Research Article

Some New Half-Linear Integral Inequalities on Time Scales and Applications

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The main aim of this paper is to establish some new half-linear integral inequalities on time scales. Our results not only extend and complement some known integral inequalities, but also provide a handy tool for the study of qualitative properties of solutions of some dynamic equations.

1. Introduction

The study of dynamic equations on time scales is now an important object of research and has been extensively studied in recent years (see [1–21]). This is due to the fact that the theory of time scales which was introduced by Stefan Hilger [22] can unify and extend the difference and differential calculus in a consistent way.

Because dynamic inequalities play an important role in the study of qualitative properties of solutions of dynamic equations on time scales, many authors have expounded on various classes of dynamic inequalities in recent years; see [23–41] and the references cited therein. For instance, in 2013, Sun and Hassan [32] investigated the nonlinear integral inequality on time scales

$$y(t) \leq f(t) + g(t) \int_{t_0}^t [a(s)y(s) + b(s)y^\alpha(\sigma(s)) - c(s)y^\beta(\sigma(s))] \Delta s, \quad (1)$$

where $y, f, g, a, b, c : \mathbb{T}^\kappa \rightarrow \mathbb{R}_+$ are rd-continuous functions and α, β are positive constants such that $0 < \alpha < 1 < \beta$.

Tian et al. [37] investigated the nonlinear integral inequality on time scales

$$y^p(t) \leq f(t) + g(t) \int_{t_0}^t [a(s)y^p(s) + b(s)y^q(s)$$

$$+ c(s) + \int_{t_0}^s m(\tau)y^r(\tau)\Delta\tau + h_1(s)y^\alpha(\sigma(s)) - h_2(s)y^\beta(\sigma(s))] \Delta s, \quad t \in \mathbb{T}^\kappa, \quad (2)$$

where $p \geq q > 0, p \geq r > 0, 0 < \alpha < p < \beta, p, q, r, \alpha$ and β are real constants and $y, f, g, a, b, c, m, h_1, h_2 : \mathbb{T}^\kappa \rightarrow \mathbb{R}_+$ are rd-continuous functions.

We note that the inequalities (1) and (2) have been proved in the cases when $0 < \alpha < 1 < \beta$ and $0 < \alpha < p < \beta$, respectively. So it would be interesting to find the explicit bound for u of (2) in the following two cases when $0 < p < \alpha < \beta$ or $0 < \beta < \alpha < p$.

In the present paper, we study some new half-linear integral inequalities on time scales. Our results not only complement the results established in [37] in the sense that the results can be applied in cases when $0 < p < \alpha < \beta$ or $0 < \beta < \alpha < p$, but also furnish a handy tool for the study of qualitative properties of solutions of some complicated dynamic equations.

2. Preliminaries

In what follows, we always assume that $\mathbb{R}_+ = [0, \infty)$, $\mathbb{T}$ is an arbitrary time scale. The following lemmas are useful in the proof of the main results of this paper.

Lemma 1. Let $m > 0, n > 0, p > 0, \alpha > 0$, and $\beta > 0$ be given; then for each $x \geq 0$,

$$mx^\alpha - nx^\beta \leq \frac{m(\beta - \alpha)}{\beta - p} \left(\frac{(\beta - p)n}{(\alpha - p)m} \right)^{(\alpha - p)/(\alpha - \beta)} x^p \quad (3)$$

holds for the cases when $0 < p < \alpha < \beta$ or $0 < \beta < \alpha < p$.

Proof. If $x = 0$, then it is easy to see that the inequality (3) holds. So we only prove that the inequality (3) holds in the case of $x > 0$. For the case $0 < p < \alpha < \beta$, set $F(x) = mx^{\alpha - p} - nx^{\beta - p}$, $x > 0$, where $m > 0$ and $n > 0$. Letting $F'(x) = 0$, we get $x_0 = (m(\alpha - p)/n(\beta - p))^{1/(\beta - \alpha)}$. Since $x \in (0, x_0)$, $F'(x) > 0$; $x \in (x_0, +\infty)$, $F'(x) < 0$, F attains its maximum at $x_0 = (m(\alpha - p)/n(\beta - p))^{1/(\beta - \alpha)}$ and $F_{\max} = F(x_0) = (m(\beta - \alpha)/(\beta - p))((\beta - p)n/(\alpha - p)m)^{(\alpha - p)/(\alpha - \beta)}$. Thus, (3) holds. For the case $0 < \beta < \alpha < p$, by a similar argument with the case $p < \alpha < \beta$, we can get that (3) holds. The proof is complete. $\square$

