Research Article

Price Discrimination in Dynamic Cournot Competition

Wei-li Zhang, Qi-Qing Song , and Yi-Rong Jiang

College of Science, Guilin University of Technology, Guilin 541004, China

Correspondence should be addressed to Qi-Qing Song; songqiqing@126.com

Received 23 March 2019; Revised 29 May 2019; Accepted 9 June 2019; Published 25 June 2019

Academic Editor: Miguel Ángel López

Copyright © 2019 Wei-li Zhang et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This paper introduces a new Cournot duopoly game and gives an applied study for price discrimination in a market by dynamic methods. One of two oligopolies has two different prices for a homogeneous product, while the other charges one kind of price. It is found that there is only one stable equilibrium for the discrete dynamic system, and a corresponding stable condition is given. Using a discriminative price is not always beneficial to a firm in equilibrium. If both oligopolies carry out price discrimination, the market's average price is lower than when only one oligopoly does it. The results are verified by numerical simulations.

1. Introduction

Cournot duopoly assumes that there are two oligopolists who compete in a market by offering a homogeneous commodity. This was introduced by Cournot [1] and has become a basic model in modern microeconomics and game theory. Cournot duopoly forms a dynamic as a consequence of oligopolists adjusting their strategies based on the result of the last period.

Since Cournot dynamic competition is related to the behavior of consumers, expectations of manufacturer demands, the number of oligopolists, and so on, there are extensive works from different points of view to study Cournot competition. For instance, see the complexity of solutions for Cournot duopoly [2–5] and Cournot games with three oligopolists [6], and see Cournot competition with incomplete information [7–9]. In [10, 11], Guirao and Rubio introduced Cournot-like models by generalizing Cournot duopoly to n players. In another study, stability results were shown for the model with three and four players who competed with their neighbors [12]. For a dynamic duopoly game with heterogeneous expectations for players, stabilities were shown by Agiza and Elsadany [13, 14], Dubiel-Teleszynski [15], and Fanti [16]. Considering bounded rationality, Bischi and Naimzada [17] studied a dynamic duopoly game model where firms adjusted their outputs along the best response direction. Meanwhile, Elsadany gave the necessary and sufficient conditions of the stable region

of the Cournot–Nash equilibrium based on relative profit maximization [18].

In a static situation, Hazledine extended the homogeneous product Cournot–Nash oligopoly model to allow for price discrimination [19], and further results were shown in [20]. Price discrimination is a widely observed phenomenon in a market. For the traditional classification of price discrimination, see the study by Pigou [21]. Typically, when the good or service does have a price tag, there are consumers who pay this amount and there are other consumers who pay less. For instance, passengers taking the same airline may have different ticket fees, and it is common that a person who books a ticket early will get a lower price. Hotels also charge different prices according to reservation times. Roughly, price discrimination implies that a service is sold at different prices to different groups according to their willingness to pay; i.e., to some extent, price discrimination is driven by heterogeneous consumers or consumers with different demand elasticities. In [19, 20], each airline i ($i = 1, 2, \dots, n$) segments the customers as K groups and sells q_i^k tickets to the group k ($\leq K$). The tickets for the fare class k are sold at price $P^k = 1 - Q^1 - Q^2 - \dots - Q^k$, where Q^k is the number of seats sold at price P^k . Following the same demand function, Kutlu gave an analysis of price discrimination in Stackelberg competition with two players [22], in which the leader only uses one kind of price and the follower executes K different prices.

In addition, in [19], the average price is independent of the extent of price discrimination. It was generalized to a larger class of demand functions as $P^k = \alpha - \beta(Q^1 + Q^2 + \dots + Q^k)^r$ by Kutlu [23] and to a situation with asymmetric costs by Mukherjee [24] and Bak and Klecz-Simon [25].

Noting that the current dynamic games have no consideration of price discrimination, in the present paper, we examine the equilibrium in a discrete dynamic Cournot game with price discrimination, where players adjust their strategies through local estimates of marginal profits. All equilibrium points of the dynamic system are unstable except for one Nash equilibrium. The sufficient condition to guarantee the local stability of the equilibrium is given. It is shown that, compared with a firm with only one kind of price, a firm does not necessarily benefit more from having two different prices. Numerical examples are presented.

The rest of this article is organized as follows. Section 2 presents the dynamic model with price discrimination. Section 3 investigates the property of all equilibrium points. Section 4 shows numerical simulation results to verify the main results in Section 3. Section 5 presents our conclusion and discussion.

