

Research Article

On Carlitz's Type Modified Degenerate q -Changhee Polynomials and Numbers

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Recently, Dolgy-Jang-Kwon-Kim introduced Carlitz's type q -Changhee polynomials. In this paper, we define Carlitz's type modified degenerate q -Changhee polynomials and investigate some interesting identities of these polynomials.

1. Introduction

Let p be a prime number with $p \equiv 1 \pmod{2}$. Throughout this paper, \mathbb{Z}_p , \mathbb{Q}_p , and \mathbb{C}_p denote the ring of p -adic integers, the field of p -adic rational numbers, and the completion of the algebraic closure of \mathbb{Q}_p , respectively. The p -adic norm $|\cdot|_p$ is normalized as $|p|_p = 1/p$. Let q be an indeterminate in \mathbb{C}_p such that $|1 - q|_p < p^{-1/(p-1)}$. The q -analogue of number x is defined as $[x]_q = (q^x - 1)/(q - 1)$. As is well known, the Euler polynomials are defined by the generating function to be

$$\frac{2}{e^t + 1} e^{xt} = \sum_{n=0}^{\infty} E_n(x) \frac{t^n}{n!} \quad (1)$$

(see [1–5]).

When $x = 0$, $E_n = E_n(0)$ ($n \geq 0$) are called the Euler numbers.

Recall that Carlitz considered the q -analogue of Euler numbers which are given by the recurrence relation as follows:

$$\begin{aligned} \mathcal{E}_{0,q} &= 1, \\ q(q\mathcal{E}_q + 1)^n + \mathcal{E}_{n,q} &= \begin{cases} [2]_q, & \text{if } n = 0, \\ 0, & \text{if } n \geq 1 \end{cases} \end{aligned} \quad (2)$$

with the usual convention about replacing \mathcal{E}_q^n by $\mathcal{E}_{n,q}$, and that he also considered q -Euler polynomials which are defined by

$$\mathcal{E}_{n,q}(x) = \sum_{l=0}^n \binom{n}{l} [x]_q^{n-l} q^{lx} \mathcal{E}_{l,q} \quad (3)$$

(see [1–5]).

Let $C(\mathbb{Z}_p)$ be the space of continuous \mathcal{C}_p -valued functions on \mathbb{Z}_p . For $f \in C(\mathbb{Z}_p)$, the fermionic p -adic q -integrals on \mathbb{Z}_p are defined by Kim to be

$$\begin{aligned} I_{-q}(f) &= \int_{\mathbb{Z}_p} f(x) d\mu_{-q}(x) \\ &= \lim_{N \rightarrow \infty} \frac{1}{[p^N]_{-q}} \sum_{x=0}^{p^N-1} f(x) (-q)^x, \end{aligned} \quad (4)$$

where $[x]_{-q} = (1 - (-q)^x)/(1 + q)$ (see [1, 3, 4, 6–14]). From (2), he derived the following formula for Carlitz's q -Euler numbers.

The Changhee polynomials are defined by the generating function to be

$$\frac{2}{2+t} (1+t)^x = \sum_{n=0}^{\infty} Ch_n(x) \frac{t^n}{n!} \quad (5)$$

(see [15–23]).

In [16, 24], the authors (2017) obtained that

$$E_n(x) = \sum_{k=0}^n S_2(n, k) Ch_k(x), \quad (6)$$

$$Ch_n(x) = \sum_{k=0}^n S_1(n, k) E_k(x), \quad (n \geq 0) \quad (7)$$

(see [15, 16, 18–23]), where $S_1(n, k)$ is the Stirling numbers of the first kind and $S_2(n, k)$ is the Stirling numbers of the second kind as follows:

$$(x)_n = x(x-1)\cdots(x-n+1) = \sum_{k=0}^n S_1(n, k) x^k, \quad (8)$$

$$(e^t - 1)^m = m! \sum_{k=m}^{\infty} S_2(k, m) \frac{t^k}{k!}, \quad (n \geq 0)$$

(see [2, 10, 25–27]).

The degenerate Euler polynomials are defined by the generating function to be

$$\frac{2}{(1+\lambda t)^{1/\lambda} + 1} (1+\lambda t)^{x/\lambda} = \sum_{n=0}^{\infty} \mathcal{E}_{n,\lambda}(x) \frac{t^n}{n!} \quad (9)$$

(see [3, 4]).

