Research Article

The Edge Connectivity of Expanded k -Ary n -Cubes

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Mass data processing and complex problem solving have higher and higher demands for performance of multiprocessor systems. Many multiprocessor systems have interconnection networks as underlying topologies. The interconnection network determines the performance of a multiprocessor system. The network is usually represented by a graph where nodes (vertices) represent processors and links (edges) represent communication links between processors. For the network G , two vertices u and v of G are said to be connected if there is a (u, v) -path in G . If G has exactly one component, then G is connected; otherwise G is disconnected. In the system where the processors and their communication links to each other are likely to fail, it is important to consider the fault tolerance of the network. For a connected network $G = (V, E)$, its inverse problem is that $G - F$ is disconnected, where $F \subseteq V$ or $F \subseteq E$. The connectivity or edge connectivity is the minimum number of $|F|$. Connectivity plays an important role in measuring the fault tolerance of the network. As a topology structure of interconnection networks, the expanded k -ary n -cube XQ_n^k has many good properties. In this paper, we prove that (1) XQ_n^k is super edge-connected ($n \geq 3$); (2) the restricted edge connectivity of XQ_n^k is $8n - 2$ ($n \geq 3$); (3) XQ_n^k is super restricted edge-connected ($n \geq 3$).

1. Introduction and Terminology

Mass data processing and complex problem solving have higher and higher demands for performance of multiprocessor systems. Many multiprocessor systems have interconnection networks (networks for short) as underlying topologies and a network is usually represented by a graph $G = (V, E)$ where nodes (vertices) represent processors and links (edges) represent communication links between processors. The network determines the performance of the system. For the network (graph) G , two vertices u and v of G are said to be connected if there is a (u, v) -path in G . Connection is an equivalence relation on the vertex set V . Thus there is a partition of V into nonempty subsets $V_1, V_2, \dots, V_\omega$ such that two vertices u and v of G are connected if and only if both u and v belong to the same set V_i . The subgraphs $G[V_1], G[V_2], \dots, G[V_\omega]$ are called the components of G . If G has exactly one component, then G is connected; otherwise G is disconnected. In the system where the processors and their communication links to each other are likely to fail, it is important to consider the fault tolerance of the network.

The fault tolerance of large networks is usually a measure of the extent to which the network can retain its original nature in the event of a certain number of nodes of failure and/or links failure in the network topology. For a connected network (graph) $G = (V, E)$, its inverse problem is that $G - F$ is disconnected, where $F \subseteq V$ or $F \subseteq E$. The connectivity or edge connectivity is the minimum number of $|F|$. Connectivity plays an important role in measuring the fault tolerance of the network.

Let $G = (V, E)$ be a simple graph. Given a nonempty vertex subset V' of V , the induced subgraph by V' in G , denoted by $G[V']$, is a graph, whose vertex set is V' and the edge set is the set of all the edges of G with both endpoints in V' . The degree $d_G(v)$ of a vertex v is the number of edges incident with v . We denote by $\delta(G)$ the minimum degrees of vertices of G . For any vertex v , we define the neighborhood $N_G(v)$ of v in G to be the set of vertices adjacent to v . If $u \in N_G(v)$, then u is called a neighbor or a neighbor vertex of v . Let $S \subseteq V$. We use $N_G(S)$ to denote the set $\cup_{v \in S} N_G(v) \setminus S$. A graph G is said to be k -regular if for any vertex v , $d_G(v) = k$. For graph-theoretical terminology and notation not defined

here we follow [1]. For a faulty subset S of edges in a connected graph G , S is a k -restricted edge cut if $G - S$ is disconnected and every component of $G - S$ has at least k vertices. If such an edge cut exists, then the k -restricted edge connectivity of G , denoted by $\lambda_k(G)$, is defined as the cardinality of a minimum k -restricted edge cut. For any positive integer k , let $\xi_k(G) = \min\{|[X, \overline{X}]| : |X| = k, G[X] \text{ is connected}\}$. For many graphs, it has been shown that $\xi_k(G)$ is an upper bound on $\lambda_k(G)$ [2–4]. A λ_k -connected graph G is λ_k -optimal if $\lambda_k(G) = \xi_k(G)$. The following is the definition on super- λ_k graphs used in our manuscript.

Definition 1 (see [5]). A λ_k -connected graph G is super k -restricted edge-connected (or super- λ_k for short) if every minimum k -restricted edge cut of G isolates one connected subgraph of order k .

In the majority of the literature, the 1-restricted edge connectivity of G is called the edge connectivity of G and is denoted by $\lambda(G)$. The 2-restricted edge connectivity of G is called the restricted edge connectivity of G and is denoted by $\lambda'(G)$. Correspondingly, if G is super 1-restricted edge-connected, then G is super edge-connected. If G is super 2-restricted edge-connected, then G is super restricted edge-connected. The sufficient conditions of super- λ_k have been studied by several authors; see [6, 7].

The k -ary n -cube has many desirable properties, such as ease of implementation of algorithms and ability to reduce message latency by exploiting communication locality found in many parallel applications [8–10]. Therefore, a number of distributed-memory parallel systems (also known as multi-computers) have been built with a k -ary n -cube forming the underlying topology, such as the Cray T3D [11], the J-machine [12], the iWarp [13], and the IBM Blue Gene [14]. In 2011, Xiang and Stewart [15] proposed the augmented k -ary n -cube. In 2017, Wang et al. [16] proposed the expanded k -ary n -cube XQ_n^k . The following is the definition.

Definition 2 (see [16]). The expanded k -ary n -cube, denoted by XQ_n^k ($n \geq 1$ and even $k \geq 6$), is a graph consisting of k^n vertices $\{u_0 u_1 \dots u_{n-1} : 0 \leq u_i \leq k-1, 0 \leq i \leq n-1\}$. Two vertices $u = u_0 u_1 \dots u_{n-1}$ and $v = v_0 v_1 \dots v_{n-1}$ are adjacent if and only if there exists an integer $j \in \{0, 1, \dots, n-1\}$ such that, for some $g \in \{1, -1, 2, -2\}$, we have $u_j = v_j + g \pmod{k}$ and $u_i = v_i$ for all $i \in \{0, 1, \dots, n-1\} \setminus \{j\}$.

