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Research Article

Analysis of Adaptive Synchronization for Stochastic Neutral-Type Memristive Neural Networks with Mixed Time-Varying Delays

Desheng Hong, Zuoliang Xiong, and Cuiping Yang

Department of Mathematics, Nanchang University, Nanchang, Jiangxi 330031, China

Correspondence should be addressed to Zuoliang Xiong; xiong1601@163.com

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Linear feedback control and adaptive feedback control are proposed to achieve the synchronization of stochastic neutral-type memristive neural networks with mixed time-varying delays. By applying the stochastic differential inclusions theory, Lyapunov functional, and linear matrix inequalities method, we obtain some new adaptive synchronization criteria. A numerical example is given to illustrate the effectiveness of our results.

1. Introduction

During the last few years, as we know neural networks have been widely researched in control, image processing, associative memory design, pattern recognition, information science, and so on (see [1–3]). Chua firstly predicted the memristor as the fourth fundamental electrical circuit element in 1971 [4]. In 2008, Hewlett-Packard research team [5] obtained a practical memristor device and exhibited its characteristic, such as nanoscale and the memory ability. It has been shown that memristors can be used to work as biological synapses in artificial neural network and replace resistor to simulate the human brain in memristor-based neural networks (MNNs) model, which would benefit many practical applications (see [6, 7]).

It is well know that time delays present complex and unpredictable behaviors in practice often caused by finite switching speeds of the amplifiers, which may affect the stability of the system and even results in oscillation, divergence, and instability phenomena. Therefore, much effort has been devoted to analyze dynamic behaviors of MNNs with various types of times delays (see [8, 9]); constant time delays and the time-varying delays have been studied in [10–12]. The investigations of MNNs discussed consider the discrete delays in [13]. However, since the neural signal propagation is often distributed during a certain time period in the presence of an amount of parallel pathways with a variety of axon sizes and lengths, hence, the authors in [14, 15] have concentrated on the mixed delays.

On the other hand, in reality, the fluctuations from the release of neurotransmitters or other probabilistic causes may affect the stability property in the nervous system and synaptic transmission. So the stability analysis with stochastic perturbation has aroused great interest of many researchers (see [16, 17]). It is natural and important that systems containing some information are not only related to the derivative of the current state, but also have a great relationship with the previous derivative, which is called neural-type neural networks (see [9, 18, 19]).

Recently synchronization and antisynchronization of memristor-based neural networks have received great attention due to their potential, such as secure communication information science and biological technology [20]. But the networks are not always able to synchronize by themselves. Then, various effective control approaches and techniques have been proposed for synchronization, such as impulsive control, feedback control, adaptive control, and intermittent control (see [21, 22]). And a lot of achievements have been made in the stability and synchronization problem of MNNs, including exponential synchronization, lag synchronization, and finite time synchronization (see [23–26]).

Motivated by the above discussion, even though the synchronization problem of stochastic MNNs has been studied, there are few studies on the synchronization problem of stochastic neutral-type MNNs. So in this paper we focus our minds on the adaptive synchronization for neutral-type MNNs with mixed time-varying delays to bridge the gap. By applying the stochastic differential inclusions theory, Lyapunov functional, and linear matrix inequalities method, we obtain some new adaptive synchronization criteria.

This paper is organized as follows. In Section 2, we introduce the model and some preliminaries. The main theoretical results are derived in Section 3. In Section 4, a numerical simulation is presented to verify our obtained results. Finally, conclusion is given in Section 5.

Throughout this paper, solutions of all the systems considered are intended in the Filippov's sense. $\mathfrak{R}^n$ and $\mathfrak{R}^{n\times n}$ denote the n-dimensional Euclidean space and the set of all $n \times n$ real matrices, respectively. The superscript T denotes matrix transposition, $\operatorname{tr}(\cdot)$ denotes the trace of the corresponding matrix, and I denotes the identity matrix. $\lambda_{\max}$ and $\lambda_{\min}$ denote the maximum and minimum eigenvalues of a real symmetric matrix. $\operatorname{diag}(\cdots)$ stands for the block diagonal matrix. $C^2_{F_0}([-\tau, 0]; R^n)$ denote the family of all F_0 measurable, $C([-\tau, 0]; R^n)$-valued stochastic variables $\xi = \{\xi(\theta) : -\tau \le \theta \le 0\}$, such that $\int_{-\tau}^{0} E[|\xi(s)|^2]ds < \infty$, where $E[\cdot]$ stands for the correspondent expectation operator with respect to the given probability measure P. $\operatorname{co}\{u, v\}$ denotes the closure of a convex hull generated by real numbers u and v or real matrices u and v.

2. Preliminaries

In this paper, the following stochastic neutral-type memristive neural network with mixed time-varying delays is described by ($i = 1, 2, \ldots, n$)

$$\begin{aligned} d\left[x_i(t) - d_i x_i(t-\tau_1(t))\right] = \Bigg[& -c_i x_i(t) \\ & + \sum_{j=1}^{n} a_{ij}(x_i(t)) \tilde{f}_j(x_j(t)) \\ & + \sum_{j=1}^{n} b_{ij}(x_i(t)) \tilde{g}_j(x_j(t-\tau_2(t))) \\ & + \sum_{j=1}^{n} w_{ij}(x_i(t)) \int_{t-\tau_3(t)}^{t} \tilde{h}_j(x_j(s))\,ds + I_i \Bigg] dt, \end{aligned} \tag{1}$$

with initial conditions $x_i(t) = \phi_i(t)$, $t \in [-\tau, 0]$, where $x_i(t)$ is the voltage of the capacitor $\mathbf{C}_i$, $\tilde{f}_j(\cdot)$, $\tilde{g}_j(\cdot)$, and $\tilde{h}_j(\cdot)$ are neuron activation functions, and I_i is the external constant input. $C = \operatorname{diag}(c_1, c_2, \ldots, c_n)$ and $D = \operatorname{diag}(d_1, d_2, \ldots, d_n)$ are self-feedback connection matrices and $c_i > 0$, $d_i > 0$ ($i = 1, 2, \ldots, n$), $a_{ij}(x_i(t))$, $b_{ij}(x_i(t))$, and $w_{ij}(x_i(t))$ represent memristor-based weights:

$$\begin{aligned} a_{ij}(x_i(t)) &= \frac{\mathbf{W}_{(1)ij}}{\mathbf{C}_i} \times \operatorname{sgin}_{ij}, \\ b_{ij}(x_i(t)) &= \frac{\mathbf{W}_{(2)ij}}{\mathbf{C}_i} \times \operatorname{sgin}_{ij}, \\ w_{ij}(x_i(t)) &= \frac{\mathbf{W}_{(3)ij}}{\mathbf{C}_i} \times \operatorname{sgin}_{ij}, \\ \operatorname{sgin}_{ij} &= \begin{cases} 1 & i \ne j, \\ -1 & i = j. \end{cases} \end{aligned} \tag{2}$$

Here $\mathbf{W}_{(k)ij}$ denote the memductances of memristors $\mathbf{R}_{(k)ij}$, $k = 1, 2, 3$. According to the pinched hysteretic loops of property of memristors, we set

$$\begin{aligned} a_{ij}(x_i(t)) &= \begin{cases} \hat{a}_{ij} & |x_i(t)| \le \gamma_i, \\ \check{a}_{ij} & |x_i(t)| > \gamma_i, \end{cases} \\ b_{ij}(x_i(t)) &= \begin{cases} \hat{b}_{ij} & |x_i(t)| \le \gamma_i, \\ \check{b}_{ij} & |x_i(t)| > \gamma_i, \end{cases} \\ w_{ij}(x_i(t)) &= \begin{cases} \hat{w}_{ij} & |x_i(t)| \le \gamma_i, \\ \check{w}_{ij} & |x_i(t)| > \gamma_i, \end{cases} \end{aligned} \tag{3}$$

