Research Article

Impact of Telemedicine on Healthcare Service System Considering Patients’ Choice

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Telemedicine is an effective way to alleviate the congestion of hospitals and improve the utilization of medical resources. In this paper, we develop a stylized queue model to study whether the hospital should adopt telemedicine service and in which form (the gatekeeper system or dual-channel service system) the hospital should provide telemedicine service. Patients’ delay sensitivity and transportation cost are also considered in our model. Our main results follow. First, in most cases, telemedicine service can help to reduce the total cost of the healthcare system, and patients treated online or offline can both benefit from the adoption of telemedicine. Second, the dual-channel healthcare system is more flexible than the traditional outpatient system and the gatekeeper system. There exists an optimal market segmentation of the two channels in the dual-channel healthcare system. Finally, we find that the hospital should implement the telemedicine when the transportation cost is high or patients have a large gap between online and offline waiting sensitivity.

1. Introduction

Aggregated by the increasing medical needs and patients’ high expectations for healthcare services, insufficient medical resources and unbalanced resources allocation have become a worldwide problem [1]. This problem is particularly acute in general hospitals of Chinese urban cities. In order to address it, the Chinese government has put forward a new medical reform program in the healthcare system and highlighted the adoption and utilization of telemedicine services. Telemedicine provides patients with access to high-quality healthcare services through advanced communication and information technology [2]. Therefore, telemedicine can greatly improve the equity and efficiency of health service delivery and increase the patients’ access to specialist expertise that was unavailable or difficult to access before [3].

In terms of service delivery methods, telemedicine can be classified into three forms: store-and-forward, remote patient monitoring, and real-time interactive services [4]. Store-and-forward telemedicine involves acquiring medical data (such as medical images and bio signals) and then transferring the data to a doctor or medical specialist across distance and time [5, 6]. Remote monitoring is that the medical experts monitor a patient remotely by using wearable devices, digital video, or other devices. This method is mainly applied to manage chronic diseases, such as heart disease, diabetes mellitus, and asthma [7, 8]. For the real-time interactive service, patients and doctors communicate through video conferencing [9–11]. Usually, patients have to go to the local telemedicine health center for this service with the help of tele-specialists [12]. By providing easier access to medical expertise, telemedicine can reduce the geographical variability of diagnosis, treatment, and clinical management [13]. The survey conducted during 2009-2010 in remote regions of the Brazilian state of Minas Gerais shows that the teleconsultations can avert 80.8% of referrals and 45.5% of transportation costs [14]. Many other studies also verified that telemedicine could greatly reduce the healthcare cost, shorten the waiting time, and improve the utilization efficiency of health resources [15–18].

Advanced information and communication technologies prompt patients to obtain health services through the website or mobile applications. This motivates an increasing number
of hospitals to devote attention to telemedicine services. For example, the division of neonatal medicine of Mayo Clinic offers newborn telemedicine consultations to 6 outpatient clinics, and nearly one-third of the infants could remain in the local hospital [19]. To provide telemedicine services, the nonprofit hospital has to face an investment to hire tele-specialists, and the system planner wants to minimize the total cost, which includes the hospital investment cost and patients’ transportation cost and waiting cost. Therefore, whether the reduction of transportation cost and waiting cost can make up for the investment becomes an important issue for the system planner.

Many service systems are arranged with a front line of gatekeepers who refer jobs to a stable of experts. For example, in the service settings of healthcare, call centers, maintenance, and restaurants, customers interact with a gatekeeper (e.g., nurse, tele-phonist, engineer, or waiter) who can decide whether to refer the customer to a specialist (e.g., doctor, service manager, or sommelier) or provide service to the customer himself [20]. Some hospitals take telemedicine as the gatekeeper before patients’ initial visit of doctors. Under this healthcare structure, all patients have to be diagnosed and treated by tele-specialists firstly. Then, they may be referred to doctors for further treatment depending on the complexity of their diseases [21]. Analogous to the gatekeeper system, the Chinese government has launched a new strategy called Regional Healthcare Association (RHA), which consists of one general hospital and several community healthcare centers. Patients are encouraged to see general practitioners in community healthcare centers, although they are also allowed to see the expert doctors in the general hospital for outpatient service [22]. In this hierarchical healthcare system, if the general practitioner cannot treat patients’ disease, they can cooperate with the expert doctors to provide high-quality service for the patient via telemedicine and avoid unnecessary geographic referrals. In other words, community healthcare centers and telemedicine services act as the gatekeeper. It benefits patients and caters to the RHA strategy. We call this hierarchical healthcare system as a dual-channel service system, since patients can choose either a community healthcare center or a general hospital for healthcare service. In this paper, we investigate the general hospital’s decision and compare the total costs of the gatekeeper system and dual-channel service system under the implementation of telemedicine.

Although telemedicine can avoid traveling and shorten the waiting time, the telemedicine service is not a better choice sometimes. For patients with complicated diseases, they cannot get cured only with the telemedicine services and still need retreatment from traditional offline outpatient. For example, in general urological care, telemedicine can obviate the initial in-person visits for nearly 90% of patients, but only 50% of urological complaints could have been managed with telemedicine alone [18]. However, patients only know the cure rate prior probability of the telemedicine service. They do not know whether they can be cured or not before getting the telemedicine service. Hospitals can influence patients’ decisions by changing the online and offline service capacity in the dual-channel service system. Thus, the market segmentation of the telemedicine service and traditional offline service is also changed.

In this paper, we focus on the following research questions:

1. Should a hospital provide telemedicine services?
2. Should a hospital take the telemedicine service as a gatekeeper or provide dual-channel service?
3. If a hospital provides the dual-channel service, how should it decide the telemedicine and traditional offline service market segmentation?
4. What are the impacts of telemedicine service on the patients’ medical behavior?

The objective of this study is to investigate how decision-makers should plan and design the healthcare system. Patients are uniformly distributed and choose the traditional offline service or telemedicine service. Several M/M/1 queueing models are formulated to depict the traditional outpatient system, the gatekeeper system, and dual-channel service system. We first study the hospital’s capacity decision and patients’ medical choice in these three healthcare systems. Then, we try to get the optimal strategy and service capacity decisions by comparing the total costs of the three systems. Finally, we focus on the dual-channel service system and provide the hospital with some suggestions on how to manage online and offline service channels. Hence, our study highlights the strategic choice before the general hospital adopts the telemedicine service.

The remainder of the paper is organized as follows. Section 2 gives a brief literature review of the related topics. Section 3 presents assumptions and notations. Sections 4 and 5 present the model and some important analytical results. Section 6 gives the concluding remarks based on the important findings of this paper. Section 7 identifies the limitations of the models and provides future research avenues. To make the paper more readable, all proofs are presented in Appendix.