For clarity of presentation, we omit writing “ $\pmod{k}$ ” in similar expressions for the remainder of the paper. In this paper, we prove that (1) XQ_n^k is super edge-connected ($n \geq 3$); (2) the restricted edge connectivity of XQ_n^k is $8n-2$ ($n \geq 3$); (3) XQ_n^k is super restricted edge-connected ($n \geq 3$).

2. The Connectivity of Expanded k -Ary n -Cubes

We can partition XQ_n^k into k disjoint subgraphs $XQ_n^k[0]$, $XQ_n^k[1]$, $\dots$, $XQ_n^k[k-1]$ (abbreviated as $XQ[0]$, $XQ[1]$, $\dots$, $XQ[k-1]$, if there is no ambiguity), where every vertex

$u = u_0 u_1 \dots u_{n-1} \in V(XQ_n^k)$ has a fixed integer i in the last position u_{n-1} for $i \in \{0, 1, \dots, k-1\}$. Let $u \in V(XQ[i])$. Then $N(u) \setminus V(XQ[i])$ is said to be outside neighbor vertices of u .

Proposition 3 (see [16]). Each $XQ[i]$ is isomorphic to XQ_{n-1}^k for $0 \leq i \leq k-1$.

Proposition 4. The expanded k -ary n -cube is the Cartesian product of n expanded k -ary 1-cubes, i.e., $XQ_n^k = \underbrace{XQ_1^k \otimes XQ_1^k \otimes \dots \otimes XQ_1^k}_n$.

Proof. We prove $XQ_n^k = \underbrace{XQ_1^k \otimes XQ_1^k \otimes \dots \otimes XQ_1^k}_n$ by the induction on n . Now define a bijection from $V(XQ_2^k)$ to $V(XQ_1^k \otimes XQ_1^k)$ given by

$$\varphi : u_1 u_0 \longrightarrow (u_1 u_0). \quad (1)$$

Note $|V(XQ_2^k)| = |V(XQ_1^k \otimes XQ_1^k)| = k^2$. Let $uv \in E(XQ_2^k)$ with $u = u_1 u_0$ and $v = v_1 v_0$. Then (1), if $u_1 = v_1$, $u_0 = v_0 + g \pmod{k}$ for $g \in \{1, -1, 2, -2\}$; (2), if $u_0 = v_0$, $u_1 = v_1 + g \pmod{k}$ for $g \in \{1, -1, 2, -2\}$. Suppose that $u_1 = v_1$, $u_0 = v_0 + g$. Then $u_0 v_0 \in E(XQ_1^k)$. By the definition of $XQ_1^k \otimes XQ_1^k$, $uv \in E(XQ_1^k \otimes XQ_1^k)$. Suppose that $u_0 = v_0$, $u_1 = v_1 + g$. Similarly, $uv \in E(XQ_1^k \otimes XQ_1^k)$.

Conversely, let $uv \in E(XQ_1^k \otimes XQ_1^k)$ with $u = (u_1 u_0)$ and $v = (v_1 v_0)$. Then $u_1 = v_1$, $u_0 v_0 \in E(XQ_1^k)$ or $u_0 = v_0$, $u_1 v_1 \in E(XQ_1^k)$. Suppose that $u_1 = v_1$ and $u_0 v_0 \in E(XQ_1^k)$. By the definition of XQ_1^k , $u_0 = v_0 + g \pmod{k}$ for $g \in \{1, -1, 2, -2\}$. By the definition of XQ_2^k , $uv \in E(Q_2^k)$. Suppose that $u_0 = v_0$ and $u_1 v_1 \in E(XQ_1^k)$. Similarly, $uv \in E(Q_2^k)$. This implies that the result is true for $n = 2$. Assume that the result holds for $n = m-1$, i.e., $XQ_{m-1}^k = \underbrace{XQ_1^k \otimes XQ_1^k \otimes \dots \otimes XQ_1^k}_{m-1}$ for $m \geq 3$.

Then $\underbrace{XQ_1^k \otimes XQ_1^k \otimes \dots \otimes XQ_1^k}_m = (\underbrace{XQ_1^k \otimes XQ_1^k \otimes \dots \otimes XQ_1^k}_{m-1}) \otimes XQ_1^k = XQ_{m-1}^k \otimes XQ_1^k$. We will prove that $XQ_m^k = XQ_1^k \otimes XQ_{m-1}^k$. Now define a bijection from $V(XQ_m^k)$ to $V(XQ_1^k \otimes XQ_{m-1}^k)$ given by

$$\varphi : u_{m-1} u_{m-2} \dots u_0 \longrightarrow (u_{m-1}, (u_{m-2} \dots u_0)). \quad (2)$$

Note $|V(XQ_m^k)| = |V(XQ_1^k \otimes XQ_{m-1}^k)| = k^m$. Let $uv \in E(XQ_m^k)$ with $u = u_{m-1} u_{m-2} \dots u_0$, $v = v_{m-1} v_{m-2} \dots v_0$, $u' = u_{m-2} \dots u_0$, and $v' = v_{m-2} \dots v_0$.

By Proposition 3, each $XQ_m^k[i]$ is isomorphic to XQ_{m-1}^k (i in the last position u_{m-1}). Suppose that $uv \in E(XQ_m^k[i])$. Then $u_{m-1} = v_{m-1} = i$. By the definition of $XQ_1^k \otimes XQ_{m-1}^k$, $uv \in E(XQ_1^k \otimes XQ_{m-1}^k)$. Suppose that $uv \in E(XQ_m^k) \setminus \sum_{i=0}^{k-1} E(XQ_m^k[i])$. Then, by the definition of $XQ_m^k[i]$, $u_{m-1} = v_{m-1} + g$ and $u_i = v_i$ for $i \in \{0, 1, \dots, m-2\}$ and $g \in \{1, -1, 2, -2\}$. Therefore, $u_{m-1} v_{m-1} \in E(XQ_1^k)$ and $u_i = v_i$ for $i \in \{0, 1, \dots, m-2\}$. By the definition of $XQ_1^k \otimes XQ_{m-1}^k$ and $uv \in E(XQ_1^k \otimes XQ_{m-1}^k)$.

