2(\lambda^*)\right)^{-1}\leq 0.\end{aligned} \tag{31}$$

Evidently, the conclusion is true. □

From Theorem 1, we know that the following conclusions are clearly true.

Theorem 5. *Under the optimal allocation strategy, if a, c_i, e_i, w_i, and α_i are fixed, then the optimal allocation amount $x_i(\lambda^*)$ decreases when λ^* increases.*

Proof. According to (28) and (29)

$$\begin{aligned}\frac{\partial x_i^1(\lambda^*)}{\partial \lambda^*} &= \frac{-(1-\alpha_i)}{w_i(a+e_i)}p_i\left(x_i^1(\lambda^*)\right)^{-1}\leq 0,\\ \frac{\partial x_i^2(\lambda^*)}{\partial \lambda^*} &= \frac{-(1-\alpha_i)}{w_i(a+e_i)}p_i\left(x_i^2(\lambda^*)\right)^{-1}\leq 0.\end{aligned} \tag{32}$$

Therefore, $\partial x_i(\lambda^*)/\partial\lambda^* \leq 0$; the conclusion is true. □

Theorem 6. *Under the optimal allocation strategy, if a, e_i, λ^*, w_i, and α_i are fixed, then the optimal allocation amount $x_i^1(\lambda^*)$ and $x_i^2(\lambda^*)$ decrease when the cost c_i increases.*

Proof. According to (28) and (29)

$$\begin{aligned}\frac{\partial x_i^1(\lambda^*)}{\partial c_i} &= \frac{-(1-\alpha_i)}{(a+e_i)}p_i\left(x_i^1(\lambda^*)\right)^{-1}\leq 0,\\ \frac{\partial x_i^2(\lambda^*)}{\partial c_i} &= \frac{-(1-\alpha_i)}{(a+e_i)}p_i\left(x_i^2(\lambda^*)\right)^{-1}\leq 0.\end{aligned} \tag{33}$$

Therefore, the conclusion is true. □

Theorem 7. *Under the optimal allocation supply strategy, when a, e_i, c_i, λ^*, and α_i are fixed, if $\lambda^* < 0$, then the optimal allocation amount $x_i^*(\lambda^*)$ decreases when the weight w_i increases; or if $\lambda^* > 0$, then the optimal allocation amount $x_i(\lambda^*)$ increases with the weight w_i.*

Proof. According to (28) and (29)

$$\begin{aligned}\frac{\partial x_i^1(\lambda^*)}{\partial w_i} &= \frac{(1-\alpha_i)\lambda^*}{w_i^2(a+e_i)}p_i\left(x_i^1(\lambda^*)\right)^{-1}>0,\\ \frac{\partial x_i^2(\lambda^*)}{\partial w_i} &= \frac{(1-\alpha_i)\lambda^*}{w_i^2(a+e_i)}p_i\left(x_i^2(\lambda^*)\right)^{-1}>0.\end{aligned} \tag{34}$$

Therefore, the conclusion is true according to (30). □

Theorem 8. *Under the optimal allocation strategy, if a, c_i, λ^*, w_i, and α_i are fixed, the optimal allocation amounts $x_i^1(\lambda^*)$ and $x_i^2(\lambda^*)$ decrease when discount price e_i increases.*

Proof. According to (28) and (29)

$$\begin{aligned}\frac{dx_i^1(\lambda^*)}{de_i} &= \frac{-(1-\alpha_i)(a-c_i-\lambda^*/w_i)}{(a+e_i)^2}p_i\left(x_i^1(\lambda^*)\right)^{-1}\\ &\leq 0,\\ \frac{dx_i^2(\lambda^*)}{de_i} &= \frac{-(1-\alpha_i)(a-c_i-\lambda^*/w_i)}{(a+e_i)^2}p_i\left(x_i^2(\lambda^*)\right)^{-1}\\ &\leq 0.\end{aligned} \tag{35}$$

Therefore, the conclusion is true. □

Theorem 9. *Under the optimal allocation strategy, if e_i, c_i, λ^*, w_i, and α_i are fixed then the optimal allocation amounts $x_i^1(\lambda^*)$ and $x_i^2(\lambda^*)$ increase with the retailer price a.*

Proof. According to (28) and (29)

$$\begin{aligned}\frac{\partial x_i^1(\lambda^*)}{\partial a} &= \frac{(1-\alpha_i)(e_i+c_i+\lambda^*/w_i)}{(a+e_i)^2}p_i\left(x_i^1(\lambda^*)\right)^{-1}\\ &\geq 0,\\ \frac{dx_i^2(\lambda^*)}{da} &= \frac{(1-\alpha_i)(e_i+c_i+\lambda^*/w_i)}{(a+e_i)^2}p_i\left(x_i^2(\lambda^*)\right)^{-1}\\ &\geq 0.\end{aligned} \tag{36}$$