2. Dynamic Cournot Model with Price Discrimination

This section aims to establish a dynamic Cournot model with price discrimination. Suppose that there are two firms in a market and they sell a homogeneous product with a constant marginal cost $c \in (0, 1)$. Firm 1 divides the consumers into two groups, group 1 and group 2, corresponding to paying high prices and low prices for the product, respectively. Firm 2 thinks that there is only one group and sells the good with a low price. The price function (inverse demand function) of the market is given as below:

$$p^k = \begin{cases} 1 - q_1^1, & k = 1; \\ 1 - q_1^1 - \sum_{i=1}^2 q_i^2, & k = 2, \end{cases} \quad (1)$$

where p^k denotes the price of the good for the group k , q_i^k indicates the sale quantity (strategy) of firm i for group k , and the inverse demand function coincides with that in [19, 20, 23] for two groups. It is clear that $p^2 \leq p^1$. In this situation, it can be assumed that firm 1 has price discrimination by setting a higher price for the same product. For the other situation, where firm 2 sells the good with a high price, the corresponding price function is

$$p'^k = \begin{cases} 1 - \sum_{i=1}^2 q_i^1, & k = 1; \\ 1 - \sum_{i=1}^2 q_i^1 - q_2^2, & k = 2. \end{cases} \quad (2)$$

Then, firm 2 also has price discrimination. The corresponding results will be discussed in the last section of this paper. According to (1), the profits of firms 1 and 2, Π_1 and Π_2 , can be written as follows:

$$\Pi_1 = q_1^1 (1 - q_1^1 - c) + q_2^2 (1 - q_1^1 - q_2^2 - c) \quad (3)$$

and

$$\Pi_2 = q_2^2 (1 - q_1^1 - q_2^2 - c). \quad (4)$$

The decision of each firm is made in discrete time. For any discrete adjustment process, the firm i adjusts its strategy to $q_i^k(t+1)$ in t period from $q_i^k(t)$. We denote IK as the set $\{(1, 1), (1, 2), (2, 2)\}$. The micro-dynamic adjustment equation is given as follows:

$$q_i^k(t+1) = q_i^k(t) + \alpha_i^k \cdot q_i^k(t) \cdot \frac{\partial \Pi_i}{\partial q_i^k(t)}, \quad (i, k) \in IK, \quad (5)$$

where α_i^k represents the rate of adjustment and generally takes a number in the interval $(0, 1)$. Due to a myopic decision and bounded rationality, in each period, the firm i adjusts its sale quantity along with the fastest direction for increasing its profit. The partial derivative, $\partial \Pi_i / \partial q_i^k(t)$, in system (5) is written as follows:

$$\begin{aligned} \frac{\partial \Pi_1}{\partial q_1^1(t)} &= 1 - c - 2q_1^1(t) - q_2^2(t) \\ \frac{\partial \Pi_1}{\partial q_1^2(t)} &= 1 - c - q_1^1(t) - 2q_1^2(t) - q_2^2(t) \\ \frac{\partial \Pi_2}{\partial q_2^2(t)} &= 1 - c - q_1^1(t) - q_2^2(t) - 2q_2^2(t). \end{aligned} \quad (6)$$

To find the stationary points of system (5), let $q_i^k(t+1) = q_i^k(t)$. Then, we have

$$q_i^k(t) \frac{\partial \Pi_i}{\partial q_i^k(t)} = 0, \quad (i, k) \in IK. \quad (7)$$

There are three cases for the solutions of (7).

Case 1 ($q_i^k(t) = 0$ for all $(i, k) \in IK$). Clearly, the solution of (7) is $(\bar{q}_1^1, \bar{q}_1^2, \bar{q}_2^2) = (0, 0, 0)$.

Case 2 ($q_i^k(t) \neq 0$ and $\partial \Pi_i / \partial q_i^k(t) = 0$ for all $(i, k) \in IK$). We have

$$\begin{aligned} 1 - c - 2q_1^1(t) - q_2^2(t) &= 0 \\ 1 - c - q_1^1(t) - 2q_1^2(t) - q_2^2(t) &= 0 \\ 1 - c - q_1^1(t) - q_2^2(t) - 2q_2^2(t) &= 0. \end{aligned} \quad (8)$$

It follows that the solution of (7) satisfies

$$(\bar{q}_1^1, \bar{q}_1^2, \bar{q}_2^2) = \left(\frac{2}{5}(1-c), \frac{1}{5}(1-c), \frac{1}{5}(1-c) \right). \quad (9)$$

This is the static Nash equilibrium for the game of the two firms with strategies (q_1^1, q_1^2) and q_2^2 . The profits of the two firms at the equilibrium are

$$\Pi_1(\bar{q}_1^1, \bar{q}_1^2, \bar{q}_2^2) = \frac{7}{5^2}(1-c)^2 \quad (10)$$

and

$$\Pi_2(\bar{q}_1^1, \bar{q}_1^2, \bar{q}_2^2) = \frac{1}{5^2}(1-c)^2. \quad (11)$$

The average price of the market is

$$p_v = \frac{\left(\left(\frac{7}{5}\right)^2(1-c)^2 + \left(\frac{1}{5}\right)^2(1-c)^2\right)}{\left(\left(\frac{4}{5}\right)(1-c)\right)} = \frac{2}{5}(1-c). \quad (12)$$

Then, the market share of firm 1 is three times that of firm 2 at the equilibrium. Moreover, firm 1 benefits much more than firm 2 from selling the good with two different prices.

