Research Article

Pricing Decisions on Reward-Based Crowdfunding with Bayesian Review System Facing Strategic Consumers

Yuting Chen, Rong Zhang, and Bin Liu

School of Economics and Management, Shanghai Maritime University, Shanghai 201306, China
Research Center of Logistics, Shanghai Maritime University, Shanghai 201306, China

Correspondence should be addressed to Bin Liu; liubin@shmtu.edu.cn

Received 31 October 2018; Revised 14 December 2018; Accepted 19 December 2018; Published 20 January 2019

Academic Editor: Lu Zhen

Copyright © 2019 Yuting Chen et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Crowdfunding marks a popular and sustainable means by which small and microentrepreneurs obtain financial resources for their innovative project. Consumers increasingly rely on online reviews to make purchase decisions. However, the crowdfunding nowadays lacks a form of review system. This paper is designed to extend research on the optimal pricing decision with review system for the reward-based crowdfunding. Firstly, a Bayesian analysis is established to construct consumers’ belief update process in presence of review system. Secondly, we take the strategies without the review system as a benchmark to explore the impacts of review system under preannounced pricing and responsive pricing. Finally, through the equilibrium analysis, we find that the review system has a positive impact on the creator under responsive pricing policy. The fraction of favorable review has a large effect on the profit of preannounced pricing. When the fraction is about 80%, the profit is the maximum. Generally speaking, the review system will make more profit for the creator.

1. Introduction

In recent years, crowdfunding has emerged as a new and sustainable channel for organizations or enterprises to receive money from a pool of individuals for different types of projects, especially for developing innovative products. Rewards-based crowdfunding constructs a funding context that enables project creators to obtain financial resources from the general public and sell the crowdfunding product at the same time [1]. According to the latest data, global crowdfunding experienced an accelerated growth in 2016 to reach $19.96 billion (from $6.1 billion in 2013) raised by some active platforms across the world. One of the leading rewards-based crowdfunding platforms is Kickstarter. In 2015 alone, 77000 projects were successfully funded there, raising around $700 million from the general public from nearly every country on the planet. Reward-crowdfunding is regarded as a marketing and financing tool. The backers can participate in the project (hereafter called period 1) at a lower price and obtain the crowdfunding product after the project is successful (period 2). Besides, the consumers can directly purchase the product on the market in period 2 when the crowdfunding project is successful. Our work develops a two-period model to discuss the optimal dynamic pricing decision for the creators and explore the effective way to reduce consumers uncertainty.

Launching a new crowdfunding product involves uncertainty; the chief uncertainty is the quality of the product. Some consumers may postpone or abandon their pledging and purchase decisions because they do not know the true quality of the crowdfunding product. Online product reviews have become an important information source for consumers to mitigate the uncertainty about the quality of a product. With the ubiquity of online shopping, the importance of social influence has never been greater; an increasing number of consumers make purchase decisions refer to online product reviews. According to a previous survey conducted by Deloitte and Touche, about 43% of consumers were reinforced of their original purchase decision by reviews, the same survey also found that 43% of consumers changed their intentions about which product to buy, and 9% of consumers even abandoned their purchase decision because of the negative reviews for the products. More recently, the advance of the Internet and online communities has
dramatically magnified the influence of reviews, as a product review online become an important information source for the consumers. While consumers initially are confuse about the true quality of the crowdfunding product online, they learn about it through some form of online reviews and adjust their estimates of its quality along the way and making their purchase decisions accordingly. Recently, Kickstarter encloses the Facebook Like button to describe crowdfunding projects and shows a counter reflecting the number of Facebook users who have previously “liked” this project. For subsequent visitors to the webpage, the button thus becomes a quality signal. It is also a form of informal online review. But there are few crowdfunding platforms containing online review systems. In this paper, we investigate how online reviews provided by previous consumers drive the dynamic pricing for crowdfunding projects.

We characterize the consumers’ behaviors as follows. Consumers arrive at the market according to a Poisson process and face the decision of either participating in the crowdfunding (period 1) with unknown quality or waiting until the crowdfunding product is successful (period 2). The consumers differ in their base valuations for the observable attributes of the crowdfunding product, which determines their willingness to participate in the crowdfunding. These base valuation parameters are assumed to be independently and identically drawn from a known distribution, which is described in Section 3. Consumers who participate in the crowdfunding product in period 1 experience its true quality. In presence of online review system, those purchasers report whether they “liked” or “disliked” the product, i.e., if their ex-post utility was positive or negative, respectively. An arriving consumer in period 2 observes the history of purchasing decisions and reviews made by former buyers, combines this information with his prior estimate, infers the associated product quality, and makes his own purchasing decision. The sequence of reviews in period 2 affects the evolution of the observable information so that the dynamic of the market responds over time.

It is typical to assume that fully rational consumers update their estimate for the unknown quality of the product through a Bayesian analysis. Two alternative classes of dynamic pricing policies employed in our study come from the spirit of the seminal paper by Besanko and Winston [2]. One is preannounced pricing, which required the project creators announce the full price path from the beginning of period 1. The other is responsive pricing which means the creators announce only the first period price (crowdfunding pricing) and delay the second period price (market pricing) announcement until the beginning of the second period.

This paper illustrates how the review system affects the profits of crowdfunding creators and proposes the optimal dynamic pricing policies when considering strategy consumers’ behaviors and the heterogeneity in consumers’ valuations. The rest of this paper proceeds as follows. Section 2 presents related literature review. The basic Bayesian analysis model of the online review system is presented in Section 3. In Sections 4 and 5, the preannounced pricing policy and responsive pricing policy are analyzed respectively, and then we study the effect of an online system using the scenario without review system as the benchmark. Section 6 concludes.

2. Literature Review

Some existing empirical and analytical works have studied crowdfunding pricing decision, including pricing and product design decisions in the context of crowdfunding [3]. Several papers have empirically investigated the consumers’ behavior caused by asymmetric quality information and observational learning from early contributions (see, e.g., [1, 4]). Chakraborty and Swinney [5] discuss how the creators may signal the quality of their projects through funding targets and how the creators’ behavior can be different under the objective of profit-maximization versus success-maximization. Buyse et al. [6] studies that crowdfunding is a group behavior in which many individuals contribute their resources to help others or organize an activity. Du et al. [7] focus on the contingent policies that creators can apply to the dynamic pledging progress after the project design has been determined. The authors demonstrate the importance of contingent policies and analyze three implementable policies. Chen et al. [8] propose an optimal pricing strategy for green crowdfunding products facing the mixed market of strategic and myopic consumers when substitutes exist. Araman and Caldentey [9] deal with the detailed modeling of the firm's optimal timing to stop the voting and start or abandon launching the product. In a different set of new product launching, Marinesi and Girotra [10] focus on forward-looking customer behavior and the intended use of the acquired information. Wessel et al. [11] assess the effects of nongenuine social information on consumers’ decision-making in the context of reward-based crowdfunding. Xu et al. [12] investigate a firm's optimal funding choice when launching an innovative product to the market with both market uncertainty and word-of-mouth (WoM) communication. Our paper yields new insights into how crowdfunding review system improves consumers’ beliefs about the quality of crowdfunding product and presents the optimal dynamic pricing policy for creators.

