Hindawi
Discrete Dynamics in Nature and Society
Volume 2018, Article ID 8535703, 15 pages
https://doi.org/10.1155/2018/8535703

Research Article

Characterizations of Certain Types of Type 2 Soft Graphs

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Received 4 March 2018; Accepted 29 August 2018; Published 20 September 2018

Academic Editor: Allan C. Peterson

The vertex-neighbors correspondence has an essential role in the structure of a graph. The type 2 soft set is also based on the correspondence of initial parameters and underlying parameters. Recently, type 2 soft graphs have been introduced. Structurally, it is a very efficient model of uncertainty to deal with graph neighbors and applicable in applied intelligence, computational analysis, and decision-making. The present paper characterizes type 2 soft graphs on underlying subgraphs (regular subgraphs, irregular subgraphs, cycles, and trees) of a simple graph. We present regular type 2 soft graphs, irregular type 2 soft graphs, and type 2 soft trees. Moreover, we introduce type 2 soft cycles, type 2 soft cut-nodes, and type 2 soft bridges. Finally, we present some operations on type 2 soft trees by presenting several examples to demonstrate these new concepts.

1. Preliminaries and Introduction

A graph $\mathcal{G} = (X, \mathcal{E})$ consists of a nonempty set of objects X, called vertices, and a set $\mathcal{E}$ of two element subsets of X called edges. Two vertices x and y are adjacent if $\{x, y\} \in \mathcal{E}$. A loop is an edge that connects a vertex to itself. A simple graph is an unweighted, undirected graph containing no multiple edges or graph loops. A graph $\mathcal{G}' = (X', \mathcal{E}')$ is said to be a subgraph of $\mathcal{G} = (X, \mathcal{E})$ if $X' \subseteq X$ and $\mathcal{E}' \subseteq \mathcal{E}$. The graph neighborhoods of a vertex x in a graph is the set of all the vertices adjacent to x including x itself. The graph neighbors of a vertex x in a graph are the set of all the vertices adjacent to x excluding x itself. The eccentricity of the vertex x is the maximum distance from x to any vertex. The distance between two vertices x and y in a graph is the number of edges in a shortest path, denoted by $d(x, y)$. The radius of a graph is the minimum eccentricity of any vertex x. A graph $\mathcal{G} = (X, \mathcal{E})$ is called a tree if it is connected and contains no cycles. Equivalently (and sometimes more useful), a tree is a connected graph on m vertices with exactly $m - 1$ edges. A forest is a disjoint union of trees. The degree of a vertex of a simple graph is the number of edges incident to the vertex. A regular graph is a graph where each vertex has the same number of neighbors. A graph that is not a regular graph is called irregular graph. A graph is called neighborly irregular graph if no two adjacent vertices have the same degree. A complete graph is a graph in which each pair of graph vertices is connected by an edge. For basic definitions of graphs see [1–3].

Soft set theory [4], firstly initiated by Molodtsov, is a new mathematical tool for dealing with uncertainties. Some fruitful operations, soft set theory, are presented by Maji et al. [5] and Ali et al. [6]. We refer to Molodtsov's soft sets as type 1 soft sets (briefly $T1SS$). Let A be a set of parameters that can have an arbitrary nature (numbers, functions, sets of words, etc.). Let U be a universe and the power set of U is denoted by $P(U)$. The soft set is defined as follows.

Definition 1 (see [5]). A pair (S, A) is called a soft set over U, where S is a mapping given by $S : A \longrightarrow P(U)$.

Note that the set of all $T1SS$ over U will be denoted by $\sigma(U)$. Many researchers take attention at applicability of soft sets in real and practical problems. In recent years, research

on soft set theory has been rapidly developed, and great progress has been achieved, including works of soft sets in graph theory. Ali et al. [7] introduced a representation of graphs based on neighborhoods. Akram et al. introduced the concepts of soft graphs [8, 9] and soft trees [10]. Let $\mathcal{G} = (X, \mathcal{E})$ be a simple graph, A be any nonempty set.

Let $R \subseteq A \times X$ be an arbitrary relation from A to X. A mapping I from A to $P(X)$ denoted as $I : A \longrightarrow P(X)$ and defined as $I(x) = \{y \in X \mid xRy\}$ and a mapping J from A to $P(\mathcal{E})$ denoted as $J : A \longrightarrow P(\mathcal{E})$ and defined as $J(x) = \{uv \in \mathcal{E} \mid \{u, v\} \subseteq I(x)\}$. Then (I, A) is a $T1SS$ over X and (J, A) is a $T1SS$ over $\mathcal{E}$. The notion of a soft graph is defined as follows.

Definition 2 (see [9]). A 4-tuple $G^{\wr} = (\mathcal{G}, I, J, A)$ is called a soft graph if it satisfies;

(i) $\mathcal{G} = (X, \mathcal{E})$ is a simple graph.

(ii) A is a non-empty set of parameters.

(iii) $[I, A]$ is a $T1SS$ over X.

(iv) $[J, A]$ is a $T1SS$ over $\mathcal{E}$.

(v) $(I(\alpha), J(\alpha))$ for all $\alpha \in A$, represents a subgraph of $\mathcal{G}$.

The soft graph $G^{\wr} = (\mathcal{G}, I, J, A)$ can also be written as $G^{\wr} = (\mathcal{G}, I, J) = \{T(\alpha) \mid \alpha \in A\}$, where $T(\alpha) = (I(\alpha), J(\alpha))\ \forall\ \alpha \in A$.

Definition 3 (see [9, 10]). Let $G^{\wr}$ be a soft graph of $\mathcal{G}$. Then $G^{\wr}$ is said to be a soft tree (resp., regular soft graph, irregular soft graph, neighborly irregular soft graph, soft cycle) if every $T(\alpha)$ is a tree (resp., regular graph, irregular graph, neighborly irregular graph, and cycle) for all $\alpha \in A$.

A soft graph $G^{\wr}$ is called a regular soft graph of degree r if $T(\alpha)$ is a regular graph of degree r for all $\alpha \in A$. In the rest of paper soft tree and soft cycle will be written as type 1 soft tree (briefly, $T1ST$) and type 1 soft cycle (briefly, $T1SC$), respectively. Some operations of $T1STs$ are defined as follows.

Definition 4 (see [10]). Let $G^{\wr} = \langle I_1, J_1, A_1\rangle$ and $G'^{\wr} = \langle I_2, J_2, A_2\rangle$ be two $T1STs$ of $\mathcal{G}$. Then $G'^{\wr}$ is a type 1 soft subtree of $G^{\wr}$ if

(i) $A_2 \subseteq A_1$;

(ii) for each $x \in A_2$, $T_2(x)$ is a subtree of $T_1(x)$.

Definition 5 (see [10]). Let $G_1^{\wr} = \langle I_1, J_1, A_1\rangle$ and $G_2^{\wr} = \langle I_2, J_2, A_2\rangle$ be two $T1STs$ of $\mathcal{G}$. The extended union of $G_1^{\wr}$ and $G_2^{\wr}$, denoted by $G_1^{\wr} \sqcup_{\varepsilon} G_2^{\wr} = G^{\wr} = \langle I, J, B\rangle$, where $B = A_1 \cup A_2$, is defined $\forall \alpha \in B$, as

$$I(\alpha) = \begin{cases} I_1(\alpha), & \text{if } \alpha \in A_1 - A_2 \\ I_2(\alpha), & \text{if } \alpha \in A_2 - A_1 \\ I_1(\alpha) \cup I_2(\alpha), & \text{if } \alpha \in A_1 \cap A_2 \end{cases}$$
$$J(\alpha) = \begin{cases} J_1(\alpha), & \text{if } \alpha \in A_1 - A_2 \\ J_2(\alpha), & \text{if } \alpha \in A_2 - A_1 \\ J_1(\alpha) \cup J_2(\alpha), & \text{if } \alpha \in A_1 \cap A_2 \end{cases} \tag{1}$$

It can be written as $G_1^{\wr} \sqcup_{\varepsilon} G_2^{\wr} = \{T(\alpha) = (I(\alpha), J(\alpha)) \mid \alpha \in B\}$.

Definition 6 (see [10]). Let $G_1^{\wr} = \langle I_1, J_1, A_1\rangle$ and $G_2^{\wr} = \langle I_2, J_2, A_2\rangle$ be two $T1STs$ in $\mathcal{G}$. The restricted intersection of $G_1^{\wr} = \langle I_1, J_1, A_1\rangle$ and $G_2^{\wr} = \langle I_2, J_2, A_2\rangle$, denoted by $G_1^{\wr} \sqcap_r G_2^{\wr} = G^{\wr} = \langle I, J, C\rangle$, where $B = A_1 \cap A_1$, is defined $\forall \alpha \in B$, as $I(\alpha) = I_1(\alpha) \cap I_2(\alpha)$, $J(\alpha) = J_1(\alpha) \cap J_2(\alpha)$, $\forall \alpha \in A_1 \cap A_2$.

It can be written as $G_1^{\wr} \sqcap_r G_2^{\wr} = \{T(\alpha) = (I(\alpha), J(\alpha)) \mid \alpha \in B\}$.

Definition 7 (see [10]). Let $G_1^{\wr} = \langle I_1, J_1, A_1\rangle$ and $G_2^{\wr} = \langle I_2, J_2, A_2\rangle$ be two $T1STs$ in $\mathcal{G}$. The AND operation of $G_1^{\wr}$ and $G_2^{\wr}$, denoted by $G_1^{\wr} \wedge G_2^{\wr} = G^{\wr} = \langle I, J, C\rangle$, where $C = A_1 \times A_2$, is defined $\forall (\alpha, \beta) \in A_1 \times A_2$, as $I(\alpha, \beta) = I_1(\alpha) \cap I_2(\beta)$, $J(\alpha, \beta) = J_1(\alpha) \cap J_2(\beta)$, $\forall (\alpha, \beta) \in A_1 \times A_2$.

Definition 8 (see [10]). Let $G_1^{\wr} = \langle I_1, J_1, A_1\rangle$ and $G_2^{\wr} = \langle I_2, J_2, A_2\rangle$ be two $T1STs$ in $\mathcal{G}$. The OR operation of $G_1^{\wr}$ and $G_2^{\wr}$, denoted by $G_1^{\wr} \vee G_2^{\wr} = G^{\wr} = \langle I, J, C\rangle$, where $C = A_1 \times A_2$, is defined $\forall (\alpha, \beta) \in A_1 \times A_2$, as $I(\alpha, \beta) = I_1(\alpha) \cup I_2(\beta)$, $J(\alpha, \beta) = J_1(\alpha) \cup J_2(\beta)$, $\forall (\alpha, \beta) \in A_1 \times A_2$.

Definition 9 (see [8]). Let $G^{\wr} = \langle I, J, A\rangle$ be a soft graph of $\mathcal{G}$. Then the complement of $G^{\wr}$ is denoted by $G^{\wr c}$ and defined by $G^{\wr c} = \langle I^c, J^c, A\rangle$, where $I^c(\alpha) = I(\alpha)$ and $J^c(\alpha)$ contain those edges which are not included in $J(\alpha)$.

A generalization of soft sets called type 2 soft sets is introduced by Chatterjee et al. [11]. Some results on type 2 soft sets are validated by Yang and Wang [12] and some new operation on typ-2 soft sets are defined by Khizar et al. [13]. Let X be universe set and E be the set of parameters. The definition of type 2 soft set is as follows:

Definition 10 (see [11]). Let (U, E) be a soft universe and $\sigma(U)$ be the collection of all $T1SS$ over (U, E). Then a mapping $: A \longrightarrow \sigma(U)$, $A \subseteq E$ is called a type 2 soft set (briefly $T2SS$) over (U, E) and it is denoted by $[S^*, A]$. In this case, corresponding to each parameter $\alpha \in A$, $S^*(\alpha)$ is a $T1SS$. Thus, for each $\alpha \in A$, there exists a $T1SS$, (S_α, L_α) such that $S^*(\alpha) = (S_\alpha, L_\alpha)$ where $S_\alpha : L_\alpha \longrightarrow P(U)$ and $L_\alpha \subset E$. In this case, we refer to the parameter set A as the "primary set of parameters", while the set of parameters $\cup L_\alpha$ is known as the "underlying set of parameters".

Let $\mathcal{G} = (X, \mathcal{E})$ be a simple graph. The set of all $T1SS$ over X is denoted by $\Gamma(X)$ and the set of all $T1SS$ over $\mathcal{E}$ is denoted by $\Gamma(\mathcal{E})$. The set of neighbors of an element $x \in X$ is denoted by $\mathcal{NB}_x$ and defined by $\mathcal{NB}_x = \{z \in X \mid xz \in \mathcal{E}\}$. Then $\mathcal{NB}_A = \bigcup_{x \in A} \mathcal{NB}_x$, where $A \subset X$. Let a subset $\mathcal{R}$ of $\mathcal{NB}_A \times X$ be an arbitrary relation from $\mathcal{NB}_A$ to X. Let $\mathcal{G} = (X, \mathcal{E})$ be a simple graph. Khizar et al. [14] introduced type 2 soft graphs by using $T2SSs$ over X and $T2SSs$ over $\mathcal{E}$.

Definition 11 (see [14]). Let $\mathcal{G} = (X, \mathcal{E})$ be a simple graph, $A \subset X$ and $\Gamma(X)$ be the collection of all $T1SS$ over X. Then a mapping $I^* : A \longrightarrow \Gamma(X)$ is called a $T2SS$ over X and it is denoted by $[I^*, A]$. In this case, corresponding to each vertex $x \in A$, $I^*(x)$ is a $T1SS$. Thus, for each $x \in A$, there exists

a $T1SS$, $(I_x, \mathcal{NB}_x)$ such that $I^*(x) = (I_x, \mathcal{NB}_x)$ where $I_x : \mathcal{NB}_x \longrightarrow P(X)$ can be defined as $I_x(z) = \{y \in X \mid z\mathcal{R}y\}$, for all $z \in \mathcal{NB}_x \subseteq X$ and $\mathcal{NB}_x$ is the set of all neighbors of $x \in A$. This $T2SS$ is also called a vertex-neighbors induced type 2 soft set (briefly, VN-type 2 soft set) over X.