Case 3 (others). The solution $(\bar{q}_1^1, \bar{q}_1^2, \bar{q}_2^2)$ of (7) consists of the following set:

$$\left\{ \left(\frac{1-c}{2}, 0, 0 \right), \left(0, \frac{1-c}{2}, 0 \right), \left(0, 0, \frac{1-c}{2} \right), \right. \\ \left. \left(\frac{1-c}{3}, \frac{1-c}{3}, 0 \right), \left(0, \frac{1-c}{3}, \frac{1-c}{3} \right), \right. \\ \left. \left(\frac{1-c}{2}, 0, \frac{1-c}{4} \right) \right\}. \quad (13)$$

To analyze the stability of these stationary points, the following two lemmas are needed.

Lemma 1. *The spectral radius $\rho(A)$ of a matrix A is not greater than any norm of A .*

Lemma 2 (see [26]). *Given a polynomial equation $ax^3 + bx^2 + ex + d = 0$ with variable x , let $A = b^2 - 3ae$, $B = be - 9ad$ and $C = e^2 - 3bd$. If $B^2 - 4AC \leq 0$, then the equation has only real roots; if $B^2 - 4AC > 0$, then the equation has complex roots.*

3. Main Results

3.1. Jacobian Matrix of System (5) at Equilibrium Points. From Section 2, we know that the stationary points of system (5) are as follows:

$$\begin{aligned} E_0 &= (0, 0, 0); \\ E_1 &= \left(\frac{1-c}{2}, 0, 0 \right); \\ E_2 &= \left(0, \frac{1-c}{2}, 0 \right); \\ E_3 &= \left(0, 0, \frac{1-c}{2} \right); \\ E_4 &= \left(\frac{1-c}{3}, \frac{1-c}{3}, 0 \right); \\ E_5 &= \left(0, \frac{1-c}{3}, \frac{1-c}{3} \right); \\ E_6 &= \left(\frac{1-c}{2}, 0, \frac{1-c}{4} \right); \\ E_7 &= \left(\frac{2(1-c)}{5}, \frac{1-c}{5}, \frac{1-c}{5} \right). \end{aligned} \quad (14)$$

System (5) can be rewritten as follows:

$$\begin{aligned} q_1^1(t+1) &= q_1^1(t) + \alpha_1^1 q_1^1(t) (1-c - 2q_1^1(t) - q_1^2(t)) \\ q_1^2(t+1) &= q_1^2(t) + \alpha_1^2 q_1^2(t) (1-c - q_1^1(t) - q_1^2(t) - 2q_2^2(t)) \end{aligned}$$

$$\begin{aligned} &= q_1^2(t) + \alpha_1^2 q_1^2(t) (1-c - q_1^1(t) - 2q_1^2(t) - q_2^2(t)) \\ q_2^2(t+1) &= q_2^2(t) + \alpha_2^2 q_2^2(t) (1-c - q_1^1(t) - q_1^2(t) - 2q_2^2(t)). \end{aligned} \quad (15)$$

Then, the Jacobian matrix of system (15) is calculated as

$$J(q_1^1, q_1^2, q_2^2) = \begin{bmatrix} \xi_1 & -\alpha_1^1 q_1^1(t) & 0 \\ -\alpha_1^2 q_1^2(t) & \xi_2 & -\alpha_2^2 q_2^2(t) \\ -\alpha_2^2 q_2^2(t) & -\alpha_2^2 q_2^2(t) & \xi_3 \end{bmatrix}, \quad (16)$$

where ξ_1, ξ_2 , and ξ_3 are given below.

$$\xi_1 = 1 + \alpha_1^1 (1-c - q_1^2(t)) - 4\alpha_1^1 q_1^1(t), \quad (17)$$

$$\xi_2 = 1 + \alpha_1^2 (1-c - q_1^1(t) - q_2^2(t)) - 4\alpha_1^2 q_1^2(t),$$

and

$$\xi_3 = 1 + \alpha_2^2 (1-c - q_1^1(t) - q_1^2(t)) - 4\alpha_2^2 q_2^2(t). \quad (18)$$

It is known that if the modulus of all the eigenvalues of J at a stationary (equilibrium) point of system (15) is less than 1, then the stationary point is locally asymptotically stable; if there exists an eigenvalue of J at a stationary point such that its modulus is greater than 1, the point is unstable.