Conversely, let $uv \in E(XQ_1^k \otimes XQ_{m-1}^k)$. Suppose that $u_{m-1} = v_{m-1}$ and $u'v' \in E(XQ_{m-1}^k)$. By the definition of XQ_{m-1}^k , there exists an integer $j \in \{0, 1, \dots, m-2\}$ such that $u_j = v_j + g \pmod{k}$ and $u_i = v_i$, for $i \in \{0, 1, \dots, m-2\} \setminus \{j\}$ and $g \in \{1, -1, 2, -2\}$. Note that $u_{m-1} = v_{m-1}$. By the definition of XQ_m^k , $uv \in E(XQ_m^k)$. Suppose that $u_{m-1}v_{m-1} \in E(XQ_1^k)$ and $u' = v'$. By the definition of XQ_1^k , $u_i = v_i + g$. Note that $u' = v'$. By the definition of XQ_m^k , $uv \in E(XQ_m^k)$. $\square$

The automorphism group of a graph G is transitive if there exists an automorphism φ to any pair u, v of vertices in G such that $\varphi(u) = v$. In this case, G is called vertex transitive.

Proposition 5 (see [16]). XQ_n^k is $4n$ -regular, vertex transitive.

Proposition 6 (see [16]). Let $u \in V(XQ[i])$. Then four outside neighbor vertices of u are in four different $XQ[j]^1$ s.

Theorem 7 (see [16]). Let XQ_n^k be the expanded k -ary n -cube with $n \geq 1$ and even $k \geq 6$. Then the connectivity $\kappa(XQ_n^k) = 4n$.

Theorem 8 (see [1]). Let G be a connected graph. Then $\kappa(G) \leq \lambda(G) \leq \delta(G)$.

By Proposition 5 and Theorems 7 and 8, we have the following corollary.

Corollary 9. $\lambda(XQ_n^k) = 4n$.

Theorem 10. Let XQ_n^k be the expanded k -ary n -cube with $n \geq 3$ and even $k \geq 6$. Then XQ_n^k is super edge-connected.

Proof. By Corollary 9, $\lambda(XQ_n^k) = 4n$. Let $F \subseteq E(XQ_n^k)$ with $|F| = 4n$ be any minimum edge cut of XQ_n^k . We can partition XQ_n^k into k disjoint subgraphs $XQ[0], XQ[1], \dots, XQ[k-1]$. By Proposition 3, each $XQ[i]$ is isomorphic to XQ_{n-1}^k for $0 \leq i \leq k-1$. Let $F_i = F \cap E(XQ[i])$ for $i \in \{0, 1, 2, \dots, k-1\}$ with $|F_0| = \max\{|F_i| : 0 \leq i \leq k-1\}$. Let $\mathcal{F}^{n-1} = F \cap (E(XQ_m^k) \setminus \sum_{i=0}^{k-1} E(XQ_m^k[i]))$. Then $F = \mathcal{F}^{n-1} \cup F_0 \cup \dots \cup F_{k-1}$. Since $n \geq 3$, $k^{n-1} > 4n$ holds, we consider the following cases.

Case 1 ($|F_0| \leq 4n-5$). Since $|F_0| = \max\{|F_i| : 0 \leq i \leq k-1\}$, $|F_i| \leq 4n-5$. By Corollary 9, $XQ[i] - F_i$ is connected. Since $k^{n-1} > 4n$ and there is a complete matching between $XQ[i]$ and $XQ[i+1]$ for $i \in \{0, 1, \dots, k-2\}$, $XQ_n^k - F$ is connected; a contradiction to that F is an edge cut of XQ_n^k .

Case 2 ($|F_0| = 4n-4$). Since $n \geq 3$, $8n-8 > 4n$ holds hence there is only one F_0 such that $|F_0| = 4n-4$. Therefore, $|F_i| \leq 4n-5$ for $i \in \{1, \dots, k-1\}$. By Corollary 9, $XQ[i] - F_i$ is connected for $i \in \{1, \dots, k-1\}$. Since $k^{n-1} > 4n$ and there is a complete matching between $XQ[i]$ and $XQ[i+1]$ for $i \in \{0, 1, \dots, k-2\}$, $XQ_n^k[\cup_{i=1}^{k-1} V(XQ[i] - F_i)]$ is connected.

Case 2.1 ($XQ[0] - F_0$ is connected). Since there is a complete matching between $XQ[i]$ and $XQ[i+1]$ for $i \in \{0, 1, \dots, k-2\}$,

$XQ_n^k - F$ is connected; a contradiction to that F is an edge cut of XQ_n^k .

Case 2.2 ($XQ[0] - F_0$ is not connected). Since $|F_0| = 4n-4$ and $|F| = 4n$, $|\mathcal{F}^{n-1}| \leq 4$ holds. Let $|\mathcal{F}^{n-1}| \leq 3$. By Proposition 6, $XQ_n^k - F$ is connected; a contradiction to that F is an edge cut of XQ_n^k . Let $|\mathcal{F}^{n-1}| = 4$. Let $B_1, \dots, B_j$ ($j \geq 2$) be the components of $XQ[0] - F_0$. Suppose $|V(B_i)| \geq 2$ for $i = 1, \dots, j$. By Proposition 6, $|V(B_i) \cap (\cup_{i=1}^{k-1} V(XQ[i]))| \geq 8$. Since $|\mathcal{F}^{n-1}| = 4$, $XQ_n^k - F$ is connected, a contradiction to that F is an edge cut of XQ_n^k . Therefore, there is a B_i such that $|B_i| = 1$. Let $V(B_i) = \{b\}$. Since $d_{XQ[0]}(b) = 4n-4 = |F_0|$, there is only a B_i such that $|B_i| = 1$. If b is incident with each of F_0 , then $XQ_n^k - F$ has two components, one of which is an isolated vertex. If b is not incident with each of F_0 , then $XQ_n^k - F$ is connected; a contradiction to that F is an edge cut of XQ_n^k .

