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Research Article

On Symmetric Identities of Carlitz's Type q-Daehee Polynomials

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In this paper, we study Carlitz's type q-Daehee polynomials and investigate the symmetric identities for them by using the p-adic q-integral on $\mathbb{Z}_p$ under the symmetry group of degree n.

1. Introduction and Preliminaries

Let p be a fixed prime number. Throughout this paper, $\mathbb{Z}_p$, $\mathbb{Q}_p$, and $\mathbb{C}_p$ will denote the ring of p-adic integers, the field of p-adic rational numbers, and the completion of the algebraic closure of $\mathbb{Q}_p$, respectively. The p-adic norm is normalized as $|p|_p = 1/p$. If $q \in \mathbb{C}_p$, we normally assume $|1-q|_p < p^{-1/(p-1)}$, so that $q^x = \exp(x \log q)$ for $|x|_p \geq 1$. The q-extension of x is defined as $[x]_q = (1-q^x)/(1-q)$ for $q \neq 1$ and x for $q = 1$.

As is well known, Carlitz's q-Bernoulli numbers are defined by

$$\beta_{0,q} = 1,$$
$$q\left(q\beta_q + 1\right)^k - \beta_{k,q} = \begin{cases} 1, & \text{if } k = 1, \\ 0, & \text{if } k > 1, \end{cases} \tag{1}$$

with the usual convention about replacing β_q^k by $\beta_{k,q}$ (see [1, 2]). Let $UD(\mathbb{Z}_p)$ be the space of uniformly differentiable functions on $\mathbb{Z}_p$. For $f \in UD(\mathbb{Z}_p)$, the p-adic q-integral on $\mathbb{Z}_p$ is defined by Kim to be

$$I_q(f) = \int_{\mathbb{Z}_p} f(x)\, d\mu_q(x) = \lim_{N \longrightarrow \infty} \frac{1}{[p^N]_q} \sum_{x=0}^{p^N-1} f(x) q^x \tag{2}$$

(see [3–6]). From (2), we note that

$$q^n I_q(f_n) - I_q(f) = q(q-1) \sum_{j=0}^{n-1} q^j f(j) + \frac{q(q-1)}{\log q} \sum_{j=0}^{n-1} f'(j) q^j, \tag{3}$$

where $f_n(x) = f(x+n)$ and $f'(j) = (d/dx) f(x)|_{x=j}$. In particular, if we take $n = 1$, then we have

$$qI_q(f_1) - I_q(f) = q(q-1) f(0) + \frac{q(q-1)}{\log q} f'(0) \tag{4}$$

(see [7]). Kim et al. [8] defined the q-Daehee polynomials by the generating function to be

$$\frac{q-1+((q-1)/\log q)\log(1+t)}{q-1+qt}(1+t)^x = \sum_{n=0}^{\infty} D_{n,q}(x) \frac{t^n}{n!}. \tag{5}$$

When $x = 0$, $D_{n,q} = D_{n,q}(0)$ are called the Daehee numbers with q-parameter. By (4), we get

$$\int_{\mathbb{Z}_p} (1+t)^{x+y} d\mu_q(y) = \frac{q-1+((q-1)/\log q)\log(1+t)}{q-1+qt}(1+t)^x. \tag{6}$$

In [9–12], we recall that the Daehee polynomials are given by the generating function to be

$$\int_{\mathbb{Z}_p} (1+t)^{x+y}\, d\mu_0(y) = \frac{\log(1+t)}{t}(1+t)^x = \sum_{n=0}^{\infty} D_n(x)\frac{t^n}{n!}, \tag{7}$$

and the q-Bernoulli polynomials are given by the generating function to be

$$\int_{\mathbb{Z}_p} e^{(x+y)t} d\mu_q(y) = \frac{q-1+((q-1)/\log q)t}{qe^t-1} = \sum_{n=0}^{\infty} B_{n,q}(x)\frac{t^n}{n!}. \tag{8}$$

When $x=0$, $D_n = D_n(0)$ are called the Daehee numbers and $B_{n,q} = B_{n,q}(0)$, $(n \ge 0)$, are called the q-Bernoulli numbers. Kim [13] proved that Carlitz's q-Bernoulli polynomials can be represented by the p-adic q-integral on $\mathbb{Z}_p$:

$$\int_{\mathbb{Z}_p} e^{([x+y]_q)t} d\mu_q(y) = \sum_{n=0}^{\infty} \beta_{n,q}(x)\frac{t^n}{n!}. \tag{9}$$

Kim-Kim-Jang [14] gave symmetric identities for degenerate Berstein and degenerate Euler polynomials and also many mathematical researchers studied symmetry identities of various polynomials (see [1, 15–17]). In this paper, we consider Carlitz's type q-Daehee polynomials and investigate the symmetry identities for them by using the p-adic q-integral on $\mathbb{Z}_p$ under the symmetry group of degree n.