Lemma 2 (see [42]). Assuming that $y \geq 0, p \geq q \geq 0$, and $p \neq 0$, then for any $K > 0$,

$$y^{q/p} \leq \frac{q}{p} K^{(q-p)/p} y + \frac{p-q}{p} K^{q/p}. \quad (4)$$

Lemma 3 (see [1, Theorem 1.117]). Suppose that for each $\varepsilon > 0$ there exists a neighborhood U of t , independent of $\tau \in [t_0, \sigma(t)]$, such that

$$\begin{aligned} & |w(\sigma(t), \tau) - w(s, \tau) - w_t^\Delta(t, \tau)(\sigma(t) - s)| \\ & \leq \varepsilon |\sigma(t) - s|, \quad s \in U, \end{aligned} \quad (5)$$

where $w : \mathbb{T} \times \mathbb{T}^\kappa \rightarrow \mathbb{R}_+$ is continuous at (t, t) , $t \in \mathbb{T}^\kappa$ with $t > t_0$, and $w_t^\Delta(t, \cdot)$ are rd-continuous on $[t_0, \sigma(t)]$. Then

$$f(t) := \int_{t_0}^t w(t, \tau) \Delta\tau \quad (6)$$

implies

$$f^\Delta(t) = \int_{t_0}^t w_t^\Delta(t, \tau) \Delta\tau + w(\sigma(t), t), \quad t \in \mathbb{T}^\kappa. \quad (7)$$

Lemma 4 (see [1, Theorem 6.1]). Suppose that y and f are rd-continuous functions and $p \in \mathcal{R}^+$. Then

$$y^\Delta(t) \leq p(t)y(t) + f(t), \quad \text{for all } t \in \mathbb{T} \quad (8)$$

implies

$$y(t) \leq y(t_0)e_p(t, t_0) + \int_{t_0}^t e_p(t, \sigma(\tau))f(\tau)\Delta\tau, \quad (9)$$

for all $t \in \mathbb{T}$.

3. Main Results

In this section, we deal with some half-line inequalities on time scales. For convenience, we always assume that $t \geq t_0, t \in \mathbb{T}^\kappa$.

Theorem 5. Assume that $x, f, a, b, c, l, m, n : \mathbb{T}^\kappa \rightarrow \mathbb{R}_+$, $g, h, k : \mathbb{T}^\kappa \rightarrow (0, \infty)$ are rd-continuous functions, $(l/g)^\Delta(t) \geq 0$, $\mu(t)F(t) < 1$ for $t \in \mathbb{T}^\kappa$, $\theta : \mathbb{T}^\kappa \rightarrow \mathbb{T}^\kappa$ is continuous function satisfying $\theta(t) \leq t$ and $\theta(t_0) = t_0$ for $t \in \mathbb{T}^\kappa$, p, q, r, α , and β are constants satisfying (i) $0 \leq q \leq p, 0 \leq r \leq p, p < \alpha < \beta$ or (ii) $0 \leq q \leq p, 0 \leq r \leq p, 0 < \beta < \alpha < p$. Suppose that x satisfies

$$\begin{aligned} x^p(t) & \leq f(t) + g(t) \int_{t_0}^t \left[a(s)x^p(s) + b(s)x^q(s) \right. \\ & \quad \left. + c(s) + m(s) \int_{\theta(t_0)}^{\theta(s)} n(\xi)x^r(\xi)\Delta\xi \right] \Delta s + l(t) \\ & \quad \cdot \int_{t_0}^t [h(s)x^\alpha(\sigma(s)) - k(s)x^\beta(\sigma(s))] \Delta s, \\ & \quad t \in \mathbb{T}^\kappa, \end{aligned} \quad (10)$$

then

$$x(t) \leq \left(f(t) + g(t) \int_{t_0}^t e_{G \oplus L}(t, \sigma(s)) A(s) \Delta s \right)^{1/p}, \quad (11)$$

for any $K_1 > 0, K_2 > 0, t \in \mathbb{T}^\kappa$,

where

$$\begin{aligned} F(t) & := \left(\frac{l}{g} \right)^\Delta(t) \int_{t_0}^t H(s)g(\sigma(s))\Delta s + \frac{l(\sigma(t))}{g(\sigma(t))} \\ & \quad \cdot H(t)g(\sigma(t)), \end{aligned} \quad (12)$$

$$G(t) := \frac{F(t)}{1 - \mu(t)F(t)}, \quad (13)$$

$$\begin{aligned} L(t) & := a(t)g(t) + \frac{q}{p}K_1^{(q-p)/p}b(t)g(t) + \frac{r}{p} \\ & \quad \cdot K_2^{(r-p)/p}m(t) \int_{\theta(t_0)}^{\theta(t)} n(\xi)g(\xi)\Delta\xi, \end{aligned} \quad (14)$$

$$\begin{aligned} M(t) & := a(t)f(t) + \frac{q}{p}K_1^{(q-p)/p}b(t)f(t) + \frac{p-q}{p} \\ & \quad \cdot K_1^{q/p}b(t) + c(t) + m(t) \\ & \quad \cdot \int_{\theta(t_0)}^{\theta(t)} n(\xi) \left[\frac{r}{p}K_2^{(r-p)/p}f(\xi) + \frac{p-r}{p}K_2^{r/p} \right] \Delta\xi \\ & \quad + \left(\frac{l}{g} \right)^\Delta(t) \int_{t_0}^t H(s)f(\sigma(s))\Delta s + \frac{l(\sigma(t))}{g(\sigma(t))}H(t) \\ & \quad \cdot f(\sigma(t)), \end{aligned} \quad (15)$$