The review systems have been studied both empirically and analytically in the literature. Many of these models focus on how the nature of a product in their market affects review outcome (e.g., positive or negative), and in turn how the reviews affect consumers’ willingness-to-pay and therefore the product demand. Banerjee [13] and Bikhchandani et al. [14] study the update process as the scenario where a consumer observes a signal and the decisions of the consumers who made a decision before him. Here, consumers are rational and update their beliefs in a Bayesian way. Ellison and Fudenberg [15, 16] consider that consumers exchange information about their experienced utility and use simple decision rules to choose between actions. Kwartk et al. [17] view the reviews as information mitigating the uncertainty in a product quality to consumers’ needs and investigate how this additional information affects upstream product competition. Hu et al. [3] consider the opposite type of social behavior, where consumers value a product more if it is more popular in the population. Roma et al. [18] consider
an entrepreneur who designs a reward-based crowdfunding campaign when the campaign provides a signal about the future demand for the product and subsequent venture capital is needed. Crapis et al. [19] present a review learning trajectory based on mean-field approximations and studied the price optimization questions in the presence of online review. Jing [20] analyze the impacts of social learning on the dynamic pricing and consumer adoption of durable goods in a two-period monopoly. Jiang and Guo [21] analyze firms' review system design and product pricing strategies. The authors formally model two review system design decisions—what rating scale cardinality to use and whether to offer granular review reports. Another stream of research has modeled product reviews as free advertising and analyzed how sellers should adjust their own marketing mix strategies in the presence of the reviews [22–24]. Fasheng Xu et al. [12] explore the interaction between social learning and network externalities. We focus on the dynamic pricing decision on reward-based crowdfunding in presence of review system and take the crowdfunding time into consider. Existing learning models assume that consumers update information signals from review system in a Bayesian framework. Ifrach et al. [25] analyze a Bayesian model where both the quality of the product and the reviews can assume only two possible values, and they provided conditions for learning. Papanastasiou et al. [26] study the social learning by a Bayesian model and investigates how social learning affects the interaction between a dynamic pricing monopolist and a forward-looking consumer population, within a simple two-period model. Based on the above researchers, we extend the model to study the updating process in presence of review system and how online reviews improve the optimal profit of crowdfunding creators. This paper is a first attempt at understanding the online crowdfunding review system when considering consumer arrival to market according to the Poisson process and taking into account the consumers' heterogeneous valuations that declined over time.

Previous economics and marketing literature has studied humans' behaviors and two classes of pricing policies. Heterogeneous consumers are present throughout the entire season and optimally select the timing of their purchases to maximize individual surplus. Stokey [27] and Lansberger and Melljöson [28] assume that a monopolist commits to a price strategy for the entire selling horizon in order to maximize expected revenue. Besanko and Winston [2] propose a contingent subgame perfect pricing strategy for the monopolist. They consider a seller facing a finite pool of rational consumers that aim to maximize their individual utilities by optimally timing their purchases. Aviv and Pazgal [29] take into account fashion-like seasonal goods in the presence of forward-looking customers. The literatures that investigate dynamic pricing typically employ either preannounced pricing (e.g., [28]) or responsive pricing [2]. Responsive price plans generate value because they allow the firm to react optimally to updated information (e.g., [30]). The general agreement in the literature is that a company will prefer a preannounced pricing when facing forward-looking consumers (e.g., [31]). Yin et al. [32] and Whang [33] deal with demand learning by preannounced pricing. Cachon and Swinney [31] examine the firm’s quantity and salvage-pricing decisions under responsive pricing. This paper is also related to the pricing of experience goods, whose quality can be determined only upon consumption (e.g., [34, 35]). Our work extends the above studies, especially Papanastasiou and Savva [36]’s model of social learning and Aviv and Pazgal [29] contingent Pricing Model, to deal with the optimal pricing of crowdfunding experience goods with online review system. We assume that consumers in our model face uncertainty regarding the intrinsic quality of a new and innovative crowdfunding product.

3. The Benchmark Model

We consider a creator posting a crowdfunding product on a platform and discuss the creator's review system choices and pricing strategies over two periods. The first period is known as the crowdfunding period, in which consumers raise money for the project. In the second period, the crowdfunding product sells on the market. The people purchased in the first period also gain product in the second period. At the beginning of the first period, the creator makes its pricing decision. Because the creator will charge different prices in the different period, first period consumers may choose to purchase in the first period or wait to purchase in the second period. If the creator chooses to host a review system, consumers who purchase in the first period may post their product reviews, and consumers in the second period can learn from these reviews before making their purchasing decision. The total number of consumers in both periods is normalized to one. Each consumer only demands one unit of the crowdfunding product during the selling cycle, and there is no activity after the second period.

3.1. Model Assumption. The sales season is $[0, H]$, which is divided into two periods, $[0, T]$ and $[T, H]$. The crowdfunding product is launched at time $t=0$. Consumers arrive thereafter according to a Passion process with rate $\lambda$, which is independent of the product's quality and consumers' preference parameters. Consumer $k$'s valuation of the crowdfunding product comprises two components: observable attributes (like color or style) and unobservable attributes (like quality or fit). The consumer $k$ has a base valuation $V_k$ for the observable attributes of crowdfunding product and is heterogeneous in terms of their preferences for different product attributes. Without loss of generality, we assume that base valuation $\{V_k\}$ is independent and identically distributed (i.i.d.) random variables drawn from a known distribution function $F$. We denote the corresponding density function by $f$. The valuation $V_k$ is discounted by a known exponential function $e^{-\alpha x}$, where $\alpha \geq 0$ represents the sensitive degree of waiting and $x$ is denoted as the waiting time. To reflect these, we use a valuation function developed by Aviv and Pazgal [29]:

$$V_k(t) = V_k e^{-\alpha x}. \quad (1)$$

Crowdfunding platforms like Kickstarter, Jingdong, and so on always require creators set a target for their crowdfunding projects. The project is deemed to be successful only if the
total funding exceeds the target. Consumers will participate in the crowdfunding in the first period by a lower price \( p_1 \) and gain products in second period if the crowdfunding is successful. On the contrary, the platform will return money to consumers. Thus, consumers face a risk (\( \mu \)) of whether the project will be successful and whether they can receive the product when taking part in the crowdfunding project in first period. Because of the risk of the crowdfunding, consumers’ valuation of participating in the crowdfunding in the first period is lower than the valuation after the project is proved to be successful. Consumers’ valuation for participating in the first period can be expressed by \( \mu V_i(t) \) (0 < \( \mu \) < 1). In the second period, the market price \( p_2 \) is higher than first period.