Case 3 ($4n-3 \leq |F_0| \leq 4n$). In this case, $|\mathcal{F}^{n-1}| \leq 3$ and $XQ_n^k[\cup_{i=1}^{k-1} V(XQ[i] - F_i)]$ is connected. By Proposition 6, $XQ_n^k - F$ is connected, a contradiction to that F is an edge cut of XQ_n^k .

By Cases 1–3, XQ_n^k is super edge-connected. $\square$

Lemma 11. Let XQ_n^k be the expanded k -ary n -cube with $n \geq 3$ and even $k \geq 6$. Then $\lambda'(XQ_n^k) \leq 8n-2$.

Proof. Let $u = \underline{00\dots0}_n$ and $v = \underline{00\dots01}_{n-1}$. Then u is adjacent to v and $u, v \in V(XQ[0])$. Let $E' = [\{u, v\}, \overline{\{u, v\}}]$. Then $|E'| = 8n-2$ and $E' \cap E(XQ[i]) = \emptyset$ for $i \in \{1, \dots, k-1\}$. Therefore, $XQ_n^k[\cup_{i=1}^{k-1} V(XQ[i] - E')]$ is connected. Let $x \in V(XQ[0]) \setminus \{u, v\}$. Then $[\{x\}, \overline{\{x\}}] \cap E' = \emptyset$. Therefore, $XQ_n^k[\{x\} \cup (\cup_{i=1}^{k-1} V(XQ[i] - E'))]$ is connected and hence $XQ_n^k - F$ has two components G_1 ($|V(G_1)| = 2$) and G_2 ($|V(G_1)| \geq 2$). By the definition of λ' , $\lambda'(XQ_n^k) \leq 8n-2$. $\square$

Proposition 12. Let XQ_1^k be the expanded k -ary 1-cube with even $k \geq 6$, and let $F \subseteq E(XQ_1^k)$ with $|F| \leq 5$. If $XQ_1^k - F$ is disconnected, then $XQ_1^k - F$ has two components, one of which is an isolated vertex.

Proof. If $|F| \leq 3$, by Corollary 9, then $XQ_2^k - F$ is connected, a contradiction. Let $|F| = 4$. By Proposition 5, XQ_1^k is 4-regular. Therefore, there is one F such that $XQ_1^k - F$ has two components, one of which is an isolated vertex. Let $|F| = 5$. Suppose that $XQ_1^k - F$ has two or more components which have order at least 2. Choose a component A of $XQ_1^k - F$ with $|V(A)| \geq 2$ so that $|V(A)|$ is as small as possible. Then $2 \leq |V(A)| \leq k/2$. In view of Proposition 5, we may assume that $V(A) = \cup_{1 \leq i \leq p} \{m_i, m_i+1, \dots, m_i+h_i-1\}$ and $\{1, \dots, k-1\} \setminus V(A) = \cup_{1 \leq i \leq p} \{m_i+h_i, m_i+h_i+1, \dots, m_{i+1}-1\}$, where $p \geq 1$ and $0 = m_1 < m_1+h_1 < m_2 < m_2+h_2 < \dots < m_p < m_p+h_p < m_{p+1} = k$. Then $|V(A)| = h_1+h_2+\dots+h_p$ and $|\{0, 1, \dots, k-1\} \setminus V(A)| = (m_2 - (m_1+h_1)) + (m_3 - (m_2+h_2)) + \dots + (k - (m_p+h_p))$. If $p \geq 3$, then the edge set $\{\{m_1+h_1-1, m_1+h_1\}, \{m_2-1, m_2\}, \{m_2+h_2-1, m_2+h_2\}$

$\{h_2\}, \{m_3 - 1, m_3\}, \{m_3 + h_3 - 1, m_3 + h_3\}, \{m_4 - 1, m_4\} \subseteq F$ (in the case where $p = 3$, we take $\{m_4 - 1, m_4\} = \{k - 1, 0\}$), and hence $|F| \geq 6$, a contradiction to that $F = 5$. Thus $1 \leq p \leq 2$. Suppose that $p = 2$. Since $|\{1, \dots, k - 1\} \setminus V(A)| \geq 3$, we have $m_2 - h_1 \geq 2$ or $k - (m_2 + h_2) \geq 2$. We may assume $m_2 - h_1 \geq 2$. Then $h_1 + 1, m_2 - 2 \in \{1, \dots, k - 1\} \setminus V(A)$, which implies $\{h_1 - 1, h_1\}, \{h_1 - 1, h_1 + 1\}, \{m_2 - 2, m_2\}, \{m_2 - 1, m_2\}, \{m_2 + h_2 - 1, m_2 + h_2\}, \{k - 1, 0\} \subseteq F$, and hence $|F| \geq 6$, a contradiction to that $F = 5$. Thus $p = 1$. Since $|\{1, \dots, k - 1\} \setminus V(A)| \geq |V(A)| \geq 2$, it follows that $h_1 \geq 2$ and $k - h_1 \geq 2$. Consequently $1, h_1 - 2 \in V(A)$ and $h_1 + 1, k - 2 \in \{1, \dots, k - 1\} \setminus V(A)$, which implies $\{h_1 - 2, h_1\}, \{h_1 - 1, h_1\}, \{h_1 - 1, h_1 + 1\}, \{k - 2, 0\}, \{k - 1, 0\}, \{k - 1, 1\} \subseteq F$. Therefore $|F| \geq 6$, a contradiction to that $F = 5$ and hence $XQ_1^k - F$ has two components, one of which is an isolated vertex. $\square$

Proposition 13. *Let XQ_1^k be the expanded k -ary 1-cube with even $k \geq 6$, and let $F \subseteq E(XQ_1^k)$ be a minimum 2-restricted edge cut of XQ_1^k . Then $|F| = 6$.*