2. Symmetry Identities for Carlitz's Type q-Daehee Polynomials

Let $t \in \mathbb{C}_p$ with $|t|_p < p^{-1/(p-1)}$. From (6), we consider Carlitz's type q-Daehee polynomials can be represented by the p-adic q-integral on $\mathbb{Z}_p$:

$$\int_{\mathbb{Z}_p} (1+t)^{[x+y]_q} d\mu_q(y) = \sum_{n=0}^{\infty} d_{n,q}(x)\frac{t^n}{n!}, \tag{10}$$

When $x=0, d_{n,q} = d_{n,q}(0)$ are called Carlitz's type q-Daehee numbers.

Theorem 1 (see [18], Witt's formula). *Let $n \ge 0$; we have*

$$d_{n,q} = \int_{\mathbb{Z}_p} \left([x+y]_q\right)_m d\mu_q(y). \tag{11}$$

Kim [19] obtained that

$$d_{n,q} = \sum_{k=0}^{n} S_1(n,k)\,\beta_{k,q}(x) \tag{12}$$

and

$$\beta_{n,q} = \sum_{k=0}^{n} S_2(n,k)\, d_{k,q}(x) \tag{13}$$

where $S_1(n,k)$ is the Stirling numbers of the first kind as follows:

$$(x)_0 = 1,$$
$$(x)_n = x(x-1)\cdots(x-n+1) = \sum_{k=0}^{n} S_1(n,k)x^k, \quad (n \ge 1) \tag{14}$$

and $S_2(n,k)$ is the Stirling numbers of the second kind as follows:

$$x^n = \sum_{l=0}^{n} S_2(n,l)(x)_l, \quad (n \ge 1). \tag{15}$$

Let S_n be the symmetry group of degree n. For positive integers $w_1, w_2, \cdots, w_n$, we consider the following integral equation for the p-adic q-integral on $\mathbb{Z}_p$.

$$\begin{aligned}
&\int_{\mathbb{Z}_p} (1+t)^{[(\prod_{i=1}^{n-1} w_i)y+(\prod_{i=1}^{n} w_i)x+w_n\sum_{i=1}^{n-1}(\prod_{n\ge j\ge 1, j\ne i} w_j)k_i]_q} dq^{w_1\cdots w_{n-1}}(y) \\
&= \lim_{N\to\infty} \frac{1}{[p^N]_{q^{w_1\cdots w_{n-1}}}} \sum_{y=0}^{p^N-1} q^{(w_1\cdots w_{n-1})y} \\
&\times (1+t)^{[(\prod_{i=1}^{n-1} w_i)y+(\prod_{i=1}^{n} w_i)x+w_n\sum_{i=1}^{n-1}(\prod_{n\ge j\ge 1, j\ne i} w_j)k_i]_q} \\
&= \lim_{N\to\infty} \frac{1}{[w_n p^N]_{q^{w_1\cdots w_{n-1}}}} \sum_{m=0}^{w_n-1}\sum_{y=0}^{p^N-1} q^{(w_1\cdots w_{n-1})(m+w_n y)} \\
&\times (1+t)^{[(\prod_{i=1}^{n-1} w_i)(m+w_n y)+(\prod_{i=1}^{n} w_i)x+w_n\sum_{i=1}^{n-1}(\prod_{n\ge j\ge 1, j\ne i} w_j)k_i]_q}
\end{aligned} \tag{16}$$