3.2. Stability Results for Stationary Points

Theorem 3. *The stationary point E_i of system (15) is unstable for each $i = 0, 1, \dots, 6$.*

Proof. (i) At the stationary point $E_0 = (0, 0, 0)$, the Jacobian matrix becomes

$$J(E_0) = \begin{bmatrix} 1 + \alpha_1^1(1-c) & 0 & 0 \\ 0 & 1 + \alpha_1^2(1-c) & 0 \\ 0 & 0 & 1 + \alpha_2^2(1-c) \end{bmatrix}. \quad (19)$$

Clearly, three eigenvalues of $J(E_0)$ are $\lambda_1 = 1 + \alpha_1^1(1-c)$, $\lambda_2 = 1 + \alpha_1^2(1-c)$, and $\lambda_3 = 1 + \alpha_2^2(1-c)$. Since $\alpha_1^1, \alpha_1^2, \alpha_2^2 \in (0, 1)$, and $c \in (0, 1)$, we have $\lambda_i > 1$ for each $i = 1, 2, 3$. Then, the point E_0 is unstable.

(ii) For the equilibrium point $E_1 = ((1/2)(1-c), 0, 0)$, the Jacobian matrix is

$$J(E_1) = \begin{bmatrix} 1 - \alpha_1^1(1-c), & -\frac{1}{2}\alpha_1^1(1-c) & 0 \\ 0 & 1 + \frac{1}{2}\alpha_1^2(1-c) & 0 \\ 0 & 0 & 1 + \frac{1}{2}\alpha_2^2(1-c) \end{bmatrix}. \quad (20)$$

The determinant of the matrix $\lambda I - J(E_1)$ (I is the 3×3 identity matrix) follows with

$$|\lambda I - J(E_1)| = \begin{vmatrix} \lambda - (1 - \alpha_1^1(1-c)) & \frac{1}{2}\alpha_1^1(1-c) & 0 \\ 0 & \lambda - \left(1 + \frac{1}{2}\alpha_1^2(1-c)\right) & 0 \\ 0 & 0 & \lambda - \left(1 + \frac{1}{2}\alpha_2^2(1-c)\right) \end{vmatrix}. \quad (21)$$

Three eigenvalues of $J(E_1)$ are λ_1, λ_2 , and λ_3 , with $\lambda_1 = 1 - \alpha_1^1(1-c)$, $\lambda_2 = 1 + (1/2)\alpha_1^2(1-c)$, and $\lambda_3 = 1 + (1/2)\alpha_2^2(1-c)$. The eigenvalues λ_2 and λ_3 are greater than 1. Hence, the equilibrium point is also unstable. Similarly, we can obtain unstable results for E_2 and E_3 .

(iii) J at the equilibrium point E_5 becomes

$$J(E_5) = \begin{bmatrix} 1 + \frac{2}{3}\alpha_1^1(1-c) & 0 & 0 \\ -\frac{1}{3}\alpha_1^2(1-c) & 1 - \frac{2}{3}\alpha_1^2(1-c) & -\frac{1}{3}\alpha_1^2(1-c) \\ -\frac{1}{3}\alpha_2^2(1-c) & -\frac{1}{3}\alpha_2^2(1-c) & 1 - \frac{2}{3}\alpha_2^2(1-c) \end{bmatrix}. \quad (22)$$

It is easy to check that one of the eigenvalues of $J(E_5)$ is $\lambda_1 = 1 + (2/3)\alpha_1^1(1-c) > 1$ by noting that $c \in (0, 1)$. Hence, the equilibrium point is also unstable. Similarly, we can get that E_4 is unstable.

(iv) J at the point E_6 is

$$J(E_6) = \begin{bmatrix} 1 - \alpha_1^1(1-c) & -\frac{1}{2}\alpha_1^1(1-c) & 0 \\ 0 & 1 + \frac{1}{4}\alpha_1^2(1-c) & 0 \\ -\frac{1}{4}\alpha_2^2(1-c) & -\frac{1}{4}\alpha_2^2(1-c) & 1 - \frac{1}{2}\alpha_2^2(1-c) \end{bmatrix}. \quad (23)$$

The determinant of the matrix $\lambda I - J(E_6)$ is

$$|\lambda I - J(E_6)| = \begin{vmatrix} \lambda - (1 - \alpha_1^1(1-c)) & \frac{1}{2}\alpha_1^1(1-c) & 0 \\ 0 & \lambda - \left(1 + \frac{1}{4}\alpha_1^2(1-c)\right) & 0 \\ \frac{1}{4}\alpha_2^2(1-c) & \frac{1}{4}\alpha_2^2(1-c) & \lambda - \left(1 - \frac{1}{2}\alpha_2^2(1-c)\right) \end{vmatrix}. \quad (24)$$

Obviously, we can check that the three eigenvalues of $J(E_6)$ are $\lambda_1 = 1 - \alpha_1^1(1-c)$, $\lambda_2 = 1 + (1/4)\alpha_1^2(1-c)$, and $\lambda_3 = 1 - (1/2)\alpha_2^2(1-c)$. Then, $\lambda_2 > 1$; hence, E_6 is unstable.