2. Literature Review

Telemedicine is relatively new in the healthcare system. There is limited research on managing the adoption and implementation of this modality of healthcare service. AlDossary et al. [1] provide a detailed review of telemedicine service. They conclude that the previous studies assess the telemedicine service from three different perspectives: clinical outcome, economics, and satisfaction. The adoption of telemedicine is mainly influenced by the economic perspective like cost saving and added revenue [23]. Many empirical papers [12, 14, 24] have emphasized the transportation cost saving of the telemedicine services. So, we consider the patients’ transportation cost as well as the waiting time costs. McConnochie et al. [25] and Xu et al. [26] further measured the threshold at which the cost of telemedicine service is the same as that of the conventional in-person service. However, they all have not made a specific comparison and analysis of telemedicine service. Expect the economics, speed and quality of the healthcare service are also very important. Saghafian et al. [27] investigate the speed-versus-quality trade-off of the telemedical physician triage systems, in which a second
The telemedical physician renders a decision on cases referred to him/her by the triage nurses. Different from this stream of literature, we use mathematical models to study whether the general hospital should adopt the telemedicine service and how the telemedicine service could influence the healthcare system.

There is a large body of literature concentrating on the effect of government regulations and welfare measures. However, the aims of the system planner or government are varied in those studies. For example, Hua et al. [28] study the effect of government fiscal policy on the capacity decision of the two-tier service system, which includes a free public service provider aimed at maximizing the patients’ net utility and a toll private service provider aimed at maximizing its expected profit. Qian and Zhuang [29] take healthcare service as a tool to achieve welfare redistribution, and the planner’s aim is to maximize the total weighted patient welfare. Zhou et al. [30] consider a medical market comprised of two hospitals with different service levels, and the aim of government is to maximize the number of patients who got treated. Additionally, Tarakci [31] considers the health condition and healthcare cost of patients with chronic diseases. In our study, the planner’s aim is to minimize the whole service system cost, which is similar to the studies of Liu et al. [32] and Lee et al. [33].

There are also some studies about the cooperation between heterogeneous service providers. They can be divided into two streams. One stream is the collaborative service system, such as consulting, financial planning, and information technology outsourcing. Those involved services usually need joint production of two or more service providers and the effort levels are complementary [34, 35]. This stream mostly studies the contract issues among a buyer and a vendor by using the principal-agent theory model. The other stream is the gatekeeper system, particularly in healthcare and call center. For example, Tarakci et al. [21] propose a queueing network in which the patients are always treated first by the general practitioners. Then a proportion of patients are divided to seek the tele-specialists’ treatment, and others are transferred to the expert doctors. Lee et al. [33] consider a one-stream referral process from the gatekeeper to the expert. They study whether the client should outsource the gatekeeper, expert process, or even the entire two-level process. Based on the study of Lee et al. [33], Liu et al. [32] examine the optimal mutual referral strategy between a city hospital and a community hospital. They form a queueing network with two stations where patients randomly enter either hospital for diagnosis. In this paper, we contribute to this stream of literature by studying both the gatekeeper system and dual-channel service system on telemedicine adoption for general hospitals.

The integration of online and offline channels to provide customers with products and services is very popular in many industries. Wang et al. [36] give an excellent review of the dual channel in a service supply chain. But the services studied in this review are mostly based on the product service supply chains, for example, retail service and delivery service [37, 38]. Literature of the dual channel in a pure service supply chain is very limited. Gao and Su [39] develop a stylized theoretical model to study the impact of online and offline self-service systems in the restaurant industry. They demonstrate that customers using self-service systems can reduce waiting cost and increase customer demand as well as employment levels. As to healthcare service, Wu and Lu [40] examine the channel effect in online health communities. They find that online health communities broaden and diversify channels for the patient–doctor interaction and provide better treatment by decreasing medical costs, making full use of available resources. Most studies in the area of dual-channel management focus on channel competition, in which price and service level are important factors. In this paper, we try to decide the optimal service capacities of both the telemedicine service and traditional offline service.

3. Problem Assumptions and Notations

This paper investigates a healthcare system providing both the offline traditional service and online telemedicine service. For the offline service, patients need to go to the hospital for treatment, which is not always necessary for the telemedicine service. We denote the distance between patients and the hospital as $x$, which is a random variable following a uniform distribution on the unit interval $[0, 1]$. We use $t$ to denote the unit transportation cost.

3.1. Notations

Model Parameters

- $x$: The distance of the patient from the hospital
- $x_0$: The market share of the traditional offline service
- $t$: The unit transportation cost
- $\Lambda$: The total arrival rate of all patients
- $\lambda_1, \lambda_2$: The arrival rate of the first diagnosis in the offline hospital/telemedicine service
- $k$: The mistreatment probability of the telemedicine service
- $\theta_1, \theta_2$: The patient waiting cost in the offline hospital/telemedicine center
- $W_1, W_2$: The patient waiting time in the offline hospital/telemedicine center
- $c_1, c_2$: The unit capacity cost in the offline hospital/telemedicine center
- $R$: The utility of the patient gets cured
- $U_1, U_2$: The patients’ net utility in the offline hospital/telemedicine center
- $p_1, p_2$: The workload of the offline hospital/telemedicine center.

Decision Variables

- $\mu_1$: The service rate of the traditional outpatient hospital
- $\mu_2$: The service rate of the telemedicine center.

3.2. Assumptions

3.2.1. Arrival Rate. A number of patients arrive at the network in a unit time sequentially. The total arrival rate is $\Lambda$. The queue discipline is first-come-first-serve. As shown in Figure 1, $\lambda_1$ denotes the arrival rate of patients who go to the...
offline hospital for the first diagnosis. \( \lambda_2 \) denotes the arrival rate of patients who go to the telemedicine center for the first diagnosis, and \( \lambda_1 + \lambda_2 = \Lambda \). All patients need to get cured. So, the total number of patients will remain unchanged. Because of technological limitation and medical level restriction, part of the patients who go to the telemedicine center for the first diagnosis may not get cured and need further offline treatment. We assume the probability of mistreatment rate as \( k \). The number of mistreated patients is assumed to be \( \lambda_m \). All the patients can heal with the offline expert doctors’ treatment.

### 3.2.2. Waiting Costs

Both the service rate and patient arrival rate can affect patients’ waiting time. Let \( W_1 \) and \( W_2 \) denote the expected waiting time (including service time) in the offline hospital and telemedicine center, respectively. The online and offline waiting costs are different; we denote them as \( \theta_1 \) and \( \theta_2 \).

### 3.2.3. Patients Utility

\( R \) refers to the utility of a patient who got cured, and it is the same for all patients who get cured no matter where they were treated. \( R \) is also assumed to be large enough so that all patients’ net utilities are nonnegative, and all patients will get services from either one of the two channels. Compared with visiting the hospital in person, the travel burden for patients going to the telemedicine center can be ignored [41]. \( U_1 \) refers to the utility that a patient located at \( x \) directly goes to the traditional offline outpatient for the first diagnosis; \( U_1 = R - tx - \theta_1 W_1 \). The patient with the telemedicine service has an expected utility \( U_2 = R - \theta_1 W_1 - k(tx + \theta_1 W_1) \). We assume that the patient located at \( x_0 \) is indifferent between the two channels, and the expected utilities of \( U_1 \) and \( U_2 \) are the same. That means \( R - tx_0 - \theta_1 W_1 = R - \theta_1 W_2 - k(tx_0 + \theta_1 W_1) \). Thus, we can derive the indifferent patients’ location:

\[
x_0 = \frac{1}{k} \left( \frac{\theta_2 W_2}{1 - k} - \theta_1 W_1 \right)
\]  

### 3.2.4. Service Rate

The offline hospital service rate \( \mu_1 \) and the telemedicine service rate \( \mu_2 \) are decision variables of the hospital. \( \mu_1 \) and \( \mu_2 \) also represent the service capacities of the two service channels. When the hospital does not provide telemedicine service, \( \mu_1 > 0 \) and \( \mu_2 = 0 \). Suppose that the offline hospital service rate cost is \( c_1 \) and the telemedicine service rate cost is \( c_2 \). Usually, the wage of the expert doctor is higher than that of the tele-specialist. Thus, we assume that \( c_1 \geq c_2 \).