Definition 12 (see [14]). Let $\mathcal{G} = (X, \mathcal{E})$ be a simple graph, $A \subset X$ and $\Gamma(\mathcal{E})$ be the collection of all $T1SS$ over $\mathcal{E}$. Let $[I^*, A]$ be a VN-type 2 soft set over X. Then a mapping $J^* : A \longrightarrow \Gamma(\mathcal{E})$ is called a $T2SS$ over $\mathcal{E}$ and it is denoted by $[J^*, A]$. In this case, corresponding to each vertex $x \in A$, $J^*(x)$ is a $T1SS$. Thus, for each $x \in A$, there exists a $T1SS$, $(J_x, \mathcal{NB}_x)$ such that $J^*(x) = (J_x, \mathcal{NB}_x)$ where $J_x : \mathcal{NB}_x \longrightarrow P(\mathcal{E})$ can be define as $J_x(z) = \{\alpha\beta \in \mathcal{E} \mid \{\alpha, \beta\} \subseteq I_x(z)\}$, for all $z \in \mathcal{NB}_x \subseteq X$ and $\mathcal{NB}_x$ is the set of all neighbors of $x \in A$. This $T2SS$ is also called a VN-type 2 soft set over $\mathcal{E}$.

Definition 13 (see [14]). A 5-tuple $\mathcal{G}^* = (\mathcal{G}, I^*, J^*, A, \mathcal{NB}_A)$ is called a type 2 soft graph (briefly, $T2SG$) if it satisfies following conditions:

(i) $\mathcal{G} = (X, \mathcal{E})$ is a simple graph.

(ii) A is a non-empty set of parameters.

(iii) $[I^*, A]$ is a VN-type 2 soft set over X.

(iv) $[J^*, A]$ is a VN-type 2 soft set over $\mathcal{E}$.

(v) $T1SS$ corresponding to $(I^*(x), J^*(x))$ $\forall x \in A$, represents a type 1 soft graph.

A $T2SG$ can also be represented by $\mathcal{G}^* = \langle I^*, J^*, A\rangle = \{T^*(x) \mid x \in A\}$, where $T^*(x) = (T_x, \mathcal{NB}_x)$ such that $T_x(z) = (I_x(z), J_x(z))$, $\forall z \in \mathcal{NB}_x$.

2. Certain Types of Type 2 Soft Graphs

In this section, we present regular type 2 soft graphs, irregular type 2 soft graphs, and type 2 soft trees. Moreover, we introduce type 2 soft cycles, type 2 soft cut-nodes, and type 2 soft bridges.

Definition 14. Let $\mathcal{G}^* = (\mathcal{G}, I^*, J^*, A, \mathcal{NB}_A)$ be a $T2SG$ of $\mathcal{G}$. Then $\mathcal{G}^*$ is said to be a regular $T2SG$ if $T1SG$ corresponding to every $T^*(\gamma)$ is a regular $T1SG$ for all $\gamma \in A$. A $T2SG$ $\mathcal{G}^*$ is called a regular $T2SG$ of degree r if $T1SG$ corresponding to $T^*(\gamma)$ is a regular $T1SG$ of degree r for all $\gamma \in A$.

Example 15. Consider a graph $\mathcal{G} = (X, \mathcal{E})$ as shown in Figure 1. Let $A = \{e_9, e_{10}\}$. It may be written that $\mathcal{NB}_{e_9} = \{e_{11}, e_8\}$ and $\mathcal{NB}_{e_{10}} = \{e_8, e_{11}\}$. Let $[I^*, A]$ and $[J^*, A]$ be two $T2SSs$ over X and $\mathcal{E}$ respectively, such that $I^*(x) = (I_x, \mathcal{NB}_x)$ and $J^*(x) = (J_x, \mathcal{NB}_x)$ for all $x \in A$. Define $I_{e_9}(z) = \{y \in X \mid z\mathcal{R}y \iff 5 \geq d(z, y) \geq 3\}$, $J_{e_9}(z) = \{\alpha\beta \in \mathcal{E} \mid \{\alpha, \beta\} \subseteq I_{e_9}(z)\}$ for all $z \in \mathcal{NB}_{e_9} \subseteq X$, and $I_{e_{10}}(z) = \{y \in X \mid z\mathcal{R}y \iff 5 \geq d(z, y) \geq 3\}$, $J_{e_{10}}(z) = \{\alpha\beta \in \mathcal{E} \mid \{\alpha, \beta\} \subseteq I_{e_{10}}(z)\}$ for all $z \in \mathcal{NB}_{e_{10}} \subseteq X$. Then $T2SSs$ $[I^*, A]$ and $[J^*, A]$ are as in the following $I^*(e_9) = \{\{e_{11}, \{e_4, e_5, e_6, e_7\}\}, \{e_8, \{e_1, e_3, e_4, e_2\}\}\}$, $J^*(e_9) = \{\{e_{11}, \{e_4e_5, e_5e_7, e_4e_6, e_7e_6\}\}, \{e_8, \{e_1e_2, e_3e_2, e_3e_4, e_1e_4, e_2e_4, e_1e_3\}\}\}$, $I^*(e_{10}) = \{\{e_{11}, \{e_4, e_5, e_6, e_7\}\}, \{e_8, \{e_1, e_3, e_4, e_2\}\}\}$, $J^*(e_{10}) = \{\{e_{11}, \{e_4e_5, e_5e_7, e_4e_6, e_7e_6\}\}, \{e_8, \{e_1e_2, e_3e_2, e_3e_4, e_1e_4, e_2e_4, e_1e_3\}\}\}$. One can check that $\mathcal{G}^* = (T^*(e_9), T^*(e_{10}))$ is a regular $T2SG$ of $\mathcal{G}$ as shown in Figure 2.

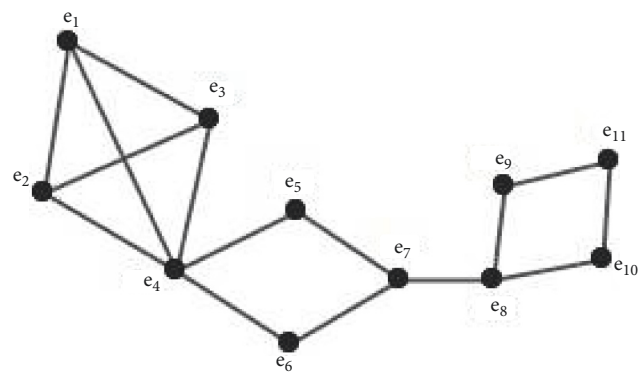

Figure 1: Simple graph.

Theorem 16. *Let $\mathcal{G}$ be a complete graph. Then every $T2SG$ of $\mathcal{G}$ is a regular $T2SG$ of $\mathcal{G}$.*

Proof. Suppose $\mathcal{G}$ is a complete graph and $\mathcal{G}^*$ be a $T2SG$ of $\mathcal{G}$. Let $(T_\alpha, \mathcal{NB}_\alpha)$ be a $T1SG$ corresponding to $T^*(\alpha)$ for all $\alpha \in A$. Then $T_\alpha(x)$ $\forall x \in \mathcal{NB}_\alpha$ is a subgraph of $\mathcal{G}$. Since every subgraph of a complete graph is complete and every complete graph is regular. Therefore, $T_\alpha(x)$ $\forall x \in \mathcal{NB}_\alpha$ is a regular subgraph. This implies that $T1SG$ corresponding to $T^*(\alpha)$ for all $\alpha \in A$ is regular $T1SG$. Hence $\mathcal{G}^*$ is a regular $T2SG$ of $\mathcal{G}$. □

Note that if $\mathcal{G}^*$ is a regular $T2SG$ of $\mathcal{G}$ then $\mathcal{G}$ need not be a complete graph.

Example 17. In Example 15, $\mathcal{G}^*$ is regular $T2SG$ but $\mathcal{G}$ is not a complete graph.

Definition 18. Let $\mathcal{G}^* = (\mathcal{G}, I^*, J^*, A, \mathcal{NB}_A)$ be a $T2SG$ of $\mathcal{G}$. Then the complement of $\mathcal{G}^*$ is denoted by $\mathcal{G}^{*c}$ and defined by $\mathcal{G}^{*c} = (T^{*c}(z_1), T^{*c}(z_2), .., T^{*c}(z_n))$ for all $x_1, x_2, .., x_n \in A$, where $T1SG$ corresponding to $T^{*c}(z_i) = (I^{*c}(z_i), J^{*c}(z_i))$ is the complement of $T1SG$ corresponding to $T^*(z_i) = (I^*(z_i), J^*(z_i))$ for all $z_i \in A$ ($i = 1$ to n).

Example 19. Consider a graph $\mathcal{G} = (X, \mathcal{E})$ as shown in Figure 3. Let $A = \{e_2, e_5\}$. It may be written that $\mathcal{NB}_{e_2} = \{e_1, e_3, e_4\}$ and $\mathcal{NB}_{e_5} = \{e_4, e_6\}$. Let $[I^*, A]$ and $[J^*, A]$ be two $T2SSs$ over X and $\mathcal{E}$ respectively, such that $I^*(x) = (I_x, \mathcal{NB}_x)$ and $J^*(x) = (J_x, \mathcal{NB}_x)$ for all $x \in A$. Define $I_{e_2}(z) = \{y \in X \mid z\mathcal{R}y \iff d(z, y) \leq 1\}$, $J_{e_2}(z) = \{\alpha\beta \in \mathcal{E} \mid \{\alpha, \beta\} \subseteq I_{e_2}(z)\}$ for all $z \in \mathcal{NB}_{e_2} \subseteq X$, and $I_{e_5}(z) = \{y \in X \mid z\mathcal{R}y \iff d(z, y) = 2\}$, $J_{e_5}(z) = \{\alpha\beta \in \mathcal{E} \mid \{\alpha, \beta\} \subseteq I_{e_5}(z)\}$ for all $z \in \mathcal{NB}_{e_5} \subseteq X$. Then $T2SSs$ $[I^*, A]$ and $[J^*, A]$ are as follows:

$$I^*(e_2) = \{\{e_1, \{e_1, e_3, e_6, e_2\}\}, \{e_3, \{e_1, e_3, e_7, e_2\}\}, \{e_4, \{e_7, e_5, e_4, e_2\}\}\},$$

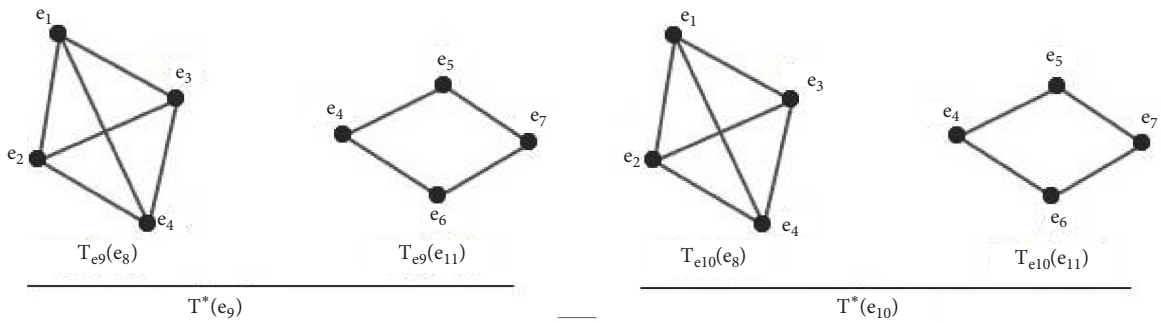

Figure 2: $\mathscr{G}^* = (T^*(e_9), T^*(e_{10}))$.

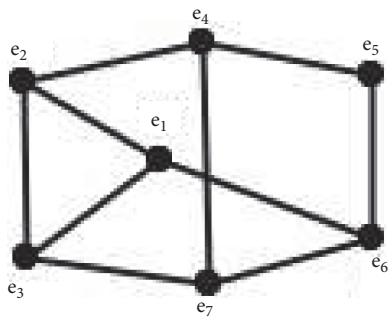

Figure 3: Simple graph.

$$\begin{aligned}
J^*(e_2) &= \{\{e_1, \{e_1e_2, e_2e_3, e_3e_1, e_1e_6\}\}, \\
&\quad \{e_3, \{e_1e_2, e_2e_3, e_3e_1, e_3e_7\}\}, \{e_4, \{e_4e_7, e_4e_5, e_4e_2\}\}\}, \\
I^*(e_5) &= \{\{e_4, \{e_6, e_1, e_3\}, \{e_6, \{e_3, e_4, e_2\}\}\}, \\
J^*(e_5) &= \{\{e_4, \{e_6e_1, e_1e_3\}\}, \{e_6, \{e_3e_2, e_4e_2\}\}\},
\end{aligned} \tag{2}$$

Then $[I^{*c}, A]$ and $[J^{*c}, A]$ are as follows:

$$\begin{aligned}
I^{*c}(e_2) &= \{\{e_1, \{e_1, e_3, e_6, e_2\}\}, \{e_3, \{e_1, e_3, e_7, e_2\}\}, \\
&\quad \{e_4, \{e_7, e_5, e_4, e_2\}\}\}, \\
J^{*c}(e_2) &= \{\{e_1, \{e_2e_6, e_6e_3\}\}, \{e_3, \{e_1e_7, e_2e_7\}\}, \\
&\quad \{e_4, \{e_2e_7, e_2e_5, e_7e_5\}\}\}, \\
I^{*c}(e_5) &= \{\{e_4, \{e_6, e_1, e_3\}, \{e_6, \{e_3, e_4, e_2\}\}\}, \\
J^{*c}(e_5) &= \{\{e_4, \{e_6e_3\}\}, \{e_6, \{e_3e_4\}\}\}.
\end{aligned} \tag{3}$$

The complement of $\mathscr{G}^* = (T^*(e_2), T^*(e_5))$ is $\mathscr{G}^{*c} = (T^{*c}(e_2), T^{*c}(e_5))$ as shown in Figure 4.