Each consumer has his private information about his arrival time and his own valuation for the product. The creator knows the product’s real attributes and consumers’ valuation distribution \( F \), but he does not know individual consumers’ valuation. Other parameters \( \lambda, \alpha, T, H, \mu \) are known to the creator and all consumers. We assume that all of consumers are strategic, and they will make their decisions by comparing the expected surplus in both periods.

3.2. Consumers’ Belief Updating Process. When the creator hosts an online review system, second period consumers can discover more about the product from review and update their beliefs on product valuation and different dimensions of quality, such as comfort or durability. We use \( s \) ∈ \( Z \) to denote the rate scales. For example, \( s=2 \) represents the case of two rating levels such as “like” and “dislike”. After consumption, consumers know the true quality of the product and how well the product fits their tastes. Park and Nicolau [37] showed that people perceive extreme ratings (positive or negative) as more useful and enjoyable than moderate ratings. We consider that a consumer who purchased the product reports that “like” if his utility is nonnegative and the true quality and their preference; he will report “dislike” if his utility is nonnegative and the true quality and their preference; he will report “dislike” if his utility is nonnegative and the true quality and their preference; he will report “dislike” if his utility is negative.

We assume that the review rate level follows a symmetric binomial distribution \( B(\theta, n) \), where \( n \) represents the review number. Samples \( X_i \) are independent and identically distributed (i.i.d.). Through the advertisement or introduction on the crowdfunding platform, consumers have prior valuations about the crowdfunding product during the first period. To simplify the analysis, we assume that consumers’ prior belief about the product follows a uniform distribution \( U[0, 1] \).

\[
q(\theta) = \begin{cases} 
1 & 0 < \theta < 1, \\
0 & \text{other}.
\end{cases}
\]  

(2)

Thanks to the review system, first period consumers rate the product based on their consumption. By observing the previous reviews, second period consumers will update their beliefs on product valuation and product fit. We use a Bayesian model to analyze how the review system interacts with strategic consumers. \( q(.) \) is the consumers’ prior belief about the product and \( q(x | .) \) is the posterior distribution of consumers’ belief.

According to the analysis above, the prior belief distribution is

\[
P(x | \theta) = C_n^x \theta^x (1 - \theta)^{n-x}.
\]  

(3)

So \( \theta \) and samples \( X_i \)’s joint probability is

\[
h(x, \theta) = q(\theta) P(x | \theta) = C_n^x \theta^x (1 - \theta)^{n-x},
\]  

(4)

where \( x=0,1,2,...,n, 0 < \theta < 1 \). The marginal density of \( X_i \) is

\[
m(x) = \int_0^1 f(x | \theta) dF^x(\theta) = \int_0^1 P(x | \theta) q(\theta) d\theta
\]

\[= \int_0^1 C_n^x \theta^x (1 - \theta)^{n-x} d\theta
\]

\[= C_n^x \frac{\Gamma(x+1) \Gamma(n-x+1)}{\Gamma(n-2)}.
\]  

(5)

Thus, according to Bayes’ rule, the posterior distribution of consumers’ belief can be expressed as follows:

\[
q(x | \theta) = \frac{h(x, \theta)}{m(x)}
\]

\[= \frac{\Gamma(x+1) \Gamma(n-x+1) \theta^x (1 - \theta)^{n-x}}{\Gamma(n-2)},
\]  

(6)

where \( \Gamma(s) = \int_0^\infty x^{s-1} e^{-x} dx \) (\( s > 0 \)) is called Gamma function and \( \Gamma(n+1) = n! \). We denote \( \gamma \) as the fraction of favorable review. Then \( x = \gamma n \). The posterior distribution is the Beta distribution, \( \beta(\gamma n+1, n-\gamma n+1) \) according to the expression above.

4. Preannounced Pricing

The creators must decide the pricing policy at the beginning of the sale season. When consumers are strategic, they may postpone the purchase in hope of more information. With such uncertainty, many consumers may end up not buying and lead to crowdfunding failure, reducing the creators’ profitability. Recent studies show that a way to mitigate this negative effect caused by strategic consumers is to use a posted or preannounced pricing policy (Correa et al. [38]). In this section, we investigate the optimal pricing decision in preannounced pricing policy. We denote \( p_1 \) as the product price in period \( i \), where \( i=1,2 \). In this policy, the creators announce \( p_1 \) and \( p_2 \) in the first period and consumers make their first period purchase decision by comparing expected surplus. In presence of review system, consumers remaining in the market update their valuation by the previous online reviews and make their second period purchase decision. Our focuses are on analyzing how the review system impact on the creator’s profit and discovering the optimal design of pricing mechanism with preannounced prices.

4.1. Benchmark: Preannounced Pricing in Absence of Review System. We discuss the preannounced pricing without review system as a benchmark. In this section, we consider a
two-period pricing problem in which the creator announces the fixed price path \(\{p_1, p_2\}\) at the beginning of the first period. The strategic consumers compare their surpluses in both periods by considering waiting cost and crowdfunding risk. As shown in Figure 1, a consumer arrives at the crowdfunding platform and notices the project at time \(t (0 < t < T)\); his valuation of attending this crowdfunding project is \(V_k e^{-\alpha(T-t)}\). Moreover, the valuation of this product will be \(V_k e^{-\alpha(T-t)}\) if the consumer waits until the second period and purchase the product at time \(T\). If a consumer notices the product at time \(t (T < t < H)\), the valuation is \(V_k\) because of no waiting. In order to describe the consumers’ purchasing decision specifically, a threshold function is provided in Theorem 1.