The degenerate q -Euler polynomials are defined by the generating function to be

$$\int_{\mathbb{Z}_p} (1+\lambda t)^{[x+y]_q/\lambda} d\mu_{-q}(y) = \sum_{n=0}^{\infty} \mathcal{E}_{n,q,\lambda}(x) \frac{t^n}{n!} \quad (10)$$

(see [3, 4]).

In [2, 26–28], Kim et al. (2017) defined the degenerate Stirling numbers of the second kind as follows:

$$\frac{1}{k!} ((1+\lambda t)^{1/\lambda} - 1)^k = \sum_{n=k}^{\infty} S_{2,\lambda}(n, k) \frac{t^n}{n!}, \quad (11)$$

where $k \in \mathbb{N} \cup \{0\}$ and $\lambda \in \mathbb{R}$. In [15], by using the fermionic p -adic q -integral on \mathbb{Z}_p , the authors defined Carlitz's type q -Changhee polynomials as follows:

$$\int_{\mathbb{Z}_p} (1+t)^{[x+y]_q} d\mu_{-q}(y) = \sum_{n=0}^{\infty} Ch_{n,q}(x) \frac{t^n}{n!} \quad (12)$$

(see [1, 2, 14, 15, 24, 29–31]).

In this paper, we define Carlitz's type modified degenerate q -Changhee polynomials and investigate some interesting identities of these polynomials.

2. Carlitz's Type Modified Degenerate q -Changhee Polynomials

In this section, we assume that $t \in \mathbb{C}_p$ with $|t|_p < p^{-1/(p-1)}$ and $|\lambda|_p < p^{-1/(p-1)}$. From (4) and (6), we note that

$$\begin{aligned} \int_{\mathbb{Z}_p} (1+t)^{x+y} d\mu_{-1}(y) &= \frac{2}{2+t} (1+t)^x \\ &= \sum_{n=0}^{\infty} Ch_n(x) \frac{t^n}{n!}, \end{aligned} \quad (13)$$

$$1+t = e^{\log(1+t)} = \lim_{\lambda \rightarrow 0} (1+\lambda \log(1+t))^{1/\lambda}. \quad (14)$$

In the viewpoint of (12) and (14), Carlitz's type modified degenerate q -Changhee polynomials are defined by

$$\begin{aligned} &\int_{\mathbb{Z}_p} (1+\lambda \log(1+t))^{[x+y]_q/\lambda} d\mu_{-q}(y) \\ &= \sum_{n=0}^{\infty} Ch_{n,q,\lambda}(x) \frac{t^n}{n!}. \end{aligned} \quad (15)$$

We observe that

$$\begin{aligned} &(1+\lambda \log(1+t))^{[x+y]_q/\lambda} \\ &= \sum_{k=0}^{\infty} \left(\frac{1}{\lambda} [x+y]_q \right)_k \lambda^k (\log(1+t))^k \\ &= \sum_{k=0}^{\infty} \left(\frac{1}{\lambda} [x+y]_q \right)_k \lambda^k \sum_{m=k}^{\infty} S_1(m, k) \frac{t^m}{m!} \\ &= \sum_{m=0}^{\infty} \sum_{k=0}^m S_1(m, k) \left(\frac{1}{\lambda} [x+y]_q \right)_k \lambda^k \frac{t^m}{m!} \\ &= \sum_{m=0}^{\infty} \sum_{k=0}^m S_1(m, k) \sum_{l=0}^k \left(\frac{1}{\lambda} \right)^l [x+y]_q^l S_1(k, l) \lambda^k \frac{t^m}{m!} \\ &= \sum_{m=0}^{\infty} \left(\sum_{k=0}^m \sum_{l=0}^k S_1(m, k) S_1(k, l) \lambda^{k-l} [x+y]_q^l \right) \frac{t^m}{m!}. \end{aligned} \quad (16)$$

From (15) and (16), we get

$$\begin{aligned} &\sum_{n=0}^{\infty} Ch_{n,q,\lambda}(x) \frac{t^n}{n!} = \int_{\mathbb{Z}_p} (1+\lambda \log(1+t))^{[x+y]_q/\lambda} d\mu_{-q}(y) \\ &= \sum_{n=0}^{\infty} \left(\sum_{k=0}^n \sum_{l=0}^k S_1(n, k) S_1(k, l) \lambda^{k-l} \right) \frac{t^n}{n!} \\ &\quad \cdot \int_{\mathbb{Z}_p} [x+y]_q^l d\mu_{-q}(y) \frac{t^n}{n!} \\ &= \sum_{n=0}^{\infty} \left(\sum_{k=0}^n \sum_{l=0}^k S_1(n, k) S_1(k, l) \lambda^{k-l} \mathcal{E}_{l,q}(x) \right) \frac{t^n}{n!}. \end{aligned} \quad (17)$$

Thus, by (17), we get the following theorem.