where the switching jumps $\gamma_i > 0$, $\hat{a}_{ij}$, $\check{a}_{ij}$, $\hat{b}_{ij}$, $\check{b}_{ij}$, $\hat{w}_{ij}$, $\check{w}_{ij}$ ($i, j = 1, 2, \ldots, n$) are constants. $A(x_i(t)) = (a_{ij}(x_i(t)))_{n\times n}$, $B(x_i(t)) = (b_{ij}(x_i(t)))_{n\times n}$, and $W(x_i(t)) = (w_{ij}(x_i(t)))_{n\times n}$ are memristive connection weights, which represent the neuron interconnection matrix, respectively. If $A(x_i(t)) = (a_{ij}(x_i(t)))_{n\times n}$, $B(x_i(t)) = (b_{ij}(x_i(t)))_{n\times n}$, and $W(x_i(t)) = (w_{ij}(x_i(t)))_{n\times n}$ are constants, system (1) will reduce to a general network. Let $\underline{a}_{ij} = \min\{\hat{a}_{ij}, \check{a}_{ij}\}$, $\overline{a}_{ij} = \max\{\hat{a}_{ij}, \check{a}_{ij}\}$, $\underline{b}_{ij} = \min\{\hat{b}_{ij}, \check{b}_{ij}\}$, $\overline{b}_{ij} = \max\{\hat{b}_{ij}, \check{b}_{ij}\}$, $\underline{w}_{ij} = \min\{\hat{w}_{ij}, \check{w}_{ij}\}$, $\overline{w}_{ij} = \max\{\hat{w}_{ij}, \check{w}_{ij}\}$, $\overline{A} = (\overline{a}_{ij})_{n\times n}$, $\overline{B} = (\overline{b}_{ij})_{n\times n}$, $\overline{W} = (\overline{w}_{ij})_{n\times n}$, $\underline{A} = (\underline{a}_{ij})_{n\times n}$, $\underline{B} = (\underline{b}_{ij})_{n\times n}$, $\underline{W} = (\underline{w}_{ij})_{n\times n}$, $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T$, $I = [I_1, I_2, \ldots, I_n]^T$, $\tilde{f}(x(t)) = [\tilde{f}(x_1(t)), \tilde{f}(x_2(t)), \ldots, \tilde{f}(x_n(t))]^T$, $\tilde{g}(x(t)) = [\tilde{g}(x_1(t)), \tilde{g}(x_2(t)), \ldots, \tilde{g}(x_n(t))]^T$, $\tilde{h}(x(t)) = [\tilde{h}(x_1(t)), \tilde{h}(x_2(t)), \ldots, \tilde{h}(x_n(t))]^T$, for $i = 1, 2, \ldots, n$. $\tau_k(t)$ ($k = 1, 2, 3$) represent the time-varying transmission delays. Since $a_{ij}(x_i(t)), b_{ij}(x_i(t)), w_{ij}(x_i(t))$ are discontinuous, in this paper, the solutions of all the following systems are illustrated in Filippov's sense. By applying theory of differential inclusions and set-valued maps in system (1), this can be written as follows:

$$d\left[x(t)-Dx\left(t-\tau_1(t)\right)\right]\in\Bigg[-Cx(t)+\sum_{j=1}^{n}\mathrm{co}\left\{a_{ij}\left(x_i(t)\right)\right\}\widetilde{f}\left(x(t)\right)+\sum_{j=1}^{n}\mathrm{co}\left\{b_{ij}\left(x_i(t)\right)\right\}\widetilde{g}\left(x\left(t-\tau_2(t)\right)\right)+\sum_{j=1}^{n}\mathrm{co}\left\{w_{ij}\left(x_i(t)\right)\right\}\int_{t-\tau_3(t)}^{t}\widetilde{h}\left(x(s)\right)ds+I\Bigg]dt, \quad (4)$$

where the set-valued maps are defined as follows:

$$\mathrm{co}\left[a_{ij}\left(x_i(t)\right)\right]=\begin{cases}\widehat{a}_{ij} & \left|x_i(t)\right|<\gamma_i,\\ \left[\underline{a}_{ij},\overline{a}_{ij}\right] & \left|x_i(t)\right|=\gamma_i,\\ \check{a}_{ij} & \left|x_i(t)\right|>\gamma_i,\end{cases}$$
$$\mathrm{co}\left[b_{ij}\left(x_i(t)\right)\right]=\begin{cases}\widehat{b}_{ij} & \left|x_i(t)\right|<\gamma_i,\\ \left[\underline{b}_{ij},\overline{b}_{ij}\right] & \left|x_i(t)\right|=\gamma_i,\\ \check{b}_{ij} & \left|x_i(t)\right|>\gamma_i,\end{cases} \quad (5)$$
$$\mathrm{co}\left[w_{ij}\left(x_i(t)\right)\right]=\begin{cases}\widehat{w}_{ij} & \left|x_i(t)\right|<\gamma_i,\\ \left[\underline{w}_{ij},\overline{w}_{ij}\right] & \left|x_i(t)\right|=\gamma_i,\\ \check{w}_{ij} & \left|x_i(t)\right|>\gamma_i,\end{cases}$$

or equivalently, there exist $a_{ij}(t)\in\mathrm{co}[a_{ij}(x_i(t))]$, $b_{ij}(t)\in\mathrm{co}[b_{ij}(x_i(t))]$, $w_{ij}(t)\in\mathrm{co}[w_{ij}(x_i(t))]$, and $\widetilde{A}(t)=(a_{ij}(t))_{n\times n}$, $\widetilde{B}(t)=(b_{ij}(t))_{n\times n}$, $\widetilde{W}(t)=(w_{ij}(t))_{n\times n}$, such that

$$d\left[x(t)-Dx\left(t-\tau_1(t)\right)\right]=\Bigg[-Cx(t)+\widetilde{A}(t)\widetilde{f}\left(x(t)\right)+\widetilde{B}(t)\widetilde{g}\left(x\left(t-\tau_2(t)\right)\right)+\widetilde{W}(t)\int_{t-\tau_3(t)}^{t}\widetilde{h}\left(x(s)\right)ds+I\Bigg]dt. \quad (6)$$

We consider system (6) as the drive system. Similarly, the response system is

$$d\left[y(t)-Dy\left(t-\tau_1(t)\right)\right]=\Bigg[-Cy(t)+\widetilde{A}(t)\widetilde{f}\left(y(t)\right)+\widetilde{B}(t)\widetilde{g}\left(y\left(t-\tau_2(t)\right)\right)+\widetilde{W}(t)\int_{t-\tau_3(t)}^{t}\widetilde{h}\left(y(s)\right)ds+I+u(t)\Bigg]dt+\sigma\left(t,y(t)-x(t),y\left(t-\tau_1(t)\right)-x\left(t-\tau_1(t)\right),y\left(t-\tau_2(t)\right)-x\left(t-\tau_2(t)\right),y\left(t-\tau_3(t)\right)-x\left(t-\tau_3(t)\right)\right)d\omega(t), \quad (7)$$

where $u(t)=[u_1(t),u_2(t),\ldots,u_n(t)]^T\in\mathfrak{R}^n$ is the controller, $\omega(t)=[\omega_1(t),\omega_2(t),\ldots,\omega_n(t)]^T$ is an n-dimensional Brownian motion defined on the complete probability space (Ω,F,P) with a natural filtration $\{F_t\}_{t\geq 0}$, and $\sigma:\mathfrak{R}^+\times\mathfrak{R}^n\times\mathfrak{R}^n\times\mathfrak{R}^n\times\mathfrak{R}^n\rightarrow\mathfrak{R}^{n\times n}$ is the noise intensity matrix, where σ satisfies $\sigma(t,0,0,0,0)\equiv 0$. Let $e(t)=[e_1(t),e_2(t),\ldots,e_n(t)]^T$ be the synchronization error, where $e_i(t)=y_i(t)-x_i(t)$, $e_i(t-\tau_2(t))=y_i(t-\tau_2(t))-x_i(t-\tau_2(t))$, $e_i(t-\tau_3(t))=y_i(t-\tau_3(t))-x_i(t-\tau_3(t))$. From (6) and (7), we can get the following synchronization error system:

$$d\left[e(t)-De\left(t-\tau_1(t)\right)\right]=\Bigg[-Ce(t)+\widetilde{A}(t)f\left(e(t)\right)+\widetilde{B}(t)g\left(e\left(t-\tau_2(t)\right)\right)+\widetilde{W}(t)\int_{t-\tau_3(t)}^{t}h\left(e(s)\right)ds+u(t)\Bigg]dt+\sigma\left(t,e(t),e\left(t-\tau_1(t)\right),e\left(t-\tau_2(t)\right),e\left(t-\tau_3(t)\right)\right)d\omega(t), \quad (8)$$

where $f(e(t))=\widetilde{f}(y(t))-\widetilde{f}(x(t))$, $g(e(t-\tau_2(t)))=\widetilde{g}(y(t-\tau_2(t)))-\widetilde{g}(x(t-\tau_2(t)))$, $h(e(t-\tau_3(t)))=\widetilde{h}(y(t-\tau_3(t)))-\widetilde{h}(x(t-\tau_3(t)))$. To prove our main results, the following assumptions and lemmas are needed.