From (16), we have

$$\begin{aligned}
&\frac{1}{[w_1\cdots w_{n-1}]_q}\sum_{k_1=0}^{w_1-1}\sum_{k_2=0}^{w_2-1}\cdots\sum_{k_{n-1}=0}^{w_{n-1}-1} q^{\sum_{i=1}^{n-1}(\prod_{n\ge j\ge 1, j\ne i} w_j)k_i} \times \int_{\mathbb{Z}_p} (1+t)^{[(\prod_{i=1}^{n-1} w_i)y+(\prod_{i=1}^{n} w_i)x+w_n\sum_{i=1}^{n-1}(\prod_{n\ge j\ge 1, j\ne i} w_j)k_i]_q} dq^{w_1\cdots w_{n-1}}(y) \\
&= \lim_{N\to\infty}\frac{1}{[w_1\cdots w_{n-1}w_n p^N]_q}\sum_{k_1=0}^{w_1-1}\sum_{k_2=0}^{w_2-1}\cdots\sum_{k_{n-1}=0}^{w_{n-1}-1}\sum_{m=0}^{w_n-1}\sum_{y=0}^{p^N-1} q^{\sum_{i=1}^{n-1}(\prod_{n\ge j\ge 1, j\ne i} w_j)k_i} \times q^{(w_1\cdots w_{n-1})(m+w_n y)}(1 \\
&+ t)^{[(\prod_{i=1}^{n-1} w_i)(m+w_n y)+(\prod_{i=1}^{n} w_i)x+w_n\sum_{i=1}^{n-1}(\prod_{n\ge j\ge 1, j\ne i} w_j)k_i]_q}.
\end{aligned} \tag{17}$$

As this expression is an invariant under any permutation $\sigma \in S_n$, we have the following theorem.

Theorem 2. *For $n \geq 1, w_1, \cdots w_n \in \mathbb{N}$, the expressions*

$$\frac{1}{[w_{\sigma(1)} \cdots w_{\sigma(n-1)}]_q} \sum_{k_1=0}^{w_{\sigma(1)}-1} \sum_{k_2=0}^{w_{\sigma(2)}-1} \cdots \sum_{k_{n-1}=0}^{w_{\sigma(n-1)}-1} q^{\sum_{i=1}^{n-1}(\prod_{n\geq j\geq 1, j\neq i} w_{\sigma(j)})k_i} \times \int_{\mathbb{Z}_p} (1 + t)^{[(\prod_{i=1}^{n-1} w_{\sigma(i)})y + (\prod_{i=1}^{n} w_{\sigma(i)})x + w_{\sigma(n)} \sum_{i=1}^{n-1}(\prod_{n\geq j\geq 1, j\neq i} w_{\sigma(j)})k_i]_q} dq^{w_{\sigma(1)}\cdots w_{\sigma(n-1)}}(y) \tag{18}$$

are the same for any $\sigma \in S_n$.

We observe that

$$\left[\left(\prod_{i=1}^{n-1} w_i\right) y + \left(\prod_{i=1}^{n} w_i\right) x + w_n \sum_{i=1}^{n-1}\left(\prod_{n\geq j\geq 1, j\neq i} w_j\right) k_i\right]_q = \left[\prod_{i=1}^{n-1} w_i\right]_q \left[y + w_n x + \sum_{i=1}^{n-1} \frac{w_n}{w_i}\right]_{q^{w_1\cdots w_{n-1}}} \tag{19}$$

From (26) and Theorem 1, we note that

$$\begin{aligned}
&\int_{\mathbb{Z}_p} (1 + t)^{[(\prod_{i=1}^{n-1} w_i)y + (\prod_{i=1}^{n} w_i)x + w_n \sum_{i=1}^{n-1}(\prod_{n\geq j\geq 1, j\neq i} w_j)k_i]_q} dq^{w_1\cdots w_{n-1}}(y) \\
&= \sum_{m=0}^{\infty} \int_{\mathbb{Z}_p} \left(\left[\left(\prod_{i=1}^{n-1} w_i\right) y + \left(\prod_{i=1}^{n} w_i\right) x + w_n \sum_{i=1}^{n-1}\left(\prod_{n\geq j\geq 1, j\neq i} w_j\right) k_i\right]_q\right)_m dq^{w_1\cdots w_{n-1}}(y) \frac{t^m}{m!} = \sum_{m=0}^{\infty} \sum_{k=0}^{m} S_1(m,k) \\
&\cdot \int_{\mathbb{Z}_p} \left(\left[\left(\prod_{i=1}^{n-1} w_i\right) y + \left(\prod_{i=1}^{n} w_i\right) x + w_n \sum_{i=1}^{n-1}\left(\prod_{n\geq j\geq 1, j\neq i} w_j\right) k_i\right]_q\right)^k dq^{w_1\cdots w_{n-1}}(y) \frac{t^m}{m!} \\
&= \sum_{m=0}^{\infty} \sum_{k=0}^{m} S_1(m,k) \left[\prod_{i=1}^{n-1} w_i\right]_q^k \sum_{j=1}^{k} S_2(k,j) \times \int_{\mathbb{Z}_p} \left(\left[y + w_n x + \sum_{i=1}^{n-1} \frac{w_n}{w_i}\right]_{q^{w_1\cdots w_{n-1}}}\right)_k dq^{w_1\cdots w_{n-1}}(y) \frac{t^m}{m!} = \sum_{m=0}^{\infty} \sum_{k=0}^{m} S_1(m,k) \\
&\cdot \left[\prod_{i=1}^{n-1} w_i\right]_q^k \sum_{j=1}^{k} S_2(k,j)\, d_{k,q^{w_1\cdots w_{n-1}}}\left(w_n x + \sum_{i=1}^{n-1} \frac{w_n}{w_i}\right).
\end{aligned} \tag{20}$$