Proposition 20. *If $\mathscr{G}^*$ is a regular $T2SG$ of $\mathscr{G}$ then $\mathscr{G}^{*c}$ is a regular $T2SG$ of $\mathscr{G}$.*

Proof. Let $\mathscr{G}^*$ be a regular $T2SG$ of $\mathscr{G}$. Let $(T_\alpha, \mathscr{N}\mathscr{B}_\alpha)$ be a $T1SG$ corresponding to $T^*(\alpha)$ for all $\alpha \in A$. Then $T_\alpha(x)$ $\forall x \in \mathscr{N}\mathscr{B}_\alpha$ is a regular subgraph of $\mathscr{G}$. Since complement of a regular graph is regular, $T_\alpha^c(x)$ $\forall x \in \mathscr{N}\mathscr{B}_\alpha$ is a regular subgraph. This implies that $T1SG$ corresponding to $T^{*c}(\alpha)$ for all $\alpha \in A$ is regular $T1SG$. Hence $\mathscr{G}^{*c}$ is a regular $T2SG$ of $\mathscr{G}$. □

Proposition 21. *Let $\mathscr{G}$ be a regular graph. Then every $T2SG$ of $\mathscr{G}$ may not be a regular $T2SG$ of $\mathscr{G}$.*

Example 22. Consider a regular graph $\mathscr{G} = (X, \mathscr{E})$, where $X = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ and $\mathscr{E} = \{e_1e_2, e_3e_2, e_3e_4, e_4e_5, e_5e_6, e_1e_6, e_1e_4, e_6e_3, e_5e_2\}$. Let $A = \{e_1, e_4\}$. It may be written that $\mathscr{N}\mathscr{B}_{e_1} = \{e_2, e_6, e_4\}$ and $\mathscr{N}\mathscr{B}_{e_5} = \{e_4, e_6, e_2\}$. Let $[I^*, A]$ and $[J^*, A]$ be two $T2SSs$ over X and $\mathscr{E}$ respectively, such that $I^*(x) = (I_x, \mathscr{N}\mathscr{B}_x)$ and $J^*(x) = (J_x, \mathscr{N}\mathscr{B}_x)$ for all $x \in A$. Define $I_{e_1}(z) = \{y \in X \mid z\mathscr{R}y \iff d(z, y) \leq 1\}$, $J_{e_1}(z) = \{\alpha\beta \in \mathscr{E} \mid \{\alpha, \beta\} \subseteq I_{e_1}(z)\}$ for all $z \in \mathscr{N}\mathscr{B}_{e_1} \subseteq X$, and $I_{e_5}(z) = \{y \in X \mid z\mathscr{R}y \iff d(z, y) \leq 1\}$, $J_{e_5}(z) = \{\alpha\beta \in \mathscr{E} \mid \{\alpha, \beta\} \subseteq I_{e_5}(z)\}$ for all $z \in \mathscr{N}\mathscr{B}_{e_5} \subseteq X$. Then $T2SSs$ $[I^*, A]$ and $[J^*, A]$ are as follows:

$$\begin{aligned}
I^*(e_1) &= \{\{e_2, \{e_2, e_1, e_5, e_3\}\}, \{e_4, \{e_4, e_1, e_5, e_3\}\}, \\
&\quad \{e_6, \{e_1, e_6, e_3, e_5\}\}\}, \\
J^*(e_1) &= \{\{e_2, \{e_1e_2, e_2e_4, e_5e_2\}\}, \{e_4, \{e_5e_4, e_4e_1, e_4e_3\}\}, \\
&\quad \{e_6, \{e_5e_6, e_6e_3, e_1e_6\}\}\}, \\
I^*(e_5) &= \{\{e_4, \{e_2, e_1, e_5, e_6, e_3\}\}, \{e_2, \{e_3, e_4, e_1, e_6, e_5\}\}, \\
&\quad \{e_6, \{e_3, e_4, e_1, e_2, e_5\}\}\}, \\
J^*(e_5) &= \{\{e_4, \{e_1e_2, e_3e_2, e_5e_6, e_1e_6, e_6e_3, e_5e_2\}\}, \\
&\quad \{e_6, \{e_1e_2, e_3e_2, e_3e_4, e_4e_5, e_1e_4, e_5e_2\}\}, \\
&\quad \{e_2, \{e_3e_4, e_4e_5, e_5e_6, e_1e_6, e_1e_4, e_6e_3\}\}\}.
\end{aligned} \tag{4}$$

Then $\mathscr{G}^* = (T^*(e_1), T^*(e_5))$ is not a regular $T2SG$ of $\mathscr{G}$ as shown in Figure 5.

Definition 23. Let $\mathscr{G}^*$ be a $T2SG$ of $\mathscr{G}$. Then $\mathscr{G}^*$ is said to be irregular $T2SG$ if $T1SG$ corresponding to $T^*(\gamma)$ is an irregular $T1SG$ for all $\gamma \in A$.

Example 24. Consider a graph $\mathscr{G} = (X, \mathscr{E})$ as shown in Figure 6. Let $A = \{e_5, e_6\}$. It may be written that $\mathscr{N}\mathscr{B}_{e_5} = \{e_1, e_6\}$ and $\mathscr{N}\mathscr{B}_{e_6} = \{e_5, e_7\}$. Let $[I^*, A]$ and $[J^*, A]$ be

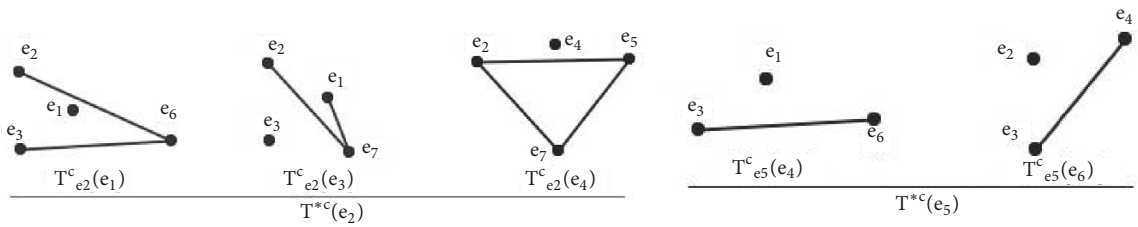

Figure 4: $\mathscr{G}^{*c} = (T^{*c}(e_2), T^{*c}(e_5))$.

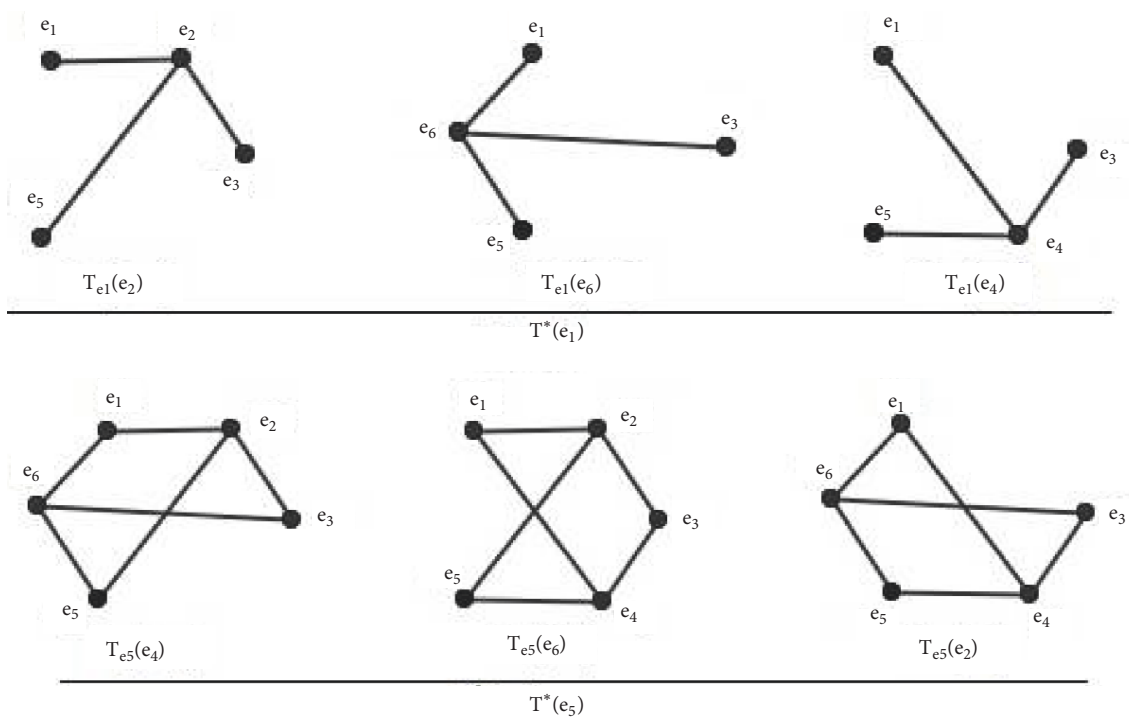

Figure 5: $\mathscr{G}^* = (T^*(e_1), T^*(e_5))$.

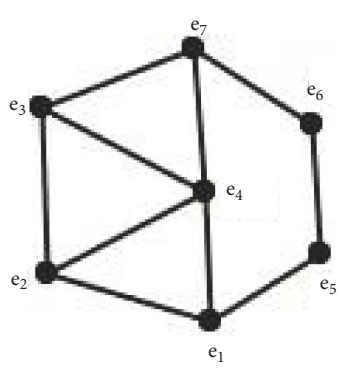

Figure 6: Simple graph.

two $T2SSs$ over X and $\mathscr{E}$ respectively, such that $I^*(x) = (I_x, \mathscr{N}\mathscr{B}_x)$ and $J^*(x) = (J_x, \mathscr{N}\mathscr{B}_x)$ for all $x \in A$. Define $I_{e_5}(z) = \{y \in X \mid z\mathscr{R}y \Longleftrightarrow d(z,y) \leq 1\}$, $J_{e_5}(z) = \{\alpha\beta \in \mathscr{E} \mid \{\alpha,\beta\} \subseteq I_{e_5}(z)\}$ for all $z \in \mathscr{N}\mathscr{B}_{e_5} \subseteq X$, and $I_{e_6}(z) = \{y \in X \mid z\mathscr{R}y \Longleftrightarrow d(z,y) \leq 1\}$, $J_{e_6}(z) = \{\alpha\beta \in \mathscr{E} \mid \{\alpha,\beta\} \subseteq I_{e_6}(z)\}$ for all $z \in \mathscr{N}\mathscr{B}_{e_6} \subseteq X$. Then $T2SSs$ $[I^*, A]$ and $[J^*, A]$ are as follows:

$$\begin{aligned}
&I^*(e_5) = \{\{e_1, \{e_1, e_4, e_5, e_2\}\}, \{e_6, \{e_5, e_6, e_7\}\}\},\\
&J^*(e_5)\\
&\quad = \{\{e_1, \{e_1e_2, e_2e_4, e_5e_1, e_1e_4\}\}, \{e_6, \{e_5e_6, e_6e_7\}\}\},\\
&I^*(e_6) = \{\{e_5, \{e_6, e_1, e_5\}\}, \{e_7, \{e_3, e_4, e_6, e_7\}\}\},\\
&J^*(e_6)\\
&\quad = \{\{e_5, \{e_6e_1, e_5e_6\}\}, \{e_7, \{e_3e_4, e_7e_3, e_7e_4, e_7e_6\}\}\}.
\end{aligned} \tag{5}$$

Then $\mathscr{G}^* = (T^*(e_6), T^*(e_5))$ is an irregular $T2SG$ of $\mathscr{G}$ as shown in Figure 7.

Definition 25. Let $\mathscr{G}^*$ be a type 2 soft graph of $\mathscr{G}$. Then $\mathscr{G}^*$ is said to be neighborly irregular $T1SG$ if $T1SG$ corresponding to $T^*(\gamma)$ is an neighborly irregular $T1SG$ for all $\gamma \in A$.

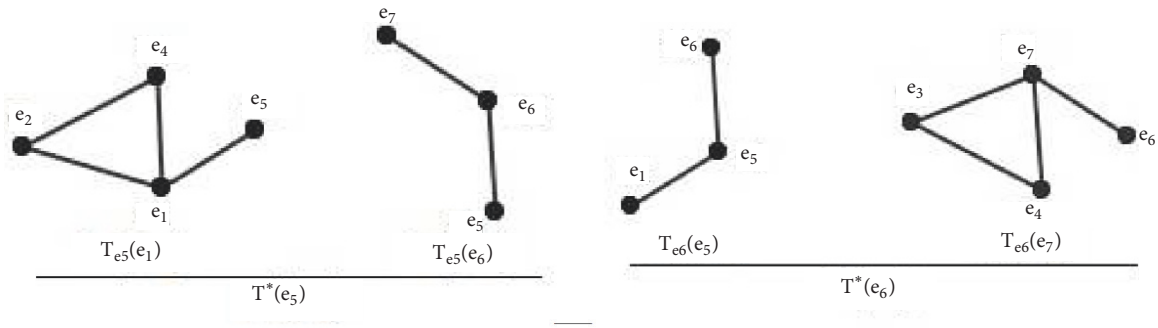

Figure 7: $\mathcal{G}^* = (T^*(e_5), T^*(e_6))$.

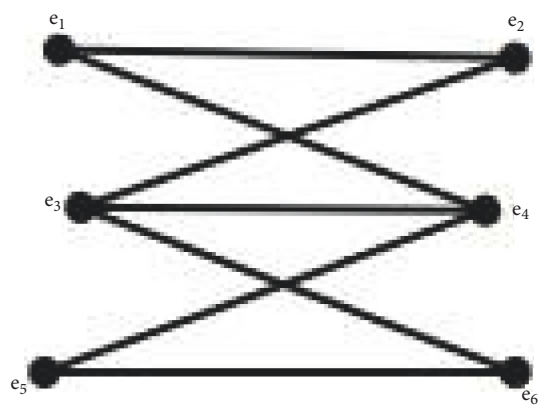

Figure 8: Simple graph.

Example 26. Consider a graph $\mathcal{G} = (X, \mathcal{E})$ as shown in Figure 8. Let $A = \{e_1, e_6\}$. It may be written that $\mathcal{NB}_{e_1} = \{e_2, e_4\}$ and $\mathcal{NB}_{e_6} = \{e_5, e_3\}$. Let $[I^*, A]$ and $[J^*, A]$ be two $T2SSs$ over X and $\mathcal{E}$ respectively, such that $I^*(x) = (I_x, \mathcal{NB}_x)$ and $J^*(x) = (J_x, \mathcal{NB}_x)$ for all $x \in A$. Define $I_{e_1}(z) = \{y \in X \mid z\mathcal{R}y \Longleftrightarrow d(z, y) \leq 1\}$, $J_{e_1}(z) = \{\alpha\beta \in \mathcal{E} \mid \{\alpha, \beta\} \subseteq I_{e_1}(z)\}$ for all $z \in \mathcal{NB}_{e_1} \subseteq X$, and $I_{e_6}(z) = \{y \in X \mid z\mathcal{R}y \Longleftrightarrow d(z, y) \leq 1\}$, $J_{e_6}(z) = \{\alpha\beta \in \mathcal{E} \mid \{\alpha, \beta\} \subseteq I_{e_6}(z)\}$ for all $z \in \mathcal{NB}_{e_6} \subseteq X$. Then $T2SSs$ $[I^*, A]$ and $[J^*, A]$ are as in the following; $I^*(e_1) = \{\{e_2, \{e_1, e_3, e_2\}\}, \{e_4, \{e_4, e_1, e_3, e_5\}\}\}$, $J^*(e_1) = \{\{e_2, \{e_1e_2, e_2e_3\}\}, \{e_4, \{e_4e_3, e_4e_5, e_4e_2\}\}\}$, $I^*(e_6) = \{\{e_3, \{e_3, e_2, e_4, e_6\}\}, \{e_5, \{e_5, e_4, e_6\}\}\}$, $J^*(e_6) = \{\{e_3, \{e_4e_3, e_3e_6, e_3e_2\}\}, \{e_5, \{e_5e_4, e_5e_6\}\}\}$, Then $\mathcal{G}^* = (T^*(e_1), T^*(e_6))$ is a neighborly irregular $T2SG$ of $\mathcal{G}$ as shown in Figure 9.