Therefore, the conclusion is true. □

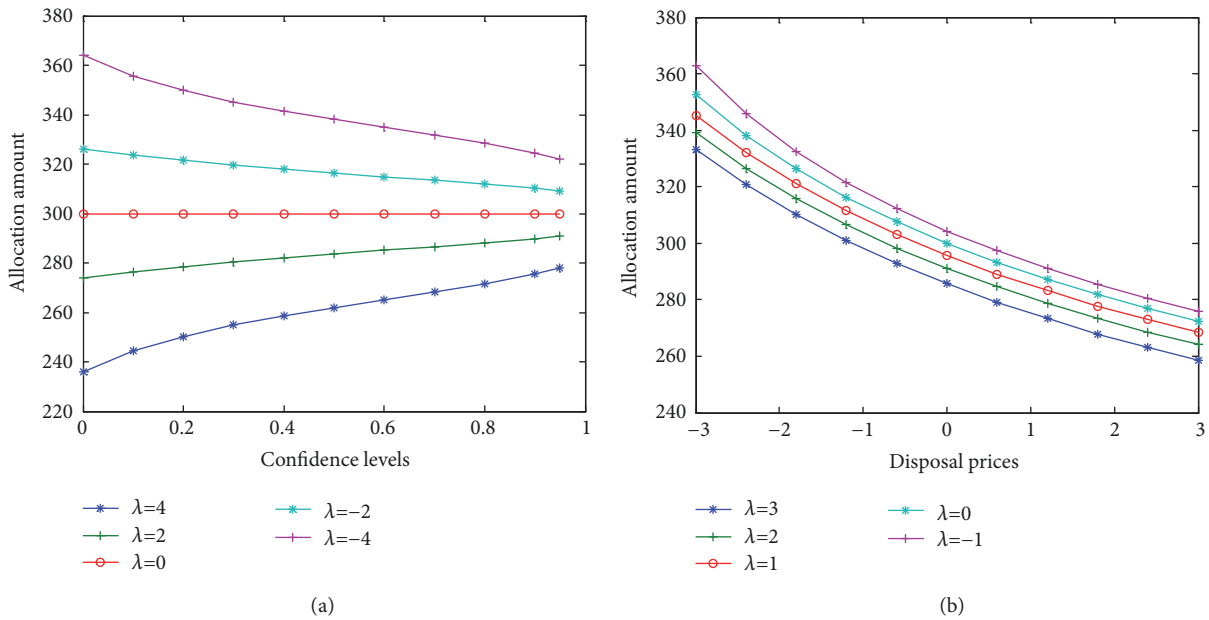

FIGURE 1: (a) Allocations at different confidence levels and multipliers λ^*. (b) Allocations at different disposal prices and multipliers λ^*.

We give the following numerical examples to illustrate Theorems 4, 5, and 8.

Example 10. Let $a = 10, w_i = 1$, and $c_i = 5$ in (28) and (29). Suppose that the demand probability density of a product satisfies a normal distribution: N $(300,50^2)$.

(1) Figure 1(a) shows the different allocation amount at different confidence levels and multipliers λ^*, where disposal price $e_i = 0$ and λ^* satisfies $-5 \le \lambda^* \le 5$. From Figure 1(a), it is understood that the allocation amount $x_i(\lambda^*)$ increases with the confidence level α_i when optimal multipliers $\lambda^* \ge 0$; and when the optimal multipliers $\lambda^* < 0$ the allocation amount $x_i(\lambda^*)$ decreases when the confidence level α_i increases.

(2) Figure 1(b) shows the allocation amount at different disposal prices and multipliers λ^*, where disposal price $e_i \in [-3, 3]$, confidence level $\alpha_i = 0.95$, and $\lambda = -1, 0, 1, 2$, and 3. From Figure 1(b), the allocation amount $x_i(\lambda^*)$ decreases when the disposal price e_i increases and decreases when the multipliers λ^* increases.

4. Algorithm of Production Allocation and Supply Strategy

According to Section 2, this section discusses the algorithm for the optimal production allocation strategy (8).

Let $\underline{\lambda} = \max\{-w_i(c_i+e_i) \mid i = 1, 2, \cdots, m\}$, $\overline{\lambda} = \min\{w_i(a-c_i) \mid i = 1, 2, \cdots, m\}$. Take $\underline{\lambda} < \lambda < \overline{\lambda}$; we solve the following equation:

$$\sum_{i=1}^{m} \frac{a-c_i}{a+e_i} P_i^{-1}\left(\frac{(1-\alpha_i)(a-c_i-\lambda/w_i)}{a+e_i}+\alpha_i\right) + \frac{c_i+e_i}{a+e_i} P_i^{-1}\left(\frac{(1-\alpha_i)(a-c_i-\lambda/w_i)}{a+e_i}\right) = Q \tag{37}$$

According to Theorem 1, if there is a solution λ^* to (37), $x_i(\lambda^*)$ $(i = 1, 2, \cdots, m)$ is the optimal allocation strategy for DCEs. $Q(\lambda)$ corresponding to the solution λ in (37) is called the optimal production amount at the given λ. We substitute $\lambda \in (\underline{\lambda}, \overline{\lambda})$ into (16), (17), and (37), so we get $(x_i(\lambda), y_i(\lambda))$ and $Q(\lambda)$, which are substituted into (*DECP*), and get the target loss function:

$$H(\lambda) = \sum_{i=1}^{m} w_i F_i(x_i(\lambda), y_i(\lambda)). \tag{38}$$

Solve

$$\begin{aligned} \min \quad & H(\lambda) = \sum_{i=1}^{m} w_i F_i(x_i(\lambda), y_i(\lambda)) \\ \text{s.t.} \quad & \underline{\lambda} < \lambda < \overline{\lambda} \end{aligned} \tag{39}$$

to obtain $Q(\lambda^*)$, an optimal solution corresponding to λ^* and an optimal total production; $x_i(\lambda^*)$ $(i = 1, 2, \cdots, m)$, an optimal allocation amount; and $y_i(\lambda^*)$, a VaR loss value. Solve the maximum loss:

$$\begin{aligned} \max \quad & H(\lambda) = \sum_{i=1}^{m} w_i F_i(x_i(\lambda), y_i(\lambda)) \\ \text{s.t.} \quad & \underline{\lambda} < \lambda < \overline{\lambda} \end{aligned} \tag{40}$$

to obtain $Q(\lambda_*)$, an optimal solution corresponding to λ_* and a minimum total production; $x_i(\lambda_*)$ $(i = 1, 2, \cdots, m)$, a minimum allocation amount; and $y_i(\lambda_*)$, a VaR loss value.