From the above (i), (ii), (iii), and (iv), the equilibrium solutions E_i are unstable for each $i = 0, 1, \dots, 6$. These complete the proof. $\square$

Theorem 4. *There exists $\alpha > 0$ such that the spectral radii of $J(E_7)$ and $\rho(J(E_7))$ satisfy $\rho(J(E_7)) < 1$ if*

$$\begin{aligned} (\alpha_1^1, \alpha_1^2, \alpha_2^2) \in \Omega_\alpha = \{(\alpha_1^1, \alpha_1^2, \alpha_2^2) \mid 0 < \alpha_1^1, \alpha_1^2, \alpha_2^2 \\ < \alpha, B^2 - 4AC \leq 0\}, \end{aligned} \quad (25)$$

where $A = b^2 - 3e$, $B = be - 9d$, and $C = e^2 - 3bd$ with $b = 4\alpha_1^1 + 2\alpha_1^2 + 2\alpha_2^2$, $e = 6\alpha_1^1\alpha_1^2 + 8\alpha_1^1\alpha_2^2 + 3\alpha_1^2\alpha_2^2$, and $d = 10\alpha_1^1\alpha_1^2\alpha_2^2$.

Proof. The Jacobian matrix J at the stationary point $E_7 = ((2/5)(1-c), (1/5)(1-c), (1/5)(1-c))$ is given below:

$$J(E_7) = \begin{bmatrix} 1 - \frac{4}{5}\alpha_1^1(1-c) & -\frac{2}{5}\alpha_1^1(1-c) & 0 \\ -\frac{1}{5}\alpha_1^2(1-c) & 1 - \frac{2}{5}\alpha_1^2(1-c) & -\frac{1}{5}\alpha_1^2(1-c) \\ -\frac{1}{5}\alpha_2^2(1-c) & -\frac{1}{5}\alpha_2^2(1-c) & 1 - \frac{2}{5}\alpha_2^2(1-c) \end{bmatrix}. \quad (26)$$

The matrix norm of $J(E_7)$ is written as

$$\begin{aligned} \|J(E_7)\|_\infty &= \max \left\{ \left|1 - \frac{4}{5}\alpha_1^1(1-c)\right| + \left|-\frac{2}{5}\alpha_1^1(1-c)\right| + 0, \right. \\ &\quad \left. \left|-\frac{1}{5}\alpha_1^2(1-c)\right| + \left|1 - \frac{2}{5}\alpha_1^2(1-c)\right| + \left|-\frac{1}{5}\alpha_1^2(1-c)\right|, \right. \\ &\quad \left. \left|-\frac{1}{5}\alpha_2^2(1-c)\right| + \left|-\frac{1}{5}\alpha_2^2(1-c)\right| + \left|1 - \frac{2}{5}\alpha_2^2(1-c)\right| \right\}. \end{aligned} \quad (27)$$

Then, it holds that

$$\|J(E_7)\|_\infty = \max \left\{ 1 - \frac{2}{5}\alpha_1^1(1-c), 1, 1 \right\} = 1. \quad (28)$$

By Lemma 1, the spectral radius $\rho(J_7)$ is not greater than the norm $\|J(E_7)\|_\infty$; that is, $\rho(J_7) \leq 1$. Hence, the modulus of each eigenvalue of $J(E_7)$ is no more than 1.

Next, we check the eigenvalues of $J(E_7)$. Consider the determinant of the matrix $|\lambda I - J(E_7)|$,

$$|\lambda I - J(E_7)| = \begin{vmatrix} \lambda - 1 + \frac{4}{5}\alpha_1^1(1-c) & \frac{2}{5}\alpha_1^1(1-c) & 0 \\ \frac{1}{5}\alpha_1^2(1-c) & \lambda - 1 + \frac{2}{5}\alpha_1^2(1-c) & \frac{1}{5}\alpha_1^2(1-c) \\ \frac{1}{5}\alpha_2^2(1-c) & \frac{1}{5}\alpha_2^2(1-c) & \lambda - 1 + \frac{2}{5}\alpha_2^2(1-c) \end{vmatrix}. \quad (29)$$

Let $\lambda - 1 = ((1-c)/5)t$. Then, the polynomial eigenvalue of $J(E_7)$ can be written as

$$|\lambda I - J(E_7)| = \frac{(1-c)^3}{5^3} \begin{vmatrix} t + 4\alpha_1^1 & 2\alpha_1^1 & 0 \\ \alpha_1^2 & t + 2\alpha_1^2 & \alpha_1^2 \\ \alpha_2^2 & \alpha_2^2 & t + 2\alpha_2^2 \end{vmatrix} \quad (30)$$

= 0.