Definition 27. Let $\mathcal{G}^*$ be a $T2SG$ of a simple graph $\mathcal{G}$. Let $(T_\alpha, \mathcal{NB}_\alpha)$ be a $T1SG$ corresponding to $T^*(\alpha)$ for all $\alpha \in A$. An ede uv in $\mathcal{G}^*$ is said to be a type 2 soft bridge if its removal disconnect the subgraph $T_\alpha(x)$, $\forall x \in \mathcal{NB}_\alpha$.

Definition 28. Let $\mathcal{G}^*$ be a $T2SG$ of a simple graph $\mathcal{G}$. Let $(T_\alpha, \mathcal{NB}_\alpha)$ be a $T1SG$ corresponding to $T^*(\alpha)$ for all $\alpha \in A$. An vertex z in $\mathcal{G}^*$ is said to be a type 2 soft cut-vertex if its removal disconnect the subgraph $T_\alpha(x)$, $\forall x \in \mathcal{NB}_\alpha$.

Example 29. Consider $T2SG$ $\mathcal{G}^*$ defined in Example 24. In the Figure 7, type 2 soft bridges of $\mathcal{G}^*$ are e_1e_5 in $T_{e_5}(e_1)$, e_7e_6, e_6e_5 in $T_{e_5}(e_6)$, e_5e_6, e_5e_1 in $T_{e_6}(e_5)$ and e_6e_7 in $T_{e_6}(e_7)$. Moreover, type 2 cut-vertices of $\mathcal{G}^*$ are e_1 in $T_{e_5}(e_1)$, e_5 in $T_{e_5}(e_6)$, e_6 in $T_{e_6}(e_5)$ and e_7 in $T_{e_6}(e_7)$.

Definition 30. Let $\mathcal{G}^* = (\mathcal{G}, I^*, J^*, A, \mathcal{NB}_A)$ be a $T2SG$ of $\mathcal{G}$. Then $\mathcal{G}^*$ is said to be a type 2 soft tree (briefly, $T2ST$) if $T1SG$ corresponding to every $T^*(\gamma)$ is a $T1ST$ for all $\gamma \in A$.

Example 31. Consider a graph $\mathcal{G} = (X, \mathcal{E})$ as shown in Figure 10. Let $A = \{c, f\} \subset X$ and $\mathcal{NB}_c = \{b, d\}$, $\mathcal{NB}_f = \{e, g\}$. Let $[I^*, A]$ and $[J^*, A]$ be two neighbor-induced $T2SSs$ over X and $\mathcal{E}$ respectively, such that $I^*(x) = (I_x, \mathcal{NB}_x)$ and $J^*(x) = (J_x, \mathcal{NB}_x)$ for all $x \in A$. We define $I_c(z) = \{y \in X \mid z\mathcal{R}y \Longleftrightarrow d(z, y) = rad(G)\}$, $J_c(z) = \{\alpha\beta \in \mathcal{E} \mid \{\alpha, \beta\} \subseteq I_c(z)\}$ for all $z \in \mathcal{NB}_c \subseteq X$ and $I_f(z) = \{y \in X \mid z\mathcal{R}y \Longleftrightarrow d(z, y) = rad(G)\}$, $J_f(z) = \{\alpha\beta \in \mathcal{E} \mid \{\alpha, \beta\} \subseteq I_f(z)\}$ for all $z \in \mathcal{NB}_f \subseteq X$. Then $T2SSs$ $[I^*, A]$ and $[J^*, A]$ are as in the following $I^*(c) = \{\{b, \{g, e\}\}, \{d, \{a, g, f\}\}\}$, $J^*(c) = \{\{b, \{ge\}\}, \{d, \{ag, gf\}\}\}$, $I^*(f) = \{\{g, \{d, b\}\}, \{e, \{a, b, c\}\}\}$, $J^*(f) = \{\{g, \{bd\}\}, \{e, \{ab, bc\}\}\}$. Therefore, $\mathcal{G}^* = (T^*(c), T^*(f))$ is a $T2ST$ of $\mathcal{G}$ as shown in Figure 11. It is also called VN-type 2 soft tree.

Hence, $\mathcal{G}^* = (T^*(c), T^*(f))$ is a $T2SG$ of $\mathcal{G}$. It is also called VN-type 2 soft graph. It may be written that $\mathcal{P} = \bigcup_{x \in A} \mathcal{NB}_x = \{b, d, e, g\}$. We may symbolize $\alpha \in \mathcal{NB}_x$, as $(x \rightarrowtail \alpha)$ and denote a set of associations of A, as $(A \rightarrowtail \mathcal{P}) = \{(x \rightarrowtail \alpha) \mid \alpha \in \mathcal{NB}_x\}$. Then tabular representation of $T2ST$ is given in Table 1.

Theorem 32. *Let $(T_\gamma, \mathcal{NB}_\gamma)$ be a $T1SS$ corresponding to $T^*(\gamma)$ $\forall \gamma \in A$. Let $T_\gamma(x)$ $\forall x \in \mathcal{NB}_\gamma$ be subgraph with $n \geq 3$ vertices of $\mathcal{G}$ and $\mathcal{G}^*$ a $T2SG$ of $\mathcal{G}$. Then $\mathcal{G}^*$ is not a complete $T2SG$ of $\mathcal{G}$.*

Proof. Let $(T_\gamma, \mathcal{NB}_\gamma)$ be a $T1SS$ corresponding to $T^*(\gamma)$ $\forall \gamma \in A$. Suppose on the contrary that $\mathcal{G}$ is a complete $T2SG$, then every subgraph $T_\gamma(x)$, for all $x \in \mathcal{NB}_\gamma$ is complete. Suppose α, β be arbitrary nodes of $(T_\gamma(x)$ and they are connected by an edge $\alpha\beta$. Since $T_\gamma(x)$ is subgraph with $n \geq 3$ vertices of $\mathcal{G}$, then we can always find at least one vertex η which is connected to α by an edge $\alpha\eta$ and to β by an edge $\beta\eta$,

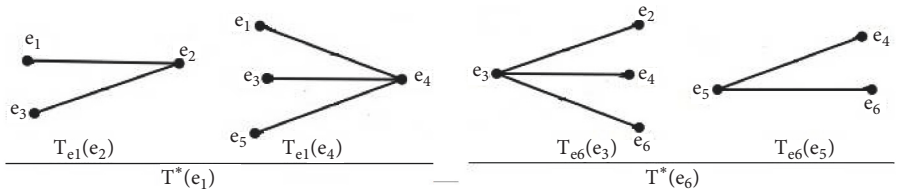

Figure 9: $\mathscr{G}^* = (T^*(e_1), T^*(e_6))$.

Table 1: Tabular representation of a type 2 soft tree.

$(A \rightarrowtail \mathscr{P}) \setminus X$	a	b	c	d	e	f	g
$c \rightarrowtail b$	0	0	0	0	1	0	1
$c \rightarrowtail d$	1	0	0	0	0	1	1
$f \rightarrowtail g$	0	1	0	1	0	0	0
$f \rightarrowtail e$	1	1	1	0	0	0	0

$(A \rightarrowtail \mathscr{P}) \setminus X$	ab	bc	bd	dc	de	ef	eg	gf	ag
$c \rightarrowtail b$	0	0	0	0	0	0	1	0	0
$c \rightarrowtail d$	0	0	0	0	0	0	0	1	1
$f \rightarrowtail g$	0	0	1	0	0	0	0	0	0
$f \rightarrowtail e$	1	1	0	0	0	0	0	0	0

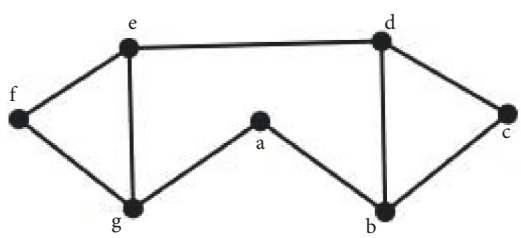

Figure 10: Simple graph.

because $T_\gamma(x)$ is a complete graph. Then there exists a cycle $\alpha\beta\eta$. Therefore, $T_\gamma(x)$ $\forall x \in \mathscr{N}\mathscr{B}_\gamma$ cannot be a $T1ST$ which contradicts the fact that $T_\gamma(x)$ is a connected subgraph of $T2SG$. Hence, $\mathscr{G}^*$ cannot be a complete $T2SG$. □

Definition 33. Let $\mathscr{G}^*$ be a $T2SG$ and $(T_\gamma, \mathscr{N}\mathscr{B}_\gamma)$ be a $T1ST$ corresponding to $T^*(\gamma)$ for any $\gamma \in A$. Then $\mathscr{G}^*$ is called type 2 soft forest if $T_\gamma(x)$ consists of more than one disconnected tree for all $x \in \mathscr{N}\mathscr{B}_\gamma$.

Definition 34. Let $\mathscr{G}^*$ be a $T2SG$ of $\mathscr{G}$. Then $\mathscr{G}^*$ is said to be a type 2 soft cycle (briefly, $T2SC$) if $T1SG$ corresponding to $T(\gamma)$ is a type 1 soft cycle of $\mathscr{G}$ for each $\gamma \in A$.

Example 35. Consider a simple graph $\mathscr{G} = (X, \mathscr{E})$, where $X = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$ and $\mathscr{E} = \{e_1e_2, e_2e_3, e_3e_1, e_3e_4, e_5e_3, e_5e_6, e_6e_4, e_7e_6, e_7e_9, e_7e_8, e_9e_{10}, e_8e_{10}\}$. Let $A = \{e_9, e_8\} \subset X$. It may be written that $\mathscr{N}\mathscr{B}_{e_9} = \{e_{10}, e_7\}$ and $\mathscr{N}\mathscr{B}_{e_8} = \{e_7, e_{10}\}$. Let $[I^*, A]$ and $[J^*, A]$ be two $T2SSs$ over X and $\mathscr{E}$, respectively, such that $I^*(x) = (I_x, \mathscr{N}\mathscr{B}_x)$ and $J^*(x) = (J_x, \mathscr{N}\mathscr{B}_x)$ for all $x \in A$. Define $I_{e_9}(z) = \{y \in X \mid z\mathscr{R}y \Longleftrightarrow 5 \geq d(z, y) \geq 3\}$, $J_{e_9}(z) = \{\alpha\beta \in \mathscr{E} \mid \{\alpha, \beta\} \subseteq I_{e_9}(z)\}$ for all $z \in \mathscr{N}\mathscr{B}_{e_9} \subseteq X$, and $I_{e_8}(z) = \{y \in X \mid z\mathscr{R}y \Longleftrightarrow 5 \geq d(z, y) \geq 3\}$, $J_{e_8}(z) = \{\alpha\beta \in \mathscr{E} \mid \{\alpha, \beta\} \subseteq I_{e_8}(z)\}$ for all $z \in \mathscr{N}\mathscr{B}_{e_8} \subseteq X$. Then $T2SSs$ $[I^*, A]$ and $[J^*, A]$ are as follows:

$$\begin{aligned}
I^*(e_9) &= \{\{e_7, \{e_2, e_3, e_1\}, \{e_{10}, \{e_5, e_3, e_4, e_6\}\}\}, \\
J^*(e_9) &= \{\{e_{10}, \{e_2e_3, e_3e_1, e_2e_1\}\}, \\
&\quad \{e_{10}, \{e_4e_3, e_3e_5, e_6e_5, e_6e_4\}\}\}, \\
I^*(e_8) &= \{\{e_7, \{e_2, e_3, e_1\}, \{e_{10}, \{e_5, e_3, e_4, e_6\}\}\}, \\
J^*(e_8) &= \{\{e_{10}, \{e_2e_3, e_3e_1, e_2e_1\}\}, \\
&\quad \{e_{10}, \{e_4e_3, e_3e_5, e_6e_5, e_6e_4\}\}\}.
\end{aligned} \tag{6}$$

One can check that $\mathscr{G}^* = (T^*(e_4), T^*(e_8))$ is a $T2SC$ of $\mathscr{G}$ as shown in Figure 12. It is also called VN-type 2 soft cycle.

Theorem 36. *If $\mathscr{G}^*$ is a $T2SC$ of $\mathscr{G}$ then $\mathscr{G}^*$ is not a $T2ST$ of $\mathscr{G}$.*

Proof. Let $\mathscr{G}^*$ be a $T2SC$ of $\mathscr{G}$. Let $(T_\gamma, \mathscr{N}\mathscr{B}_\gamma)$ be a type 1 soft cycle corresponding to $T^*(\gamma)$ $\forall \gamma \in A$. By definition, tree does not contain cycle. Then $T_\gamma(x)$ is not a tree for all $x \in \mathscr{N}\mathscr{B}_\gamma$, so that $(T_\gamma, \mathscr{N}\mathscr{B}_\gamma)$ is not a type 1 soft tree. Hence $\mathscr{G}^*$ is not a $T2ST$ of $\mathscr{G}$. □

The converse of above theorem is not true in general; i.e., if $\mathscr{G}^*$ is not a $T2ST$ then $\mathscr{G}$ need not be a $T2SC$. The following example illustrates it.

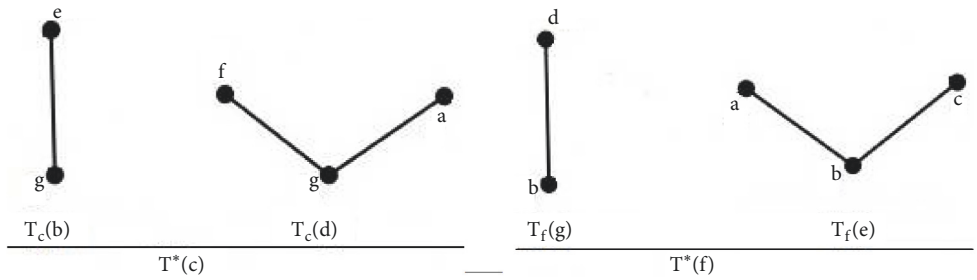

Figure 11: $\mathscr{G}^* = (T^*(c), T^*(f))$.

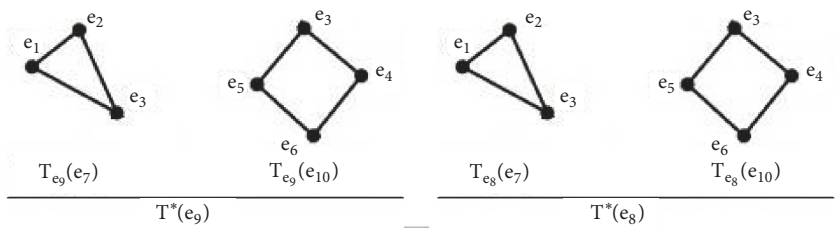

Figure 12: $\mathscr{G}^* = (T^*(e_9), T^*(e_8))$.