We have the following approximate **algorithm**:

(0) Give the initial value: $N, a, e_i, c_i, w_i, \alpha_i$ $(i = 1, 2, \cdots, m)$, where the weight is given as follows:

$$w_i = \frac{E\xi_i}{\sum_{i=1}^{m} E\xi_i}, \quad i = 1, 2, \cdots, m. \tag{41}$$

(1) Calculate $\lambda_1 = \max\{-w_i(c_i + e_i) \mid i = 1, 2, \cdots, m\}$, $\lambda_2 = \min\{w_i(a - c_i) \mid i = 1, 2, \cdots, m\}$, then split $[\underline{\lambda}, \overline{\lambda}]$ into N segments: $\triangle\lambda = (\lambda_2 - \lambda_1)/N$.

(2) Calculate $H(\lambda_k) = \sum_{i=1}^{m} w_i F_i(x_i(\lambda_k), y_i(\lambda_k))$, where $\lambda_k = \lambda_1 + k * \triangle\lambda$, $k = 1, 2, \cdots, N$, then get $(x_i(\lambda_k), y_i(\lambda_k))$ and $Q(\lambda_k)$ $(i = 1, 2, \cdots, m)$ from (16), (17), and (37).

(3) Solve $\lambda^* = \arg\min\{H(\lambda_k) = \sum_{i=1}^{m} w_i F_i(x_i(\lambda_k), y_i(\lambda_k)) \mid k = 1, 2, \cdots, N\}$, and get an approximate optimal total production amount $Q(\lambda^*)$ and optimal allocation amount $x_i = x_i(\lambda^*)$ $(i = 1, 2, \cdots, m)$.

(4) Solve $\lambda_* = \arg\max\{H(\lambda_k) = \sum_{i=1}^{m} w_i F_i(x_i(\lambda_k), y_i(\lambda_k)) \mid k = 1, 2, \cdots, N\}$, and get the approximate minimum total supply amount $Q(\lambda_*)$ and minimum allocation amount $x_i = x_i(\lambda_*)$ $(i = 1, 2, \cdots, m)$.

We have $Q(\underline{\lambda}) = +\infty$, $Q(\lambda) \in [Q(\overline{\lambda}), +\infty)$ when $\lambda \in (\underline{\lambda}, \overline{\lambda}]$. Evidently, the larger N, the more accurate $H(\lambda_k)$. If we can get the corresponding sample discrete points $\{\xi_i\}$: $\{\eta_i^{j_i}\}$ $(j_i = 1, 2, \cdots, J_i)$ with the corresponding probability distributions $\{P_i(\eta_i^{j_i})\}(\sum_{j_i=1}^{J_i} P_i(\eta_i^{j_i}) = 1)$, $H(\lambda_k)$ is approximately defined by

$$\begin{aligned}\widetilde{H}(\lambda_k) = {} & \sum_{i=1}^{m} w_i \left(y_i(\lambda_k) + (1 - \alpha_i)\right)^{-1} \\ & \cdot \sum_{j_i=1}^{J_i} \Big[(c_i + e_i)\left(x_i(\lambda_k) - \eta_i^{j_i}\right)^+ \\ & + (a - c_i)\left(\eta_i^{j_i} - x_i(\lambda_k)\right)^+ - y_i(\lambda_k)\Big]^+ P_i\left(\eta_i^{j_i}\right).\end{aligned} \tag{42}$$

We substitute (42) into $\lambda^* = \arg\min\{\widetilde{H}(\lambda_k) \mid k = 1, 2, \cdots, N\}$ in the second step of algorithm.

Divide (42) into two parts: oversupply loss $\widetilde{H}^+(x(\lambda_k))$ and undersupply loss $\widetilde{H}^-(x(\lambda_k))$:

$$\begin{aligned}\widetilde{H}^+(x(\lambda_k)) = {} & \sum_{i=1}^{m} w_i \left(y_i(\lambda_k) + (1 - \alpha_i)\right)^{-1} \\ & \cdot \sum_{j_i=1}^{J_i} \Big[(c_i + e_i)\left(x_i(\lambda_k) - \eta_i^{j_i}\right)^+ - y_i(\lambda_k)\Big]^+ P_i\left(\eta_i^{j_i}\right),\end{aligned} \tag{43}$$

$$\begin{aligned}\widetilde{H}^-(x(\lambda_k)) = {} & \sum_{i=1}^{m} w_i \left(y_i(\lambda_k) + (1 - \alpha_i)\right)^{-1} \\ & \cdot \sum_{j_i=1}^{J_i} \Big[(a - c_i)\left(\eta_i^{j_i} - x_i(\lambda_k)\right)^+ - y_i(\lambda_k)\Big]^+ P_i\left(\eta_i^{j_i}\right).\end{aligned} \tag{44}$$

Oversupply losses for DCEs are reduced by lowering retail prices after the normal sale period, when the remaining products are sold at discount prices. But the loss from undersupply is difficult to deal with, unless more products are supplied. According to Theorem 5, the optimal allocation $x_i(\lambda^*)$ and oversupply loss decrease when λ^* increases, and the loss of undersupply increases with λ^*.