Assumption 1 (see [27]). There exist diagonal matrices $L_i^-=\mathrm{diag}(l_{i1}^-,l_{i2}^-,\ldots,l_{in}^-)$ and $L_i^+=\mathrm{diag}(l_{i1}^+,l_{i2}^+,\ldots,l_{in}^+)$, $i=1,2,3$, satisfying $l_{1j}^-\leq(\widetilde{f}_j(u)-\widetilde{f}_j(v))/(u-v)\leq l_{1j}^+$, $l_{2j}^-\leq(\widetilde{g}_j(u)-\widetilde{g}_j(v))/(u-v)\leq l_{2j}^+$, $l_{3j}^-\leq(\widetilde{h}_j(u)-\widetilde{h}_j(v))/(u-v)\leq l_{3j}^+$, for all $u,v\in\mathfrak{R}$, $u\neq v$, $j=1,2,\ldots,n$.

Assumption 2. There exist positive constants $\tau_1,\tau_2,\tau_3,\mu_1,\mu_2$, and μ_3, such that

$$\begin{aligned}0&\leq\tau_1(t)\leq\tau_1,\\ 0&\leq\tau_2(t)\leq\tau_2,\\ 0&\leq\tau_3(t)\leq\tau_3,\\ \dot{\tau}_1(t)&\leq\mu_1<1,\\ \dot{\tau}_2(t)&\leq\mu_2<1,\\ \dot{\tau}_3(t)&\leq\mu_3<1.\end{aligned} \quad (9)$$

Remark 3. The assumption strong condition can be weaken; please refer to [28, 29] Assumptions 2 and 1.

Assumption 4. $\forall a, b \in R^N$, there exist positive constants $\underline{L}_i, \overline{L}_i, \underline{M}_i, \overline{M}_i, \underline{N}_i, \overline{N}_i$, such that

$$\begin{aligned} \left|\tilde{f}_i(\cdot)\right| &\le \underline{L}_i, \\ \left|\tilde{g}_i(\cdot)\right| &\le \underline{M}_i, \\ \left|\tilde{h}_i(\cdot)\right| &\le \underline{N}_i, \\ \left|\tilde{f}_i(a) - \tilde{f}_i(b)\right| &\le \overline{L}_i\,|a-b|, \\ \left|\tilde{g}_i(a) - \tilde{g}_i(b)\right| &\le \overline{M}_i\,|a-b|, \\ \left|\tilde{h}_i(a) - \tilde{h}_i(b)\right| &\le \overline{N}_i\,|a-b|, \end{aligned} \tag{10}$$

where $\tilde{f}_i(0) = \tilde{g}_i(0) = \tilde{h}_i(0) = 0$ and $i, j = 1, 2, \ldots, n$.

Assumption 5 (see [30]). There exist positive matrices R_1, R_2, R_3, and R_4, such that

$$\begin{aligned} &\operatorname{tr}\left[\sigma^T(t, x_1, x_2, x_3, x_4)\,\sigma(t, x_1, x_2, x_3, x_4)\right] \\ &\quad \le x_1^T R_1 x_1 + x_2^T R_2 x_2 + x_3^T R_3 x_3 + x_4^T R_4 x_4, \end{aligned} \tag{11}$$

for all $x_1, x_2, x_3, x_4 \in \mathfrak{R}^n$ and $t \in \mathfrak{R}^+$.

Assumption 6. The matrix D satisfies $\rho(D) < 1$, where $\rho(D)$ is the spectral radius of D.

Definition 7 (see [31]). The two coupled memristive neural networks (6) and (7) are said to be stochastic synchronization for almost every initial data if for every $\xi \in C^2_{F_0}([\tau, 0]; \mathfrak{R}^n)$, $\lim_{x\to\infty} e(t;\xi) = 0$ a.s.

Lemma 8 (see [32]). *For any vectors $a, b \in \mathfrak{R}^n$, the inequality $\pm 2a^T b \le a^T S a + b^T S^{-1} b$ holds, in which S is any matrix with $S > 0$.*

Lemma 9 (see [33]). *For any positive definite matrix $M \in \mathfrak{R}^{n\times n}$, scalar $\gamma > 0$, and vector function $\eta : [0,\gamma] \to \mathfrak{R}^n$ such that the integration concerned is well defined, then*

$$\begin{aligned} &\left(\int_0^\gamma \eta(s)\,ds\right)^T M \left(\int_0^\gamma \eta(s)\,ds\right) \\ &\quad \le \gamma \int_0^\gamma \eta(s)^T M \eta(s)\,ds. \end{aligned} \tag{12}$$

Lemma 10 (see [34]). *If $a_1 \ge a_2 \ge a_3, b_1 \ge b_2 \ge b_3$, then $2(a_1 b_1 + a_2 a_2 + b_3 b_3) \ge a_1 b_2 + a_1 b_3 + a_2 b_1 + a_2 b_3 + a_3 b_1 + a_3 b_2$, $\forall a_i, b_i\ (i = 1, 2, 3) \in R$.*

Remark 11. There are some other convenient and useful inequality techniques; refer to [1] Lemmas 2, 3, 7, and 8.

Lemma 12 (see [35]). *Given matrices $\Omega_1, \Omega_2, \Omega_3$, where $\Omega_1 = \Omega_1^T$ and $\Omega_2 > 0$, then $\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0$ if and only if*

$$\begin{aligned} &\begin{bmatrix} \Omega_1 & \Omega_3^T \\ \Omega_3 & -\Omega_2 \end{bmatrix} < 0 \\ \text{or } &\begin{bmatrix} -\Omega_2 & \Omega_3 \\ \Omega_3^T & \Omega_1 \end{bmatrix} < 0. \end{aligned} \tag{13}$$

3. Main Results

In this section, the stochastic synchronization for the two coupled memristive neural networks (6) and (7) is investigated under Assumptions 1–6.

3.1. Stochastic Adaptive Synchronization for the Two Coupled Memristive Neural Networks via the Adaptive Feedback Control

Theorem 13. *Under Assumptions 1–6, the two coupled memristive neural networks (6) and (7) can be synchronized for almost every initial data, if there exist positive diagonal matrices $H_1, H_2, H_3, P = \operatorname{diag}(p_1, p_2, \ldots, p_n)$, positive definite matrices $Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7, Q_8, S_1, S_2$, and a positive scalar λ such that the LMIs hold:*

$$P \le \lambda I, \tag{14}$$

$$\tau_3 (S_1 + S_2) \le Q_8, \tag{15}$$

$$\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} & 0 & 0 & \Psi_{15} & 0 & 0 & \Psi_{18} & 0 \\ * & \Psi_{22} & 0 & 0 & \Psi_{25} & 0 & 0 & \Psi_{28} & 0 \\ * & * & \Psi_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Psi_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Psi_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Psi_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \Psi_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \Psi_{88} & 0 \\ * & * & * & * & * & * & * & * & \Psi_{99} \end{bmatrix} < 0, \tag{16}$$

where

$$\begin{aligned} \Psi_{11} &= -2PC + 2P\alpha + P\overline{W}S_1^{-1}\overline{W}^T P^T + L_1 H_1 L_1 \\ &\quad + L_2 H_2 L_2 + L_3 H_3 L_3 + Q_1 + Q_2 + Q_3 \\ &\quad + \lambda (R_1 + R_4), \\ \Psi_{12} &= DPC - \alpha PD, \\ \Psi_{15} &= P\overline{A}, \end{aligned}$$

$$\Psi_{18} = P\overline{B},$$
$$\Psi_{22} = \lambda R_2 + (\mu_1 - 1) Q_1 + DP\overline{W}S_2^{-1}\overline{W}^T P^T D^T,$$
$$\Psi_{25} = -DP\overline{A},$$
$$\Psi_{28} = -DP\overline{B},$$
$$\Psi_{33} = \lambda R_3 + (\mu_2 - 1) Q_2,$$
$$\Psi_{44} = (\mu_3 - 1) Q_3,$$
$$\Psi_{55} = -H_1 + \tau_1 Q_6 + \tau_2 Q_7,$$
$$\Psi_{66} = -H_2 + Q_4,$$
$$\Psi_{77} = \tau_3 Q_8 - H_3 + Q_5,$$
$$\Psi_{88} = (\mu_2 - 1) Q_4,$$
$$\Psi_{99} = (\mu_3 - 1) Q_5. \tag{17}$$