Proof. By Proposition 12, $|F| \geq 6$. Let $[\{0, 1\}, \overline{\{0, 1\}}]$. Then $[\{0, 1\}, \overline{\{0, 1\}}]$ is a 2-restricted edge cut of XQ_1^k and $|\{0, 1\}, \overline{\{0, 1\}}| = 6$. Therefore, $|F| = 6$. $\square$

Proposition 14. *Let XQ_2^k be the expanded k -ary 2-cube with even $k \geq 6$, and let $F \subseteq E(XQ_2^k)$ with $|F| \leq 13$. If $XQ_2^k - F$ is disconnected, then $XQ_2^k - F$ has two components, one of which is an isolated vertex.*

Proof. We can partition XQ_2^k into k disjoint subgraphs $XQ_2^k[0], XQ_2^k[1], \dots, XQ_2^k[k - 1]$ (abbreviated as $XQ[0], XQ[1], \dots, XQ[k - 1]$, if there is no ambiguity), where every vertex $u_0 u_1 \in V(XQ_2^k)$ has a fixed integer i in the last position u_1 for $i \in \{0, 1, \dots, k - 1\}$. By Proposition 3, each $XQ[i]$ is isomorphic to XQ_{n-1}^k for $0 \leq i \leq k - 1$. By Corollary 9, $\lambda(XQ[i]) = 4$. Let $F_i = F \cap V(XQ[i])$ for $i \in \{0, 1, 2, \dots, k - 1\}$ with $|F_0| = \max\{|F_i| : 0 \leq i \leq k - 1\}$. Let $\mathcal{F}^{n-1} = F \cap (E(XQ_2^k) \setminus \sum_{i=0}^{k-1} E(XQ_2^k[i]))$. Then $F = \mathcal{F}^{n-1} \cup F_0 \cup \dots \cup F_{k-1}$. We consider the following cases.

Case 1 ($|F_0| \leq 3$). Since $|F_0| = \max\{|F_i| : 0 \leq i \leq k - 1\}$, $|F_i| \leq 3$. By Corollary 9, $XQ[i] - F_i$ is connected. By Proposition 6 and $13 < 24$, $XQ_2^k - F$ is connected, a contradiction to that F is a cut of XQ_2^k .

Case 2 ($|F_0| = 4$). Since $|F_0| = \max\{|F_i| : 0 \leq i \leq k - 1\}$, $|F_i| \leq 4$. Since $|F_1| + \dots + |F_5| \leq 9$, there are at most three F_i 's such that $|F_i| = 4$ for $i \in \{1, 2, \dots, k - 1\}$.

Case 2.1 ($|F_0| = 4$ and $|F_i| \leq 3$ for $i \in \{1, 2, \dots, k - 1\}$). By Corollary 9, $XQ[i] - F_i$ is connected for $i \in \{1, 2, \dots, k - 1\}$. By Proposition 6 and $13 < 24$, $XQ_2^k[V(XQ[1] - F_1) \cup \dots \cup V(XQ[k - 1] - F_{k-1})]$ is connected. By Theorem 10, $XQ[0] - F_0$ is connected or $XQ[0] - F_0$ has two components, one of which is an isolated vertex. Since $13 < 20$, $XQ_2^k - F$ is connected (a contradiction) or $XQ_2^k - F$ has two components, one of which is an isolated vertex.

Case 2.2 ($|F_0| = 4$, $|F_{i_0}| = 4$ and $|F_i| \leq 3$ for $i \in \{1, 2, \dots, k - 1\} \setminus \{i_0\}$). In this case, $|\mathcal{F}^{n-1}| \leq 5$. Similarly to Case 2.1, $XQ_2^k - F$ is connected (a contradiction) or $XQ_2^k - F$ has two components, one of which is an isolated vertex.

Case 2.3 ($|F_0| = 4$, $|F_{i_0}| = 4$, $|F_{j_0}| = 4$ and $|F_i| \leq 1$ for $i \in \{1, 2, \dots, k - 1\} \setminus \{i_0, j_0\}$). In this case, $|\mathcal{F}^{n-1}| \leq 1$. Similarly to Case 2.1, $XQ_2^k - F$ is connected, a contradiction.

Case 3 ($|F_0| = 5$). Since $|F_0| = \max\{|F_i| : 0 \leq i \leq k - 1\}$, $|F_i| \leq 5$. Since $|F_1| + \dots + |F_5| \leq 8$, there are at most two F_i 's such that $|F_i| = 4$ for $i \in \{1, 2, \dots, k - 1\}$.

Case 3.1 ($|F_0| = 5$ and $|F_i| \leq 3$ for $i \in \{1, 2, \dots, k - 1\}$). By Corollary 9, $XQ[i] - F_i$ is connected for $i \in \{1, 2, \dots, k - 1\}$. By Proposition 6 and $13 < 24$, $XQ_2^k[V(XQ[1] - F_1) \cup \dots \cup V(XQ[k - 1] - F_{k-1})]$ is connected. By Proposition 12, $XQ[0] - F_0$ is connected or $XQ_2^k - F$ has two components, one of which is an isolated vertex. Since $13 < 20$, $XQ_2^k - F$ is connected (a contradiction) or $XQ_2^k - F$ has two components, one of which is an isolated vertex.

Case 3.2 ($|F_0| = 5$, $|F_{i_0}| = 4$ and $|F_i| \leq 3$ for $i \in \{1, 2, \dots, k - 1\} \setminus \{i_0\}$). In this case, $|\mathcal{F}^{n-1}| \leq 4$. Similarly to Case 3.1, $XQ_2^k - F$ is connected (a contradiction) or $XQ_2^k - F$ has two components, one of which is an isolated vertex.

Case 3.3 ($|F_0| = 5$, $|F_{i_0}| = 5$ and $|F_i| \leq 3$ for $i \in \{1, 2, \dots, k - 1\} \setminus \{i_0\}$). In this case, $|\mathcal{F}^{n-1}| \leq 3$. By Proposition 12, $XQ[0] - F_0$ is connected or $XQ_2^k - F$ has two components, one of which is an isolated vertex. Therefore, $XQ_2^k - F$ is connected, a contradiction.