Let θ $(\theta > 0)$ be a risk loss threshold. Due to capacity constraints, let Q_0 be the total production by DCEs. The DCE should adopt the following production allocation strategies:

(1) If $\widetilde{H}^+(x(\lambda_k)) < \widetilde{H}^-(x(\lambda_k)) - \theta$ $(k = 1, 2, \cdots, N)$, i.e., undersupply risk loss is greater than oversupply risk loss, the k-th retail store should adopt the optimal production strategies: ① If $Q(\lambda^*) \le Q_0$, $x_i(\lambda^*)$ $(i = 1, 2, \cdots, m)$ is chosen as allocation strategy, where $Q(\lambda^*)$ is total production. Oversupply strategy is used to avoid the risk of loss due to insufficient supply. ② If $Q(\lambda^*) > Q_0 \ge Q(\lambda_*)$, $x_i(\lambda^*)$ $(i = 1, 2, \cdots, m)$ is not chosen as allocation strategies, so $x_i(\lambda_k^0)$ $(i = 1, 2, \cdots, m)$ is chosen corresponding to $\lambda_k^0 = \arg\min\{|Q(\lambda_k) - Q_0| \mid Q_0 \ge Q(\lambda_k), k = 0, 1, 2, \cdots, N\}$ as the allocation strategy, where $Q(\lambda_k^0)$ is total production; i.e., DCEs can find a production strategy very close to Q_0 but smaller than Q_0. ③ If $Q_0 < Q(\lambda_*)$, i.e., the capacity is less than the demands under the maximum risk loss, DCEs' profit is low. The allocation strategy is calculated according to the following:

$$x_i^0 = x_i(\lambda_*) - w_i\left(Q(\lambda_*) - Q_0\right) \quad (i = 1, 2, \cdots, m), \tag{45}$$

where w_i is calculated by (41). In this case, the business risk is mainly the loss caused by insufficient supply.

(2) If $\widetilde{H}^+(x(\lambda_k)) > \widetilde{H}^-(x(\lambda_k)) + \theta$ $(k = 1, 2, \cdots, N)$, i.e., insufficient risk loss is lower than oversupply risk loss, the k-th retail store should adopt the minimum production strategy. If $Q_0 \ge Q(\lambda_*)$, DCEs choose $x_i(\lambda_*)$ $(i = 1, 2, \cdots, m)$ as allocation strategy, where $Q(\lambda_*)$ is total production, since the less total production avoids oversupply risk. If $Q_0 < Q(\lambda_*)$, the oversupply loss is avoided. Then the allocation strategy is obtained by (45).

(3) If $\widetilde{H}^-(x(\lambda_k)) - \theta \le \widetilde{H}^+(x(\lambda_k)) \le \widetilde{H}^-(x(\lambda_k)) + \theta$ $(k = 1, 2, \cdots, N)$, i.e., the oversupply loss is very close to the insufficient supply loss, DCEs should adopt a decentralized production supply strategy. ① If $Q(0) \le Q_0$, DCEs choose $x_i(0)$ $(i = 1, 2, \cdots, m)$ as the allocation strategy, where $Q(0)$ is total production, to avoid the risk losses by oversupply or insufficient supply. ② If $Q(0) > Q_0 \ge Q(\lambda_*)$, $x_i(\lambda_k^0)$ $(i = 1, 2, \cdots, m)$ corresponding to $\lambda_k^0 = \arg\min\{|Q(\lambda_k) - Q_0| \mid Q_0 \ge Q(\lambda_k), k = 0, 1, 2, \cdots, N\}$ is the allocation supply strategy, where $Q(\lambda_k^0)$ is a total production; i.e., DCEs can take a production strategy very close to Q_0 but smaller than Q_0. ③ If $Q_0 < Q(\lambda_*)$, i.e., the production capacity is less than demands under the lowest risk loss, the allocation strategy is solved by (45).

At the end of every normal sales period, the chain stores should check its inventory on hand and decide whether

Table 1: **Allocation,** VaR, CVaR, and t**he excepted profit of chain stores in decentralized supply** ($\lambda = 0$).

Disposal price	Optimal allocation	Production	VaR	CVaR	Excepted profit
(-3,-3,-3,-3,-3,-3,-3)	(138,185,224,355,239,465,560)	2165	22.07	2930.30	6590.85
(-1, -1, -1, -1, -1, -1, -1)	(132,180,221,349,233,457,553)	2125	15.31	4995.05	6504.40
(0, 0, 0, 0, 0, 0, 0)	(130,178,220,346,231,455,551)	2111	13.87	5735.81	6472.08
(1, 1, 1, 1, 1, 1, 1)	(128,177,219,344,230,453,549)	2100	12.91	6352.11	6444.26
(3, 3, 3, 3, 3, 3, 3)	(126,174,217,341,227,449,546)	2082	11.70	7323.74	6399.28

Table 2: **Allocation,** VaR, CVaR, and t**he excepted profit of chain stores in the centralized supply** (λ^*).

Disposal price	λ^*	Optimal allocation	Production	VaR	CVaR	Excepted profit
(-3,-3,-3,-3,-3,-3,-3)	-0.12	(144,187,224,356,240,466,561)	2178	22.78	2639.86	6604.89
(-1, -1, -1, -1, -1, -1, -1)	-0.24	(138,182,222,350,235,458,554)	2140	15.71	4562.96	6525.52
(0, 0, 0, 0, 0, 0, 0)	-0.30	(136,181,221,348,233,456,552)	2126	14.18	5240.69	6495.04
(1, 1, 1, 1, 1, 1, 1)	-0.36	(135,179,220,346,232,454,550)	2116	13.14	5818.27	6470.84
(3, 3, 3, 3, 3, 3, 3)	-0.48	(132,177,219,343,230,451,547)	2099	11.81	6690.41	6430.88

discount price should apply. The following discount strategy is given as per the above algorithm.