And the adaptive feedback controller is designed as

$$u(t) = -Ke(t), \tag{18}$$

where the feedback strength $K = \mathrm{diag}(k_1, k_2, \ldots, k_n)$ *is updated by the following law:*

$$\dot{k}_i = \varphi_i e_i^2(t) - \varphi_i d_i e_i(t) e_i(t - \tau_1(t)), \tag{19}$$

with arbitrary constant $\varphi_i > 0$ $(i = 1, 2, \ldots, n)$.

Proof. We consider the following Lyapunov-Krasovskii functions:

$$V(t, e(t)) = \sum_{i=1}^{10} V_i(t, e_i(t)), \tag{20}$$

where

$$V_1(t, e(t)) = [e(t) - De(t - \tau_1(t))]^T P [e(t) - De(t - \tau_1(t))],$$
$$V_2(t, e(t)) = \int_{t-\tau_1(t)}^{t} e(s)^T Q_1 e(s)\, ds,$$
$$V_3(t, e(t)) = \int_{t-\tau_2(t)}^{t} e(s)^T Q_2 e(s)\, ds,$$
$$V_4(t, e(t)) = \int_{t-\tau_3(t)}^{t} e(s)^T Q_3 e(s)\, ds,$$
$$V_5(t, e(t)) = \int_{t-\tau_2(t)}^{t} g^T(e(s)) Q_4 g(e(s))\, ds,$$
$$V_6(t, e(t)) = \int_{t-\tau_3(t)}^{t} h^T(e(s)) Q_5 h(e(s))\, ds,$$
$$V_7(t, e(t)) = \int_{-\tau_1(t)}^{0} \int_{t+r}^{t} f^T(e(s)) Q_6 f(e(s))\, ds\, dr,$$
$$V_8(t, e(t)) = \int_{-\tau_2(t)}^{0} \int_{t+r}^{t} f^T(e(s)) Q_7 f(e(s))\, ds\, dr,$$
$$V_9(t, e(t)) = \int_{-\tau_3(t)}^{0} \int_{t+r}^{t} h^T(e(s)) Q_8 h(e(s))\, ds\, dr,$$
$$V_{10}(t, e(t)) = \sum_{i=1}^{n} \frac{p_i (k_i + \alpha)^2}{\varphi_i}. \tag{21}$$

By Itô formula, it follows that

$$\begin{aligned} LV(t, e(t)) &= V_t(t, e(t)) + V_e(t, e(t)) \Big[-Ce(t) \\ &\quad + \widetilde{A}(t) f(e(t)) + \widetilde{B}(t) g(e(t - \tau_2(t))) + \widetilde{W}(t) \\ &\quad \cdot \int_{t-\tau_3(t)}^{t} h(e(s))\, ds + u(t)\Big] + \frac{1}{2}\mathrm{tr}\big[\sigma^T(t, e(t), \\ &\quad e(t - \tau_1(t)), e(t - \tau_2(t)), e(t - \tau_3(t))) V_{ee} \sigma(t, e(t), \\ &\quad e(t - \tau_1(t)), e(t - \tau_2(t)), e(t - \tau_3(t)))\big], \end{aligned} \tag{22}$$

where $V_t(t, e(t)) = \partial V(t, e(t))/\partial t$, $V_e(t, e(t)) = (\partial V(t, e(t))/\partial e_1, \ldots, \partial V(t, e(t))/\partial e_n)$, $V_{ee}(t, e(t)) = (\partial^2 V(t, e(t))/\partial e_i \partial e_j)_{n\times n}$, and

$$\begin{aligned} LV_1(t, e(t)) &= 2[e(t) - De(t - \tau_1(t))]^T P \Big[-Ce(t) \\ &\quad + \widetilde{A}(t) f(e(t)) + \widetilde{B}(t) g(e(t - \tau_2(t))) + \widetilde{W}(t) \\ &\quad \cdot \int_{t-\tau_3(t)}^{t} h(e(s))\, ds - Ke(t)\Big] + \mathrm{tr}\big[\sigma^T(t, e(t), \\ &\quad e(t - \tau_1(t)), e(t - \tau_2(t)), e(t - \tau_3(t))) P \times \sigma(t, \\ &\quad e(t), e(t - \tau_1(t)), e(t - \tau_2(t)), e(t - \tau_3(t)))\big] \\ &= e^T(t)[-2PC]e(t) + e^T(t)\big[2P\widetilde{A}(t)\big] f(e(t)) \\ &\quad + e^T(t)\big[2P\widetilde{B}(t)\big] g(e(t - \tau_2(t))) + e^T(t) \\ &\quad \cdot \big[2P\widetilde{W}(t)\big] \int_{t-\tau_3(t)}^{t} h(e(s))\, ds + e^T(t - \tau_1(t)) \\ &\quad \cdot [2DPC] e(t) + e^T(t - \tau_1(t)) \big[-2DP\widetilde{A}(t)\big] \\ &\quad \cdot f(e(t)) + e^T(t - \tau_1(t)) \big[-2DP\widetilde{B}(t)\big] g(e(t \\ &\quad - \tau_2(t))) + e^T(t - \tau_1(t)) \big[-2DP\widetilde{W}(t)\big] \\ &\quad \cdot \int_{t-\tau_3(t)}^{t} h(e(s))\, ds - 2\sum_{i=1}^{n} p_i k_i e_i^2(t) \\ &\quad + 2\sum_{i=1}^{n} d_i p_i k_i e_i(t) e_i(t - \tau_1(t)) + \mathrm{tr}\big[\sigma^T(t, e(t), \end{aligned}$$

$$e(t-\tau_1(t)), e(t-\tau_2(t)), e(t-\tau_3(t))) P\sigma(t, e(t),$$
$$e(t-\tau_1(t)), e(t-\tau_2(t)), e(t-\tau_3(t)))]. \tag{23}$$

From Lemma 8, we get

$$\begin{aligned}
&e^T(t)\left[2P\widetilde{W}(t)\right]\int_{t-\tau_3(t)}^{t} h(e(s))\,ds \\
&\leq e^T(t)\left[P\overline{W}S_1^{-1}\right]\overline{W}^T P^T e(t) \\
&\quad + \left[\int_{t-\tau_3(t)}^{t} h(e(s))\,ds\right]^T S_1 \left[\int_{t-\tau_3(t)}^{t} h(e(s))\,ds\right], \\
&e^T(t-\tau_1(t))\left[-2DP\widetilde{W}(t)\right]\int_{t-\tau_3(t)}^{t} h(e(s))\,ds \\
&\leq e^T(t-\tau_1(t))\left[DP\overline{W}S_2^{-1}\right]\overline{W}^T P^T D^T e(t-\tau_1(t)) \\
&\quad + \left[\int_{t-\tau_3(t)}^{t} h(e(s))\,ds\right]^T S_2 \left[\int_{t-\tau_3(t)}^{t} h(e(s))\,ds\right].
\end{aligned} \tag{24}$$

Utilizing Lemma 9 yields

$$\begin{aligned}
&\left[\int_{t-\tau_3(t)}^{t} h(e(s))\,ds\right]^T (S_1+S_2)\left[\int_{t-\tau_3(t)}^{t} h(e(s))\,ds\right] \\
&\leq \tau_3(t)\int_{t-\tau_3(t)}^{t} h^T(e(s))(S_1+S_2)h(e(s))\,ds \\
&\leq \int_{t-\tau_3(t)}^{t} h^T(e(s))\left[\tau_3(S_1+S_2)\right]h(e(s))\,ds.
\end{aligned} \tag{25}$$