### 4. The Model Formulation

According to the structure of the healthcare system, this paper derives the equilibrium solution for the healthcare systems in three cases: (1) traditional outpatient system, where only the offline channel provides healthcare service; (2) gatekeeper system, where the telemedicine center is adopted as the gatekeeper, and all patients need to go to the telemedicine center for the first diagnosis; for those patients who were not cured online, they will be referred to the offline channel for further treatment; (3) dual-channel service system, where both offline and online channels provide the first diagnosis. Patients can choose either channel for the first diagnosis according to their net utility.

#### 4.1. Traditional Outpatient System

In this section, we consider the case where the hospital only provides offline services, and all the patients need to visit the hospital in person no matter where they are located. This case can be set as a benchmark. We build an M/M/1 queueing model to describe this process. According to the average waiting time (including service time) function in Hua et al. [28], we set the expected waiting time for a patient in the offline channel as \( W_1^F = 1/(\mu_1^F - \Lambda) \) and set the consumer net utility as \( U_1^F = R - tx - \theta_1 W_1^F \). The system cost includes two parts. One is patients’ total cost, denoted as \( PC^F \). It includes patients’ transportation cost and waiting cost. The other part is the investment of the hospital in the service capacity. The decision problem for the system planner is to determine the optimal capacity \( \mu_1^F \) and minimize the total cost of the traditional outpatient system, denoted by \( TC^F (\mu_1^F) \). The object function is

\[
\min \ TC^F (\mu_1^F) = \int_0^1 (tx + \theta_1 W_1^F) \lambda dx + c_1 \mu_1^F
\]
Lemma 1. For the case of the traditional outpatient system, there exist equilibrium solutions, where $\mu_1^{\text{Fs}} = \Lambda + \sqrt{\theta_1 \Lambda / c_1}$ and $W_1^{\text{Fs}} = \sqrt{c_1 / \theta_1 \Lambda}, TC^{\text{Fs}} = 2 \sqrt{c_1 \Lambda + (1/2)t \Lambda + c_1 \Lambda}$.

By substituting the optimal solution into the first part of (2) and workload equation $\rho_1^{\text{Fs}} = \Lambda / \mu_1^{\text{Fs}}$, we can get the patients’ total cost $PC^{\text{Fs}} = \sqrt{\theta_1 \Lambda c_1 + (1/2)t \Lambda}$ and the hospital workload $\rho_1^{\text{Fs}} = \sqrt{c_1 / (\sqrt{c_1 \Lambda} + \sqrt{\theta_1 \Lambda})}$.

Different from the previous study of healthcare, in this paper, we consider a certain number of patients with heterogeneous transportation cost. We analyze the related parameters in Propositions 2 and 3. All proofs are in the Appendix.

Proposition 2. For the case of the traditional outpatient system, the unit transportation cost $t$ can influence the system and patients’ total cost, where $\partial TC^{\text{Fs}} / \partial t = \partial PC^{\text{Fs}} / \partial t = (1/2) \Lambda$. However, $t$ does not affect the capacity investment and the hospital workload since $\partial \mu_1^{\text{Fs}} / \partial t = \partial \rho_1^{\text{Fs}} / \partial t = 0$.

Proposition 2 shows that the cost of the traditional outpatient system is very high, especially when the unit transportation cost is high or the population served by the general hospital is large. We assume that the distance between the patient and the hospital is uniformly distributed along [0,1] in Section 3. That means the unit transportation cost does not refer to the actual unit distance cost. When the hospital serves a widely scattered population, the unit distance in this assumption refers to a larger actual distance. Besides, when the district the hospital served has poor transportation facilities, the unit transportation cost is also comparatively high. In short, for the district with a large and widely scattered population or poor transportation facilities, the telemedicine service is very necessary, whereas both the hospital’s service capacity and workload have nothing to do with the unit transportation cost, and patients bear the high transportation cost. So, it is very necessary for the government to introduce some related policies to promote the adoption and implementation of telemedicine in this situation. The empirical study of Kituyi et al. (2011) also confirms the importance of telemedicine policy.

Proposition 3. (1) $\partial TC^{\text{Fs}} / \partial c_1 > \partial PC^{\text{Fs}} / \partial c_1 > 0$; (2) $\partial \mu_1^{\text{Fs}} / \partial c_1 < 0$, $\partial \rho_1^{\text{Fs}} / \partial c_1 > 0$, and $\partial W_1^{\text{Fs}} / \partial c_1 > 0$.

Proposition 3(1) implies that, in traditional outpatient system, as the unit service capacity cost increases, both system total cost and patients’ total cost increase, while system total cost increased more rapidly. Proposition 3(2) implies that the hospital will reduce the service capacity in response to the costlier service capacity. As a result, the system workload becomes heavier and the patients’ waiting time becomes longer. Even so, the hospital’s investment of service capacity still increases; that is why the system total cost increases more rapidly than the patients’ total cost. It is usually hard to reduce the service capacity cost of the traditional offline channel since expert doctors are always in shortage.

4.2. Gatekeeper System. This subsection considers the case that the hospital sets the telemedicine service as the gatekeeper, and all patients go to the online telemedicine center for the first diagnosis. Some patients may get cured with the tele-specialist treatment. Their mean waiting time is $W_1^{T} = 1/(\mu_1^{T} - k \Lambda)$. Other patients still need further treatment from the expert doctors. The further traditional offline treatment waiting time is $W_1^{T} = 1/(\mu_1^{T} - k \Lambda)$. It is easy to see that the offline and online service rate can be expressed as $\mu_1^{T} = k \Lambda + 1/W_1^{T}$ and $\mu_2^{T} = \Lambda + 1/W_1^{T}$. The decision problem for the gatekeeper system can be written as

$$\min TC^{T} (\mu_1^{T}, \mu_2^{T})$$

$$= \int_0^1 \left[ \theta_2 W_2^{T} + k (tx + \theta_1 W_1^{T}) \right] \Lambda dx + c_1 \mu_1^{T} + c_2 \mu_2^{T}$$

The first term denotes the patients’ total cost. It includes the definite online waiting cost and the contingent offline waiting time and transportation cost. The second and third terms are the investments of offline and online service capacities.