Theorem 1. For $n \geq 0$, one has

$$Ch_{n,q,\lambda}(x) = \sum_{k=0}^n \sum_{l=0}^k S_1(n, k) S_1(k, l) \lambda^{k-l} \mathcal{E}_{l,q}(x). \quad (18)$$

Note that

$$\begin{aligned} \lim_{\lambda \rightarrow 0} Ch_{n,q,\lambda}(x) &= \lim_{\lambda \rightarrow 0} \sum_{k=0}^n \sum_{l=0}^k S_1(n, k) \lambda^{k-l} \mathcal{E}_{l,q}(x) \\ &= \sum_{k=0}^n S_{n,k} \mathcal{E}_{k,q}(x) = Ch_{n,q}(x), \end{aligned} \quad (19)$$

$$\int_{\mathbb{Z}_p} (1+t)^{[x+y]_q} d\mu_{-q}(y) = \sum_{n=0}^{\infty} Ch_{n,q}(x) \frac{t^n}{n!}. \quad (20)$$

Replacing t by $(1 + \lambda \log(1+t))^{1/\lambda} - 1$ in (20), we observe that

$$\begin{aligned} &\int_{\mathbb{Z}_p} (1 + \lambda \log(1+t))^{(1/\lambda)[x+y]_q} d\mu_{-q}(y) \\ &= \sum_{m=0}^{\infty} Ch_{m,q}(x) \frac{1}{m!} \left((1 + \lambda \log(1+t))^{1/\lambda} - 1 \right)^m \\ &= \sum_{m=0}^{\infty} Ch_{m,q}(x) \sum_{k=m}^{\infty} S_{2,\lambda}(k, m) \frac{(\log(1+t))^k}{k!} \\ &= \sum_{k=0}^{\infty} \sum_{m=0}^k Ch_{m,q}(x) S_{2,\lambda}(k, m) \sum_{n=k}^{\infty} S_1(n, k) \frac{t^n}{n!} \\ &= \sum_{n=0}^{\infty} \left(\sum_{k=0}^n \sum_{m=0}^k Ch_{m,q}(x) S_{2,\lambda}(k, m) S_1(n, k) \right) \frac{t^n}{n!}. \end{aligned} \quad (21)$$

From (15) and (21), we get the following theorem.

Theorem 2. For $n \geq 0$, one has

$$Ch_{n,q,\lambda}(x) = \sum_{k=0}^n \sum_{m=0}^k Ch_{m,q}(x) S_{2,\lambda}(k, m) S_1(n, k). \quad (22)$$

Replacing t by $e^t - 1$ in (15), we get

$$\begin{aligned} &\int_{\mathbb{Z}_p} (1 + \lambda t)^{(1/\lambda)[x+y]_q} d\mu_{-q}(y) \\ &= \sum_{k=0}^{\infty} Ch_{k,q,\lambda}(x) \frac{1}{k!} (e^t - 1)^k \\ &= \sum_{k=0}^{\infty} Ch_{k,q,\lambda}(x) \sum_{n=k}^{\infty} S_2(n, k) \frac{t^n}{n!} \\ &= \sum_{n=0}^{\infty} \left(\sum_{k=0}^n Ch_{k,q,\lambda}(x) S_2(n, k) \right) \frac{t^n}{n!}. \end{aligned} \quad (23)$$

Note that

$$\int_{\mathbb{Z}_p} (1 + \lambda t)^{(1/\lambda)[x+y]_q} d\mu_{-q}(y) = \sum_{n=0}^{\infty} \mathcal{E}_{n,q,\lambda}(x) \frac{t^n}{n!}. \quad (24)$$

From (23) and (24), we get the following theorem.