**Theorem 1.** The threshold function \(\chi(t)\) for preannounced pricing without review system can be expressed as follows:

\[
\chi(t) = \begin{cases} 
  \frac{p_1 - p_2}{\mu - 1} e^{\alpha(T-t)} & 0 < t < T, \\
  p_2 & T \leq t \leq H.
\end{cases}
\] (7)

**Proof.** See the appendix.

The consumer \(k\) will take part in the crowdfunding or purchase the product if \(V_k \geq \chi(t)\) in both periods. In addition, if the consumer visits the platform before time \(T\) and \(V_k < \chi(t)\), he will revisit the platform at time \(T\) and purchases the product if the valuation is larger than the threshold at this time. However, in the preannounced pricing policy, the strategic consumers arriving at \([0, T]\) will weigh their valuation and threshold according to the fixed price path \(\{p_1, p_2\}\). Thus, they can decide to purchase or postpone or even abandon the product immediately. According to Theorem 1, there are four types of decisions about the product. The first one is that the consumers arriving in the first period and \(V_k \geq \chi(t)\); this type of consumers will buy it in the first period. The second type is that the consumers arrive in the first period, but their valuation is lower than the threshold \(V_k < \chi(t)\); this type of consumers will postpone their purchasing decision until the second period. And the third type is that the consumers notice the product in the second period; if \(V_k \geq \chi(t)\), they will buy one immediately. The rest of the consumers will not buy anyone during the selling season. Then we discuss the expected number of each type as follows:

As the assumption above, consumers arrive at the platform and notice the project according to a Poisson process with a rate of \(\lambda\). This process is independent of the valuation or the pricing policy. And consumers are heterogeneous in their common valuation as well as in their arrival time. The common valuation is drawn from a distribution \(F\).

The condition that consumers take part in the crowdfunding in the first period should be satisfied:

\[
0 < t < T, \\
V_k \geq \chi(t).
\] (8)

![Figure 1: The consumer’s valuation without review system.](image)

Thus, the expected population of the first type without review system can be expected as

\[
n_1^{pb} = \lambda \int_0^T F\left(p_1 - p_2 e^{\alpha(T-t)}\right) dt.
\] (9)

The case that consumers arrive in the first period and purchase the product in the second period is satisfied:

\[
0 < t < T, \\
V_k < \chi(t),
\] (10)

The expected population of the second type is

\[
n_2^{pb} = \lambda \int_0^T \left[F\left(p_2 e^{\alpha(T-t)}\right) - F\left(p_1 - p_2 e^{\alpha(T-t)}\right)\right] dt.
\] (11)

The third kind of consumers arrives in the second period and purchases the product as long as their valuation is larger than the price.

\[
T < t < H, \\
V_k \geq p_2.
\] (12)

And the expected number of the third type of consumers is

\[
n_3^{pb} = \lambda \int_T^H F(p_2) dt.
\] (13)

According to the analysis above, the total profit of creator in preannounced pricing policy without review system is

\[
\max_{\{p_1, p_2\}} n^{pb} = (p_1 - c) n_1^{pb} + (p_2 - c) (n_2^{pb} + n_3^{pb})
\] (14)

s.t. \(p_1 n_1^{pb} \geq K\).

Here, \(p_1, p_2\) are decision variables and \(c\) is a constant. \(n_1^{pb}, n_2^{pb}, n_3^{pb}\) are the expected number of each kind of consumer, which is a function of \(p_1, p_2\) and impacted by \(\alpha, \mu, F\), and so on. \(K\) is a fixed cost and \(K\) is the target of the crowdfunding.
When the funds reach the target value $K$, the crowdfunding is successful. Otherwise, the platform will return all funds to the funders.

The creator’s optimal pricing strategy without review system is as follows.

**Proposition 2.** Suppose that $F$ follows a uniform distribution; the optimal pricing strategy of preannounced pricing in absence of online review system is

$$
p_1^* = c,
$$

$$
p_2^* = c\left(\frac{H\alpha + T\mu - T\mu e^{\alpha T} + T\alpha - T\mu}{2(H\alpha + T\mu - T\mu e^{\alpha T} + T\alpha - T\mu)}\right).
$$

**Proof.** See the appendix.

It is obvious that the first period price should be low enough to ensure successful crowdfunding. And when the risk of crowdfunding is small $\mu \rightarrow 1$, the prices of first period and second period tend to be the same under preannounced pricing without review system. In absence of review system, with the risk of crowdfunding, the valuations of product in both periods tend to be consistent. The consumers waiting until the second period also are confused about the unobservable attribute of the product.

### 4.2. Preannounced Pricing in Presence of Review System

The consumers tend to make purchase decision refer to online product reviews. Smith [39] states that 60% of consumers consider ratings and reviews important when researching products. The consumers update their valuations of the crowdfunding product through the online reviews from previous consumers. Through the review system, the consumers will reduce uncertainty to the crowdfunding product.

In presence of review system, the consumers purchased in first period post their reviews on the platform after experiencing the product. If the consumer’s utility is larger than the price he paid, he will announce his review as “Like”, otherwise, he will announce “Dislike”. As discussed in the previous section, the reviews from consumers follow a binomial distribution. To simplify to the model, we assume that the consumers experience the product and post the review as soon as they receive the product. The rest of the consumers will update their valuations through those reviews. Similarly, as shown in Figure 2, the consumer $k$’s valuation of participating in crowdfunding in the first period is $\mu V_k e^{-\alpha(T-t)}$ (where $\mu$ is the risk of crowdfunding). However, if the consumer postpones his purchasing decision until the second period, the valuation of the product will be $V_k e^{-\alpha(T-t)} + \theta$ where $\theta$ means the belief updating from the review system. $\pi(x | \theta)$ is deduced from the Bayesian model in the previous section. If a consumer arrives at the platform in the second period, his valuation of the product will be $V_k + \theta$.

Thanks to the review system, the consumers remaining in the market gain more information about the product. However, more consumers will tend to postpone their purchase to the second period, which leads to gathering less information and failing in crowdfunding. Theorem 3 expresses the equilibrium of consumers’ decision and the creators’ pricing strategy.

**Theorem 3.** The threshold function $\phi(t)$ for preannounced pricing with review system is

$$
\phi(t) = \begin{cases} 
\phi(t) & 0 < t < T, \\
p_2 & T \leq t \leq H.
\end{cases}
$$

where $\phi(t)$ is the unique solution to the implicit equation:

$$
\mu \phi e^{-\alpha(T-t)} - p_1 = E\left(\max\{\phi e^{-\alpha(T-t)} + \theta - p_2, 0\}\right)
$$

$$
= \int_{p_2 - \phi e^{-\alpha(T-t)}}^{1} (\phi e^{-\alpha(T-t)} + \theta - p_2) q(x | \theta) \, d\theta.
$$

**Proof.** See the appendix.

For any given pricing path $[p_1, p_2]$, it is optimal for consumers to make their purchasing decision according to a threshold function $\phi(t)$. If consumers’ valuation of time $t$ $V(t) \geq \phi(t)$, they will purchase the product (or attend in the crowdfunding in the first period). If a consumer arrives in the first period and the valuation $V_k < \phi(t)$, but $V_k e^{-\alpha(T-t)} \geq p_2$, he will postpone the purchase until the second period. The left-hand side of the equation represents the consumer’s surplus of participating in the crowdfunding in the first period. The right-hand side of the equation represents the expected surplus of purchasing a product in the second period. $\pi(x | \theta)$ is deduced from the Bayesian analysis in the previous section. According to the Bayesian analysis of review system, the posterior distribution of consumers’ belief is a Beta distribution, $\beta (\gamma n + 1, n - \gamma n + 1)$, and $q(x | \theta) = (\Gamma(x + 1)\Gamma(n - x + 1)/\Gamma(n - 2))\theta^x(1-\theta)^{n-x}$.