Lemma 4. For the gatekeeper system, there exists an equilibrium solution, where $\mu_1^{T} = k \Lambda + \sqrt{\theta_1 k \Lambda / c_1}, \mu_2^{T} = \Lambda + \sqrt{\theta_2 k \Lambda / c_2}, W_1^{T} = \sqrt{c_1 / k \Lambda}, W_2^{T} = \sqrt{c_2 / k \Lambda}$, and the minimum total cost is $TC^{T} = 2 \sqrt{\theta_2 k \Lambda c_2 + 2 \sqrt{\theta_1 c_1 k \Lambda} + (1/2) tk \Lambda + c_1 \Lambda + c_2 \Lambda}$. Then, the offline and online workloads of the system are $\rho_1^{T} = \sqrt{c_1 k \Lambda / (\sqrt{c_1 k \Lambda} + \sqrt{\theta_1})}$ and $\rho_2^{T} = \sqrt{c_2 k \Lambda / (\sqrt{c_2 k \Lambda} + \sqrt{\theta_2})}$, respectively, and the patients’ total cost is $PC^{T} = \sqrt{\theta_1 k \Lambda c_1} + \sqrt{\theta_2 k \Lambda c_2} + (1/2) tk \Lambda$.

Proposition 5. When the hospital sets the telemedicine service as the gatekeeper, $\partial TC^{T} / \partial k > 0, \partial PC^{T} / \partial k > 0, \partial W_1^{T} / \partial k > 0, \partial \mu_1^{T} / \partial k > 0, \partial \rho_1^{T} / \partial k > 0, \partial \mu_2^{T} / \partial k = 0$, and $\partial W_1^{T} / \partial k = \partial \mu_2^{T} / \partial k = 0$.

Proposition 5 indicates that if the hospital sets the telemedicine service as the gatekeeper, the mistreatment rate only affects the offline channel and has no influence on the online channel. This means that the tele-specialists are not responsible for their mistreatments, although tele-specialists’ ability and effort are essential factors for the mistreatment. This is the natural shortage of gatekeeper in many service systems. Some studies introduce contract mechanisms to overcome this shortage. Shumsky and Pinker [20] examine the effects of two incentive bonuses. One is based on the referral rate, and the other is based on the number of patients treated successfully. Liu et al. [32] investigate the mistreatment cost-sharing scheme in a mutual referral healthcare system. In the next section, we design a dual-channel healthcare system where patients can freely choose online tele-specialists or offline expert doctor for their first diagnosis. To some extent, the dual-channel service system automatically overcomes the shortcoming of the gatekeeper system without any additional contract.
4.3. Dual-Channel Service System. In this section, we consider the case where both the telemedicine center and the traditional offline outpatient provide the first diagnosis. Patients make decisions based on their utility, and there are three cases: (a) all patients go to the offline channel directly for the first diagnosis; (b) all patients go to the online channel for the first diagnosis; (c) patients belonging to \([0, x_0)\) go to offline channel for the first diagnosis, and the remaining patients belonging to \((x_0, 1]\) will turn to online channel for the first diagnosis. In other words, \(x_0\) is the market share of the offline channel. The output process of an M/M/1 system is a Poisson process at the same rate as the arrival rate \([42]\).

As the arrival rate of the online channel is \(\lambda\), the expected arrival rate of mistreatment is \(\theta\), and the online service rates are \(\mu^D_1\) and \(\mu^D_2\). From (1) and (4), we know that all patients will go to the online channel for the first diagnosis when \(\theta_2W^D_2/(1 - k) - \theta_1W^D_1 > t\). When \(\theta_2W^D_2/(1 - k) - \theta_1W^D_1 < 0\), all patients go to the offline channel for service, which is similar to the traditional outpatient system. When \(0 < \theta_2W^D_2/(1 - k) - \theta_1W^D_1 < t\), patients go to both the online and offline channels for the first diagnosis. The total cost of the dual-channel service system is expressed as

\[
(\lambda^D_1, \lambda^D_2) = \begin{cases} 
(\Lambda, 0) & x_0 \geq 1 \\
(\Lambda x_0 + k\Lambda (1 - x_0), \Lambda (1 - x_0)) & 0 < x_0 < 1 \\
(k\Lambda, \Lambda) & x_0 \leq 0
\end{cases}
\]  

A patient’s expected offline and online waiting times are assumed to be \(W^D_1\) and \(W^D_2\), respectively, and the offline and online service rates are \(\mu^D_1\) and \(\mu^D_2\). From (1) and (4), we know that all patients will go to the online channel for the first diagnosis when \(\theta_2W^D_2/(1 - k) - \theta_1W^D_1 > t\). When \(\theta_2W^D_2/(1 - k) - \theta_1W^D_1 < 0\), all patients go to the offline channel for service, which is similar to the traditional outpatient system. When \(0 < \theta_2W^D_2/(1 - k) - \theta_1W^D_1 < t\), patients go to both the online and offline channels for the first diagnosis. The total cost of the dual-channel service system is

\[
\min TC^D(D_1, \mu^D_2) = \int_0^{x_0} (tx + \theta_1W^D_1) \Lambda dx + \int_0^{x_0} [\theta_2W^D_2 + k(tx + \theta_1W^D_1)] \Lambda dx + c_2\mu^D_2
\]

\[
st. 0 < \frac{1}{1 - k}\theta_2W^D_2 - \theta_1W^D_1 < t
\]

We can easily get

\[
W^D_1 = 1/(\mu^D_1 - \lambda^D_1) = 1/(\mu^D_1 - \lambda x_0 - k\Lambda(1 - x_0))
\]

\[
W^D_2 = 1/(\mu^D_2 - \lambda^D_2) = 1/(\mu^D_2 - \lambda(1 - x_0))
\]

in the M/M/1 queues. Thus, the offline service rate is \(\mu^D_1 = \lambda x_0 + k\Lambda(1 - x_0) + 1/W^D_1\), and the online service rate is \(\mu^D_2 = \lambda (1 - x_0) + 1/W^D_2\). Submit \(x_0 = (1/t)(\theta_2W^D_2/(1 - k) - \theta_1W^D_1)\) to the expressions of \(\mu^D_1\) and \(\mu^D_2\); then we get

\[
\mu^D_1 = k\Lambda + (\theta_2\Lambda/t)W^D_2 - ((1 - k)\Lambda\theta_1/t)W^D_1 + 1/W^D_1
\]

\[
\mu^D_2 = \Lambda + (\theta_2\Lambda/t)W^D_2 - (\Lambda\theta_1/t(1 - k))W^D_1 + 1/W^D_2
\]

Thus, there exists a unique \((\mu^D_1, \mu^D_2)\) for every \((W^D_1, W^D_2)\), and we can use \((W^D_1, W^D_2)\) to replace the decision variable \((\mu^D_1, \mu^D_2)\).