Therefore, by Theorem 2 and (20), we obtain the following theorem.

Theorem 3. *For $n \geq 1, w_1, \cdots w_n \in \mathbb{N}$, the expressions*

$$\begin{aligned}
&\sum_{k_1=0}^{w_{\sigma(1)-1}} \sum_{k_2=0}^{w_{\sigma(2)}-1} \cdots \sum_{k_{n-1}=0}^{w_{\sigma(n-1)}-1} q^{\sum_{i=1}^{n-1}(\prod_{n\geq j\geq 1, j\neq i} w_{\sigma(j)})k_i} \\
&\times \sum_{m=0}^{\infty} \sum_{k=0}^{m} S_1(m,k) \left[\prod_{i=1}^{n-1} w_{\sigma(i)}\right]_q^k \\
&\cdot \sum_{j=1}^{k} S_2(k,j)\, d_{k,q^{w_{\sigma(1)}\cdots w_{\sigma(n-1)}}}\left(w_{\sigma(n)} x + \sum_{i=1}^{n-1} \frac{w_{\sigma(n)}}{w_{\sigma(i)}}\right)
\end{aligned} \tag{21}$$

are the same for any $\sigma \in S_n$.

We observe that

$$\begin{aligned}
&\left[y + w_n x + \sum_{i=1}^{n-1} \frac{w_n}{w_i} k_i\right]_q \\
&= \frac{1 - q^{w_1\cdots w_{n-1} y + w_1\cdots w_n x + w_n \sum_{i=1}^{n-1}(\prod_{n\geq j\geq 1, j\neq i} w_j)k_i}}{1 - q^{w_1\cdots w_{n-1}}} \\
&= \frac{[w_n]_q}{[w_1 \cdots w_{n-1}]_q} \left[\sum_{i=1}^{n-1}\left(\prod_{n\geq j\geq 1, j\neq i} w_j\right) k_i\right]_{q^{w_n}} \\
&\times q^{\sum_{i=1}^{n-1}(\prod_{n\geq j\geq 1, j\neq i} w_j)k_i} [y + w_n x]_{q^{w_1\cdots w_{n-1}}}.
\end{aligned} \tag{22}$$