Case 4 ($6 \leq |F_0| \leq 13$). In this case, $|\mathcal{F}^{n-1}| \leq 7$ and $|F_1| + \dots + |F_{k-1}| \leq 7$. Therefore, $XQ[0] - F_0$ is connected (a contradiction) or $XQ_2^k - F$ has two components, one of which is an isolated vertex. $\square$

Proposition 15. *Let XQ_n^k be the expanded k -ary n -cube with even $k \geq 6$, and let $F \subseteq E(XQ_n^k)$ with $|F| \leq 8n - 3$. If $XQ_n^k - F$ is disconnected, then $XQ_n^k - F$ has two components, one of which is an isolated vertex.*

Proof. We can partition XQ_n^k into k disjoint subgraphs $XQ_n^k[0], XQ_n^k[1], \dots, XQ_n^k[k - 1]$ (abbreviated as $XQ[0], XQ[1], \dots, XQ[k - 1]$, if there is no ambiguity), where every vertex $u_0 u_1 \dots u_{n-1} \in V(XQ_n^k)$ has a fixed integer i in the last position u_{n-1} for $i \in \{0, 1, \dots, k - 1\}$. By Proposition 3, each $XQ[i]$ is isomorphic to XQ_{n-1}^k for $0 \leq i \leq k - 1$. Let $F \subseteq E(XQ_n^k)$ with $|F| \leq 8n - 3$ and let $XQ_n^k - F$ be disconnected. Let $F_i = F \cap E(XQ[i])$ for $i \in \{0, 1, 2, \dots, k - 1\}$ with $|F_0| = \max\{|F_i| : 0 \leq i \leq k - 1\}$. Let $\mathcal{F}^{n-1} = F \cap (E(XQ_n^k) \setminus \sum_{i=0}^{k-1} E(XQ_n^k[i]))$. When $n = 2$, the result holds by Proposition 14. We proceed by induction on n . Our induction hypothesis is that $XQ_{n-1}^k - F$ has two components, one of which is an isolated vertex for $|F| \leq 8n - 11$ and

$n \geq 3$. By Proposition 3, each $XQ[i]$ is isomorphic to XQ_{n-1}^k for $0 \leq i \leq k-1$. We consider the following cases.

Case 1 ($|F_0| \leq 4n-5$). Since $|F_0| = \max\{|F_i| : 0 \leq i \leq k-1\}$, $|F_i| \leq 4n-5$ for $i \in \{1, 2, \dots, k-1\}$. By Corollary 9, $XQ[i] - F$ is connected for $i \in \{0, 1, \dots, k-1\}$. Since $k^{n-1} > 8n-13$ and there is a complete matching between $XQ[i]$ and $XQ[i+1]$, for $i \in \{0, 1, \dots, k-2\}$, $XQ_n^k - F$ is connected; a contradiction to that F is a cut of XQ_n^k .

Case 2 ($4n-4 \leq |F_0| \leq 4n+1$). In this case, $|F_1| + \dots + |F_{k-1}| \leq 8n-3 - (4n-4) = 4n+1$.

Case 2.1 ($|F_i| \leq 4n-5$ for $i \in \{1, 2, \dots, k-1\}$). By Corollary 9, $XQ[i] - F_i$ is connected for $i \in \{1, 2, \dots, k-1\}$. Since $k^{n-1} > 8n-13$, $XQ_n^k[V(XQ[1] - F_1) \cup \dots \cup V(XQ[k-1] - F_{k-1})]$ is connected. By Corollary 9, $XQ[0] - F_0$ is connected or $XQ[0] - F_0$ has two components, one of which is an isolated vertex. Since $k^{n-1} - 1 > 8n-13$, $XQ_n^k - F$ is connected (a contradiction) or $XQ_n^k - F$ has two components, one of which is an isolated vertex.

Case 2.2 ($4n-4 \leq |F_{i_0}| \leq 4n+1$ for $1 \leq i_0 \leq k-1$). In this case, $|\mathcal{F}^{n-1}| \leq 5$ and $|F_i| \leq 5$ for $i \in \{1, \dots, k-1\} \setminus \{i_0\}$. Since $|F_0| = \max\{|F_i| : 0 \leq i \leq k-1\}$, $4n-4 \leq |F_{i_0}| \leq 4n-2$. By Corollary 9, $XQ[i] - F_i$ is connected for $i \in \{1, \dots, k-1\} \setminus \{i_0\}$. Therefore, $XQ_n^k[\cup_{i=1}^{i_0-1} V(XQ[i] - F_i) \cup \cup_{i=i_0+1}^{k-1} V(XQ[i] - F_i)]$ is connected.

Case 2.2.1 ($XQ[0] - F_0$ is connected and $XQ[i_0] - F_{i_0}$ is connected). Since $k^{n-1} > 5$, XQ_n^k is connected; a contradiction to that F is an edge cut of XQ_n^k .

Case 2.2.2 ($XQ[0] - F_0$ is connected and $XQ[i_0] - F_{i_0}$ is disconnected). By the induction hypothesis, $XQ[i_0] - F_{i_0}$ has two components, one of which is an isolated vertex. Since $k^{n-1} - 1 > 4n-5$, $XQ_n^k - F$ is connected (a contradiction) or $XQ_n^k - F$ has two components, one of which is an isolated vertex.

Case 2.2.3 ($XQ[0] - F_0$ is disconnected and $XQ[i_0] - F_{i_0}$ is disconnected). By the induction hypothesis, $XQ[0] - F_0$ has two components, one of which is an isolated vertex u and $XQ[i_0] - F_{i_0}$ has two components, one of which is an isolated vertex v . Suppose that u is adjacent to v in $XQ_n^k - F$. Since $|\mathcal{F}^{n-1}| \leq 5$, by Proposition 6, XQ_n^k is connected; a contradiction to that F is an edge cut of XQ_n^k . Suppose that u is not adjacent to v in $XQ_n^k - F$. Since $|\mathcal{F}^{n-1}| \leq 5$, $XQ[0] - F_0$ has two components, one of which is an isolated vertex or XQ_n^k is connected; a contradiction to that F is an edge cut of XQ_n^k .