The constraint condition (6) becomes the domain. Equation (5) is equivalent to

\[
TC^D(D_1, \mu^D_2) = -\Lambda \frac{1 - k}{2t} \left( \frac{1}{1 - k} \theta_2W^D_2 - \theta_1W^D_1 \right)^2 + \Lambda \frac{1}{t} (c_1 - c_2 - c_2) \left( \frac{1}{1 - k} \theta_2W^D_2 - \theta_1W^D_1 \right) + \theta_2W^D_2 + k\theta_1W^D_1 + c_2 \frac{1}{W^D_2} + \frac{1}{2} k\Lambda
\]

\[
+ c_1 k\Lambda + c_2\Lambda
\]

Lemma 6. Given \(tk > c_1 - c_1k - c_2\), if the stationary point \((W^D_1, W^D_2)\) of (7) satisfies the conditions \(\Lambda\theta_2^2/2c_2(1 - k))W^D_3 + (\Lambda\theta_2^2(1 - k)/2c_1)W^D_3 < t\) and \(0 < \theta_2W^D_2/(1 - k) - \theta_1W^D_1 < t\), there exists an optimal solution for the dual-channel service system which can be gotten by solving

\[
\frac{\partial TC^D}{\partial W^D_1} = -\frac{\Lambda\theta_1}{t} \left[ (1 - k) \theta_1W^D_1 - \theta_2W^D_2 + c_1 - c_2 \right] + k\theta_1\Lambda - c_2 \frac{1}{W^D_2} = 0
\]

\[
\frac{\partial TC^D}{\partial W^D_2} = \frac{\Lambda\theta_2}{t(1 - k)} \left[ (1 - k) \theta_1W^D_1 - \theta_2W^D_2 + c_1 - c_2 \right] + \theta_2\Lambda - c_2 \frac{1}{W^D_2} = 0
\]

The two conditions in Lemma 6 can be both met when the unit transportation cost is large enough. That means the optimal solution exists under some condition. The Hessian matrix of function (7) is not always positive definite, and the optimal solution may get on the boundary of the domain. In some situations, patients will only choose one channel for the first diagnosis, although the hospital provides both online and offline channels for the first diagnosis. We note that it is virtually impossible to find the closed-form solutions, so we use numerical analysis to further discuss this problem.

Figure 2(b) shows that when \(k = 0.51\), the total cost of the dual-channel service system first decreases and then increases with \(W^D_1\) and \(W^D_2\). We observe that the shape of the curved surface is like a “hammock,” and there indeed exists a unique extremum that consists of the optimal waiting times of the two channels. However, Figure 2(b) only describes the partial properties of the function. The entire surface of the objective function is decreased firstly and then increased and finally decreased. So, the surface may have another minimum value. But, from the figure, we know that the offline or online waiting times of other minima are much longer, which is not practical. Figures 2(a) and 2(c) reflect the cases where the optimal solutions are on the boundary.

**Proposition 7.** Denote these solutions (if they exist) as \(W^D_1\) and \(W^D_2\), when \(tk > c_1 - c_1k - c_2\); then the system total cost
Figure 2

first decreases and then increases and at last decreases with 
\((W_1^D)W_2^D\).

There are some policies to ensure the offline (online) 
waiting time below a certain threshold [43], and the optimal 
solution is usually gotten on the threshold. When offline 
(online) waiting time is a constant, as shown in Proposition 7, 
the hospital can change both online and offline service capac-
ities to adjust online (offline) waiting time and minimize 
the system total cost. To some extent, this means that the 
dual-channel service system is more flexible than both the 
traditional outpatient system and the gatekeeper system.

**Proposition 8.** Denote these solutions (if they exist) 
as \(W_1^{D*}\) and \(W_2^{D*}\); then 
\((c_1/\Lambda\theta_1)(1/W_1^{D*2}) + ((1-k)c_2/\Lambda\theta_2)(1/W_2^{D*2}) = 1).\)

Proposition 8 implies that the two channels influence 
each other. If one of the two waiting times increased, the 
other one must decrease. Besides, when the parameters of 
one channel change, both two channels are affected. For 
example, when the total arrival rate increases, offline and 
online waiting times decrease. That is because the hospital 
has to treat more patients per unit time, so the waiting 
time becomes shorter. When mistreatment rate increases, 
the optimal offline and online waiting times decrease. In other 
words, the hospital will increase the investment of service 
capacity as the response to the higher mistreatment rate.

5. **Total Cost Implications**

The total costs of the three service systems are affected by var-
ious factors. We compare the three systems analytically and 
present numerical examples to make a further comparison.

5.1. **Comparison of the Three Systems.** We first compare the 
minimum total costs of the traditional outpatient system and 
the gatekeeper system. The large value of \(TC^{F*} - TC^{T*}\) 
means that the implementation of telemedicine can save significant 
cost for the healthcare system. Because we consider the
nonprofit general hospitals and try to find the optimal strategy from the whole healthcare system perspective, our analysis below focuses on the changes of the systems’ total costs which include both the hospital’s investment and patients’ cost.

**Proposition 9.** Comparing Lemmas 1 and 4, we can get the following:

(i) If \((1/2)\sqrt{\Lambda}(1/2)(t(1-k) + (1-k)c_1 - c_2) > \sqrt{\theta_2 c_2} + (\sqrt{K} - 1)(\sqrt{\theta_1 c_1})\), then \(TC^{D^*}_c > TC^{T^*}_c\); the gatekeeper system is more cost-saving. If \((1/2)\sqrt{\Lambda}(1/2)(t(1-k) + (1-k)c_1 - c_2) \leq \sqrt{\theta_2 c_2} + (\sqrt{K} - 1)(\sqrt{\theta_1 c_1})\), then \(TC^{D^*}_c \leq TC^{T^*}_c\); the traditional outpatient system is more cost-saving.

(ii) \(\partial(TC^{D^*}_c - TC^{T^*}_c)/\partial t > 0, \partial(TC^{D^*}_c - TC^{T^*}_c)/\partial k < 0, \) and \(\partial(TC^{D^*}_c - TC^{T^*}_c)/\partial c < 0\).

Proposition 9(1) is presented to show the conditions under which the hospital should set the telemedicine service as the gatekeeper. Proposition 9(2) implies that the gap of total cost between the traditional outpatient system and gatekeeper system becomes greater with the increase of transportation cost. This means that high transportation cost can drive the system planner to adopt telemedicine, just as Proposition 2 shows. However, the gap becomes smaller with the increases of mistreatment rate and online service capacity cost. So, lower online service capacity cost and mistreatment rate also contribute to the implementation of telemedicine service. Therefore, government or system planner who promotes the development of telemedicine technology should try to provide effective telemedicine services at low price.

We next compare the total costs of the gatekeeper system and dual-channel service system. Currently, the Chinese government has introduced many policies to carry out the hierarchical medical system, and telemedicine becomes an important consideration. Telemedicine promotes more patients to get healthcare service at home or community healthcare center and relieves the congestion in general hospitals. But in what way should the hospital adopt the telemedicine service? This remains a real question because implementing telemedicine involves substantial financial investment and significant changes in the healthcare system. Previous analysis and prediction can help the system planner make better decisions.

**Proposition 10.** When the optimal solution of \(TC^{D^*}_c\) is gotten on the boundary \(x_0 = 0\), then \(TC^{D^*}_c \geq TC^{T^*}_c\). In other situations, there exists \(TC^{D^*}_c < TC^{T^*}_c\).

Patients are free to choose either channel in the dual-channel service system. If all patients choose the online telemedicine channel for the first diagnosis, the condition \(\theta_1 W_2/(1-k) - \theta_2 W_1 \leq 0\) must be met, and the service capacity of telemedicine must be higher than a certain value. In the gatekeeper system, regulations are established to make patients get the first diagnosis on telemedicine center, and the decision of telemedicine service capacity is not affected by patients’ choice in this scenario. The total cost of the gatekeeper system can be lower than that of the dual-channel service system. On the other hand, when the transportation cost is relatively low or patients’ diseases are complex, it is better for parts of patients to go to the offline channel for the first diagnosis. The dual-channel service system can reduce unnecessary online treatment and investments of telemedicine. Conclusively, in some cases, the gatekeeper system is better than the dual-channel service system, but, in other cases, it is worse than the dual-channel service system.