Based on the analysis above, the consumers can divide into four kinds. If the consumer would like to participate in the crowdfunding in the first period and obtain the product...
in the second period, his valuation and arriving time should be satisfied:
\[0 < t < T,
V_k \geq \varphi(t) .\]  \hfill (18)

The expected number of consumers joining in the crowdfunding in the first period is
\[n_1^{ps} = \lambda \int_{0}^{T} F(\varphi(t)) \, dt.\]  \hfill (19)

The condition that the consumer will postpone his purchase until the second period should be satisfied:
\[0 < t < T,
V_k < \varphi(t),\]  \hfill (20)
\[V_k e^{-\alpha(T-t)} \geq p_2.\]

The expected population of consumers postponing their purchase until the second period is
\[n_2^{ps} = \lambda \int_{0}^{T} \left[F\left(p_k e^{\alpha(T-t)}\right) - F(\varphi(t))\right] \, dt.\]  \hfill (21)

The expected population of consumers arriving in the second period and buying the product immediately is
\[n_3^{ps} = \lambda \int_{T}^{H} F(p_2) \, dt.\]  \hfill (22)

Thus, the creator's total profit in both periods with the review system can be expressed in the following maximization problem:
\[
\max_{(p_1, p_2)} \pi^{ps} = (p_1 - c) n_1^{ps} + (p_2 - c)(n_2^{ps} + n_3^{ps})
\]  \hfill (23)
\[\text{s.t. } p_1 n_1^{ps} \geq K,
\]
where \(K\) is a fixed cost of this production process and also the funding target of the crowdfunding. \(n_1^{ps}\) is the demand in the first period, and \(n_2^{ps} + n_3^{ps}\) is the demand in the second period.

### 5. Responsive Pricing

Under the responsive pricing, a creator announces first period price \(p_1\) according to the actual product attribute and estimated demand. The second period \(p_2\) pricing decision is constrained by the realized demand of first period and impacted by the review rate level generated by the first period in present of the review system. In short, \(p_2\) here is endogenous and posts at the beginning of second period according to sales feedback in first period. With responsive pricing, a creator can adjust their pricing strategy in response to market in time. We consider the subgame that begins after the first period price is fixed. Similarly, our study begins with the optimal responsive pricing strategy without a review system.

#### 5.1. Benchmark: Responsive Pricing in Absence of Review System

Under the responsive policy, the creator announces the first period price at the beginning of the first period. The consumers participating in the crowdfunding would get the product until time \(T\). The second period price \(p_2\) is announced at time \(T\) according to the demand in the first period. Actually, there are some intervals between the end of the first period and the beginning of the second period. To simplify the model, we ignore the intervals. The second period price \(p_2\) and the purchase decision by consumers form a Nash equilibrium in the subgame. In this section, we take the pricing without review system as the benchmark as well.

First, we consider the creator's profit of second period. Similar to the preannounced pricing, the profit in the second period is
\[\pi_2^{rb} = (p_2 - c)\left(n_2^{rb} + n_3^{rb}\right).\]  \hfill (24)

where \(n_2^{rb}\) and \(n_3^{rb}\) are the expected number of second and third type consumers, respectively. To analyze the demand for the second period, the purchase strategies of consumers should be considered first. The consumers’ purchase strategies are described by the threshold function as well, as shown in Theorem 4.

**Theorem 4.** The threshold function \(\psi(t)\) for responsive pricing without review system is
\[
\psi(t) = \begin{cases} 
\nu_1(t) & 0 < t < T, \\
\nu_2(t) & T \leq t \leq H,
\end{cases}
\]  \hfill (25)

where \(\nu_1(t)\) and \(\nu_2(t)\) are unique solutions to the implicit equations
\[\mu \nu_1 e^{-\alpha(T-t)} - p_1 = E\left(\max\left\{\nu_1 e^{-\alpha(T-t)} - p_2^*, 0\right\}\right),\]  \hfill (26)
\[\nu_2 - p_2^* = 0.\]

**Proof.** See the appendix. \(\square\)

Under the responsive pricing policy without review system, \(\psi(t)\) is denoted as the threshold function. A consumer \(k\) arrives at the platform in the first period; he will take part in the crowdfunding if \(V_k \geq \nu_1(t)\); he will not purchase a product until the second period if \(V_k < \nu_1(t)\) and \(V(t) \geq \nu_2(t)\). If the consumer \(k\) arrives in the second period, he will purchase the product immediately if \(V_k \geq \nu_2(t)\). In the first equation, the left-hand side is the consumer surplus of taking part in the crowdfunding in the first period; the right-hand side is the expected surplus of buying the product in the second period. In the second equation, \(\nu_2 - p_2^*\) means the surplus of the consumer arriving in the second period. The second equation is the critical condition of consumers buying a product in the second period immediately. \(p_2^*\) is deduced from \(\pi_2^{rb}\). In order to analyze \(p_2^*\), we consider the expected demands of the four types firstly.
Similar to the preannounced pricing, the condition that the first period consumer will fund the crowdfunding in the first period is

\[ 0 < t < T, \]
\[ V_k \geq v_1(t). \]

Thus, the expected demand of the first period with responsive pricing in absence of review system is

\[ n_{1b}^r = \lambda \int_0^T (F(v_1(t))) \, dt. \]  

(28)

Similarly, the condition that consumers will postpone their purchase until the second period should be satisfied:

\[ 0 < t < T, \]
\[ V_k < v_1(t), \]
\[ V_k e^{-\alpha(T-t)} \geq v_2(t). \]

(29)

And the expected demand of this type of consumers is

\[ n_{2b}^r = \lambda \int_0^T \left[ (F(v_2(t)) e^{\alpha(T-t)}) - F(v_1(t)) \right] \, dt. \]

(30)

The condition that the second period consumer will buy the product immediately should be satisfied:

\[ T < t < H, \]
\[ V_k \geq p_2. \]

(31)

The expected number of the third type is

\[ n_{3b}^r = \int_T^H F(v_2(t)) \, dt. \]

(32)

\[ n_{2b}^r + n_{3b}^r \] is the demand of the second period. The optimal second period price \( p_2^* \) is derived from

\[ \frac{\partial \pi^{rb}}{\partial p_2} = 0. \]

(33)

Here, \( p_2^* \) is a function of \( v_1 \) and \( v_2 \), we denote the optimal second period price deduced from the subgame is \( p_2^* (v_1, v_2) \). From the implicit equations above, we can obtain the threshold functions \( v_1 \) and \( v_2 \).