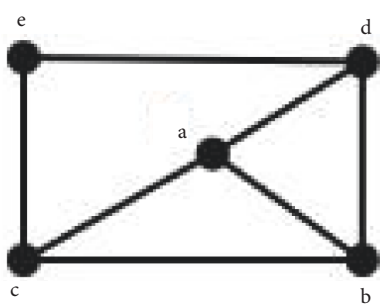

Figure 13: Simple graph.

Example 37. Consider a graph $\mathscr{G} = (X, \mathscr{E})$ as shown in Figure 13.

Let $= \{a, b\} \subset X$. Then $\mathscr{N}\mathscr{B}_a = \{b, c, d\}$, $\mathscr{N}\mathscr{B}_b = \{a, c, d\}$. Let $[I^*, A]$ and $[J^*, A]$ be two *T2SSs* over X and $\mathscr{E}$, respectively, such that $I^*(x) = (I_x, \mathscr{N}\mathscr{B}_x)$ and $J^*(x) = (J_x, \mathscr{N}\mathscr{B}_x)$ for all $x \in A$. Let $I_a(\alpha) = \{y \in X \mid \alpha\mathscr{R}y \Longleftrightarrow d(\alpha, y) \leq 1\}$, $J_a(\alpha) = \{uv \in \mathscr{E} \mid \{u, v\} \subseteq I_a(\alpha)\}$ $\forall\alpha \in \mathscr{N}\mathscr{B}_a$ and $I_b(\beta) = \{y' \in X \mid \beta\mathscr{R}y' \Longleftrightarrow d(\beta, y') = 1\}$, $J_b(\beta) = \{u'v' \in \mathscr{E} \mid \{u', v'\} \subseteq I_b(\beta)\}$ $\forall\beta \in \mathscr{N}\mathscr{B}_b$. Then

$$\begin{aligned}
I^*(a) &= \{\{b, \{a, b, d, c\}\}, \{c, \{a, b, c, e\}\}, \{d, \{a, b, d, e\}\}\},\\
J^*(a) &= \{\{b, \{ab, ad, ac, bc, bd\}\}, \{c, \{ab, ac, ce, bc\}\},\\
&\quad \{d, \{ab, ad, de, db\}\}\},\\
I^*(b) &= \{\{a, \{b, c, d\}\}, \{c, \{a, b, e\}\}, \{d, \{a, b, e\}\}\},\\
J^*(b) &= \{\{a, \{bc, bd\}\}, \{c, \{ab\}\}, \{d, \{ab\}\}\}
\end{aligned} \tag{7}$$

Figure 14 shows the respective *T1SGs* corresponding to $T^*(a) = (I^*(a), J^*(a))$ and $T^*(b) = (I^*(b), J^*(b))$ respectively. One can check that $T_a(b) = (I_a(b), J_a(b))$, $T_a(c) = (I_a(c), J_a(c))$, $T_a(d) = (I_a(d), J_a(d))$, $T_b(c) = (I_b(c), J_b(c))$ and $T_b(d) = (I_b(d), J_b(d))$ are not trees. This implies that $\mathscr{G}^* = (T^*(a), T^*(b))$ is not a *T2ST* of $\mathscr{G}$. But $\mathscr{G}^*$ is not a *T2SC* of $\mathscr{G}$.

Proposition 38. *Every T2SC of $\mathscr{G}$ is a regular T2SG of $\mathscr{G}$.*

Proof. Suppose that $\mathscr{G}^*$ is a *T2SC*. Let $(T_\gamma, \mathscr{N}\mathscr{B}_\gamma)$ be a *T1SC* corresponding to $T^*(\gamma)$ for any $\gamma \in A$. Then $T_\gamma(x)$ is a cycle for all $x \in \mathscr{N}\mathscr{B}_\alpha$. Since cycle is closed path and each vertex has degree 2, this implies that $T_\gamma(x)$ is a regular graph for all $x \in \mathscr{N}\mathscr{B}_\gamma$. Therefore $(T_\gamma, \mathscr{N}\mathscr{B}_\gamma)$ is regular *T1SG*. Since γ was taken to be arbitrary, thus it holds for all $\gamma \in A$. Hence $\mathscr{G}^*$ is a regular *T2SG* of $\mathscr{G}$. □

3. Operations on Type 2 Soft Trees

In this section, we present type 2 soft subtree of *T2ST*, union, intersection, *OR* operation, and *AND* operation of *T2ST*.

Definition 39. Let $\mathscr{G}_1^* = \langle I_1^*, J_1^*, A_1\rangle$ and $\mathscr{G}_2^* = \langle I_2^*, J_2^*, A_2\rangle$ be two *T2STs* of $\mathscr{G}$. Then $\mathscr{G}_2^*$ is a type 2 soft subtree of $\mathscr{G}_1^*$ if

(i) $A_2 \subseteq A_1$,

(ii) for each $x \in A_2$, *T1ST* corresponding to $T_2^*(x) = (I_2^*(x), J_2^*(x))$ is a type 1 soft subtree of *T1ST* corresponding to $T_1^*(x) = (I_1^*(x), J_1^*(x))$.

Example 40. Consider a simple graph $\mathscr{G} = (X, \mathscr{E})$, where $X = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ and $\mathscr{E} = \{e_1e_2, e_2e_3, e_3e_4, e_5e_4, e_5e_6, e_6e_7, e_7e_1\}$. Let $A = \{e_2, e_4\}$, $B = \{e_2, e_4, e_7\}$. It may be written that $\mathscr{N}\mathscr{B}_{e_2} = \{e_3, e_1\}$, $\mathscr{N}\mathscr{B}_{e_4} = \{e_3, e_5\}$ and $\mathscr{N}\mathscr{B}_{e_7} = \{e_1, e_6\}$.

Let $[I^*, A]$ and $[J^*, A]$ be two *T2SSs* over X and $\mathscr{E}$, respectively, such that $I^*(x) = (I_x, \mathscr{N}\mathscr{B}_x)$ and $J^*(x) = (J_x, \mathscr{N}\mathscr{B}_x)$ for all $x \in A$. Define $I_{e_2}(z) = \{y \in X \mid z\mathscr{R}y \Longleftrightarrow$

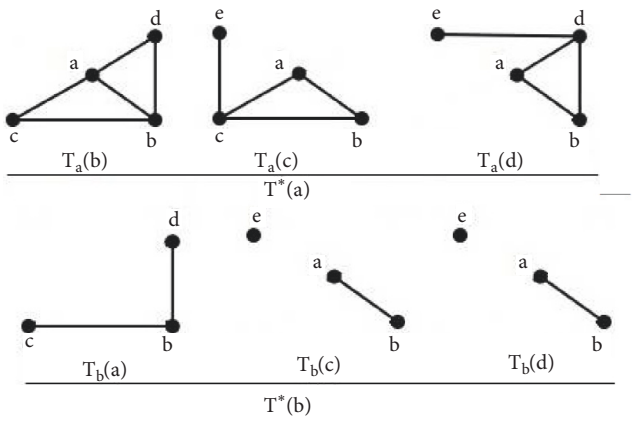

Figure 14: $\mathscr{G}^* = (T^*(a), T^*(b))$.

$d(z, y) \leq 1\}$, $J_{e_1}(z) = \{\alpha\beta \in \mathscr{E} \mid \{\alpha, \beta\} \subseteq I_{e_1}(z)\}$ for all $z \in \mathscr{N}\mathscr{B}_{e_2} \subseteq X$, $I_{e_4}(z) = \{y \in X \mid z\mathscr{R}y \Longleftrightarrow d(z, y) \leq 1\}$, $J_{e_4}(z) = \{\alpha\beta \in \mathscr{E} \mid \{\alpha, \beta\} \subseteq I_{e_4}(z)\}$ for all $z \in \mathscr{N}\mathscr{B}_{e_4} \subseteq X$. Then $T2SSs$ $[I^*, A]$ and $[J^*, A]$ are as follows:

$$\begin{aligned} I^*(e_2) &= \{\{e_1, \{e_1, e_2, e_7\}\}, \{e_3, \{e_3, e_2, e_4\}\}\}, \\ J^*(e_2) &= \{\{e_1, \{e_1e_2, e_7e_1\}\}, \{e_3, \{e_2e_3, e_3e_4\}\}\}, \\ I^*(e_4) &= \{\{e_3, \{e_4, e_2, e_3\}\}, \{e_5, \{e_6, e_5, e_4\}\}\}, \\ J^*(e_4) &= \{\{e_3, \{e_3e_4, e_3e_2\}\}, \{e_5, \{e_6e_5, e_5e_4\}\}\}. \end{aligned} \tag{8}$$

Then $\mathscr{G}^* = (T^*(e_2), T^*(e_4))$ is a $T2ST$ of $\mathscr{G}$ as shown in Figure 15.

Let $[I'^*, B]$ and $[J'^*, B]$ be two $T2SSs$ over X and $\mathscr{E}$, respectively, such that $I'^*(x) = (J'_x, \mathscr{N}\mathscr{B}_x)$ and $J'^*(x) = (J'_x, \mathscr{N}\mathscr{B}_x)$ for all $x \in B$. Define

$$\begin{aligned} I'_{e_2}(z) &= \{y \in X \mid z\mathscr{R}y \Longleftrightarrow d(z, y) \leq 1\}, \\ J'_{e_2}(z) &= \{\alpha\beta \in \mathscr{E} \mid \{\alpha, \beta\} \subseteq I'_{e_2}(z)\} \\ &\qquad \text{for all } z \in \mathscr{N}\mathscr{B}_{e_2} \subseteq X, \\ I'_{e_4}(z) &= \{y \in X \mid z\mathscr{R}y \Longleftrightarrow d(z, y) \leq 2\}, \\ J'_{e_4}(z) &= \{\alpha\beta \in \mathscr{E} \mid \{\alpha, \beta\} \subseteq I'_{e_4}(z)\} \\ &\qquad \text{for all } z \in \mathscr{N}\mathscr{B}_{e_4} \subseteq X, \\ I'_{e_7}(z) &= \{y \in X \mid z\mathscr{R}y \Longleftrightarrow d(z, y) \leq 1\}, \\ J'_{e_7}(z) &= \{\alpha\beta \in \mathscr{E} \mid \{\alpha, \beta\} \subseteq I'_{e_7}(z)\} \\ &\qquad \text{for all } z \in \mathscr{N}\mathscr{B}_{e_7} \subseteq X. \end{aligned} \tag{9}$$

Then $T2SS$ $[I'^*, B]$ and $[J'^*, B]$ are as follows:

$$\begin{aligned} I'^*(e_2) &= \{\{e_1, \{e_1, e_2, e_7\}\}, \{e_3, \{e_3, e_2, e_4\}\}\}, \\ J'^*(e_2) &= \{\{e_1, \{e_1e_2, e_7e_1\}\}, \{e_3, \{e_2e_3, e_3e_4\}\}\}, \end{aligned}$$

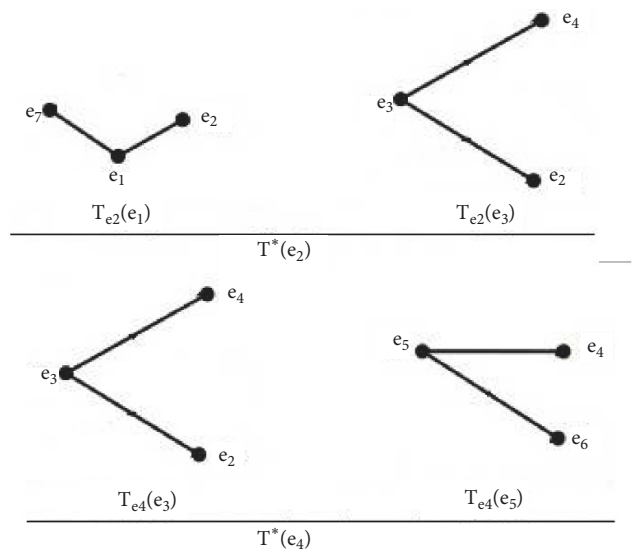

Figure 15: $\mathscr{G}^* = (T^*(e_2), T^*(e_4))$.

$$\begin{aligned} I'^*(e_4) &= \{\{e_3, \{e_4, e_2, e_3, e_1, e_5\}\}, \\ &\quad \{e_5, \{e_6, e_5, e_4, e_7, e_3\}\}\}, \\ J'^*(e_4) &= \{\{e_3, \{e_1e_2, e_2e_3, e_3e_4, e_5e_4\}\}, \\ &\quad \{e_5, \{e_3e_4, e_4e_5, e_5e_6, e_6e_7\}\}\}, \\ I'^*(e_7) &= \{\{e_1, \{e_1, e_2, e_7\}\}, \{e_6, \{e_5, e_6, e_7\}\}\}, \\ J'^*(e_7) &= \{\{e_1, \{e_1e_2, e_7e_1\}\}, \{e_6, \{e_5e_6, e_6e_7\}\}\}, \end{aligned} \tag{10}$$

Then $\mathscr{G}'^* = (T'^*(e_2), T'^*(e_4), T'^*(e_7))$ is a $T2ST$ of $\mathscr{G}$ as shown in Figure 16.

One can check that $A \subset B$ and $T^*(e_2)\tilde{\subseteq}T'^*(e_2)$, $T^*(e_4)\tilde{\subseteq}T'^*(e_4)$. Hence $\mathscr{G}^*$ is a type 2 subtree of $\mathscr{G}'^*$.

Theorem 41. *Let $\mathscr{G}_1^* = \langle I_1^*, J_1^*, A_1\rangle$ and $\mathscr{G}_2^* = \langle I_2^*, J_2^*, A_2\rangle$ be two $T2STs$ of $\mathscr{G}$. Then $\mathscr{G}_2^*$ is a type 2 soft subtree of $\mathscr{G}_1^*$ if and only if $I_2^*(x)\tilde{\subseteq}I_1^*(x)$ and $J_2^*(x)\tilde{\subseteq}J_1^*(x)$ for all $x \in A_2$.*

Proof. Suppose $\mathscr{G}_2^*$ is a type 2 soft subtree of $\mathscr{G}_1^*$. Then, by the definition of type 2 soft subtree,

(i) $A_2 \subseteq A_1$,

(ii) For each $x \in A_2$, $T1ST$ corresponding to $T_2^*(x) = (I_2^*(x), J_2^*(x))$ is a type 1 soft subtree of $T1ST$ corresponding to $T_1^*(x) = (I_1^*(x), J_1^*(x))$.