This is equivalent to

$$\begin{vmatrix} t + 4\alpha_1^1 & 2\alpha_1^1 & 0 \\ \alpha_1^2 & t + 2\alpha_1^2 & \alpha_1^2 \\ \alpha_2^2 & \alpha_2^2 & t + 2\alpha_2^2 \end{vmatrix} = 0. \quad (31)$$

Furthermore, the left-hand side of (31) is as follows:

$$\begin{aligned} & \begin{vmatrix} t + 4\alpha_1^1 & 2\alpha_1^1 & 0 \\ \alpha_1^2 & t + 2\alpha_1^2 & \alpha_1^2 \\ \alpha_2^2 & \alpha_2^2 & t + 2\alpha_2^2 \end{vmatrix} \\ &= (t + 4\alpha_1^1) \begin{vmatrix} t + 2\alpha_1^2 & \alpha_1^2 \\ \alpha_2^2 & t + 2\alpha_2^2 \end{vmatrix} - 2\alpha_1^1 \begin{vmatrix} \alpha_1^2 & \alpha_1^2 \\ \alpha_2^2 & t + 2\alpha_2^2 \end{vmatrix} \\ &= (t + 4\alpha_1^1) [(t + 2\alpha_1^2)(t + 2\alpha_2^2) - \alpha_1^2\alpha_2^2] \\ &\quad - 2\alpha_1^1 [\alpha_1^2(t + 2\alpha_2^2) - \alpha_1^2\alpha_2^2] \\ &= t^3 + (4\alpha_1^1 + 2\alpha_1^2 + 2\alpha_2^2)t^2 \\ &\quad + (6\alpha_1^1\alpha_1^2 + 8\alpha_1^1\alpha_2^2 + 3\alpha_1^2\alpha_2^2)t + 10\alpha_1^1\alpha_1^2\alpha_2^2. \end{aligned} \quad (32)$$

Let $a = 1$, $b = 4\alpha_1^1 + 2\alpha_1^2 + 2\alpha_2^2$, $e = 6\alpha_1^1\alpha_1^2 + 8\alpha_1^1\alpha_2^2 + 3\alpha_1^2\alpha_2^2$, and $d = 10\alpha_1^1\alpha_1^2\alpha_2^2$. Then, from (32), (31) can be written as

$$at^3 + bt^2 + et + d = 0. \quad (33)$$

The next steps, (i) and (ii), show that the eigenvalues are not 1 and -1 .

(i) Suppose that $\lambda = 1$; i.e., $t = 0$. Equation (33) is

$$d = 10\alpha_1^1\alpha_1^2\alpha_2^2. \quad (34)$$

This contradicts $\alpha_1^1\alpha_1^2\alpha_2^2 > 0$. Thus, $\lambda \neq 1$.

(ii) Assuming that $\lambda = -1$, we have $t = -10/(1-c)$. In this case, (33) is

$$100(1-c)b - 10(1-c)^2e + d(1-c)^3 = 1000. \quad (35)$$

When α_1^2 , α_1^1 , and α_2^2 take the value sufficiently near 0, the left-hand side of (35) is less than 1000. Therefore, there is a small enough $\alpha > 0$ (for instance, α can take 1) such that the value on the left-hand side of (35) is smaller than 1000 if $\alpha_1^2, \alpha_1^1, \alpha_2^2 < \alpha$. This contradicts (35). Therefore, we have $\lambda \neq -1$.

Let $A = b^2 - 3e$, $B = be - 9d$, and $C = e^2 - 3bd$. From Lemma 2, it can be deduced that when $\Delta = B^2 - 4AC \leq 0$, (33) has only real roots. Noting (i) and (ii), we can conclude that for each $(\alpha_1^1, \alpha_1^2, \alpha_2^2) \in \Omega_\alpha$, it holds that $\rho(J(E_7)) < 1$. $\square$

Noting that $\alpha_i^k \in (0, 1)$ for any $(i, k) \in IK$ by the settings in Section 2, from the proof of Theorem 4, we can take $\alpha = 1$ in Theorem 4. Then, the following result related to the stability of the equilibrium point E_7 can be obtained.

Theorem 5. *The stationary point E_7 is locally asymptotically stable for system (15) if $(\alpha_1^1, \alpha_1^2, \alpha_2^2) \in \Omega_1$.*

4. Numerical Simulations

From Theorem 4, there is a range Ω_α for the adjustment rate $(\alpha_1^1, \alpha_1^2, \alpha_2^2)$ that guarantees the stability of system (15) near the stationary point E_7 , where

$$\Omega_\alpha = \{(\alpha_1^1, \alpha_1^2, \alpha_2^2) \mid 0 < \alpha_1^1, \alpha_1^2, \alpha_2^2 < \alpha, \Delta \leq 0\}. \quad (36)$$

Figure 1 shows the simulation result for the value of $\Delta = B^2 - 4AC$ when $(\alpha_1^1, \alpha_1^2, \alpha_2^2) \in \Omega_1$. + symbols represent the cases for $\Delta \leq 0$; * symbols denote cases for $\Delta > 0$. It can be seen that when $\alpha_1^2 < \alpha_1^1$, there is $\Delta = B^2 - 4AC \leq 0$. Then, there are many cases such that $(\alpha_1^1, \alpha_1^2, \alpha_2^2) \in \Omega_1$, which means that system (15) is locally asymptotically stable at stationary point E_7 for parameters with a large range of values.