The optimal pricing decision \( (p_1^*, p_2^*) \) and the maximum income of the creator \( (\pi^{rb}) \) are derived from the following nonlinear programming:

\[ \max_{p_1, p_2} \pi^{rb} = (p_1 - c) n_{1b}^r + (p_2 - c) (n_{2b}^r + n_{3b}^r) \]

s.t. \( p_1 n_{1b}^r \geq K. \)

Proposition 5. Suppose that \( F \) follows a uniform distribution. In the absence of review system, the optimal pricing decision of the creator with common valuation following a uniform distribution is

\[ p_1^r = \frac{c \left( 1 + 2 T \alpha + 2 H \alpha (\mu - 1) - 2 \mu - 2 T \alpha \mu + e^{2\alpha} (2\mu - 1) \right)}{3 + 4 T \alpha + 4 H \alpha (\mu - 1) - 4 \mu - 4 T \alpha \mu + e^{2\alpha} (4\mu - 3)}, \]

\[ p_2^r = \frac{1}{6} c \left( 3 + \frac{2}{\mu} + \frac{2}{4\mu - 3} \right) - \frac{(H - T) \alpha (\mu - 1)}{\mu ((H - T) \alpha (\mu - 1) + (e^{2\alpha} - \mu)) - (4\mu - 3) (3 + 4 (H - T) \alpha (\mu - 1) - 4 \mu + e^{2\alpha} (4\mu - 3))}. \]

(35)

Proof. See the appendix.

With the review system, the optimal pricing is impacted by the crowding time, the risk sensitive degree, and the sensitive degree of waiting. The larger \( \mu \) and \( \alpha \) are, the higher price and profit are.

5.2. Responsive Pricing in Presence of Review System. In presence of review system, the first period consumers announce their review on the platform, and the rest of the consumers will update their valuation in the second period. We assume that all consumers taking part in the crowdfunding in the first period will post their review at the same time as they get the product. Suppose that, in the second period, consumers and creator are able to obtain all reviews for the consumers in the first period. Thus, the creator will respond to the review level of the previous consumers.

Similar to the strategy without review system, we consider the second period profit first. The profit is

\[ \pi_{2r}^r = (p_2 - c) (n_{2r}^r + n_{3r}^r). \]

(36)

Theorem 6. The threshold function \( \psi^r(t) \) for responsive pricing with review system is

\[ \psi^r(t) = \begin{cases} \psi_1(t) & 0 < t < T, \\ \psi_2(t) & T \leq t \leq H \end{cases} \]

(37)

where \( \psi_1(t) \) and \( \psi_2(t) \) are unique solutions to the implicit equations

\[ \mu \psi_1(t) e^{-\alpha(T-t)} - \psi_1(t) = E \left( \max \left\{ \psi_1(t) e^{-\alpha(T-t)} + \theta - p_2^*, \right\} \right) \]

\[ = \int_{p_2^*(\psi_2(t) - \psi_1(t))}^{1} \left( d \theta e^{-\alpha(T-t)} \right) f(x, \theta) \, d\theta, \]

(38)

where

\[ \psi_2(t) = \psi_2. \]

Proof. To see the appendix.

In presence of review system, the consumers purchasing strategies are based on the threshold function \( \psi^r(t) \). When \( 0 < t < T \) and \( V_k \geq \psi_1(t) \), the consumer will take part in the crowdfunding in the first period; if here \( V_k < \psi_1(t) \) and \( V_k e^{-\alpha(T-t)} < \psi_2(t) \), the consumer will not buy a
product until the second period. If \( T \leq t \leq H \) and \( V_k \geq \gamma^2(t) \), the consumer arriving in the second period would buy the product. In the first equation, the left-hand side is the consumer surplus of attending in the crowdfunding in the first period; the right-hand side is the expected surplus of postponing the consumer’s purchase. \( p_1(\gamma_1) \) and \( p_2(\gamma_2) \) are optimal second period prices derived by the second period profit. \( \theta \) is deduced by Bayesian analysis in the previous section. The expected demands of each type are

\[
\begin{align*}
n_1^{rs} &= \lambda \int_0^T \left( F\left( \gamma_1(t) \right) \right) dt. \\
n_2^{rs} &= \lambda \int_0^T \left[ \left( F\left( \gamma_2(t) e^{\alpha(T-t)} \right) \right) - F\left( \gamma_1(t) \right) \right] dt. \\
n_3^{rs} &= \lambda \int_T^H F\left( \gamma_2(t) \right) dt.
\end{align*}
\]

Substitute \( n_2^{rs} \) and \( n_3^{rs} \) into \( n_1^{rs} \). From \( \delta n_2^{rs} / \delta p_2 = 0 \), we can get \( p_2(\gamma_1, \gamma_2) \). Despite the implicit equations above, \( \gamma_1 \) and \( \gamma_2 \) are deduced and expressed by \( p_1, \mu, \alpha \), and so on. The maximum profit and optimal pricing decision can be gotten by the following nonlinear programming:

\[
\begin{align*}
\max_{p_1, p_2} & \quad \pi^{rs} = (p_1 - c) n_1^{rs} + (p_2 - c) (n_2^{rs} + n_3^{rs}) \\
\text{s.t.} & \quad p_1 n_1^{rs} \geq K.
\end{align*}
\]

6. Equilibrium Analysis

In this section, we take a closer look at the impacts of the review system and pricing policy by the equilibrium analysis. In order to explore the expected profit performance impacted by the sensitive degree of waiting and the good review level, a numerical study is conducted. Suppose that the common valuation \( \{V_k\} \) follows a uniform distribution; the Passion process rate \( \lambda = 1 \); the risk of participating in the crowdfunding \( \mu = 0.5 \). The length of the sale season \( H = 1 \). To simplify the parameter, we set \( \rho = e^\alpha \). The comparison of profits under preannounced pricing and responsive pricing and the impact of the review system is shown by Figures 3–7.