Discount Pricing Strategy. Let $\tilde{x}_i$ $(i = 1, 2, \cdots, m)$ be the inventory at the end of the normal sales period and $\eta \in (0, 1)$ be the minimum oversupply inventory rate set by DCEs. There exist three cases. (1) If DCEs take the optimal allocation strategy at the beginning of sales, the discount price $\tilde{e}_i \leq \min\{-(1 - x_i(\lambda^*)/\tilde{x}_i)a, e_i\}$ is applied for $\tilde{x}_i/x_i(\lambda^*) > \eta$ in the remaining sales cycle; (2) If DCEs take the decentralized allocation strategy at the beginning of sales, the discount price $\tilde{e}_i \leq -(1 - x_i(0)/\tilde{x}_i)a$ is applied for $\tilde{x}_i/x_i(0) > \eta$ in the remaining sales cycle; otherwise no discount applies. (3) If DCEs take the minimum allocation amount at the beginning of sales, the discount price $\tilde{e}_i \leq -(1 - x_i(\lambda_*)/\tilde{x}_i)a$ is taken for $\tilde{x}_i/x_i(\lambda_*) > \eta$ in the remaining sales cycle; otherwise no discount applies.

5. Numerical Analysis

Based on the sales data collected from a direct food chain manufacturer, we use the above algorithm and get the following allocation strategy for its seven chain stores.

The number of chain stores m is 7, retailer price a is 10, and cost prices c for each store are [5, 5.5, 6, 5.2, 5.3, 5.2, 5.7], respectively. The product has a shelf life of seven days. The data from the seven stores is converted into the approximately normal distribution: N(130.00, 7.56^2), N(180.00, 7.56^2), N(221.86,4.83^2), N(347.14, 8.97^2), N(232.43,7.43^2), N(455.71, 9.87^2), and N(554.29,10.47^2). From (41) weight w is [0.0613, 0.0848, 0.1046, 0.1636, 0.1096, 0.2148, 0.2613], where the greater the demand mean, the greater the weight. As the chain stores operate in a single cycle, the normal retail price and cost price do not change. So, the effect of disposal price, confidence level, and multiplier λ on the allocation strategy should be taken into consideration. The numerical analysis shows the effects of disposal price, confidence level, and multiplier below.

When the production allocation strategy $(x_1, x_2, \cdots, x_m)$ is given, we can calculate the excepted profit $\sum_{i=1}^{m} w_i E h_i(x_i, \xi_i)$, where the profit of the i-th retailer store is

$$h_i(x_i, \xi_i) = (a - c_i)\min(\xi_i, x_i) - (c_i + e_i)(x_i - \xi_i)^+. \quad (46)$$

(1) The Effect of Discounted Prices. Tables 1–3 show the production allocation, VaR loss, CVaR loss, and excepted profit obtained from the decentralized supply, the centralized optimal supply, and the centralized minimum supply, respectively, at different disposal prices under confidence level $\alpha = 0.95$, with Table 4 a summary.

Table 1 is the approximate total production, allocation, CVaR, and the expected return in the decentralized supply under five disposal prices. Table 1 shows when disposal price increases the total production decreases, CVaR increases, VaR decreases, and the expected profit decreases. Evidently, DCEs can use the disposal price at the end of the normal sales period in order to reduce risk and raise the return.

Table 2 gives the approximate total production amount, allocation, CVaR, and the expected return in the centralized supply under five disposal prices. Compared with Table 1, the total optimal production in the centralized supply is higher than that in the decentralized supply, but the risk loss in the centralized supply is lower than that in the decentralized supply, and the expected profit in the centralized supply is higher than that in the decentralized supply. That is to say, the centralized production allocation is better than the decentralized allocation under the same disposal prices. At the same time, it is shown from Table 2 that, when disposal prices increase, the total production decreases, risk loss increases, and VaR and the expected profit decrease. It indicates that the goods sold at proper disposal prices at the end of the sales period reduce the risk and increase the profit.

Table 3: **Allocation,** VaR, CVaR, and t**he excepted profit of chain stores in the centralized supply** (λ_*).

Disposal price	λ_*	Optimal allocation	Production	VaR	CVaR	Excepted profit
(-3,-3,-3,-3,-3,-3,-3)	0.30	(132,182,222,353,236,463,559)	2146	22.78	3466.91	6562.77
(-1, -1, -1, -1, -1, -1, -1)	0.30	(127,177,219,347,231,456,552)	2108	15.71	5486.06	6476.90
(0, 0, 0, 0, 0, 0, 0)	0.29	(125,175,218,345,229,454,550)	2095	14.18	6241.64	6444.61
(1, 1, 1, 1, 1, 1, 1)	0.29	(123,174,217,343,228,452,548)	2084	13.14	6860.28	6416.45
(3, 3, 3, 3, 3, 3, 3)	0.29	(120,171,216,340,226,448,545)	2066	11.81	7841.90	6371.81

Table 4: A comparison of the above three.

Disposal price	Decentralized allocation $\lambda = 0$.		Centralized optimal allocation λ^*.		Centralized minimum allocation λ_*.	
	Oversupply loss	undersupply loss	Oversupply loss	undersupply loss	Oversupply loss	undersupply loss
(-3,-3,-3,-3,-3,-3,-3)	48.26	2904.11	62.46	2600.18	32.86	3456.38
(-1, -1, -1, -1, -1, -1, -1)	59.78	4950.57	91.05	4487.61	42.38	5459.45
(0, 0, 0, 0, 0, 0, 0)	52.92	5696.77	85.49	5169.38	33.34	6222.64
(1, 1, 1, 1, 1, 1, 1)	41.34	6323.67	73.56	5757.84	24.59	6849.06
(3, 3, 3, 3, 3, 3, 3)	27.34	7308.09	49.74	6652.48	18.10	7835.96

Table 3 gives the approximate minimum total production, allocation, VaR, CVaR, and the expected return in centralized supply under five disposal prices. Compared with Table 1, the minimum total optimal production in the centralized supply is lower than that in the decentralized supply, but the risk loss in the centralized supply is higher than that in the decentralized supply, and the expected profit in the centralized supply is lower than that in the decentralized supply.