5.2. The Effect of Parameters on the Systems’ Total Costs. We use numerical analysis to compare the total costs of the three service systems and the impact of various parameters. With Lemma 6, we know that does not always exist an optimal solution for the dual-service system, but there indeed exists a minimum total cost. As the objective function has two boundary restrictions, the minimum value can be gotten in the domain or on the boundary. The healthcare service process is affected by many different factors, and we analyze four groups of key affecting factors: transportation cost, telemedicine mistreatment rate, offline and online waiting costs, and service capacity costs.

As for the admission rates of the general hospital in China, it is reasonable to refer to the historical data of Xiangya Hospital (1.1 million per year), Peking Union Medical College Hospital (2.26 million per year), and Tongji Hospital (4.8 million per year), which is averaged to 14.5 patients per minute in a single general hospital [32]. So, we set \(\Lambda = 10\) persons per minute to denote the total arrival rate of the healthcare system for general analysis. Service capacity costs are mainly reflected in the tele-specialist’s salary and expert doctors’ salary. According to the study of Lee et al. (2012), we artifically set the expert doctors’ salary twice of the tele-specialists’; that is, \(c_1 = 3\) and \(c_2 = 1.5\). Offline and online waiting costs are varied for different patients. Generally, online waiting time cost is lower than offline waiting cost, and we set \(\theta_1 = 0.5\) and \(\theta_2 = 0.4\). Transportation cost includes both the fee of taking a bus or other vehicle and the cost of the time spent on transportation, so we set the transportation cost \(t = 0.4\). The mistreatment rate is affected by many different factors, and we set \(k = 0.51\). We set the same initial value for all the three systems and compare the total costs.

5.2.1. Impact of Transportation Cost, \(t\). Figure 3 reflects the effect of transportation cost on the total costs of the three systems. When transportation cost is relatively low, the traditional outpatient system is better; as the transportation cost increases, the dual-channel service system becomes optimal. But when the transportation cost increases to a certain value, the cost of the gatekeeper system is the same as that of the dual-channel service system, and both are better than the traditional outpatient system. Combining with Proposition 2, it can be concluded that when the transportation cost is low and the population of the hospital served is very small, the hospital should only provide offline channel service. Otherwise, it is better to provide dual-channel service. The changes in transportation cost will not make the gatekeeper system the optimal strategy in this situation.
5.2.2. Impact of Mistreatment Rate, $k$. Mistreatment rate of telemedicine has no effect on the traditional outpatient system. However, it can increase the total cost of both the gatekeeper system and the dual-channel service system. From Figure 4, we know that the total cost of the dual-channel service system $TC^D$ does not increase linearly as the mistreatment rate increases. Initially, at a low level of mistreatment rate ($k < 0.6$), the increase of $TC^D$ slows down progressively. It is because the hospital reduces the investment on telemedicine, and more patients go to offline channel for the first diagnosis. Once the mistreatment rate reaches a critical value (for instance, when $k \geq 0.6$), the increase of $TC^D$ will accelerate due to the larger investment of telemedicine in the dual-channel service system. In this situation, it is better for the hospital to give up online service channel.

Telemedicine is helpful for most chronic diseases (like diabetes, stroke, heart disease, and hypertension), and the cure rates of telemedicine for some simple diseases (like skin diseases, urinary system diseases, and allergies) are also high. So, for those diseases, telemedicine should be widely adopted. Furthermore, with the development of telemedicine technology, the mistreatment rate will reduce gradually, and more diseases can be cured through the online channel. Combining with Figure 4, we can predict that the gatekeeper system may be the future trend, but in the implementation stage of telemedicine service, the dual-channel service system will be the better choice.

5.2.3. Impact of Waiting Costs, $\theta_1$ and $\theta_2$. Here, we analyze how the patients’ waiting costs influence the systems’ total costs. From Figure 5(a), we know that the total costs of the three systems are increased as the offline waiting cost increases but not linearly. This is because the hospital will increase the investment of the offline channel to reduce the waiting time, and more patients turn to online channel for healthcare service. However, in Figure 5(b), with the increase of online waiting cost, the total cost of the traditional outpatient system remains the same, while the gatekeeper system increases linearly. At the same time, the dual-channel service system increases slowly and is higher than the traditional outpatient system when $\theta_2 > 0.53$.

Generally, online waiting time is lower than offline waiting time, especially for office workers; that is, $\theta_1 > 0.4$ in Figure 5(a) or $\theta_2 < 0.5$ in Figure 5(b). Obviously, gatekeeper system and dual-channel service system are better choices for this kind of patients. For chronic patients, time is not the dominated decision factor, and both online waiting cost and offline waiting cost are low. However, we should notice that Figures 5(a) and 5(b) describe the effect of waiting time of one channel, while the other one is fixed. They reflect the relative changes in offline or online waiting costs. So, for both the chronic and emergency patients, we should inspect the point when $\theta_1 = \theta_2 = 0.4$ in Figure 5(a) and the point when $\theta_2 = \theta_1 = 0.5$ in Figure 5(b). From these two points, we know the dual-channel service system is the best strategy. Telemedicine benefits chronic diseases because most chronic diseases need long-term but usually uncomplicated treatment. Telemedicine service can greatly meet chronic patients’ treatment needs and save transportation cost. So, governments and healthcare institutions can implement the telemedicine service first for chronic diseases, postoperative recovery and prognosis, and elderly healthcare.

5.2.4. Impact of Service Capacity Costs, $c_1$ and $c_2$. From Figures 6(a) and 6(b), we know that as offline service rate cost increases or online service rate cost decreases, the optimal strategy varies from the traditional outpatient system to the dual-channel service system. At last, the total cost of the gatekeeper system is the same as the dual-channel service
system. In reality, online service capacity cost is generally lower than offline service rate for the following two reasons. Firstly, online healthcare service mainly focuses on ailments and chronic diseases. Thus, telemedicine center often hires general practitioners whose salaries are much lower than the offline expert doctors. Secondly, online healthcare service is more effective. It can break the geographic restrictions and make full use of healthcare resources. The results, present in Figures 6(a) and 6(b), show that when the online service capacity cost is lower than the offline one to some extent, it is advantageous for the hospital to open the online channel.

5.3. The Effect of Parameters on the Dual-Channel Service System. Numerical analysis is focused on the impact of various parameters on the dual-channel service system with fixed service capacities. This analysis is applicable since the service
capacity of the system is not easy to adjust in the short run. For our baseline model parameters, we use similar values as above; that is, \( \Lambda = 10 \), \( t = 0.2 \), \( k = 0.51 \), \( c_1 = 3 \), \( c_2 = 1.5 \), \( \theta_1 = 0.5 \), \( \theta_2 = 0.2 \), \( \mu_1 = 8 \), and \( \mu_2 = 5 \). The results and detailed analysis are given below.