Theorem 3. For $n \geq 0$, one has

$$\mathcal{E}_{n,q,\lambda}(x) = \sum_{k=0}^{\infty} Ch_{k,q,\lambda}(x) S_2(n, k). \quad (25)$$

When $x = 0$, $Ch_{n,q,\lambda} = Ch_{n,q,\lambda}(0)$ are called Carlitz's type modified degenerate q -Changhee number. We also observe that

$$\begin{aligned} &\left((1 + \lambda \log(1+t))^{1/\lambda} + 1 - 1 \right)^{[x+y]_q/\lambda} \\ &= \sum_{k=0}^{\infty} \left(\frac{[x+y]_q}{\lambda} \right)_k \left((1 + \lambda \log(1+t))^{1/\lambda} - 1 \right)^k \\ &= \sum_{k=0}^{\infty} \left(\frac{[x+y]_q}{\lambda} \right)_k \frac{1}{k!} \\ &\cdot \left((1 + \lambda \log(1+t))^{1/\lambda} - 1 \right)^k = \sum_{k=0}^{\infty} \left(\frac{[x+y]_q}{\lambda} \right)_k \\ &\cdot \sum_{m=k}^{\infty} S_{2,\lambda}(m, k) \frac{1}{m!} (\log(1+t))^m \\ &= \sum_{m=0}^{\infty} \left(\sum_{k=0}^m \left(\frac{[x+y]_q}{\lambda} \right)_k S_{2,\lambda}(m, k) \sum_{n=m}^{\infty} S_1(n, m) \right) \frac{t^n}{n!} \\ &\cdot \frac{t^n}{n!} \\ &= \sum_{n=0}^{\infty} \left(\sum_{m=0}^n \sum_{k=0}^m S_{2,\lambda}(m, k) \sum_{l=0}^k S_1(k, l) \lambda^{-l} [x+y]_q^l \right) \\ &\cdot \frac{t^n}{n!} \\ &= \sum_{n=0}^{\infty} \left(\sum_{m=0}^n \sum_{k=0}^m \sum_{l=0}^k S_{2,\lambda}(m, k) S_1(k, l) \lambda^{-l} [x+y]_q^l \right) \\ &\cdot \frac{t^n}{n!}. \end{aligned} \quad (26)$$

From (26), we get

$$\begin{aligned} \sum_{n=0}^{\infty} Ch_{n,q,\lambda}(x) \frac{t^n}{n!} &= \int_{\mathbb{Z}_p} \left((1 + \lambda \log(1+t))^{1/\lambda} + 1 - 1 \right)^{[x+y]_q/\lambda} d\mu_{-q}(y) \\ &= \sum_{n=0}^{\infty} \left(\sum_{m=0}^n \sum_{k=0}^m \sum_{l=0}^k S_{2,\lambda}(m, k) S_1(k, l) \lambda^{-l} [x+y]_q^l \right) \\ &\cdot \int_{\mathbb{Z}_p} [x+y]_q^l d\mu_{-q}(y) \frac{t^n}{n!}. \end{aligned} \quad (27)$$

By (27), we get the following theorem.

Theorem 4. For $n \geq 0$, one has

$$Ch_{n,q,\lambda}(x) = \sum_{m=0}^n \sum_{k=0}^m \sum_{l=0}^k S_{2,\lambda}(m, k) S_{k,l} \lambda^{-l} \mathcal{E}_{l,q}(x). \quad (28)$$

3. Results and Discussions

This study was to define the modified degenerate q -Changhee polynomials in (15). Theorem 1 is an interesting identity between the modified degenerate q -Changhee polynomials and the q -Euler polynomials. Theorem 2 is that the modified degenerate q -Changhee polynomials is represented by a sum of products of the degenerate Stirling numbers of the second kind, the Stirling numbers of the first kind, and q -Changhee polynomials. Theorem 3 is an identity between the degenerate q -Euler polynomials and the modified degenerate q -Changhee polynomials. Theorem 4 is an identity between the modified degenerate q -Changhee polynomials and the q -Euler polynomials. In the future, we will study to define and to investigate the higher-order modified degenerate q -Changhee polynomials (see [3, 11]) and to investigate the symmetric identities of the modified degenerate q -Changhee polynomials (see [3, 11]).

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors contributed equally to this work. All authors read and approved the final manuscript.

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