Table 4 compares the oversupply CVaR loss and insufficient supply CVaR loss in decentralized supply, the centralized optimal allocation, and the centralized minimum allocation under five disposal prices. The results show that, with the increase in disposal prices, oversupply loss increases, and the loss of oversupply is far greater than the loss of undersupply. In this case, DCEs should try to avoid the loss caused by insufficient supply, use the oversupply strategy, and sell the remaining products at a discount price at the end of the sales cycle. For instance, the bakery checks the inventory at around 6pm to decide whether discount price applies.

It is clear that the oversupply loss is much less than the loss of insufficient supply, DCEs should use the optimal allocation strategy $x_i(\lambda^*)$ (8), with the total supply amount $Q(\lambda^*)$. For example, when disposal prices are (0, 0, 0, 0, 0, 0, 0), if the DCE's capacity is 2111, he should adopt the allocation strategy of (130,178,220,346,231,455,551) from Table 2. When the capacity is less than 2095, the minimum total production, the supply plan is obtained by (45).

(2) The Effect of $\lambda \in (\underline{\lambda}, \overline{\lambda}]$. Figures 2–4 compare CVaR, the expected profit, and total production allocation when λ changes, at a disposal price 0 and confidence level 0.95. Figure 2 shows the change of CVaR and of the expected profit with the change of $\lambda \in (\underline{\lambda}, \overline{\lambda}]$, at the disposal price 0, where the total expected loss increases and the total profit decreases when λ increases. Figure 3 shows the changes of oversupply loss and undersupply loss with the change of $\lambda \in (\underline{\lambda}, \overline{\lambda}]$, at the disposal price 0, where the total expected oversupply risk loss decreases and total expected insufficient supply risk loss increases when λ increases. The risk loss of total insufficient supply is greatly higher than that of oversupply, so DCEs should choose the optimal allocation strategy. As the product has seven days of shelf life, the inventory is to be checked on around 5th-6th day. If there is actually an excessive inventory, the chain stores need to do some promotions and offer discount price in the final part of the sales cycle.

The total production decreases when λ increases as shown in Figure 4.

(3) The Effect of Confidence Levels under Different $\lambda \in (\underline{\lambda}, \overline{\lambda}]$. Figures 5 and 6 show CVaR increases and total expected profit decreases when the confidence level increases. As λ increases, the CVaR risk loss gap does not decrease, and the expected return gap tends to decrease. The change in total production, CVaR, the expected profit, oversupply loss, insufficient supply loss, and allocation are discussed under the three different confidence levels 0.90, 0.95, and 0.99 as follows.

Figures 7 and 8 show that the expected oversupply loss decreases and expected insufficient supply increases when the confidence level increases. As λ increases, their loss gaps gradually decreases, and trend becomes almost identical, as shown in Figure 9.

So in this case, the disposal price is necessary. When the insufficient supply loss is large, the optimal production allocation strategy is adopted. During the sales period, the oversupply risk is balanced by adjusting the disposal price. If the oversupply risk is too large, the minimum production allocation strategy is to be adopted. The total production

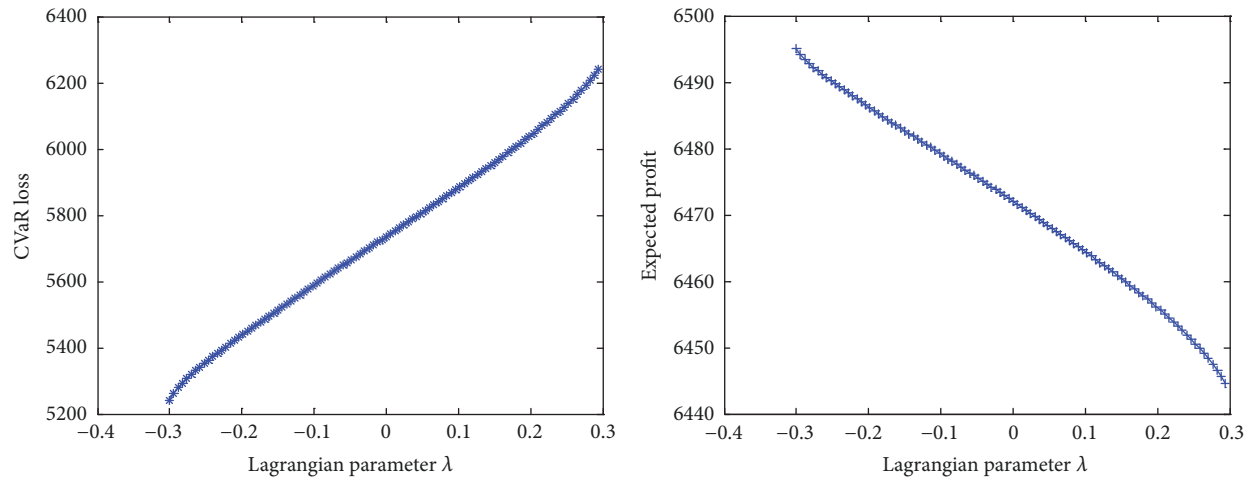

Figure 2: CVaR and the expected profit under different $\lambda \in (\underline{\lambda}, \overline{\lambda}]$.