It follows from Assumption 5 and (14) that

$$\begin{aligned}
&\operatorname{tr}\left[\sigma^T(t, e(t), e(t-\tau_1(t)), e(t-\tau_2(t)), e(t-\tau_3(t)))P\right. \\
&\times \sigma(t, e(t), e(t-\tau_1(t)), e(t-\tau_2(t)), \\
&\left. e(t-\tau_3(t)))\right] \leq \lambda_{\max}(P)\operatorname{tr}\left[\sigma^T(t, e(t),\right. \\
&e(t-\tau_1(t)), e(t-\tau_2(t)), e(t-\tau_3(t))) \times \sigma(t, e(t), \\
&\left. e(t-\tau_1(t)), e(t-\tau_2(t)), e(t-\tau_3(t)))\right] \leq \lambda\left[e^T(t)\right. \\
&\cdot R_1(t)e(t) + e^T(t-\tau_1(t))R_2(t)e(t-\tau_1(t)) \\
&+ e^T(t-\tau_2(t))R_3(t)e(t-\tau_2(t)) + e^T(t-\tau_3(t)) \\
&\left.\cdot R_4(t)e(t-\tau_3(t))\right].
\end{aligned} \tag{26}$$

By Itô formula, we have

$$\begin{aligned}
&LV_2(t, e(t)) = e^T(t)Q_1 e(t) \\
&\quad - (1-\dot{\tau}_1(t))e^T(t-\tau_1(t))Q_1 e(t-\tau_1(t)) \\
&\quad \leq e^T(t)Q_1 e(t) \\
&\quad + (\mu_1 - 1)e^T(t-\tau_1(t))Q_1 e(t-\tau_1(t)). \\
&LV_3(t, e(t)) = e^T(t)Q_2 e(t) \\
&\quad - (1-\dot{\tau}_2(t))e^T(t-\tau_2(t))Q_2 e(t-\tau_2(t)) \\
&\quad \leq e^T(t)Q_2 e(t) \\
&\quad + (\mu_2 - 1)e^T(t-\tau_2(t))Q_2 e(t-\tau_2(t)), \\
&LV_4(t, e(t)) = e^T(t)Q_3 e(t) \\
&\quad - (1-\dot{\tau}_3(t))e^T(t-\tau_3(t))Q_3 e(t-\tau_3(t)) \\
&\quad \leq e^T(t)Q_3 e(t) \\
&\quad + (\mu_3 - 1)e^T(t-\tau_3(t))Q_3 e(t-\tau_3(t)), \\
&LV_5(t, e(t)) = g^T(e(t))Q_4 g(e(t)) \\
&\quad - (1-\dot{\tau}_2(t))g^T(e(t-\tau_2(t)))Q_4 g(e(t-\tau_2(t))) \\
&\quad \leq g^T(e(t))Q_4 g(e(t)) \\
&\quad + g^T(e(t-\tau_2(t)))\left[(\mu_2-1)Q_4\right]g(e(t-\tau_2(t))), \\
&LV_6(t, e(t)) = h^T(e(t))Q_5 h(e(t)) \\
&\quad - (1-\dot{\tau}_3(t))h^T(e(t-\tau_2(t)))Q_5 h(e(t-\tau_3(t))) \\
&\quad \leq h^T(e(t))Q_5 h(e(t)) \\
&\quad + h^T(e(t-\tau_3(t)))\left[(\mu_3-1)Q_5\right]h(e(t-\tau_3(t))), \\
&LV_7(t, e(t)) = \tau_1(t)f^T(e(t))Q_6 f(e(t)) \\
&\quad - \int_{t-\tau_1(t)}^{t} f^T(e(s))Q_6 f(e(s))\,ds \\
&\quad \leq f^T(e(t))\left[\tau_1 Q_6\right]f(e(t)), \\
&LV_8(t, e(t)) = \tau_2(t)f^T(e(t))Q_7 f(e(t)) \\
&\quad - \int_{t-\tau_2(t)}^{t} f^T(e(s))Q_7 f(e(s))\,ds \\
&\quad \leq f^T(e(t))\left[\tau_2 Q_7\right]f(e(t)),
\end{aligned} \tag{27}$$

From Assumption 1, it follows that

$$\begin{aligned}
f^T(e(t))H_1 f(e(t)) &\leq e^T(t)L_1 H_1 L_1 e(t), \\
g^T(e(t))H_2 g(e(t)) &\leq e^T(t)L_2 H_2 L_2 e(t), \\
h^T(e(t))H_3 h(e(t)) &\leq e^T(t)L_3 H_3 L_3 e(t),
\end{aligned} \tag{28}$$

where H_1, H_2, H_3 are positive diagonal matrices and $L_j = \operatorname{diag}(l_{j1}, l_{j2}, \ldots, l_{jn})$, $l_{ji} = \max\{l_{ji}^-, l_{ji}^+\}$ $(j = 1, 2, 3)$ for $i = 1, 2, \ldots, n$.

$$\begin{aligned}
&LV_9(t, e(t)) \\
&\quad = \tau_3(t)h^T(e(t))Q_8 h(e(t)) \\
&\qquad - \int_{t-\tau_3(t)}^{t} h^T(e(s))Q_8 h(e(s))\,ds
\end{aligned}$$

$$\leq h^T(e(t))[\tau_3 Q_8]h(e(t)) - \int_{t-\tau_3(t)}^{t} h^T(e(s))Q_8 h(e(s))\,ds,$$

$$LV_{10}(t,e(t)) = 2\sum_{i=1}^{n}\frac{p_i}{\varphi_i}(k_i+\alpha)\dot{k}_i = 2\sum_{i=1}^{n}p_i(k_i+\alpha)\left(e_i^2(t) - d_i e_i(t)e_i(t-\tau_1(t))\right). \tag{29}$$

Condition (15) yields

$$\int_{t-\tau_3(t)}^{t} h^T(e(s))[\tau_3(S_1+S_2)]h(e(s))\,ds - \int_{t-\tau_3(t)}^{t} h^T(e(s))Q_8 h(e(s))\,ds \leq 0. \tag{30}$$

Substituting inequalities (23)–(30) into (22), we obtain

$$\begin{aligned} LV(t,e(t)) &= e^T(t)\left[-2PC + 2P\alpha + P\overline{W}S_1^{-1}\overline{W}^T P^T + L_1H_1L_1 + L_2H_2L_2 + L_3H_3L_3 + Q_1 + Q_2 + Q_3 + \lambda(R_1+R_4)\right]e(t) + e^T(t)\left[2P\overline{A}\right]f(e(t)) + e^T(t) \\ &\cdot \left[2P\overline{B}\right]g(e(t-\tau_2(t))) + e^T(t-\tau_1(t))[2DPC - 2\alpha PD]e(t) + e^T(t-\tau_1(t))\left[-2DP\overline{A}\right]f(e(t)) \\ &+ e^T(t-\tau_1(t))\left[-2DP\overline{B}\right]g(e(t-\tau_2(t))) + e^T(t-\tau_1(t))\left[\lambda R_2 + (\mu_1-1)Q_1 + DP\overline{W}S_2^{-1}\overline{W}^T P^T D^T\right] \\ &\cdot e(t-\tau_1(t)) + e^T(t-\tau_2(t))[\lambda R_3 + (\mu_2-1)Q_2] \cdot e(t-\tau_2(t)) + e^T(t-\tau_3(t))[(\mu_3-1)Q_3]e(t-\tau_3(t)) \\ &+ h^T(e(t))[\tau_3Q_8 - H_3 + Q_5]h(e(t)) + f^T(e(t))[\tau_1Q_6 - H_1 + \tau_2Q_7]f(e(t)) \\ &+ g^T(e(t))[-H_2+Q_4]g(e(t)) + h^T(e(t-\tau_3(t))) \cdot [(\mu_3-1)Q_5]h(e(t-\tau_3(t))) + g^T(e(t-\tau_2(t))) \\ &\cdot [(\mu_2-1)Q_4]g(e(t-\tau_2(t))) - e^T(t)[\lambda R_4]e(t) + e^T(t-\tau_3(t))[\lambda R_4]e(t-\tau_3(t)) = \Phi^T(t)\Psi\Phi(t) \\ &- e^T(t)[\lambda R_4]e(t) + e^T(t-\tau_3(t))[\lambda R_4]e(t-\tau_3(t)), \end{aligned} \tag{31}$$