From (22), we note that

$$\begin{aligned}
&\int_{\mathbb{Z}_p}\left[y+w_nx+\sum_{i=1}^{n-1}\left(\prod_{n\geq j\geq 1,j\neq i}w_j\right)k_i\right]_{q^{w_1\cdots w_{n-1}}}^{n} dq^{w_1\cdots w_{n-1}} \\
&\cdot (y)=\sum_{m=0}^{n}\binom{n}{m}\left(\frac{[w_n]_q}{[w_1\cdots w_{n-1}]_q}\right)^{n-m} \\
&\cdot\left[\sum_{i=1}^{n-1}\left(\prod_{n\geq j\geq 1,j\neq i}w_j\right)k_i\right]_{q^{w_n}}^{n-m} \\
&\times q^{m\sum_{i=1}^{n-1}(\prod_{n\geq j\geq 1,j\neq i}w_j)k_i}\int_{\mathbb{Z}_p}[y \\
&+w_nx]_{q^{w_1\cdots w_{n-1}}}^{m} d\mu_{q^{w_1\cdots w_{n-1}}}(y) \\
&=\sum_{m=0}^{n}\binom{n}{m}\left(\frac{[w_n]_q}{[w_1\cdots w_{n-1}]_q}\right)^{n-m} \\
&\cdot\left[\sum_{i=1}^{n-1}\left(\prod_{n\geq j\geq 1,j\neq i}w_j\right)k_i\right]_{q^{w_n}}^{n-m} \\
&\times q^{m\sum_{i=1}^{n-1}(\prod_{n\geq j\geq 1,j\neq i}w_j)k_i}\sum_{l=0}^{m}S_2(m,l)\int_{\mathbb{Z}_p}([y \\
&+w_nx]_{q^{w_1\cdots w_{n-1}}})_l\, d\mu_{q^{w_1\cdots w_{n-1}}}(y) \\
&=\sum_{m=0}^{n}\binom{n}{m}\left(\frac{[w_n]_q}{[w_1\cdots w_{n-1}]_q}\right)^{n-m} \\
&\cdot\left[\sum_{i=1}^{n-1}\left(\prod_{n\geq j\geq 1,j\neq i}w_j\right)k_i\right]_{q^{w_n}}^{n-m} \\
&\times q^{m\sum_{i=1}^{n-1}(\prod_{n\geq j\geq 1,j\neq i}w_j)k_i}\sum_{l=0}^{m}S_2(m,l)\,d_{l,q^{w_1\cdots w_{n-1}}}(w_nx).
\end{aligned} \tag{23}$$

By (24), we get

$$\begin{aligned}
&[w_1\cdots w_{n-1}]_q^{n-1}\sum_{k_1=0}^{w_1-1}\sum_{k_2=0}^{w_2-1}\cdots\sum_{k_{n-1}=0}^{w_{n-1}-1}q^{\sum_{i=1}^{n-1}(\prod_{n\geq j\geq 1,j\neq i}w_j)k_i} \\
&\quad\times\int_{\mathbb{Z}_p}\left[y+w_nx+\sum_{i=1}^{n-1}\left(\prod_{n\geq j\geq 1,j\neq i}w_j\right)\right. \\
&\quad\left.\cdot k_i\right]_{q^{w_1\cdots w_{n-1}}}^{n} dq^{w_1\cdots w_{n-1}}(y) \\
&=\sum_{m=0}^{n}\binom{n}{m}[w_1\cdots w_{n-1}]_q^{n-1}[w_n]^{n-m}\sum_{l=0}^{m}S_2(m, \\
&\quad l)\,d_{l,q^{w_1\cdots w_{n-1}}}(w_nx)\,T_{n,q^{w_n}}(w_1\cdots w_{n-1}\mid m),
\end{aligned} \tag{24}$$

where

$$\begin{aligned}
&T_{n,q^{w_n}}(w_1\cdots w_{n-1}\mid m) \\
&\quad\cdot\sum_{k_1=0}^{w_1-1}\sum_{k_2=0}^{w_2-1}\cdots\sum_{k_{n-1}=0}^{w_{n-1}-1}q^{(m+1)\sum_{i=1}^{n-1}(\prod_{n\geq j\geq 1,j\neq i}w_j)k_i} \\
&\quad\cdot\left[\sum_{i=1}^{n-1}\left(\prod_{n\geq j\geq 1,j\neq i}w_j\right)k_i\right]_{q^{w_n}}^{m-1}.
\end{aligned} \tag{25}$$

As this expression is an invariant under any permutation $\sigma\in S_n$, we have the following theorem.

Theorem 4. *For $n\geq 1, w_1,\cdots w_n\in\mathbb{N}$, the expressions*

$$\begin{aligned}
&\sum_{m=0}^{n}\binom{n}{m}[w_{\sigma(1)}\cdots w_{\sigma(n-1)}]_q^{n-1}[w_{\sigma(n)}]^{n-m} \\
&\quad\times\sum_{l=0}^{m}S_2(m,l)\,d_{l,q^{w_{\sigma(1)}\cdots w_{\sigma(n-1)}}}(w_nx)\,T_{n,q^{w_{\sigma(n)}}}(w_{\sigma(1)}\cdots w_{\sigma(n-1)}\mid m)
\end{aligned} \tag{26}$$

are the same for any $\sigma\in S_n$.

Data Availability

The numerical simulation data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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