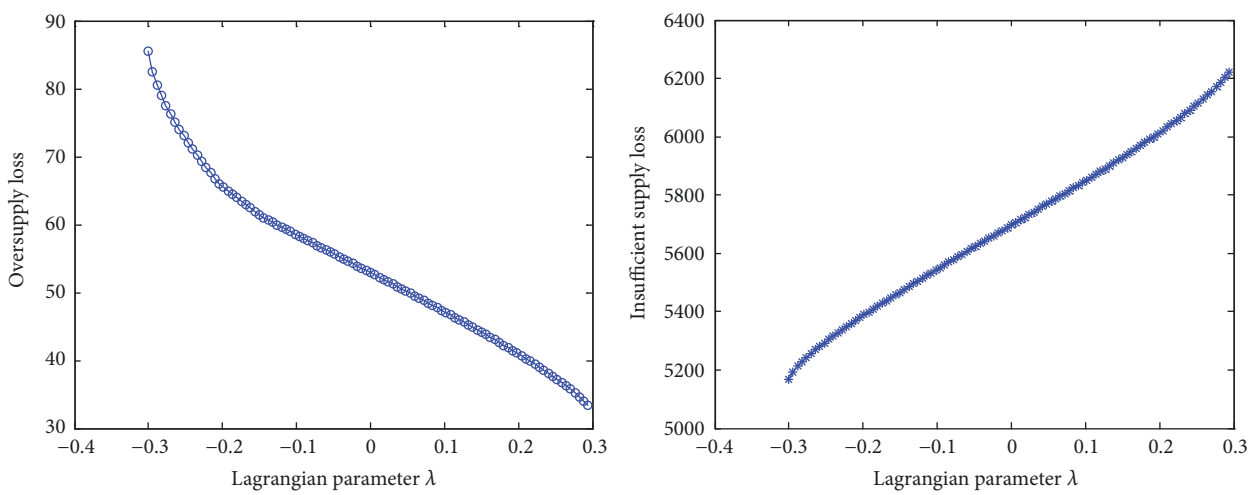

Figure 3: Oversupply loss and insufficient supply loss under different λ.

amount of DCEs should be controlled within the best range $[Q(\lambda_*), Q(\lambda^*)]$.

6. Conclusion

In this paper, the optimal allocation strategy based on CVaR is derived for DCEs. Then, the algorithm of the centralized supply is given, where approximate total optimal production and allocation and minimum total production and allocation are obtained. And the discount pricing strategy in the sales cycle is obtained. Numerical examples show the production and allocation of a product of a food direct chain company. The numerical results corroborate that the risk of the optimal production allocation strategy is less than that of the decentralized supply, with the discounted sales which help gain more.

We have the following conclusions:

(1) With the probability distribution of historical sales data, the oversupply loss and undersupply loss are obtained so as to determine whether the centralized optimal supply strategy, decentralized strategy, or the centralized minimum supply strategy should be used.

(2) Based on the inventory on hand during the sales period, DCEs should determine discounts to reduce losses. In practice, chain stores give different discounts based on the levels of inventory, no discount when the remaining inventory is low; or they should sell the remaining products bundled with best sales to ensure that the remaining inventory is sold out at the end of the sales period.

(3) An effective discount strategy can reduce the risk of overstocking and increase the profit. Practice has shown that discount pricing is very effective for chain stores, and customers are particularly sensitive to price reduction at the end of the sales period.

(4) In theory, we have proved that the centralized supply can improve the risk of total production compared with the decentralized supply. In practice, DCEs improve their production capacity to meet the needs

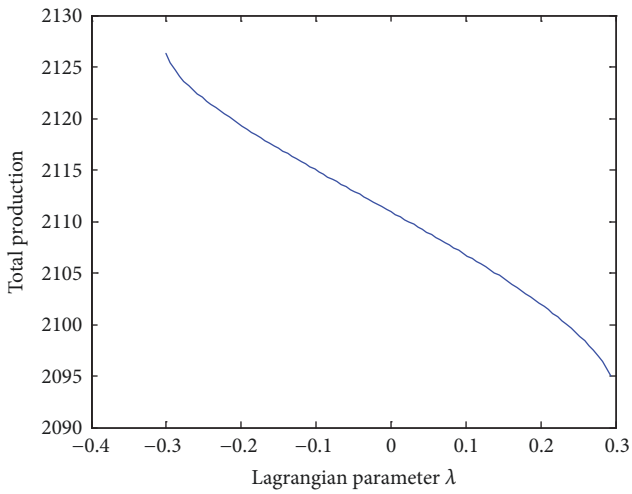

Figure 4: Total production under different λ.

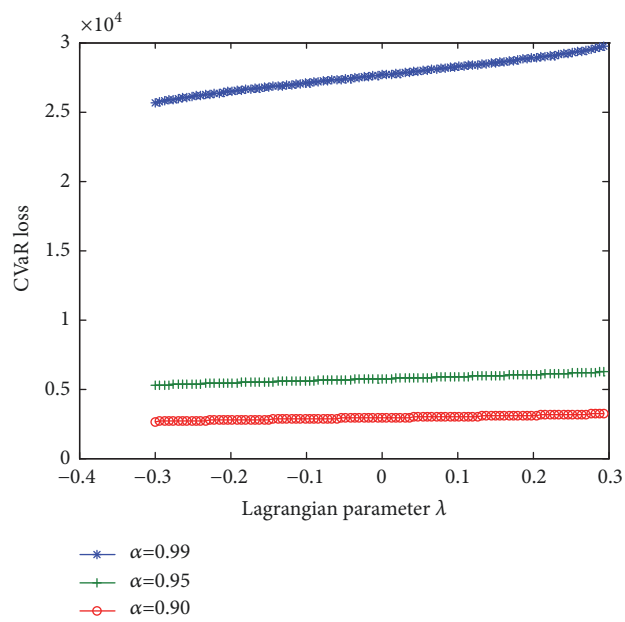

Figure 5: CVaR loss under different λ.

of all their chain stores based on the requirement from their chain stores. This phenomenon is particularly prominent in apparel companies. The paper suggests when the oversupply risk is greater than the insufficient loss DCEs should reduce the total production.