where

$$\Phi(t) = \left[e(t), e(t-\tau_1(t)), e(t-\tau_2(t)), e(t-\tau_3(t)), f(e(t)), g(e(t)), h(e(t)), g(e(t-\tau_2(t))), h(e(t-\tau_2(t)))\right]^T,$$

$$\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} & 0 & 0 & \Psi_{15} & 0 & 0 & \Psi_{18} & 0 \\ * & \Psi_{22} & 0 & 0 & \Psi_{25} & 0 & 0 & \Psi_{28} & 0 \\ * & * & \Psi_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Psi_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Psi_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Psi_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \Psi_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \Psi_{88} & 0 \\ * & * & * & * & * & * & * & * & \Psi_{99} \end{bmatrix}, \tag{32}$$

with

$$\begin{aligned} \Psi_{11} &= -2PC + 2P\alpha + P\overline{W}S_1^{-1}\overline{W}^T P^T + L_1H_1L_1 + L_2H_2L_2 + L_3H_3L_3 + Q_1 + Q_2 + Q_3 + \lambda(R_1+R_4), \\ \Psi_{12} &= DPC - \alpha PD, \\ \Psi_{15} &= P\overline{A}, \\ \Psi_{18} &= P\overline{B}, \\ \Psi_{22} &= \lambda R_2 + (\mu_1-1)Q_1 + DP\overline{W}S_2^{-1}\overline{W}^T P^T D^T, \\ \Psi_{25} &= -DP\overline{A}, \\ \Psi_{28} &= -DP\overline{B}, \\ \Psi_{33} &= \lambda R_3 + (\mu_2-1)Q_2, \\ \Psi_{44} &= (\mu_3-1)Q_3, \\ \Psi_{55} &= -H_1 + \tau_1Q_6 + \tau_2Q_7, \\ \Psi_{66} &= -H_2 + Q_4, \\ \Psi_{77} &= \tau_3Q_8 - H_3 + Q_5, \\ \Psi_{88} &= (\mu_2-1)Q_4, \\ \Psi_{99} &= (\mu_3-1)Q_5. \end{aligned} \tag{33}$$

Using Lemma 12, if $\Psi < 0$, let $\zeta = \lambda_{\min}(-\Psi)$, and, clearly, the constant $\zeta > 0$. This fact together with (31) gives

$$\begin{aligned} LV(t,e(t)) &\leq -e^T(t)(\lambda R_4 + \zeta I)e(t) + e^T(t-\tau_3(t))(\lambda R_4 - \zeta I)e(t-\tau_3(t)) \\ &= -\omega_1(e(t)) + \omega_2(e(t-\tau_3(t))), \end{aligned} \tag{34}$$

where $\omega_1(e(t)) = e^T(t)(\lambda R_4 + \zeta I)e(t)$ and $\omega_2(e(t)) = e^T(t)(\lambda R_4 - \zeta I)e(t)$.
It is obvious that $\omega_1(e(t)) > \omega_2(e(t))$ for any $e(t) \neq 0$. Therefore, applying LaSalle-type invariance principle (see [27, 36]) for the stochastic differential delay equations, we can conclude that the coupled memristive neural networks (6) and (7) can be synchronized for almost every initial data. This completes the proof. □

When $D = 0$, from Theorem 13, we obtain the following corollary.

Corollary 14. *Under Assumptions 1–5, the two coupled memristive neural networks (6) and (7) with $D = 0$ can be synchronized for almost every initial data, if there exist positive diagonal matrices $H_1, H_2, H_3, P = \mathrm{diag}(p_1, p_2, \ldots, p_n)$, positive definite matrices $Q_2, Q_3, Q_4, Q_5, Q_7, Q_8, S_1$, and a positive scalar λ such that the LMIs hold:*

$$\begin{aligned}
&P \leq \lambda I, \\
&\tau_3 S_1 \leq Q_8, \\
&\Theta = \begin{bmatrix}
\Theta_{11} & 0 & 0 & \Psi_{15} & 0 & 0 & \Psi_{18} & 0 \\
* & \Psi_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & \Psi_{44} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & \Theta_{55} & 0 & 0 & 0 & 0 \\
* & * & * & * & \Psi_{66} & 0 & 0 & 0 \\
* & * & * & * & * & \Psi_{77} & 0 & 0 \\
* & * & * & * & * & * & \Psi_{88} & 0 \\
* & * & * & * & * & * & * & \Psi_{99}
\end{bmatrix} \\
&< 0,
\end{aligned} \tag{35}$$

where

$$\begin{aligned}
\Theta_{11} = {}& -2PC + 2P\alpha + P\overline{W}S_1^{-1}\overline{W}^T P^T + L_1 H_1 L_1 \\
&+ L_2 H_2 L_2 + L_3 H_3 L_3 + Q_2 + Q_3 \\
&+ \lambda \left(R_1 + R_4\right). \\
\Theta_{55} = {}& -H_1 + \tau_2 Q_7,
\end{aligned} \tag{36}$$

And the adaptive feedback controller is designed as

$$u(t) = -Ke(t), \tag{37}$$

where the feedback strength $K = \mathrm{diag}(k_1, k_2, \ldots, k_n)$ is updated by the following law:

$$\dot{k}_i = \varphi_i e_i^2(t), \tag{38}$$

with arbitrary constant $\varphi_i > 0$ $(i = 1, 2, \ldots, n)$.

Remark 15. When we remove the stochastic perturbations, our models become the model in [37], so our models are the extension of the model in [37]. Because the stochastic perturbations are unavoidable in practice, our models are more general and useful in practice.

Remark 16. If $A(x_i(t)) = (a_{ij}(x_i(t)))_{n\times n}$, $B(x_i(t)) = (b_{ij}(x_i(t)))_{n\times n}$ and $W(x_i(t)) = (w_{ij}(x_i(t)))_{n\times n}$ are constants, system (1) will reduce to a general network. What is more, when we remove the neutral terms, our models become the model in [10, 27], so our models are the extension of the model in [10, 27]. Because the neutral terms are important and complicated, our models are more general and useful in practice.

3.2. Stochastic Adaptive Synchronization for the Two Coupled Memristive Neural Networks via the Linear Feedback Control

Theorem 17. *Under Assumptions 1–6, two coupled memristive neural networks (6) and (7) can be synchronized for almost every initial data, if there exist positive diagonal matrices $H_1, H_2, H_3, P = \mathrm{diag}(p_1, p_2, \ldots, p_n)$, positive definite matrices $Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7, Q_8, S_1, S_2$, and a positive scalar λ such that the LMIs hold:*

$$\begin{aligned}
&P \leq \lambda I, \\
&\tau_3 \left(S_1 + S_2\right) \leq Q_8, \\
&X \\
&= \begin{bmatrix}
X_{11} & X_{12} & 0 & 0 & X_{15} & 0 & 0 & X_{18} & 0 \\
* & X_{22} & 0 & 0 & X_{25} & 0 & 0 & X_{28} & 0 \\
* & * & X_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & X_{44} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & X_{55} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & X_{66} & 0 & 0 & 0 \\
* & * & * & * & * & * & X_{77} & 0 & 0 \\
* & * & * & * & * & * & * & X_{88} & 0 \\
* & * & * & * & * & * & * & * & X_{99}
\end{bmatrix} \\
&< 0,
\end{aligned} \tag{39}$$

where

$$\begin{aligned}
X_{11} = {}& -2PC - 2PK + P\overline{W}S_1^{-1}\overline{W}^T P^T + L_1 H_1 L_1 \\
&+ L_2 H_2 L_2 + L_3 H_3 L_3 + Q_1 + Q_2 + Q_3 \\
&+ \lambda \left(R_1 + R_4\right), \\
X_{12} = {}& DPC + KPD, \\
X_{15} = {}& P\overline{A}, \\
X_{18} = {}& P\overline{B}, \\
X_{22} = {}& \lambda R_2 + \left(\mu_1 - 1\right) Q_1 + DP\overline{W}S_2^{-1}\overline{W}^T P^T D^T, \\
X_{25} = {}& -DP\overline{A}, \\
X_{28} = {}& -DP\overline{B}, \\
X_{33} = {}& \lambda R_3 + \left(\mu_2 - 1\right) Q_1,
\end{aligned}$$

$$\begin{aligned} X_{44} &= (\mu_3 - 1) Q_3, \\ X_{55} &= -H_1 + \tau_1 Q_6 + \tau_2 Q_7, \\ X_{66} &= -H_2 + Q_4, \\ X_{77} &= \tau_3 Q_8 - H_3 + Q_5, \\ X_{88} &= (\mu_2 - 1) Q_4, \\ X_{99} &= (\mu_3 - 1) Q_5. \end{aligned} \tag{40}$$