Case 3 ($4n+2 \leq |F_0| \leq 8n-11$). In this case, $|F_1| + \dots + |F_{k-1}| \leq 8n-3 - (4n+2) = 4n-5$. Therefore, $|F_i| \leq 4n-5$ for $i \in \{1, 2, \dots, k-1\}$. By Corollary 9, $XQ[i] - F_i$ is connected for $i \in \{1, 2, \dots, k-1\}$. Since $k^{n-1} > 4n-5$, $XQ_n^k[V(XQ[1] - F_1) \cup \dots \cup V(XQ[k-1] - F_{k-1})]$ is connected. By the induction

hypothesis, $XQ[0] - F_0$ has two components, one of which is an isolated vertex. Since $k^{n-1} - 1 > 4n-5$, $XQ_n^k - F$ is connected (a contradiction) or $XQ_n^k - F$ has two components, one of which is an isolated vertex.

Case 4 ($8n-10 \leq |F_0| \leq 8n-3$). In this case, $|F_1| + \dots + |F_{k-1}| \leq 7$ and $|\mathcal{F}^{n-1}| \leq 7$. Similarly to Case 3, $XQ_n^k[V(XQ[1] - F_1) \cup \dots \cup V(XQ[k-1] - F_{k-1})]$ is connected. Since $|\mathcal{F}^{n-1}| \leq 7$, $XQ_n^k - F$ is connected (a contradiction) or $XQ_n^k - F$ has two components, one of which is an isolated vertex. $\square$

Theorem 16. Let XQ_n^k be the expanded k -ary n -cube with $n \geq 3$ and even $k \geq 6$. Then $\lambda'(XQ_n^k) = 8n-2$.

Proof. By Lemma 11, $\lambda'(XQ_n^k) \leq 8n-2$. By Proposition 15, $\lambda'(XQ_n^k) \geq 8n-2$. Therefore, $\lambda'(XQ_n^k) = 8n-2$. $\square$

Theorem 17. Let XQ_n^k be the expanded k -ary n -cube with $n \geq 3$ and even $k \geq 6$. Then XQ_n^k is super restricted edge-connected.

Proof. We can partition XQ_n^k into k disjoint subgraphs $XQ_n^k[0], XQ_n^k[1], \dots, XQ_n^k[k-1]$ (abbreviated as $XQ[0], XQ[1], \dots, XQ[k-1]$, if there is no ambiguity), where every vertex $u_0 u_1 \dots u_{n-1} \in V(XQ_n^k)$ has a fixed integer i in the last position u_{n-1} for $i \in \{0, 1, \dots, k-1\}$. By Proposition 3, each $XQ[i]$ is isomorphic to XQ_{n-1}^k for $0 \leq i \leq k-1$. Let F be a minimum restricted edge cut of XQ_n^k . By Theorem 16, $|F| = 8n-2$. Let $F_i = F \cap E(XQ[i])$ for $i \in \{0, 1, 2, \dots, k-1\}$ with $|F_0| = \max\{|F_i| : 0 \leq i \leq k-1\}$. Let $\mathcal{F}^{n-1} = F \cap (E(XQ_n^k) \setminus \sum_{i=0}^{k-1} E(XQ_n^k[i]))$. We consider the following cases.

Case 1 ($|F_0| \leq 4n-5$). Since $|F_0| = \max\{|F_i| : 0 \leq i \leq k-1\}$, $|F_i| \leq 4n-5$ for $i \in \{1, 2, \dots, k-1\}$. By Corollary 9, $XQ[i] - F_i$ is connected for $i \in \{0, 1, \dots, k-1\}$. Since $4k^{n-1} > 8n-2$ ($n \geq 2$), $XQ_n^k - F$ is connected; a contradiction to that F is an edge cut of XQ_n^k .

Case 2 ($4n-4 \leq |F_0| \leq 4n+2$). In this case, $|F_1| + \dots + |F_{k-1}| \leq 8n-2 - (4n-4) = 4n+2$.

Case 2.1 ($|F_i| \leq 4n-5$ for $i \in \{1, 2, \dots, k-1\}$). By Corollary 9, $XQ[i] - F_i$ is connected for $i \in \{1, 2, \dots, k-1\}$. Since $4k^{n-1} > 8n-2$, $XQ_n^k[V(XQ[1] - F_1) \cup \dots \cup V(XQ[k-1] - F_{k-1})]$ is connected. By Proposition 15, $XQ[0] - F_0$ is connected or $XQ[0] - F_0$ has two components, one of which is an isolated vertex. Since $4(k^{n-1} - 1) > 8n-2$, $XQ_n^k - F$ is connected or $XQ_n^k - F$ has two components, one of which is an isolated vertex, a contradiction to that F is a restricted edge cut of XQ_n^k .

Case 2.2 ($4n-4 \leq |F_{i_0}| \leq 4n+2$ for $1 \leq i_0 \leq k-1$). In this case, $|\mathcal{F}^{n-1}| \leq 6$ and $|F_i| \leq 6$ for $i \in \{1, \dots, k-1\} \setminus \{i_0\}$. By Corollary 9, $XQ[i] - F_i$ is connected for $i \in \{1, \dots, k-1\} \setminus \{i_0\}$. Therefore, $XQ_n^k[\cup_{i=1}^{i_0-1} V(XQ[i] - F_i) \cup \cup_{i=i_0+1}^{k-1} V(XQ[i] - F_i)]$ is connected.

Case 2.2.1 ($XQ[0] - F_0$ is connected and $XQ[i_0] - F_{i_0}$ is connected). Since $k^{n-1} > 6$, XQ_n^k is connected; a contradiction to that F is an edge cut of XQ_n^k .