5.3.1. Impact of Transportation Cost \( t \) with Fixed Capacity. Figure 7(a) presents that, with the increasing of transportation cost, the total cost increases and the offline market share decreases. Figure 7(b) presents that the offline waiting time decreases and the online waiting time increases as transportation cost grows. From Figures 7(a) and 7(b), we know that some patients will turn to online channel for the first diagnosis when transportation cost is relatively high. Meanwhile, the number of mistreated patients is also increased, so the online waiting time increasing rate is higher than the offline waiting time decreasing rate.

5.3.2. Impact of Mistreatment Rate \( k \) with Fixed Capacity. Figure 8(a) illustrates that when the mistreatment rate is relatively low, the total cost increases slowly, and the corresponding waiting time also increases slowly. This is because the system is relatively idle with low mistreatment rate, and the dual-channel service system can adjust itself automatically.
Some patients transfer to the offline channel for treatment with the increase of mistreatment rate. The total cost and waiting time increase rapidly when the mistreatment rate increases to a certain degree. However, we should notice that the offline market share decreased with the increase of online mistreatment rate. It is mainly because the market share $x_0$ in our model refers to the proportion of patients going to the offline channel for the first diagnosis. As the mistreatment rate increases, there are more patients going to the offline channel for retreatment. To avoid the congestion of the offline channel, some patients transfer to online channel for the first diagnosis. So, the offline market share decreases with the increase of mistreatment rate. In one word, mistreatment rate does not always have great influence on the total cost of the dual-channel service system, and it is of great benefit for the hospital to keep the mistreatment rate under a certain value.

5.3.3. Impact of Total Arrival Rate $\Lambda$ with Fixed Capacity.

As the total arrival rate increases, both online and offline arrival rates increase. Figure 9(a) indicates that offline market share is increased, and this means more patients go to offline channel for the first diagnosis. The total cost and waiting times increase slowly at the early stage due to the dual-channel service system's automatic adjustability. When the total arrival rate reaches a certain value, the system reaches a saturated state and loses its adjustability, and the total cost increases rapidly, while the market share stays the same.

6. Conclusions

With the development of information technology, telemedicine service becomes more and more popular in the healthcare system. In this paper, we analyze the impact of telemedicine on healthcare service system and compare three healthcare systems. In particular, we consider the patients' choice between the offline and online channels for the first diagnosis. Our analysis reveals that telemedicine in some cases can help reduce the total cost of the healthcare system as well as the patients' waiting time.

We find that the hospital should not provide telemedicine service in some situations, which is similar to the findings of Tarakci et al. [21]. However, their results that a policy treating all patients via telemedicine is never optimal can be extended under some conditions. Our results show that, in some given situations, the gatekeeper system can be the optimal strategy. In most cases, the dual-channel service system is the optimal strategy, and it is a synthesis of the traditional outpatient system and the gatekeeper system.

The dual-channel service system allows patients to choose medical services according to their utility, while the hospital can decide the market segmentation by balancing the offline and online service capacity. The results of our study suggest that there is an optimal market segmentation between the online and offline systems. So, the hospital can adjust the online and offline service capacities to achieve the lowest total cost of the dual-channel service system.

There are many factors affecting the total cost of the healthcare system. Telemedicine service offers patients more convenience by allowing them to get access to medical service in the community healthcare center or even at their homes through digital devices. Data from many studies show that more than half of the outpatients currently treated in general hospitals can be well treated with telemedicine services [14, 17, 44]. Our results show that a higher transportation cost or a large gap between online and offline waiting sensitivities tilts the hospital's optimal choice in favor of telemedicine services. However, if the telemedicine service has a high mistreatment rate as well as high service capacity cost, it is better not to implement telemedicine services.
7. Limitations and Future Scope of Work

There are several limitations in this work. For example, in real life, patients can prognosticate the complexity of their disease and choose the proper service channel rather than make choices only according to their transportation cost and waiting cost. In addition, we take the mistreatment rate as an exogenous variable and ignore the improvement of telemedicine technical level since it can also affect the mistreatment rate. At last, we have not considered the effect of doctors’ workloads during the performance. These simplifications are made to obtain tractable analytical results.

There are several avenues for future research. First, since many commercial companies providing telemedicine service are independent of any general hospital, we can further discuss the competition between hospitals and telemedicine companies. Second, we suppose that all patients have the same priority in the queue, and the hospital provides service to patients based on the first-come-first-serve strategy. In practice, the referral patients may have higher priority and can be served in negligible waiting time.

Appendix

A.

A.1. Proof of Lemma 1. Submit $W_1^F = 1/(\mu_1^F - \Lambda)$ into (2) and replace the variable $\mu_1^F$ with $W_1^F$; then we get $TC^F(W_1^F) = \theta_1 \Lambda W_1^F + c_1/W_1^F + c_1 \Lambda + (1/2) \Lambda$. Taking the second derivative of $TC^F(W_1^F)$ with respect to $W_1^F$, we get $\partial^2 TC^F(W_1^F)/\partial W_1^F = 2c_1/W_1^F > 0$, which indicates that the function $TC^F(W_1^F)$ is convex to $W_1^F$. Then, we obtain the minimum total cost

$$
TC^F = 2\sqrt{\theta_1 c_1 / \Lambda} + (1/2) t \Lambda + c_1 \Lambda \quad \text{when} \quad W_1^F = \sqrt{c_1 / \theta_1 / \Lambda},
$$

and the optimal offline service capacity $\mu_1^{F*} = \Lambda + \sqrt{c_1 / \theta_1 / \Lambda}$.

A.2. Proof of Proposition 2. Taking the first-order derivatives of $TC^F*$ and $PC^F*$ with respect to $c_1$, we have $\partial TC^F*/\partial c_1 = 2(\sqrt{\theta_1 c_1 / \Lambda}/\partial c_1) + c_1$ and $\partial PC^F*/\partial c_1 = \partial(\theta_1 c_1 / \Lambda)/\partial c_1$. Hence, $\partial TC^F*/\partial c_1 > \partial PC^F*/\partial c_1 > 0$.

A.3. Proof of Lemma 4. Replace the decision variables $(\mu_1^*, \mu_2^*)$ with $(W_1^*, W_2^*)$; thus we get

$$\min \quad TC^T(W_1^T, W_2^T)$$

$$= \theta_2 \Lambda W_2^T + c_2/W_2^T + k \theta_1 W_1^T + c_1/W_1^T + 1/2 k t \Lambda \quad (A.1)$$

$$+ c_k \Lambda + c_2 \Lambda.$$

Hence, we obtain $\partial TC^T(W_1^T, W_2^T)/\partial W_1^T > 0$, $\partial TC^T(W_1^T, W_2^T)/\partial W_2^T > 0$, and $\partial TC^T(W_1^T, W_2^T)/\partial W_1^T \partial W_2^T = \partial TC^T(W_1^T, W_2^T)/\partial W_2^T \partial W_1^T = 0$. The Hessian matrix of $TC^T(W_1^T, W_2^T)$ is positive definite. Hence, $TC^T(W_1^T, W_2^T)$ is jointly convex in $W_1^T$ and $W_2^T$. Thus, there exists a minimum solution $W_1^{T*} = \sqrt{c_1 / \theta_1 / k \Lambda}$ and $W_2^{T*} = \sqrt{c_2 / \theta_2 / \Lambda}$. Then we have $\mu_1^{T*} = k \Lambda + \sqrt{c_1 / \theta_1 / k \Lambda / c_1}$, $\mu_2^{T*} = \Lambda + \sqrt{c_2 / \theta_2 / \Lambda / c_2}$, and $TC^{T*}_{\text{min}} = 2 \sqrt{\theta_1 c_1 / \Lambda} + 2 \sqrt{\theta_2 c_2 / \Lambda} + (1/2) k t \Lambda + c_k \Lambda + c_2 \Lambda$.