And the linear feedback controller is designed as

$$u(t) = -Ke(t), \tag{41}$$

where $K = \mathrm{diag}(k_1, k_2, \ldots, k_n)$, $k_i > 0$ *is the feedback gain.*

Proof. We consider the following Lyapunov-Krasovskii functions:

$$V(t, e(t)) = \sum_{i=1}^{9} V_i(t, e_i(t)), \tag{42}$$

where

$$\begin{aligned} &V_1(t, e(t)) \\ &\quad = [e(t) - De(t - \tau_1(t))]^T P [e(t) - De(t - \tau_1(t))], \\ &V_2(t, e(t)) = \int_{t-\tau_1(t)}^{t} e(s)^T Q_1 e(s)\, ds, \\ &V_3(t, e(t)) = \int_{t-\tau_2(t)}^{t} e(s)^T Q_2 e(s)\, ds, \\ &V_4(t, e(t)) = \int_{t-\tau_3(t)}^{t} e(s)^T Q_3 e(s)\, ds, \\ &V_5(t, e(t)) = \int_{t-\tau_2(t)}^{t} g^T(e(s)) Q_4 g(e(s))\, ds, \\ &V_6(t, e(t)) = \int_{t-\tau_3(t)}^{t} h^T(e(s)) Q_5 h(e(s))\, ds, \\ &V_7(t, e(t)) = \int_{-\tau_1(t)}^{0} \int_{t+r}^{t} f^T(e(s)) Q_6 f(e(s))\, ds\, dr, \\ &V_8(t, e(t)) = \int_{-\tau_2(t)}^{0} \int_{t+r}^{t} f^T(e(s)) Q_7 f(e(s))\, ds\, dr, \\ &V_9(t, e(t)) = \int_{-\tau_3(t)}^{0} \int_{t+r}^{t} h^T(e(s)) Q_8 h(e(s))\, ds\, dr. \end{aligned} \tag{43}$$

Then, the proof of Theorem 17 is similar to Theorem 13, so the proof process is omitted here. □

Corollary 18. *Under Assumptions 1–5, two coupled memristive neural networks (6) and (7) with* $D = 0$ *can be synchronized for almost every initial data, if there exist positive diagonal matrices* $H_1, H_2, H_3, P = \mathrm{diag}(p_1, p_2, \ldots, p_n)$, *positive definite matrices* $Q_2, Q_3, Q_4, Q_5, Q_7, Q_8, S_1$, *and a positive scalar* λ *such that the LMIs hold:*

$$\begin{aligned} &P \le \lambda I, \\ &\tau_3 S_1 \le Q_8, \\ &Z = \begin{bmatrix} Z_{11} & 0 & 0 & \Psi_{15} & 0 & 0 & \Psi_{18} & 0 \\ * & \Psi_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Psi_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & Z_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & \Psi_{66} & 0 & 0 & 0 \\ * & * & * & * & * & \Psi_{77} & 0 & 0 \\ * & * & * & * & * & * & \Psi_{88} & 0 \\ * & * & * & * & * & * & * & \Psi_{99} \end{bmatrix} \\ &\quad < 0, \end{aligned} \tag{44}$$

where

$$\begin{aligned} Z_{11} &= -2PC - 2PK + P\overline{W} S_1^{-1} \overline{W}^T P^T + L_1 H_1 L_1 \\ &\quad + L_2 H_2 L_2 + L_3 H_3 L_3 + Q_2 + Q_3 \\ &\quad + \lambda (R_1 + R_4), \\ Z_{55} &= -H_1 + \tau_2 Q_7. \end{aligned} \tag{45}$$

And the linear feedback controller is designed as

$$u(t) = -Ke(t), \tag{46}$$

where $K = \mathrm{diag}(k_1, k_2, \ldots, k_n)$, $k_i > 0$ *is the feedback gain.*

Remark 19. When $D = 0$, the systems are no longer neutral-type neural networks. We find that adaptive synchronization of other types of neural networks model has been researched (see [38, 39]). We can also get the synchronization results from Theorem 17 when $D = 0$.

Remark 20. When $\widetilde{W}(t) = 0$, the systems no longer have distributed time-varying delays. Our models become the model in [40]; we can also get the synchronization results, so our models are the extension of the model in [40] and they are more general than that.

4. Numerical Simulation

In this section, a numerical example is given to illustrate the effectiveness of Theorem 13. Consider a two-dimensional synchronization error system (8) with $u(t) = -Ke(t)$ such that $\dot{k}_i = \varphi_i e_i^2(t) - \varphi_i d_i e_i(t) e_i(t - \tau_1(t))$.

Take $f(e(t)) = g(e(t)) = h(e(t)) = [\tan h(e_1(t)), \tan h(e_1(t))]^T, \tau_1(t) = 0.6, \tau_2(t) = 0.1, \tau_3(t) = 0.2,\ L_1 = L_2 = L_3 = I$, and $\sigma(t, e(t), e(t-\tau_1(t)), e(t-\tau_2(t)), e(t-\tau_3(t))) =$

$$\begin{bmatrix} 0.5e_1 t + 0.2e_1(t-\tau_1(t)) & 0 \\ 0 & 0.5e_2(t-\tau_2(t)) + 0.2e_2(t-\tau_3(t)) \end{bmatrix},$$

$$\widetilde{A}(x(t)) = \begin{bmatrix} a_{11}(x_1(t)) & a_{12}(x_1(t)) \\ a_{21}(x_2(t)) & a_{22}(x_2(t)) \end{bmatrix},$$

$$\widetilde{B}(x(t)) = \begin{bmatrix} b_{11}(x_1(t)) & b_{12}(x_1(t)) \\ b_{21}(x_2(t)) & b_{22}(x_2(t)) \end{bmatrix},$$

$$\widetilde{W}(x(t)) = \begin{bmatrix} w_{11}(x_1(t)) & w_{12}(x_1(t)) \\ w_{21}(x_2(t)) & w_{22}(x_2(t)) \end{bmatrix},$$

$$a_{11}(x_1(t)) = \begin{cases} 0.3 & |x_1(t)| \le 1, \\ -0.3 & |x_1(t)| > 1, \end{cases}$$

$$a_{12}(x_1(t)) = \begin{cases} 0.2 & |x_1(t)| \le 1, \\ -0.2 & |x_1(t)| > 1, \end{cases}$$

$$a_{21}(x_2(t)) = \begin{cases} 0.2 & |x_2(t)| \le 1, \\ -0.2 & |x_2(t)| > 1, \end{cases}$$

$$a_{22}(x_2(t)) = \begin{cases} 1 & |x_2(t)| \le 1, \\ -1 & |x_2(t)| > 1, \end{cases}$$

$$b_{11}(x_1(t)) = \begin{cases} 0.4 & |x_1(t)| \le 1, \\ -0.4 & |x_1(t)| > 1, \end{cases} \quad (47)$$

$$b_{12}(x_1(t)) = \begin{cases} 0.3 & |x_1(t)| \le 1, \\ -0.3 & |x_1(t)| > 1, \end{cases}$$

$$b_{21}(x_2(t)) = \begin{cases} 0.5 & |x_2(t)| \le 1, \\ -0.5 & |x_2(t)| > 1, \end{cases}$$

$$b_{22}(x_2(t)) = \begin{cases} 0.2 & |x_2(t)| \le 1, \\ -0.2 & |x_2(t)| > 1, \end{cases}$$