Numerical solutions for system (15) are shown in Figure 2, where we set the following parameters: $c = 0.075$, $(\alpha_1^1, \alpha_1^2, \alpha_2^2) = (0.01, 0.02, 0.01) \in \Omega_1$.

As shown in Figures 2(a), 2(b), and 2(c), when the initial points are near the corresponding equilibrium points $E_0 = (0, 0, 0)$, $E_1 = ((1/2)(1-c), 0, 0)$, $E_5 = (0, (1/3)(1-c), (1/3)(1-c))$, the solutions deviate from the equilibrium points. This is consistent with Theorem 3. For $E_7 = ((2/5)(1-c), (1/5)(1-c), (1/5)(1-c))$, the solution tends to the equilibrium E_7 , as seen in Figure 2(d), which verifies Theorems 4 and 5.

FIGURE 1: Blue + signs denote cases for $\Delta = B^2 - 4AC \leq 0$; black * signs indicate cases for $\Delta = B^2 - 4AC > 0$. Here, A , B , and C are from Theorem 4.

5. Conclusion and Discussion

We assume that one firm sells the product with a certain price while another firm sells the same good for two different prices. This paper contributes to applied research on price discrimination by introducing a new dynamic Cournot competition model. For the dynamic system, each firm makes a decision with bounded rationality in each discrete period to maximize its profit. The stability analysis shows that only one static Nash equilibrium is stable for the dynamic in a large parameter range. The seven other stationary points of the dynamic system are all unstable. At the stable Nash equilibrium, the firm with two prices benefits greatly due to having a major market share; the firm with only a low price has one-third of the market share of the other firm.

If firm 2 only charges a high price p'^1 in (2), then, eight stationary points $(\bar{q}_1^1, \bar{q}_1^2, \bar{q}_2^1)$ can be obtained for the corresponding dynamic system as follows:

$$\begin{aligned}
 E'_0 &= E_0; \\
 E'_1 &= E_1; \\
 E'_2 &= E_2; \\
 E'_3 &= E_3; \\
 E'_4 &= E_4; \\
 E'_5 &= \left(\frac{1-c}{3}, 0, \frac{1-c}{3} \right); \\
 E'_6 &= \left(0, \frac{1-c}{4}, \frac{1-c}{2} \right); \\
 E'_7 &= \left(\frac{1-c}{5}, \frac{1-c}{5}, \frac{2(1-c)}{5} \right).
 \end{aligned} \tag{37}$$

Similar to the results presented in Sections 2 and 3, E'_7 is the only stable equilibrium point for the dynamic system. The profits of the two firms at E'_7 are

$$\Pi'_1(E'_7) = \frac{3}{5^2} (1-c)^2 \tag{38}$$

and

$$\Pi'_2(E'_7) = \frac{4}{5^2} (1-c)^2. \tag{39}$$

Then, although each firm has the same market share $(2/5)(1-c)$ at the equilibrium E'_7 , firm 1 cannot benefit more than firm 2. The average price is as follows:

$$\begin{aligned}
 p'_v &= \frac{\left(\left(\frac{3}{5^2} \right) (1-c)^2 + \left(\frac{4}{5^2} \right) (1-c)^2 \right)}{\left(\frac{4}{5} \right) (1-c)} \\
 &= \frac{7}{20} (1-c).
 \end{aligned} \tag{40}$$

Comparing (10) and (11) with $\Pi'_1(E'_7)$ and $\Pi'_2(E'_7)$, we see that having two different prices is not always beneficial to a firm. From p_v in (12) and the above p'_v , it holds that $p'_v < p_v$. Therefore, if both firms have a high (discriminative) price for the homogeneous product, the market's average price is lower than when only one firm has a higher price.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflicts of interest.

FIGURE 2: Subgraphs (a), (b), and (c) show the solutions of (15) with an initial value near E_0 , E_2 , and E_6 , respectively. Subgraph (d) displays the solution of (15) with an initial value near E_7 . It can be observed that E_7 is stable and E_i is unstable for $i = 0, 2, 6$. Subgraph titles clarify the initial points of system (15). The parameters are set as follows: $c = 0.075$, $\alpha_1^1 = 0.01$, $\alpha_1^2 = 0.02$, and $\alpha_2^2 = 0.01$.

Acknowledgments

This project is supported by the National Natural Science Foundation of China (11661030, 61763008) and Guangxi Natural Science Foundation (2016GXNSFAA380059).