Case 2.2.2 ($XQ[0] - F_0$ is connected and $XQ[i_0] - F_{i_0}$ is disconnected). By Proposition 15, $XQ[i_0] - F_{i_0}$ has two components, one of which is an isolated vertex. Therefore, $XQ_n^k - F$ is connected or $XQ_n^k - F$ has two components, one of which is an isolated vertex; a contradiction to that F is a restricted edge cut of XQ_n^k .

Case 2.2.3 ($XQ[0] - F_0$ is disconnected and $XQ[i_0] - F_{i_0}$ is disconnected). By Proposition 15, $XQ[0] - F_0$ has two components, one of which is an isolated vertex u and $XQ[i_0] - F_{i_0}$ has two components, one of which is an isolated vertex v . Suppose that u is adjacent to v in $XQ_n^k - F$. Since $|\mathcal{F}^{n-1}| \leq 6$, by Proposition 6, XQ_n^k is connected (a contradiction) $XQ[0] - F_0$ has two components, one of which is an isolated vertex (a contradiction) or $XQ_n^k - F$ has two components, one of which is a K_2 . Suppose that u is not adjacent to v in $XQ_n^k - F$. Since $|\mathcal{F}^{n-1}| \leq 6$, $XQ[0] - F_0$ has two components, one of which is an isolated vertex or XQ_n^k is connected, a contradiction to that F is a restricted edge cut of XQ_n^k .

Case 3 ($4n + 3 \leq |F_0| \leq 8n - 11$). In this case, $|F_1| + \dots + |F_{k-1}| \leq 8n - 2 - (4n + 3) = 4n - 5$. Therefore, $|F_i| \leq 4n - 5$ for $i \in \{1, 2, \dots, k-1\}$. By Corollary 9, $XQ[i] - F_i$ is connected for $i \in \{1, 2, \dots, k-1\}$. Since $k^{n-1} > 4n - 5$, $XQ_n^k[V(XQ[1] - F_1) \cup \dots \cup V(XQ[k-1] - F_{k-1})]$ is connected. By Proposition 15, $XQ[0] - F_0$ has two components, one of which is an isolated vertex. Since $k^{n-1} - 1 > 4n - 5$, $XQ_n^k - F$ is connected or $XQ_n^k - F$ has two components, one of which is an isolated vertex; a contradiction to that F is a restricted edge cut of XQ_n^k .

Case 4 ($|F_0| = 8n - 10$). In this case, $|F_1| + \dots + |F_{k-1}| \leq 8$ and $|\mathcal{F}^{n-1}| \leq 8$.

XQ_n^k is connected (a contradiction), $XQ[0] - F_0$ has two components, one of which is an isolated vertex (a contradiction), or $XQ_n^k - F$ has two components, one of which is a K_2 .

Case 5 ($8n - 9 \leq |F_0| \leq 8n - 2$). In this case, $|F_1| + \dots + |F_{k-1}| \leq 7$ and $|\mathcal{F}^{n-1}| \leq 7$. Similarly to Case 3, $XQ_n^k[V(XQ[1] - F_1) \cup \dots \cup V(XQ[k-1] - F_{k-1})]$ is connected. Since $|\mathcal{F}^{n-1}| \leq 7$, $XQ_n^k - F$ is connected or $XQ_n^k - F$ has two components, one of which is an isolated vertex; a contradiction to that F is a restricted edge cut of XQ_n^k . $\square$

A super- λ graph G to be m -super- λ if $G - S$ is still super- λ for any edge subset S of G with $|S| \leq m$. The maximum integer of such m , written as $S_\lambda(G)$, is said to be the edge fault tolerance of G with respect to the super- λ property.

Theorem 18 (see [17]). *Let G be a regular graph. If G is super- λ' and $\delta(G) \geq 4$, then $S_\lambda(G) = \delta(G) - 1$.*

By Theorems 17 and 18, we have the following theorem.

Theorem 19. *Let XQ_n^k be the expanded k -ary n -cube with $n \geq 3$ and even $k \geq 6$. Then $S_\lambda(XQ_n^k) = 4n - 1$.*

Remarks on Theorem 19. Note that $S_\lambda(G) \leq \delta(G) - 1$. Since $\delta(XQ_n^k) = 4n$, $S_\lambda(XQ_n^k)$ is maximum.

A spanning subgraph of a graph G is a subgraph H with $V(H) = V(G)$.

Proposition 20. *The k -ary n -cube Q_n^k is a spanning subgraph of the expanded k -ary n -cube XQ_n^k .*

Proof. By the definitions of Q_n^k and XQ_n^k , $V(Q_n^k) = V(XQ_n^k)$. Let $uv \in E(Q_n^k)$ with $u = u_0u_1 \dots u_{n-1}$ and $v = v_0v_1 \dots v_{n-1}$. Then there exists an integer $j \in \{0, 1, \dots, n-1\}$ such that $u_j = v_j + g \pmod{k}$ and $u_i = v_i$, for $i \in \{0, 1, \dots, n-1\} \setminus \{j\}$ and $g \in \{1, -1\}$ in Q_n^k . By the definition of XQ_n^k , $uv \in E(XQ_n^k)$. $\square$

A graph G is conditional faulty if each vertex of G is incident with at least two healthy edges.

Theorem 21 (see [18]). *Let $n \geq 2$ and let $k \geq 4$ be even. Then the conditional faulty Q_n^k with at most $4n - 5$ faulty edges is Hamiltonian.*

By Theorem 21 and Proposition 20, we have the following corollary.

Corollary 22. *The expanded k -ary n -cube is Hamiltonian.*

3. Conclusions

In this paper, we investigate the problem of the super connectivity of the expanded k -ary n -cube XQ_n^k . It is proved that XQ_n^k is super edge-connected and super restricted edge-connected ($n \geq 3$). The work will help engineers to develop more different measures of the super connectivity based on application environment, network topology, network reliability, and statistics related to fault patterns.

Data Availability

The data cited in the manuscript titled “the Edge Connectivity of Expanded k -Ary n -Cubes” are all published articles. There are no other data.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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