$$w_{11}(x_1(t)) = \begin{cases} 0.3 & |x_1(t)| \le 1, \\ 0.2 & |x_1(t)| > 1, \end{cases}$$

$$w_{12}(x_1(t)) = \begin{cases} 0.5 & |x_1(t)| \le 1, \\ 0.3 & |x_1(t)| > 1, \end{cases}$$

$$w_{21}(x_2(t)) = \begin{cases} 0.5 & |x_2(t)| \le 1, \\ 0.3 & |x_2(t)| > 1, \end{cases}$$

$$w_{22}(x_2(t)) = \begin{cases} 0.3 & |x_2(t)| \le 1, \\ 0.2 & |x_2(t)| > 1, \end{cases}$$

and $R_1 = 0.3I,\ R_2 = 0.2I,\ R_3 = 0.3I,\ R_4 = 0.1I,\ \mu_1 = \mu_2 = \mu_3 = 0.2$. Then

$$\overline{A} = \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 1 \end{bmatrix},$$

$$\overline{B} = \begin{bmatrix} 0.4 & 0.3 \\ 0.5 & 0.2 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.7 \end{bmatrix},$$

$$D = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.4 \end{bmatrix},$$

$$\overline{W} = \begin{bmatrix} 0.3 & 0.5 \\ 0.5 & 0.3 \end{bmatrix}. \quad (48)$$

Letting $\alpha = 20$, using LMI toolbox in MATLAB, we obtain the following feasible solutions to LMIs in Theorem 13:

$$P = \begin{bmatrix} 0.9183 & -0.3941 \\ -0.3941 & 0.4683 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} 0.3760 & -0.1907 \\ -0.1907 & 0.1717 \end{bmatrix},$$

$$H_2 = \begin{bmatrix} 0.3823 & -0.1868 \\ -0.1868 & 0.1367 \end{bmatrix},$$

$$H_3 = \begin{bmatrix} 0.7747 & -0.4052 \\ -0.4052 & 0.2813 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 0.3465 & -0.1538 \\ -0.1538 & 0.4013 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 0.2118 & -0.0897 \\ -0.0897 & 0.1025 \end{bmatrix},$$

$$Q_3 = \begin{bmatrix} 0.1834 & -0.0959 \\ -0.0959 & 0.0666 \end{bmatrix},$$

$$Q_4 = \begin{bmatrix} 0.2613 & -0.1224 \\ -0.1224 & 0.0927 \end{bmatrix}, \quad (49)$$

$$Q_5 = \begin{bmatrix} 0.2087 & -0.1090 \\ -0.1090 & 0.0760 \end{bmatrix},$$

$$Q_6 = \begin{bmatrix} 0.2036 & -0.1067 \\ -0.1067 & 0.0716 \end{bmatrix},$$

$$Q_7 = \begin{bmatrix} 0.1283 & -0.0672 \\ -0.0672 & 0.0453 \end{bmatrix},$$

$$Q_8 = \begin{bmatrix} 0.2313 & -0.1210 \\ -0.1210 & 0.0839 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} 1.0402 & 1.5085 \\ 1.5085 & 2.8776 \end{bmatrix},$$

$$S_2 = \begin{bmatrix} 1.0402 & 1.5085 \\ 1.5085 & 2.8776 \end{bmatrix},$$

$\lambda = 1.2143$.

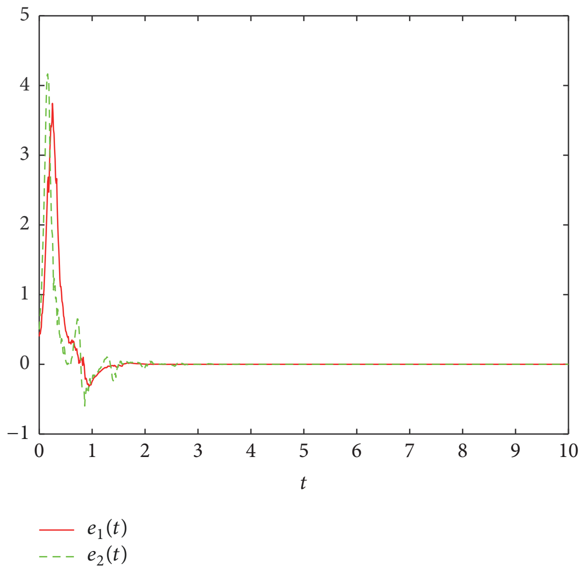

Figure 1: The curve of the synchronization errors e_1, e_2.

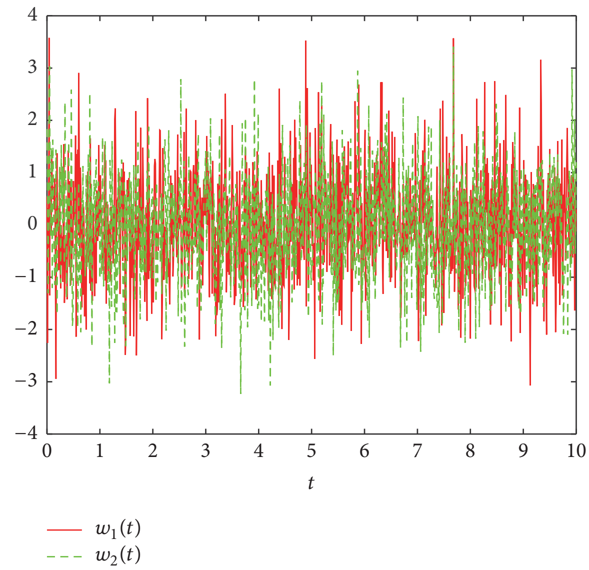

Figure 3: The evolution graph of the Brownian motions ω_1, ω_2.

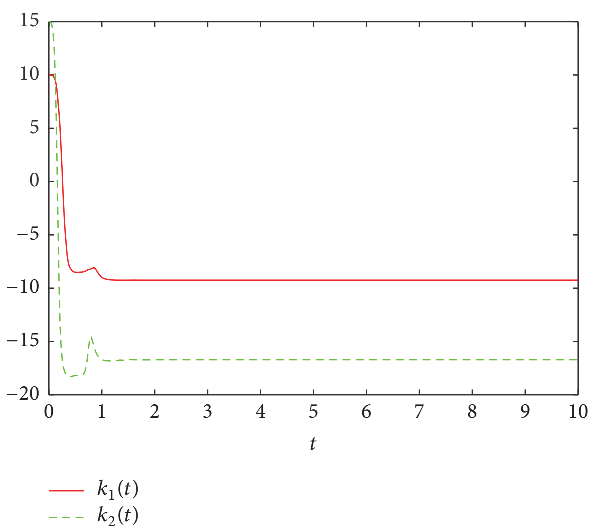

Figure 2: The evolution graph of the adaptive coupling strengths k_1, k_2.

So the conditions of Theorem 13 are satisfied, and we conclude that two coupled memristive neural networks (6) and (7) can be synchronized for almost every initial data.

Now by taking the initial date as $e(0) = [0.4, 0.5]^T$, $K(0) = [10, 15]^T$ and $\varphi_1 = 0.1$, $\varphi_2 = 0.2$, we can draw the dynamic curves of the error system, the evolution of adaptive coupling strength k_1, k_2, and the Brownian motion $\omega(t)$, respectively, as Figures 1–3. Figure 1 shows that two coupled memristive neural networks (6) and (7) are synchronized.

5. Conclusions and Discussion

In this paper, by applying LaSalle-type invariance principle for stochastic differential delays equations, the stochastic differential inclusions theory, Lyapunov functional, and linear matrix inequalities method, linear feedback control and adaptive feedback control are proposed to achieve the synchronization of stochastic neutral-type memristive neural networks with mixed time-varying delays. Even though the synchronization problem of stochastic MNNs has been studied, there are few studies on the synchronization problem of stochastic neutral-type MNNs. Neutral terms are taken into account in this paper, which make the model have wider application and make research more meaningful. So we generalized the synchronization problem of MNNs. The effectiveness of our results has been illustrated by a numerical example. Furthermore, exponential synchronization and passivity of this model can be discussed in the near future.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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