1) Under Preannounced Pricing. In the preannounced pricing policy, creator announces \( p_1 \) and \( p_2 \) at the beginning of the first period. The strategic consumers will make their purchasing decision through the threshold function based on their common valuation and sensitive degree of waiting. The consumers funding in the crowdfunding will gain the product at the beginning of the second period. In presence of review system, the consumers of the first period will post their review on the platform; the rest of consumers update their belief for the product by the reviews. Thus, the rest consumers learn more about the product’s unobservable attribute and their uncertainty of the product is mitigated, while, in absence of the review system, the consumers in the second period are still confused about the product’s attribute. To visualize the impact of the review system, we compare the profits in Figure 3.
Under the preannounced policy, as shown in the Figure 3, the advantage of the review system is not obvious. \( \rho = e^\alpha \) is small, which means that the consumers are not sensitive about waiting, and the valuation is higher than large \( \rho \). That is why the profit is decreased by the increasing \( \rho \). When \( \rho \) is small, as shown in Figure 3, the preannounced pricing without review system makes more profit. In absence of review system, the consumers cannot learn more information about the product. When the sensitive degree coefficient \( \rho \) is small, the utility difference of purchasing in the first and second period is small, while, in presence of review system, if \( \rho \) is small, the consumers in the second period will learn more information about the product and the valuation in the second period is higher than in the first period. Thus, with the review system, consumers tend to purchase in the second period if they are not sensitive for waiting, which may lead to failure in the crowdfunding. When the consumers are sensitive to waiting, the profit of creator will be lower either with the review system or without review system. But, when \( \rho \) is larger enough, the valuation in both periods is lower, but the profit in review system is a little larger, because the valuation of the product in the second period is larger with the review system. In summary, under preannounced pricing, the impact of the review system is not significant. Whether to disclose the review system on the crowdfunding platform mainly depends on the sensitive degree of waiting.

(2) Under the Responsive Pricing. Under the responsive pricing, creator announces the first period price at the beginning of the first period and announces the second period price based on the demand in the first period or review rating level at the beginning of the second period. With the review system, the consumers of the first period post their review online and the rest of the consumers get more information about the product and the creator learn more about the valuation of the consumers. Thus, both consumers and creator can respond to the reviews online under responsive pricing policy. The impact of the review system is shown in Figure 4.

Observe that the review system has superiority under the responsive pricing policy. As shown in Figure 4 the profit is larger with the review system than without review system when the sensitive degree of waiting is large or small. If the consumers are not sensitive for waiting, the creator can gain considerable profits in both cases; and here the advantage of review system is not obvious. When \( \rho \) is approximately 0.2, the policy without review system will make more profit for the creator. If the sensitive degree is larger, the profit of the creator will be decreased, but the policy with the review system is better than without it. Generally speaking, under the responsive pricing policy, the review system has a positive impact on improving the creator’s profit.

However, most of the crowdfunding platform still only provide the single selling period for the product. The crowdfunding platform always takes a certain percent of the funds from the creator as the fee. Thus, the more profit creator will get the more platform will get. According to the results, providing two periods and disclosing the review system for the crowdfunding product can improve the profit for the creator. We suggest that the platform ought to provide the second period selling channel for the creator, which can also provide a platform for the consumers to follow the crowdfunding product in case those consumers regret no funding in the first period. At the same time, the first period consumers have an effective way to feedback their experience and the second period consumers are familiar with the product through those previous reviews. Besides, the review system helps the creators better satisfy consumers’ preference, which benefits both consumers and creators.

(3) Pricing Policy Comparison. Preannounced pricing and responsive pricing have their own advantages. It is necessary to adopt different pricing policies in the different social environment, which will make more profit for the creator and platform. To analyze the superiority of each policy with the review system, a numerical study is required. Next, we discuss the impacts of the review rating level and the sensitive degree of waiting, respectively.
(a) The Review Rating Level. As mentioned above, we set \( \gamma \) as the fraction of favorable review which is a standard to measure the level of review ratings. The fraction of favorable review reflects not only the true quality of the product but also the consumer’s valuation distribution through the product. According to the previous analysis, consumers’ review follows a binomial distribution, and the posterior distribution of valuation is the Beta distribution. The profit of different policies with increasing \( \gamma \) is shown in Figure 5.

Observe that the profit of responsive pricing policy almost makes no difference with increasing favorable review. If the fraction of favorable review is small, responsive pricing seems to be a better choice, but if \( \gamma \) is larger, preannounced pricing is dominant. As shown in Figure 5, when the product is satisfied with the consumers’ preference, the profit of preannounced pricing is dozens of times than responsive pricing. This result is consistent with previous studies (e.g., Papanastasiou et al. [36] and Aviv and Pazgal [29]). The creator’s profit under the preannounced pricing policy increases with the increasing \( \gamma \), especially when \( \gamma \) is about 0.8, is the most. As Vogt and Fesenmaier’s study [40] showed, the positive reviews have higher positive influence than negative reviews. However, the profit of preannounced pricing policy does not increase monotonically with \( \gamma \); when \( \gamma \) is too high, the profit begins to decrease. Park and Nicolai [37], Kahneman and Tversky [41], and Hu, Pavlou, and Zhang [42] suggest that rational consumers tend to focus on negative reviews more seriously. While positive reviews satisfy people’s enjoyment, they make little impact on usefulness. Additionally, it is important to recognize that when online reviews are not helpful for consumers to make an online purchase decision, individuals would not be likely to revisit the website [37]. Those studies are consistent with our result and show that the fraction of favorable review is not the higher the better.

(b) The Sensitive Degree of Waiting. With the increasing of consumers sensitive degree of waiting, the profit of the creator comes down. In presence of review system, two policies have their own advantages in different cases. The result of equilibrium analysis is shown in Figure 6.

In presence of review system, when the consumers are extremely sensitive or little sensitive about waiting, the responsive pricing is dominant. When \( \rho \) is about 0.2 and 0.45, the preannounced pricing is better. Responsive pricing policy can set the second period price in response to the consumers’ valuation, demand, and review levels. Generally speaking, with the impact of \( \rho \), the responsive pricing has more advantages, but it is not obvious.

(c) The Length of Funding Time (T). In this part, we discuss the profit impacted by the length funding time (the length of the first period \( T \)) under preannounced pricing and responsive pricing, respectively. If the funding time is too short, the crowdfunding will fail because of lacking funders; if the funding time is too long, the total profit may be lower because the first period price is less than the second period price and the consumers purchasing in the first period has to wait a long time to receive the product. Figure 7 shows the impacts of \( T \) in different pricing policies.

Obviously, the length of funding time has fewer impacts on the preannounced pricing; the curve is gentle. However, the curve of responsive pricing is volatile; when \( T \) is about 0.3, the responsive pricing is a dominant strategy. If the length of funding time in the range of \([0.5,0.85]\), the preannounced pricing can make more profit.