Since $T1ST$ corresponding to $T_2^*(x)$ is a type 1 soft subtree of $T1ST$ corresponding to $T_1^*(x)$ for all $x \in A_2$. Then $I_2^*(x)\tilde{\subseteq}I_1^*(x)$ and $J_2^*(x)\tilde{\subseteq}J_1^*(x)$ for all $x \in A_2$.

Conversely, given that $I_2^*(x)\tilde{\subseteq}I_1^*(x)$ and $J_2^*(x)\tilde{\subseteq}J_1^*(x)$ for all $x \in A_2$. As $\mathscr{G}_1^*$ is a $T2ST$ of $\mathscr{G}$, $T1SS$ corresponding to $T_1^*(x)$ is a $T1ST$ of $\mathscr{G}$ for all $x \in A_1$. Also, $\mathscr{G}_2^*$ is a $T2ST$ of $\mathscr{G}$, $T1SS$ corresponding to $T_2^*(x)$ is a $T1ST$ of $\mathscr{G}$ for all $x \in A_2$. This implies that $T1ST$ corresponding to $T_2^*(x)$ is a type 1 soft

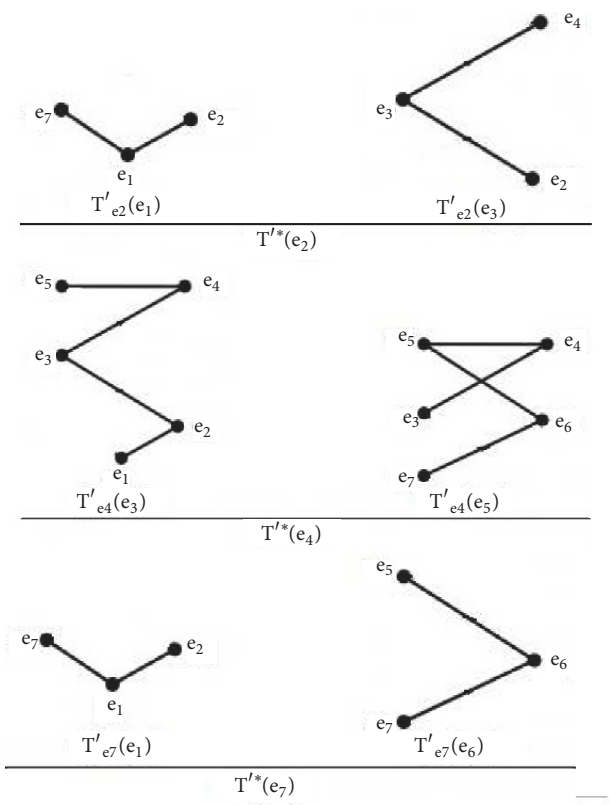

Figure 16: $\mathscr{G}'^{*} = (T'^{*}(e_2), T'^{*}(e_4), T'^{*}(e_7))$.

subtree of $T1ST$ corresponding to $T_1^*(x)$ for all $x \in A_2$. Thus, $\mathscr{G}_2^*$ is a type 2 soft subtree of $\mathscr{G}_1^*$. □

Definition 42. Let $\mathscr{G}_1^* = \langle I_1^*, J_1^*, A_1\rangle$ and $\mathscr{G}_2^* = \langle I_2^*, J_2^*, A_2\rangle$ be two $T2STs$ of $\mathscr{G}$. The *union* of $\mathscr{G}_1^*$ and $\mathscr{G}_2^*$, is denoted by $\mathscr{G}_1^* \sqcup \mathscr{G}_2^* = \mathscr{G}^* = \langle I^*, J^*, C\rangle$, where $C = A_1 \cup A_2$ is defined $\forall \alpha \in C$, as

$$I^*(\alpha) = \begin{cases} I_1^*(\alpha), & \text{if } \alpha \in A_1 - A_2 \\ I_2^*(\alpha), & \text{if } \alpha \in A_2 - A_1 \\ I_1^*(\alpha) \tilde{\cup} I_2^*(\alpha), & \text{if } \alpha \in A_1 \cap A_2 \end{cases} \tag{11}$$

where $I_1^*(\alpha)\tilde{\cup}I_2^*(\alpha)$ for all $\alpha \in A_1 \cap A_2$ refers to the usual type 1 soft union between the respective $T1ST$ corresponding to $I_1^*(\alpha)$ and $I_2^*(\alpha)$, respectively. And,

$$J^*(\alpha) = \begin{cases} J_1^*(\alpha), & \text{if } \alpha \in A_1 - A_2 \\ J_2^*(\alpha), & \text{if } \alpha \in A_2 - A_1 \\ J_1^*(\alpha) \tilde{\cup} J_2^*(\alpha), & \text{if } \alpha \in A_2 \cap A_1 \end{cases} \tag{12}$$

where $J_1^*(\alpha)\tilde{\cup}J_2^*(\alpha)$ for all $\alpha \in A_2 \cap A_1$ refers to the usual type 1 soft extended union between the respective $T1ST$ corresponding to $J_1^*(\alpha)$ and $J_2^*(\alpha)$, respectively.

It can be written as $\mathscr{G}_1^* \sqcup \mathscr{G}_2^* = \{T^*(\alpha) = (I^*(\alpha), J^*(\alpha)) \mid \alpha \in C\}$.

Theorem 43. *Let $\mathscr{G}_1^* = \langle I_1^*, J_1^*, A_1\rangle$ and $\mathscr{G}_2^* = \langle I_2^*, J_2^*, A_2\rangle$ be two $T2STs$ of $\mathscr{G}$ with $A_1 \cap A_2 = \emptyset$. Then $\mathscr{G}_1^* \sqcup \mathscr{G}_2^*$ is a $T2ST$ of G.*

Proof. The *union* of $\mathscr{G}_1^* = \langle I_1^*, J_1^*, A_1\rangle$ and $\mathscr{G}_2^* = \langle I_2^*, J_2^*, A_2\rangle$ is defined as $\mathscr{G}_1^* \sqcup \mathscr{G}_2^* = \mathscr{G}^* = \langle I^*, J^*, C\rangle$ where $C = A_1 \cup A_2$ for all $\alpha \in C$,

$$I^*(\alpha) = \begin{cases} I_1^*(\alpha), & \text{if } \alpha \in A_1 - A_2 \\ I_2^*(\alpha), & \text{if } \alpha \in A_2 - A_1 \\ I_1^*(\alpha) \tilde{\cup} I_2^*(\alpha), & \text{if } \alpha \in A_1 \cap A_2 \end{cases} \tag{13}$$

where $I_1^*(\alpha)\tilde{\cup}I_2^*(\alpha)$ for all $\alpha \in A_1 \cap A_2$ refers to the usual type 1 soft union between the respective $T1SS$ corresponding to $I_1^*(\alpha)$ and $I_2^*(\alpha)$, respectively. And,

$$J^*(\alpha) = \begin{cases} J_1^*(\alpha), & \text{if } \alpha \in A_1 - A_2 \\ J_2^*(\alpha), & \text{if } \alpha \in A_2 - A_1 \\ J_1^*(\alpha) \tilde{\cup} J_2^*(\alpha), & \text{if } \alpha \in A_1 \cap A_2 \end{cases} \tag{14}$$

where $J_1^*(\alpha)\tilde{\cup}J_2^*(\alpha)$ for all $\alpha \in A_1 \cap A_2$ refers to the usual type 1 soft extended union between the respective $T1SS$ corresponding to $J_1^*(\alpha)$ and $J_2^*(\alpha)$, respectively.

Since $\mathscr{G}_1^*$ is a $T2ST$ of G. Then $T1ST$ corresponding to $(I_1^*(x), J_1^*(x))$ is a $T1ST$ of $\mathscr{G}$ for all $x \in A_1 - A_2$.

Since $\mathscr{G}_2^*$ is a $T2ST$ of G. Then $T1ST$ corresponding to $(I_2^*(x), J_2^*(x))$ is a $T1ST$ of $\mathscr{G}$ for all $x \in A_2 - A_1$.

It is given that $A_1 \cap A_2 = \emptyset$. Thus, $\mathscr{G}_1^* \sqcup \mathscr{G}_2^* = \mathscr{G}^* = \langle I^*, J^*, A_1 \cup A_2\rangle$ is a $T2ST$ of $\mathscr{G}$. □

If $A_1 \cap A_2 \neq \emptyset$ then union of two $T2ST$ may not be a $T2ST$ as it can be seen in the following example.

Example 44. Consider a graph $\mathscr{G}$ defined in Example 40. Let $A = \{e_2, e_4\}$,$B = \{e_2, e_7\}$. It may be written that $\mathscr{N}\mathscr{B}_{e_2} = \{e_3, e_1\}$, $\mathscr{N}\mathscr{B}_{e_4} = \{e_3, e_5\}$ and $\mathscr{N}\mathscr{B}_{e_7} = \{e_1, e_6\}$.

Let $[I^{\circ *}, A]$ and $[J^{\circ *}, A]$ be two $T2SSs$ over X and $\mathscr{E}$, respectively, such that $I^{\circ *}(x) = (I_x^{\circ}, \mathscr{N}\mathscr{B}_x)$ and $J^{\circ *}(x) = (J_x^{\circ}, \mathscr{N}\mathscr{B}_x)$ for all $x \in A$. Define $I_{e_2}^{\circ}(z) = \{y \in X \mid z\mathscr{R}y \Longleftrightarrow d(z, y) \leq 1\}$, $J_{e_2}^{\circ}(z) = \{\alpha\beta \in \mathscr{E} \mid \{\alpha, \beta\} \subseteq I_{e_2}^{\circ}(z)\}$ for all $z \in \mathscr{N}\mathscr{B}_{e_2} \subseteq X$ and $I_{e_4}^{\circ}(z) = \{y \in X \mid z\mathscr{R}y \Longleftrightarrow d(z, y) \leq 1\}$, $J_{e_4}^{\circ}(z) = \{\alpha\beta \in \mathscr{E} \mid \{\alpha, \beta\} \subseteq I_{e_4}^{\circ}(z)\}$ for all $z \in \mathscr{N}\mathscr{B}_{e_4} \subseteq X$. Then $T2SSs$ $[I^{\circ *}, A]$ and $[J^{\circ *}, A]$ are as follows:

$$\begin{aligned} I^{\circ *}(e_2) &= \{\{e_1, \{e_1, e_2, e_7\}\}, \{e_3, \{e_3, e_2, e_4\}\}\}, \\ J^{\circ *}(e_2) &= \{\{e_1, \{e_1e_2, e_7e_1\}\}, \{e_3, \{e_2e_3, e_3e_4\}\}\}, \\ I^{\circ *}(e_4) &= \{\{e_3, \{e_4, e_2, e_3\}\}, \{e_5, \{e_6, e_5, e_4\}\}\}, \\ J^{\circ *}(e_4) &= \{\{e_3, \{e_3e_4, e_3e_2\}\}, \{e_5, \{e_6e_5, e_5e_4\}\}\}. \end{aligned} \tag{15}$$

Then $\mathscr{G}^* = (T^{\circ *}(e_2), T^{\circ *}(e_4))$ is a $T2ST$ of $\mathscr{G}$.

Let $[I'^{*}, B]$ and $[J'^{*}, B]$ be two $T2SSs$ over X and $\mathscr{E}$, respectively, such that $I'^{*}(x) = (I_x', \mathscr{N}\mathscr{B}_x)$ and $J'^{*}(x) =$

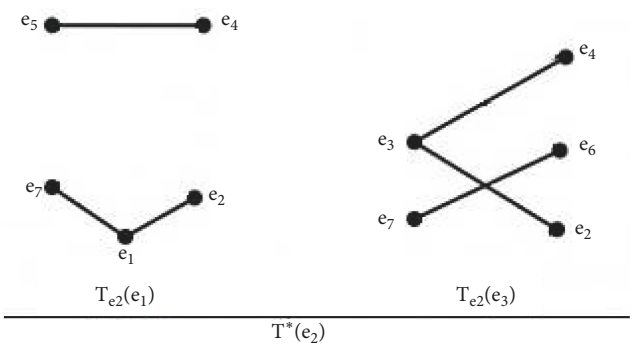

Figure 17: $T^*(e_2)$.

$(J'_x, \mathcal{N}\mathcal{B}_x)$ for all $x \in B$. Define $I'_{e_2}(z) = \{y \in X \mid z\mathcal{R}y \Longleftrightarrow d(z,y) \geq 2\}$, $J'_{e_1}(z) = \{\alpha\beta \in \mathcal{E} \mid \{\alpha,\beta\} \subset I'_{e_2}(z)\}$ for all $z \in \mathcal{N}\mathcal{B}_{e_2} \subseteq X$ and $I'_{e_7}(z) = \{y \in X \mid z\mathcal{R}y \Longleftrightarrow d(z,y) \geq 1\}$, $J'_{e_7}(z) = \{\alpha\beta \in \mathcal{E} \mid \{\alpha,\beta\} \subset I'_{e_7}(z)\}$ for all $z \in \mathcal{N}\mathcal{B}_{e_7} \subseteq X$. Then $T2SSs$ $[I'^*, B]$ and $[J'^*, B]$ are as follows:

$$\begin{aligned}
&I'^*(e_2) = \{\{e_1, \{e_4, e_5\}\}, \{e_3, \{e_6, e_7\}\}\},\\
&J'^*(e_2) = \{\{e_1, \{e_4e_5\}\}, \{e_3, \{e_6e_7\}\}\},\\
&I'^*(e_7) = \{\{e_1, \{e_3, e_4, e_5, e_6\}\}, \{e_6, \{e_1, e_2, e_3, e_4\}\}\},\\
&J'^*(e_7)\\
&\quad = \{\{e_1, \{e_3e_4, e_4e_5, e_5e_6\}\}, \{e_6, \{e_1e_2, e_2e_3, e_3e_4\}\}\}.
\end{aligned} \tag{16}$$

Then $\mathcal{G}'^* = (T'^*(e_2), T'^*(e_4), T'^*(e_7))$ is a $T2ST$ of $\mathcal{G}$. By the definition of *union* of $T2STs$,

$$\begin{aligned}
&I^*(e_4) = I^{\circ*}(e_4)\\
&\text{and } J^*(e_4) = J^{\circ*}(e_4),\\
&\qquad\qquad e_4 \in A - B,\\
&I^*(e_7) = I'^*(e_7)\\
&\text{and } J^*(e_7) = J'^*(e_7),\\
&\qquad\qquad e_7 \in B - A,\\
&I^*(e_2) = I^{\circ*}(e_2) \cup I'^*(e_2)\\
&\text{and } J^*(e_2) = J^{\circ*}(e_2) \tilde{\cup} J'^*(e_2),\\
&\qquad\qquad e_2 \in B \cap A.
\end{aligned} \tag{17}$$

By routine calculations, it is easy to see that $T1SGs$ corresponding to $T^*(e_4) = T^{\circ*}(e_4)$ and $T^*(e_7) = T'^*(e_7)$ are $T1STs$. But $T1SG$ corresponding to $T^*(e_2) = T^{\circ*}(e_2)\tilde{\cup}T'^*(e_2)$ is a disconnected type 2 soft forest, as shown in Figure 17. Therefore, $\mathcal{G}_1^* \sqcup \mathcal{G}_2^* = \mathcal{G}^* = \langle I^*, J^*, A \cup B\rangle$ is not a $T2ST$ of $\mathcal{G}$.