References

- [1] A. Cournot, *Rsrches sur les Principes Mathmatiques de la thorie de la richesse*, Hachette, Paris, France, 1938.
- [2] D. Rand, "Exotic phenomena in games and duopoly models," *Journal of Mathematical Economics*, vol. 5, no. 2, pp. 173–184, 1978.
- [3] T. Puu, "Chaos in duopoly pricing," *Chaos, Solitons & Fractals*, vol. 1, no. 6, pp. 573–581, 1991.
- [4] G. I. Bischi, C. Mammana, and L. Gardini, "Multistability and cyclic attractors in duopoly games," *Chaos, Solitons & Fractals*, vol. 11, no. 4, pp. 543–564, 2000.
- [5] R.-A. Dana and L. Montrucchio, "Dynamic complexity in duopoly games," *Journal of Economic Theory*, vol. 40, no. 1, pp. 40–56, 1986.

- [6] T. Puu, "Complex dynamics with three oligopolists," *Chaos, Solitons & Fractals*, vol. 7, no. 12, pp. 2075–2081, 1996.
- [7] E. Galor, "Information sharing in oligopoly," *Econometrica*, vol. 53, no. 2, pp. 329–343, 1985.
- [8] J. N. Lagerlof, "Equilibrium uniqueness in a Cournot model with demand uncertainty," *The B.E. Journal of Theoretical Economics*, vol. 6, no. 1, pp. 1–6, 2006.
- [9] E. Einy, O. Haimanko, D. Moreno, and B. Shitovitz, "On the existence of Bayesian Cournot equilibrium," *Games and Economic Behavior*, vol. 68, no. 1, pp. 77–94, 2010.
- [10] J. L. G. Guirao and R. G. Rubio, "Detecting simple dynamics in Cournot-like models," *Journal of Computational and Applied Mathematics*, vol. 233, no. 4, pp. 1091–1095, 2009.
- [11] J. L. Guirao and R. G. Rubio, "Extensions of Cournot duopoly: an applied mathematical view," *Applied Mathematics Letters*, vol. 23, no. 8, pp. 836–838, 2010.
- [12] J. L. G. Guirao, M. A. López, J. Llibre, and R. Martínez, "A note on the equilibria of an economic model with local competition "à la cournot"," *Journal of Computational & Applied Mathematics*, vol. 236, no. 12, pp. 3052–3057, 2012.
- [13] H. N. Agiza and A. A. Elsadany, "Chaotic dynamics in nonlinear duopoly game with heterogeneous players," *Applied Mathematics and Computation*, vol. 149, no. 3, pp. 843–860, 2004.
- [14] A. A. Elsadany, "Dynamics of a delayed duopoly game with bounded rationality," *Mathematical and Computer Modelling*, vol. 52, no. 9–10, pp. 1479–1489, 2010.
- [15] T. Dubiel-Teleszynski, "Nonlinear dynamics in a heterogeneous duopoly game with adjusting players and diseconomies of scale," *Communications in Nonlinear Science and Numerical Simulation*, vol. 16, no. 1, pp. 296–308, 2011.
- [16] L. Fanti, "The dynamics of a banking duopoly with capital regulations," *Economic Modelling*, vol. 37, pp. 340–349, 2014.
- [17] G. I. Bischi and A. Naimzada, "Global analysis of a dynamic duopoly game with bounded rationality," in *Advances in Dynamic Games and Application*, J. A. Filar, V. Gaitsgory, and K. Mizukami, Eds., vol. 5, pp. 361–385, Birkhäuser, Basel, Switzerland, 2000.
- [18] A. A. Elsadany, "Dynamics of a Cournot duopoly game with bounded rationality based on relative profit maximization," *Applied Mathematics and Computation*, vol. 294, pp. 253–263, 2017.
- [19] T. Hazledine, "Price discrimination in Cournot-Nash oligopoly," *Economics Letters*, vol. 93, no. 3, pp. 413–420, 2006.
- [20] T. Hazledine, "Oligopoly price discrimination with many prices," *Economics Letters*, vol. 109, no. 3, pp. 150–153, 2010.
- [21] A. C. Pigou, *The Economics of Welfare*, Macmillan, London, UK, 1920.
- [22] L. Kutlu, "Price discrimination in stackelberg competition*," *The Journal of Industrial Economics*, vol. 57, no. 2, p. 364, 2009.
- [23] L. Kutlu, "Price discrimination in Cournot competition," *Economics Letters*, vol. 117, no. 3, pp. 540–543, 2012.
- [24] A. Mukherjee, "Price discrimination in oligopoly with asymmetric firms," *Economics Bulletin*, vol. 30, pp. 2668–2670, 2010.
- [25] B. Bakó and A. rásKálec-Simon, "Price discrimination in asymmetric Cournot oligopoly," *Economics Letters*, vol. 116, no. 3, pp. 301–303, 2012.
- [26] S. Fan, "A new extracting formula and a new distinguishing means on the one variable cubic equation," *Journal of Hainan Teachers College*, vol. 2, pp. 91–98, 1989 (Chinese).