(5) Theorem 1 shows that the increase in the number of chain stores can improve the total production and reduce the risk of loss. In practice, DCEs have been increasing the number of chain stores in order to expand market share.

The above conclusions provide a theoretical reference for DCEs' production and allocation strategy.

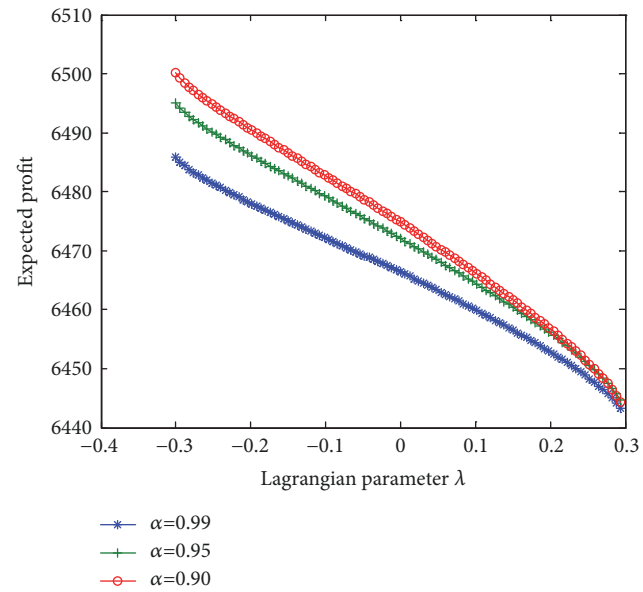

Figure 6: The expected returns under different λ.

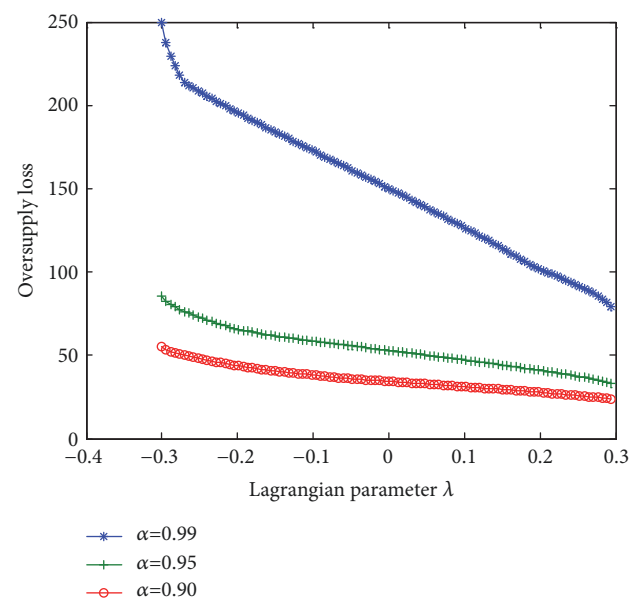

Figure 7: Oversupply loss under the different λ.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

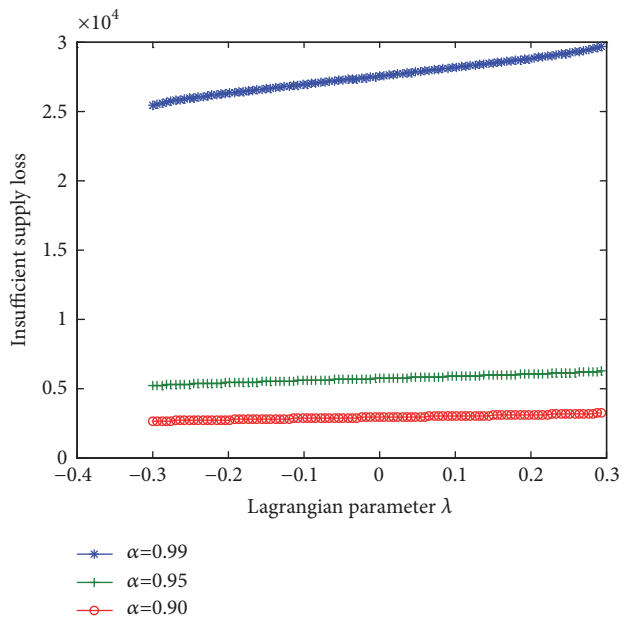

Figure 8: Insufficient supply under different λ.

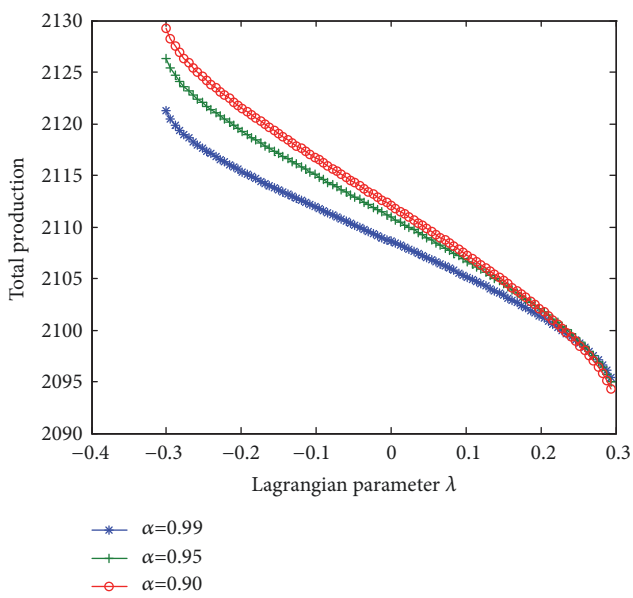

Figure 9: Total production amount under different λ.

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