Lemma 45. *Let $\mathcal{G}_1^* = \langle S_1^*, T_1^*, A_1\rangle$ and $\mathcal{G}_2^* = \langle S_2^*, T_2^*, A_2\rangle$ be two $T2STs$ of $\mathcal{G}$. If $A_1 \cap A_2 = \emptyset$, then their union is a $T2ST$ of $\mathcal{G}$.*

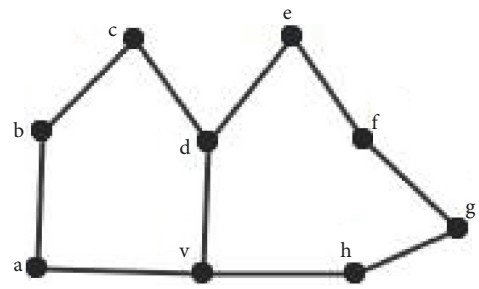

Figure 18: Simple graph.

Definition 46. Let $\mathcal{G}_1^* = \langle I_1^*, J_1^*, A_1\rangle$ and $\mathcal{G}_2^* = \langle I_2^*, J_2^*, A_2\rangle$ be two $T2STs$ of $\mathcal{G}$. The *intersection* of $\mathcal{G}_1^*$ and $\mathcal{G}_2^*$, denoted by $\mathcal{G}_1^* \sqcap \mathcal{G}_2^* = \mathcal{G}^* = \langle I^*, J^*, C\rangle$, where $C = A \cap B$ is defined as $I^*(\alpha) = I_1^*(\alpha)\tilde{\cap}I_2^*(\alpha)$, for all $\alpha \in A \cap B$, where $I_1^*(\alpha)\tilde{\cap}I_2^*(\alpha)$ for all $\alpha \in A \cap B$ refers to the usual type 1 soft intersection between the respective $T1SS$ corresponding to $I_1^*(\alpha)$ and $I_2^*(\alpha)$, respectively. And $J^*(\alpha) = J_1^*(\alpha)\tilde{\cap}J_2^*(\alpha)$ for all $\alpha \in A \cap B$ where $J_1^*(\alpha)\tilde{\cap}J_2^*(\alpha)$ for all $\alpha \in A \cap B$ refers to the usual type 1 soft intersection between the respective $T1SS$ corresponding to $J_1^*(\alpha)$ and $J_2^*(\alpha)$, respectively.

It can be written as $\mathcal{G}_1^* \sqcap \mathcal{G}_2^* = \{T^*(\alpha) = (I^*(\alpha), J^*(\alpha)) \mid \alpha \in C\}$.

The intersection of two $T2STs$ may not be $T2ST$ as it can be seen in the following example.

Example 47. Consider a simple graph $\mathcal{G}$ shown in Figure 18. Let $A = \{a, h\}$ and $B = \{b, h\}$. It may be written that $\mathcal{N}\mathcal{B}_a = \{v, b\}$, $\mathcal{N}\mathcal{B}_h = \{v, g\}$ and $\mathcal{N}\mathcal{B}_b = \{a, c\}$.

Let $[I^{\circ*}, A]$ and $[J^{\circ*}, A]$ be two $T2SSs$ over X and $\mathcal{E}$, respectively, such that $I^{\circ*}(x) = (I_x^\circ, \mathcal{N}\mathcal{B}_x)$ and $J^{\circ*}(x) = (J_x^\circ, \mathcal{N}\mathcal{B}_x)$ for all $x \in A$. Define $I_a^\circ(z) = \{y \in X \mid z\mathcal{R}y \Longleftrightarrow d(z,y) \leq 1\}$, $J_a^\circ(z) = \{\alpha\beta \in \mathcal{E} \mid \{\alpha,\beta\} \subset I_a^\circ(z)\}$ for all $z \in \mathcal{N}\mathcal{B}_a \subseteq X$ and $I_h^\circ(z) = \{y \in X \mid z\mathcal{R}y \Longleftrightarrow d(z,y) \leq 1\}$, $J_h^\circ(z) = \{\alpha\beta \in \mathcal{E} \mid \{\alpha,\beta\} \subset I_h^\circ(z)\}$ for all $z \in \mathcal{N}\mathcal{B}_h \subseteq X$. Then $T2SSs$ $[I^{\circ*}, A]$ and $[J^{\circ*}, A]$ are as follows:

$$\begin{aligned}
&I^{\circ*}(a) = \{\{b, \{a, b, c\}\}, \{v, \{b, v, h, d\}\}\},\\
&J^{\circ*}(a) = \{\{b, \{ab, bc\}\}, \{v, \{vb, vh, vd\}\}\},\\
&I^{\circ*}(h) = \{\{v, \{a, d, h, v\}\}, \{g, \{g, f, h\}\}\},\\
&J^{\circ*}(h) = \{v, \{vd, va, vh\}\}, \{g, \{gf, gh\}\}\}.
\end{aligned} \tag{18}$$

Then $\mathcal{G}^* = (T^{\circ*}(a), T^{\circ*}(g))$ is a $T2ST$ of $\mathcal{G}$.

Let $[I'^*, B]$ and $[J'^*, B]$ be two $T2SSs$ over X and $\mathcal{E}$, respectively, such that $I'^*(x) = (I'_x, \mathcal{N}\mathcal{B}_x)$ and $J'^*(x) = (J'_x, \mathcal{N}\mathcal{B}_x)$ for all $x \in B$. Define $I'_b(z) = \{y \in X \mid z\mathcal{R}y \Longleftrightarrow d(z,y) \geq 3\}$, $J'_b(z) = \{\alpha\beta \in \mathcal{E} \mid \{\alpha,\beta\} \in I'_b(z)\}$ for all $z \in \mathcal{N}\mathcal{B}_b \subseteq X$ and $I'_h(z) = \{y \in X \mid z\mathcal{R}y \Longleftrightarrow 1 \leq d(z,y) \leq 3\}$, $J'_h(z) = \{\alpha\beta \in \mathcal{E} \mid \{\alpha,\beta\} \in I'_h(z)\}$ for all $z \in \mathcal{N}\mathcal{B}_h \subseteq X$. Then $T2SSs$ $[I'^*, B]$ and $[J'^*, B]$ are as follows:

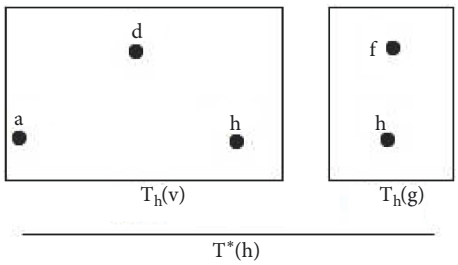

FIGURE 19: $\mathcal{G}_1^* \sqcap \mathcal{G}_2^* = \mathcal{G}^* = \langle I^*, J^*, A \cap B\rangle$.

$$\begin{aligned} &I'^*(b) = \{\{a, \{e, f, g\}\}, \{c, \{f, g, h\}\}\}, \\ &J'^*(b) = \{\{a, \{ef, fg\}\}, \{c, \{fg, gh\}\}\}, \\ &I'^*(h) \\ &\quad = \{\{v, \{e, f, g, d, a, b, h, c\}\}, \{g, \{d, e, f, h, v, a\}\}\}, \\ &J'^*(h) \\ &\quad = \{\{v, \{ef, fg, ed, vd, dc\}\}, \{g, \{ef, ed, vh, va, vd\}\}\}. \end{aligned} \tag{19}$$

Then $\mathcal{G}'^* = (T'^*(b), T'^*(h))$ is a $T2ST$ of $\mathcal{G}$. By the definition of *intersection* of $T2ST$,

$$\begin{aligned} I^*(h) &= I^{\circ *}(h) \tilde{\cap} I'^*(h) \\ \text{and } J^*(h) &= J^{\circ *}(h) \tilde{\cap} J'^*(h), \\ \{h\} &= B \cap A. \end{aligned} \tag{20}$$

By routine calculations, it is easy to see that $T1SG$ corresponding to $T^*(h) = T^{\circ *}(h) \tilde{\cap} T'^*(h)$ is a disconnected $T1SG$, as shown in Figure 19. Therefore, $\mathcal{G}_1^* \sqcap \mathcal{G}_2^* = \mathcal{G}^* = \langle I^*, J^*, A \cup B\rangle$ is not a $T2ST$ of $\mathcal{G}$.

Definition 48. Let $\mathcal{G}_1^* = \langle I_1^*, J_1^*, A_1\rangle$ and $\mathcal{G}_2^* = \langle I_2^*, J_2^*, A_2\rangle$ be two $T2STs$ of $\mathcal{G}$. The *AND* operation of $\mathcal{G}_1^*$ and $\mathcal{G}_2^*$, denoted by $\mathcal{G}_1^* \bigwedge \mathcal{G}_2^* = \mathcal{G}^* = \langle I^*, J^*, A \times B\rangle$ defined by $I^*(\gamma, \eta) = I_1^*(\gamma) \tilde{\wedge} I_2^*(\eta)$, $J^*(\gamma, \eta) = J_1^*(\gamma) \tilde{\wedge} J_2^*(\eta)$ for all $(\gamma, \eta) \in A \times B$, where $I_1^*(\gamma) \tilde{\wedge} I_2^*(\eta)$ for all $(\gamma, \eta) \in A \times B$ refers to the usual type 1 soft *AND* operation between the respective $T1SS$ corresponding to $I_1^*(\gamma)$ and $I_2^*(\eta)$ respectively and $J_1^*(\gamma) \tilde{\wedge} J_2^*(\eta)$ for all $(\gamma, \eta) \in A \times B$ refers to the usual type 1 soft *AND* operation between the respective $T1SS$ corresponding to $J_1^*(\gamma)$ and $J_2^*(\eta)$ respectively.

Example 49. Consider a simple graph $\mathcal{G} = (X, \mathcal{E})$, where $X = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$ and $\mathcal{E} = \{e_1e_2, e_2e_3, e_3e_4, e_5e_4, e_5e_6, e_6e_7, e_7e_8, e_8e_1\}$. Let $A = \{e_1, e_8\}$,$B = \{e_5\}$. It may be written that $\mathcal{NB}_{e_1} = \{e_8, e_2\}$, $\mathcal{NB}_{e_8} = \{e_7, e_1\}$ and $\mathcal{NB}_{e_5} = \{e_4, e_6\}$.

Let $[I^{\circ *}, A]$ and $[J^{\circ *}, A]$ be two $T2SSs$ over X and $\mathcal{E}$, respectively, such that $I^{\circ *}(x) = (I_x^\circ, \mathcal{NB}_x)$ and $J^{\circ *}(x) = (J_x^\circ, \mathcal{NB}_x)$ for all $x \in A$. Define $I_{e_1}^\circ(z) = \{y \in X \mid z\mathcal{R}y \Longleftrightarrow d(z, y) \le 1\}$, $J_{e_1}^\circ(z) = \{\alpha\beta \in \mathcal{E} \mid \{\alpha, \beta\} \subset I_{e_1}^\circ(z)\}$ for all $z \in \mathcal{NB}_{e_1} \subseteq X$ and $I_{e_8}^\circ(z) = \{y \in X \mid z\mathcal{R}y \Longleftrightarrow d(z, y) \le 1\}$, $J_{e_8}^\circ(z) = \{\alpha\beta \in \mathcal{E} \mid \{\alpha, \beta\} \subset I_{e_8}^\circ(z)\}$ for all $z \in \mathcal{NB}_{e_8} \subseteq X$. Then $T2SSs$ $[I^{\circ *}, A]$ and $[J^{\circ *}, A]$ are as follows:

$$\begin{aligned} &I^{\circ *}(e_1) = \{\{e_2, \{e_1, e_2, e_3\}\}, \{e_8, \{e_1, e_8, e_7\}\}\}, \\ &J^{\circ *}(e_1) = \{\{e_2, \{e_1e_2, e_2e_3\}\}, \{e_8, \{e_1e_8, e_8e_7\}\}\}, \\ &I^{\circ *}(e_8) = \{\{e_1, \{e_8, e_1, e_7\}\}, \{e_7, \{e_6, e_7, e_8\}\}\}, \\ &J^{\circ *}(e_8) = \{\{e_1, \{e_2e_1, e_1e_8, e_8e_7, e_7e_6\}\}, \\ &\quad \{e_7, \{e_1e_8, e_7e_8, e_6e_7, e_6e_5\}\}\}. \end{aligned} \tag{21}$$

Then $\mathcal{G}^* = (T^{\circ *}(e_1), T^{\circ *}(e_8))$ is a $T2ST$ of $\mathcal{G}$.