A.4. Proof of Lemma 6. The second-order partial derivatives $\partial^2 TC^D*/\partial W_1^D = (-\theta_1^2 / t)(1 - k) + 2c_1(1/W_1^D)$ and $\partial^2 TC^D*/\partial W_2^D = (-\theta_1^2 / t)(1 - k) + 2c_2(1/W_2^D)$ are reduction functions. In the constraint feasible domain $0 < (1/(1 - k)) \theta_1 W_2^D - \theta_1 W_1^D < t$, for any $W_1^D$, there exists $\partial TC^D*/\partial W_2^D > 0$. This means that the system total cost first decreases and then increases and at last decreases along with the increase of $W_2^D$. When $k > c_1 - c_2 - c_2$, for any $W_2^D$, there exists $\partial TC^D*/\partial W_2^D > 0$, so the system cost first decreases and then increases and at last decreases along with the increase of $W_2^D$. If $k > c_1 - c_2 - c_2$, there exists a unique minimum that consists of the optimal service rates of the dual-channel service and the corresponding minimal system cost for all possible values of $\mu_1^D$ and $\mu_2^D$. As $\partial^2 TC^D*/\partial W_1^D \partial W_2^D = \partial^2 TC^D*/\partial W_2^D \partial W_1^D = \theta_1 \theta_2 / t > 0$, with
the increasing of $W_1^D(W_2^D)$, the first-order partial derivative of $TC^{D^*}$ in $W_2^D(W_1^D)$ increases as well; Hessian matrix of the function $TC_{dual}(W_1, W_2)$ is

$$H_{W_1,W_2} = \begin{bmatrix} \frac{\theta_1 \theta_2}{t} & \frac{\theta_1 \theta_2}{t} \\ \frac{\theta_1 \theta_2}{t} & \frac{\theta_1 \theta_2}{t} \end{bmatrix},$$

(A.4)

and the determinant of the Hessian is given by

$$\det(H) = \frac{\Lambda \theta_2^2}{t} (1-k) + 2c_1 \frac{1}{W_1^3}.$$  

(A.5)

If the Hessian is positive definite, then we know that

$$-\frac{\Lambda \theta_2^2}{t} (1-k) + 2c_1 \frac{1}{W_1^3} > 0$$

(A.6)

Simplifying the equation set, we get $(\Lambda \theta_2^2/2c_1 t(1-k))W_3^3 + (\Lambda \theta_2^2 (1-k)/2c_1)W_1^3 < 1$. Since the optimal solution also must meet the constraints condition $0 < \theta_2 W_2^D/(1-k) - \theta_1 W_1^D < t$, obviously,

$$\frac{\Lambda \theta_2^2}{2c_1 (1-k)} W_2^3 + \frac{\Lambda \theta_2^2}{2c_1 (1-k)} W_1^3 < t$$

(A.7)

and there exists a feasible domain. Then there is an optimal solution if the stagnation point is in this feasible domain.

A.5. Proof of Proposition 7. From (8), we know that the first-order conditions $\partial TC^{D^*}/\partial W_1^D = 0$ and $\partial TC^{D^*}/\partial W_2^D = 0$ both have positive solutions when $tk > (1-k)c_1 - c_2$; denote these solutions as $W_1^{D^*}$ and $W_2^{D^*}$.

A.6. Proof of Proposition 8. From Lemmas 1 and 4, we have

$$TC^{F^*} - TC^{T^*} = 2\sqrt{\theta_2 c_1 (1/2)t + c_1 - 2\sqrt{\theta_2 c_1 k\Lambda + (1/2)t}\Lambda + c_1 k\Lambda + c_2\Lambda}.$$  

If $\sqrt{\Lambda/2}[(1/2)(1-k) + (1-k)k - c_1] > \sqrt{\theta_2 c_1 (1-k)}$, then $TC^{F^*} - TC^{T^*}$, otherwise, $TC^{F^*} < TC^{T^*}$.

(ii) Hence, $\partial(TC^{F^*} - TC^{T^*})/\partial c_2 = -\sqrt{\theta_2 c_1 c_2} - \Lambda < 0$, $\partial(TC^{F^*} - TC^{T^*})/\partial \Lambda = (1/2)(1-k) > 0$, and $\partial(TC^{F^*} - TC^{T^*})/\partial k = -(\sqrt{\theta_2 c_1 k\Lambda + (1/2)t}\Lambda + c_1\Lambda) < 0$.

A.7. Proof of Proposition 10. With Lemmas 4 and 6, we have

$$TC^{D^*} - TC^{T^*}$$

$$= -\Lambda \frac{1-k}{2t} \left( \frac{1}{1-k} \theta_2 W_2^D - \theta_1 W_1^D \right)^2 - \Lambda \frac{1}{t} \left( c_1 - c_2 \right) \left( \frac{1}{1-k} \theta_2 W_2^D - \theta_1 W_1^D \right) + \theta_2 W_2^D \Lambda + k \theta_1 W_1^D \Lambda + c_1 \frac{1}{W_1^D} + c_2 \frac{1}{W_2^D}$$

(A.8)

$$- \left( 2\sqrt{\theta_2 c_1 c_2} + 2\sqrt{\theta_1 c_1 k\Lambda} \right) \geq 0.$$  

On one hand, we can see the optimal solution of $TC^{D^*}$ may be got on the boundary $x_0 = 0$, where $(1/(1-k))\theta_2 W_2^{D^*} - \theta_1 W_1^{D^*} = 0$. In this case, all patients go to the telemedicine center for the first diagnosis. Then we have

$$TC^{D^*} - TC^{T^*} = -\Lambda \frac{1-k}{2t} x_t^2$$

(A.9)

$$+ \Lambda \frac{1}{t} \left( c_1 - c_2 \right) x_t \theta_2 W_2^D + k \theta_1 W_1^D \Lambda + c_1 \frac{1}{W_1^D} + c_2 \frac{1}{W_2^D}$$

Thus, the cost of the dual-channel service system is no less than the gatekeeper system.

From (1), we obtain $(1/(1-k))\theta_2 W_2^{D^*} - \theta_1 W_1^{D^*} = tx$; then

$$TC^{D^*} - TC^{T^*} = -\Lambda \frac{1-k}{2t} x_t^2$$

(A.10)

This equation is a univariate quadratic equation with variable $t$, and the coefficient of quadratic term is negative. Hence, there may exist $TC^{D^*} < TC^{T^*}$.

Data Availability

The data used to support the findings of this study are included within the article.
Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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