7. Conclusion

In recent years, crowdfunding has been regarded as a sustainable and viable alternative channel for entrepreneurs to fund their early stage businesses. However, the uncertainty of the crowdfunding product leads to the lower success rate of crowdfunding and the lower profit of the creator. Hofstede [43] shows that people feel anxious and threatened when they are faced with uncertainty and they rely on their shared beliefs about the current situation. Thus, it is necessary to disclose a review system on the crowdfunding platform so that the latter consumers will mitigate the uncertainty. In this paper, we discuss the impact of the review system and explore the optimal pricing decision of the reward-based crowdfunding. We propose the Bayesian analysis to describe the consumers’ valuations update process with the review system. Besides, in order to investigate the performance of the review system, the model without the review system is provided as the benchmark. What is more, the preannounced pricing and the responsive pricing are compared in our model. Thanks to the equilibrium analysis, the following results are discovered. First, under the preannounced pricing policy, the review system does not have the obvious effect on the creator’s profit. When the sensitive degree of waiting is low, the strategy without review system makes more profit. Second, under the responsive pricing policy, the strategy with the review system has a dominant advantage in most situations. Third, under the responsive pricing, the impact of the fraction of favorable review is not obvious and the curve of responsive pricing is gentle, while the curve of preannounced pricing is volatile. Finally, the length of funding time has some effect on the responsive pricing; when the cycle time (\( T \)) is at the range of 0.25 to 0.35 of the total season, the profit is maximum.

According to the conclusion above, we provide the following suggestions. Firstly, the creator should estimate the product’s attributes accurately and do some investigation about the crowdfunding product, which can help to understand the consumers’ valuation, demand, and sensitive degree of waiting. Next, generally speaking, the review system has a positive impact on improving the creator’s profit. Thus, the crowdfunding platform can gain more profit and hence the consumers’ loyalty to the platform through the review system as well. We suggest that the platform should disclose an online review system to provide more utilities for consumers, creators, and itself as well. Besides, the consumers will mitigate their uncertainty of the crowdfunding product. Finally, the second period selling channel should be opened on the crowdfunding platform, which can provide a way for the consumers not purchasing in the first period to follow the product and help the rest of the consumers to obtain first-hand reviews from the platform.
However, there are some limitations to this study. Firstly, we only take the strategic consumers into account, but the real market is mixed with strategic and myopic consumers. Secondly, the review rating is diversification, but we only follow Park and Nicolau [37]'s research to consider the extreme review ratings. Finally, in order to simplify the model, the interval between the first and second period is ignored. Those limitations provide some direction for further researches and the model of review system can be used in other areas.

Appendix

See Theorem 1.

Proof of Theorem 1. The consumer $k$ will take part in the crowdfunding or purchase the product if $V_k \geq \chi(t)$ in both periods. In addition, if the consumer visits the platform before time $T$ and $V_k < \chi(t)$, he will revisit the platform at time $T$ and purchases the product if the valuation is larger than the threshold at this time. However, in the pre-announced pricing policy, the strategic consumers arriving at $[0, T]$ will weigh their valuation and threshold according to the fixed price path $\{p_1, p_2\}$.

The consumer's surplus in period 1 $s_1$ and in period 2 $s_2$ is

\[
s_1 = \mu V_k e^{-\alpha(T-t)} - p_1 \tag{A.1}
\]
\[
s_2 = V_k e^{-\alpha(T-t)} - p_2
\]

At time $T$ ($0 < T < 1$), the condition consumers take part in the crowdfunding in the first period should be satisfied:

\[
0 < t < T, \quad s_1 \geq s_2. \tag{A.2}
\]

So, $V_k \geq ((p_1 - p_2)/\mu - 1) e^{\alpha(T-t)}$. Similarly, we obtain that $\chi(t) = p_2$ when $T < t < H$. \hfill \Box

See Proposition 2.

Proof of Proposition 2.

\[
n_1 = \lambda^* T * \left( \int_0^T p_2 * e^{\alpha(T-t)} dt \right) - n_1^b
\]
\[
n_2 = \lambda * T * \left( \int_0^T p_2 * e^{\alpha(T-t)} dt \right) - n_1^b
\]
\[
\pi^b = (p_1 - c) n_1^b + (p_2 - c) (n_2^b + n_3^b) \tag{A.3}
\]

Let the first-order condition of $\pi^b$ be 0.\hfill \Box

\[
\partial \pi^b / \partial p_1 = 0.
\]

\[
p_1 = c
\]
\[
p_2
\]
\[
= c \left( T - e^{\alpha T - H\alpha + \alpha T\alpha + \alpha T\mu + e^{\alpha T} H\alpha + \alpha T\mu} \right) \left( -H\alpha + \alpha T\alpha + \alpha T\mu \right)
\]
\[
\partial \pi^b / \partial p_2 = 0.
\]

See Proposition 5.

Proof of Proposition 5.

\[
\pi^r^b = (p_2 - c) \left( n_2^b + n_3^b \right)
\]
\[
= \left( p_2 - \frac{p_1 - p_2}{\mu} \right) * \left( \int_0^T e^{\alpha(T-t)} dt + p_2 * (H - T) \right)
\]
\[
\partial \pi^b / \partial n_1^b = 0
\]
\[
p_2 = \frac{-p_1 + e^{\alpha T} p_1 - c H\alpha + c T\alpha - c\mu + e^{\alpha T} c\mu + c H\alpha + c T\mu}{2 (\alpha T\alpha + \alpha T\mu - e^{\alpha T}\alpha T\mu + H\alpha + T\alpha H\alpha)} \tag{A.5}
\]
\[
\gamma_1 = \frac{p_1 - p_2}{\mu - 1} e^{\alpha(T-t)}
\]
\[
n_1 = \int_0^T \gamma_1 dt
\]
\[
\pi^b = (p_1 - c) n_1^b + n_2^b
\]

Let

\[
\partial \pi^b / \partial n_1^b = 0 \tag{A.6}
\]

We obtain that

\[
p_1^* = \frac{c (1 + 2 T \alpha + 2 H \alpha (1 + \mu) - 2 \mu - 2 T T \alpha \mu + e^{\alpha T} (1 + 2 \mu))}{3 + 4 T \alpha + 4 H \alpha (1 + \mu) - 4 \mu - 4 T T \alpha \mu + e^{\alpha T} (3 + 4 \mu)}
\]
\[
p_2^* = \frac{1}{6} c \left( 3 + \frac{1}{\mu} + \frac{2}{-3 + 4 \mu} - (H - T) \alpha (1 + \mu)}{\mu (H - T) \alpha (1 + \mu) + (1 + e^{\alpha T}) \mu}
\]
\[
- \frac{8 (H - T) \alpha (1 + \mu)}{(-3 + 4 \mu) (3 + 4 (H - T) \alpha (1 + \mu) - 4 \mu + e^{\alpha T} (3 + 4 \mu))} \tag{A.7}
\]
**Data Availability**

The data used to support the findings of this study are included within the article.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

**Acknowledgments**

The authors gratefully acknowledge the support from the National Science Foundation of China through Grant 71571117 and Human and Social Science Foundation of Education Committee of China through Grant no. 18YJA630143.

**References**