Let $[I'^*, B]$ and $[J'^*, B]$ be two $T2SSs$ over X and $\mathcal{E}$, respectively, such that $I'^*(x) = (J_x', \mathcal{NB}_x)$ and $J'^*(x) = (J_x', \mathcal{NB}_x)$ for all $x \in B$. Define $I_{e_5}'(z) = \{y \in X \mid z\mathcal{R}y \Longleftrightarrow d(z, y) \ge 1\}$, $J_{e_5}'(z) = \{\alpha\beta \in \mathcal{E} \mid \{\alpha, \beta\} \subset I_{e_5}'(z)\}$ for all $z \in \mathcal{NB}_{e_5} \subseteq X$. Then $T2SSs$ $[I'^*, B]$ and $[J'^*, B]$ are as follows:

$$\begin{aligned} &I'^*(e_5) = \{\{e_6, \{e_4, e_2, e_3, e_1, e_5, e_8, e_7\}\}, \\ &\quad \{e_4, \{e_6, e_5, e_1, e_2, e_3, e_8, e_7\}\}\}, \\ &J'^*(e_5) = \{\{e_6, \{e_1e_2, e_2e_3, e_3e_4, e_5e_4, e_1e_8, e_7e_8\}\}, \\ &\quad \{e_4, \{e_3e_2, e_2e_1, e_7e_6, e_6e_5, e_5e_4, e_1e_8, e_5e_4, e_7e_8\}\}\}, \end{aligned} \tag{22}$$

Then $\mathcal{G}'^* = (T'^*(e_5))$ is a $T2ST$ of $\mathcal{G}$. The *AND* operation on $\mathcal{G}^*$ and $\mathcal{G}'^*$ is defined as in the following:

$$\begin{aligned} &I^*(e_1, e_5) = I^{\circ *}(e_1) \tilde{\wedge} I'^*(e_5) \\ &\quad = \{\{(e_2, e_6), \{e_2, e_3, e_1\}\}, \{(e_2, e_4), \{e_1, e_3, e_2\}\}, \\ &\quad \{(e_8, e_6), \{e_1, e_8, e_7\}\}, \{(e_8, e_4), \{e_1, e_8, e_7\}\}\}, \\ &J^*(e_1, e_5) = J^{\circ *}(e_1) \tilde{\wedge} J'^*(e_5) \\ &\quad = \{\{(e_2, e_6), \{e_2e_3, e_2e_1\}\}, \{(e_2, e_4), \{e_1e_2, e_3e_2\}\}, \\ &\quad \{(e_8, e_6), \{e_1e_8, e_8e_7\}\}, \{(e_8, e_4), \{e_1e_8, e_8e_7\}\}\}, \\ &I^*(e_8, e_5) = I^{\circ *}(e_8) \tilde{\wedge} I'^*(e_5) \\ &\quad = \{\{(e_1, e_6), \{e_7, e_8, e_1\}\}, \{(e_1, e_4), \{e_1, e_8, e_7\}\}, \\ &\quad \{(e_7, e_6), \{e_8, e_7\}\}, \{(e_7, e_4), \{e_6, e_7, e_8\}\}\}, \\ &J^*(e_8, e_5) = J^{\circ *}(e_8) \tilde{\wedge} J'^*(e_5) \\ &\quad = \{\{(e_1, e_6), \{e_7e_8, e_1e_8\}\}, \{(e_1, e_4), \{e_1e_8, e_7e_8\}\}, \\ &\quad \{(e_7, e_6), \{e_8e_7\}\}, \{(e_7, e_4), \{e_6e_7, e_8e_7\}\}\}. \end{aligned} \tag{23}$$

The *AND* operation on $\mathcal{G}^*$ and $\mathcal{G}'^*$ is shown in Figure 20.

Definition 50. Let $\mathcal{G}_1^* = \langle I_1^*, J_1^*, A_1\rangle$ and $\mathcal{G}_2^* = \langle I_2^*, J_2^*, A_2\rangle$ be two $T2STs$ of $\mathcal{G}$. The *OR* operation of $\mathcal{G}_1^*$ and $\mathcal{G}_2^*$, denoted by $\mathcal{G}_1^* \bigvee \mathcal{G}_2^* = \mathcal{G}^* = \langle I^*, J^*, A \times B\rangle$ defined by $I^*(\gamma, \eta) = I_1^*(\gamma) \tilde{\vee} I_2^*(\eta)$, $J^*(\gamma, \eta) = J_1^*(\gamma) \tilde{\vee} J_2^*(\eta)$ for all $(\gamma, \eta) \in$

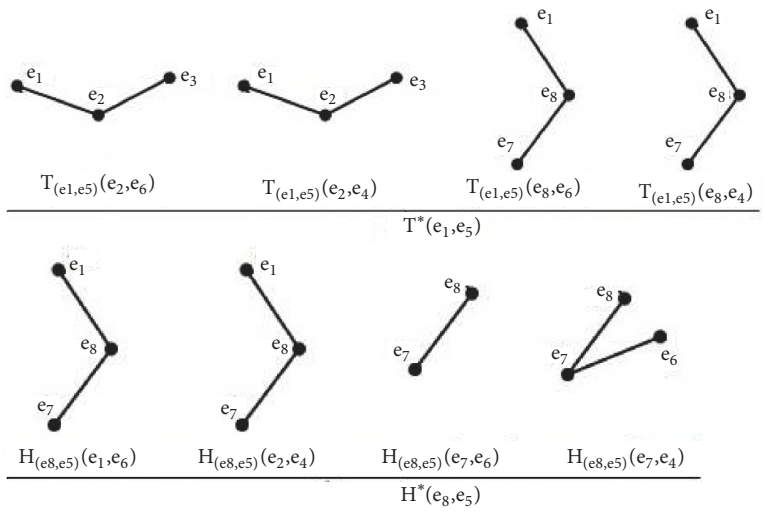

Figure 20: *AND* operation.

$A \times B$, where $I_1^*(\gamma)\tilde{\vee}I_2^*(\eta)$ for all $(\gamma,\eta) \in A \times B$ refers to the usual type 1 soft *OR* operation between the respective $T1SS$ corresponding to $I_1^*(\gamma)$ and $I_2^*(\eta)$ respectively and $J_1^*(\gamma)\tilde{\vee}J_2^*(\eta)$ for all $(\gamma,\eta) \in A\times B$ refers to the usual type 1 soft *OR* operation between the respective $T1SS$ corresponding to $J_1^*(\gamma)$ and $J_2^*(\eta)$ respectively.

Example 51. Consider a simple graph $\mathcal{G} = (X, \mathcal{E})$ defined in Example 49. Let $A = \{e_3, e_5\}$, $B = \{e_6\}$. It may be written that $\mathcal{N}\mathcal{B}_{e_3} = \{e_4, e_2\}$, $\mathcal{N}\mathcal{B}_{e_6} = \{e_7, e_5\}$ and $\mathcal{N}\mathcal{B}_{e_5} = \{e_4, e_6\}$.

Let $[I^{\circ *}, A]$ and $[J^{\circ *}, A]$ be two $T2SSs$ over X and $\mathcal{E}$ respectively, such that $I^{\circ *}(x) = (I_x^\circ, \mathcal{N}\mathcal{B}_x)$ and $J^{\circ *}(x) = (J_x^\circ, \mathcal{N}\mathcal{B}_x)$ for all $x \in A$. Define $I_{e_3}^\circ(z) = \{y \in X \mid z\mathcal{R}y \Longleftrightarrow d(z,y) \le 1\}$, $J_{e_3}^\circ(z) = \{\alpha\beta \in \mathcal{E} \mid \{\alpha,\beta\} \subset I_{e_3}^\circ(z)\}$ for all $z \in \mathcal{N}\mathcal{B}_{e_3} \subseteq X$, $I_{e_5}^\circ(z) = \{y \in X \mid z\mathcal{R}y \Longleftrightarrow d(z,y) \ge 3\}$, $J_{e_5}^\circ(z) = \{\alpha\beta \in \mathcal{E} \mid \{\alpha,\beta\} \subset I_{e_5}^\circ(z)\}$ for all $z \in \mathcal{N}\mathcal{B}_{e_5} \subseteq X$. Then $T2SSs$ $[I^{\circ *}, A]$ and $[J^{\circ *}, A]$ are as follows:

$$\begin{aligned}
I^{\circ *}(e_3) &= \{\{e_2, \{e_1, e_2, e_3\}\}, \{e_4, \{e_3, e_4, e_5\}\}\},\\
J^{\circ *}(e_3) &= \{\{e_2, \{e_1e_2, e_2e_3\}\}, \{e_4, \{e_3e_4, e_4e_5\}\}\},\\
I^{\circ *}(e_5) &= \{\{e_4, \{e_8, e_1, e_7\}\}, \{e_6, \{e_1, e_2, e_3\}\}\},\\
J^{\circ *}(e_5) &= \{\{e_4, \{e_8e_7, e_1e_8\}\}, \{e_6, \{e_1e_2, e_2e_3\}\}\}.
\end{aligned} \tag{24}$$

Then $\mathcal{G}^* = (T^{\circ *}(e_2), T^{\circ *}(e_4))$ is a $T2ST$ of $\mathcal{G}$.

Let $[I'^*, B]$ and $[J'^*, B]$ be two $T2SSs$ over X and $\mathcal{E}$, respectively, such that $I'^*(x) = (I'_x, \mathcal{N}\mathcal{B}_x)$ and $J'^*(x) = (J'_x, \mathcal{N}\mathcal{B}_x)$ for all $x \in B$. Define $I'_{e_6}(z) = \{y \in X \mid z\mathcal{R}y \Longleftrightarrow d(z,y) \le 1\}$, $J'_{e_6}(z) = \{\alpha\beta \in \mathcal{E} \mid \{\alpha,\beta\} \subset I'_{e_6}(z)\}$ for all $z \in \mathcal{N}\mathcal{B}_{e_6} \subseteq X$. Then $T2SSs$ $[I'^*, B]$ and $[J'^*, B]$ are as follows:

$$\begin{aligned}
I'^*(e_6) &= \{\{e_7, \{e_4, e_2, e_3, e_1, e_5\}\},\\
&\quad \{e_5, \{e_1, e_2, e_3, e_8, e_7\}\}\},\\
J'^*(e_6) &= \{\{e_7, \{e_1e_2, e_2e_3, e_3e_4, e_5e_4\}\},\\
&\quad \{e_5, \{e_3e_2, e_2e_1, e_1e_8, e_7e_8\}\}\},
\end{aligned} \tag{25}$$

Then $\mathcal{G}'^* = (T'^*(e_6))$ is a $T2ST$ of $\mathcal{G}$. The *OR* operation on $\mathcal{G}^*$ and $\mathcal{G}'^*$ is defined as in the following:

$$\begin{aligned}
I^*(e_3, e_6) &= I^{\circ *}(e_3)\tilde{\wedge}I'^*(e_6)\\
&= \{\{(e_2, e_7), \{e_4, e_2, e_3, e_1, e_5\}\},\\
&\quad \{(e_2, e_5), \{e_7, e_2, e_3, e_1, e_8\}\},\\
&\quad \{(e_4, e_7), \{e_4, e_2, e_3, e_1, e_5\}\},\\
&\quad \{(e_4, e_5), \{e_4, e_2, e_3, e_1, e_5, e_7, e_8\}\}\},\\
J^*(e_3, e_6) &= J^{\circ *}(e_3)\tilde{\wedge}J'^*(e_6)\\
&= \{\{(e_2, e_7), \{e_1e_2, e_2e_3, e_3e_4, e_5e_4\}\},\\
&\quad \{(e_2, e_5), \{e_1e_2, e_2e_3, e_1e_8, e_7e_8\}\},\\
&\quad \{(e_4, e_7), \{e_1e_2, e_3e_2, e_4e_3, e_4e_5\}\},\\
&\quad \{(e_4, e_5), \{e_1e_2, e_2e_3, e_3e_4, e_5e_4, e_1e_8, e_8e_7\}\}\},\\
I^*(e_5, e_6) &= I^{\circ *}(e_5)\tilde{\wedge}I'^*(e_6)\\
&= \{\{(e_4, e_7), \{e_4, e_2, e_3, e_1, e_5, e_7, e_8\}\},\\
&\quad \{(e_4, e_5), \{e_7, e_2, e_3, e_1, e_8\}\},\\
&\quad \{(e_6, e_7), \{e_1, e_2, e_3, e_4, e_5\}\},\\
&\quad \{(e_6, e_5), \{e_1, e_2, e_3, e_8, e_7\}\}\},\\
J^*(e_8, e_5) &= J^{\circ *}(e_8)\tilde{\wedge}J'^*(e_5)\\
&= \{\{(e_4, e_7), \{e_7e_8, e_1e_8, e_4e_5, e_3e_4, e_3e_2, e_1e_2\}\},
\end{aligned}$$

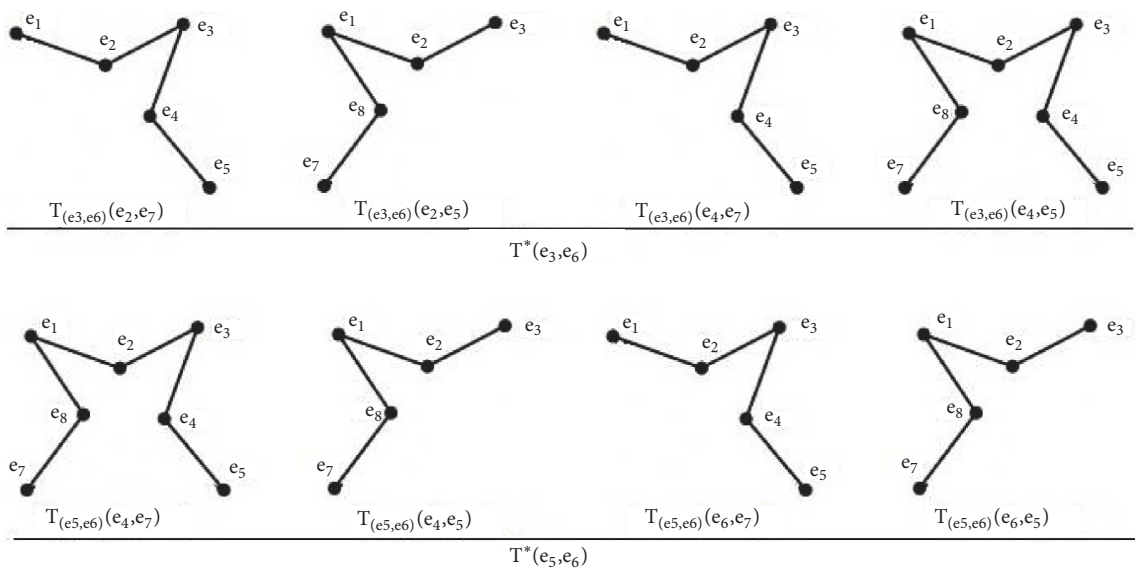

Figure 21: *OR* operation.

$$\{(e_4, e_5), \{e_1e_2, e_2e_3, e_1e_8, e_8e_7\}\},$$
$$\{(e_6, e_7), \{e_1e_2, e_2e_3, e_3e_4, e_4e_5\}\},$$
$$\{(e_6, e_5), \{e_1e_2, e_2e_3, e_7e_8, e_1e_8\}\}\}. \tag{26}$$

The *OR* operation on $\mathscr{G}^*$ and $\mathscr{G}'^*$ is shown in Figure 21.

4. Conclusion

In above study, we have characterized type 2 soft graphs on underlying subgraphs (regular subgraphs, irregular subgraphs, cycles, trees) of a simple graph. We have presented regular type 2 soft graphs, irregular type 2 soft graphs, and type 2 soft trees. Moreover, we have introduced type 2 soft cycles, type 2 soft cut-nodes, and type 2 soft bridges. Finally, we have presented some operations on type 2 soft trees by presenting several examples to demonstrate these new concepts. In future work, we will extend our work in following areas of research:

(i) Applications of type 2 soft graphs in computer networks and social networks

(ii) Fuzzy type 2 soft graphs and their applications.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This paper is supported by the High Level Construction Fund of Guangzhou University China. Also this work was supported by the Natural Science Foundation of Guangdong Province (2016A030313552), the Guangdong Provincial Government to Guangdong International Student Scholarship (yuejiao [2014] 187), and Guangzhou Vocational College of Science and Technology (no. 2016TD03).

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