$$H(t) := \frac{h(t)(\beta - \alpha)}{\beta - p} \left(\frac{(\beta - p)k(t)}{(\alpha - p)h(t)} \right)^{(\alpha - p)/(\alpha - \beta)}, \quad (16)$$

$$A(t) := (1 + \mu(t)G(t))M(t). \quad (17)$$

Proof. From Lemma 1 and (10), we get that

$$\begin{aligned} x^p(t) &\leq f(t) + g(t) \int_{t_0}^t \left[a(s)x^p(s) + b(s)x^q(s) \right. \\ &\quad \left. + c(s) + m(s) \int_{\theta(t_0)}^{\theta(s)} n(\xi)x^r(\xi)\Delta\xi \right] \Delta s + l(t) \\ &\quad \cdot \int_{t_0}^t \left[h(s)x^\alpha(\sigma(s)) - k(s)x^\beta(\sigma(s)) \right] \Delta s \\ &\leq f(t) + g(t) \int_{t_0}^t \left[a(s)x^p(s) + b(s)x^q(s) + c(s) \right. \\ &\quad \left. + m(s) \int_{\theta(t_0)}^{\theta(s)} n(\xi)x^r(\xi)\Delta\xi \right] \Delta s + l(t) \\ &\quad \cdot \int_{t_0}^t H(s)x^p(\sigma(s))\Delta s, \quad t \in \mathbb{T}^\kappa, \end{aligned} \quad (18)$$

where $H(t)$ is defined as in (16). Denote

$$\begin{aligned} z(t) &= \int_{t_0}^t \left[a(s)x^p(s) + b(s)x^q(s) + c(s) \right. \\ &\quad \left. + m(s) \int_{\theta(t_0)}^{\theta(s)} n(\xi)x^r(\xi)\Delta\xi \right] \Delta s + \frac{l(t)}{g(t)} \\ &\quad \cdot \int_{t_0}^t H(s)x^p(\sigma(s))\Delta s, \quad t \in \mathbb{T}^\kappa. \end{aligned} \quad (19)$$

From the assumptions on $x, a, b, c, m, n, \theta, g, l, H$, (18) and (19), we obtain that z is nondecreasing and

$$x(t) \leq (f(t) + g(t)z(t))^{1/p}, \quad t \in \mathbb{T}^\kappa. \quad (20)$$

Combining (19) and (20), we have that

$$\begin{aligned} z^\Delta(t) &= a(t)x^p(t) + b(t)x^q(t) + c(t) + m(t) \\ &\quad \cdot \int_{\theta(t_0)}^{\theta(t)} n(\xi)x^r(\xi)\Delta\xi + \left(\frac{l}{g} \right)^\Delta(t) \end{aligned}$$

$$\begin{aligned} &\cdot \int_{t_0}^t H(s)x^p(\sigma(s))\Delta s + \frac{l(\sigma(t))}{g(\sigma(t))}H(t)x^p(\sigma(t)) \\ &\leq a(t)(f(t) + g(t)z(t)) + b(t) \\ &\quad \cdot (f(t) + g(t)z(t))^{q/p} + c(t) + m(t) \\ &\quad \cdot \int_{\theta(t_0)}^{\theta(t)} n(\xi)(f(\xi) + g(\xi)z(\xi))^{r/p}\Delta\xi + \left(\frac{l}{g} \right)^\Delta(t) \\ &\quad \cdot \int_{t_0}^t H(s)(f(\sigma(s)) + g(\sigma(s))z(\sigma(s)))\Delta s \\ &\quad + \frac{l(\sigma(t))}{g(\sigma(t))}H(t)(f(\sigma(t)) + g(\sigma(t))z(\sigma(t))), \\ &\quad t \in \mathbb{T}^\kappa. \end{aligned} \quad (21)$$

Applying Lemma 2 on the right side of (21), we get that

$$\begin{aligned} z^\Delta(t) &\leq a(t)(f(t) + g(t)z(t)) + b(t) \left[\frac{q}{p} \right. \\ &\quad \cdot K_1^{(q-p)/p}(f(t) + g(t)z(t)) + \frac{p-q}{p}K_1^{q/p} \left. \right] + c(t) \\ &\quad + m(t) \int_{\theta(t_0)}^{\theta(t)} n(\xi) \left[\frac{r}{p}K_2^{(r-p)/p}(f(\xi) + g(\xi)z(\xi)) \right. \\ &\quad \left. + \frac{p-r}{p}K_2^{r/p} \right] \Delta\xi + \left(\frac{l}{g} \right)^\Delta(t) \int_{t_0}^t H(s) \\ &\quad \cdot (f(\sigma(s)) + g(\sigma(s))z(\sigma(s)))\Delta s \\ &\quad + \frac{l(\sigma(t))}{g(\sigma(t))}H(t)(f(\sigma(t)) + g(\sigma(t))z(\sigma(t))) \\ &\leq a(t)f(t) + a(t)g(t)z(t) + \frac{q}{p}K_1^{(q-p)/p}b(t) \\ &\quad \cdot (f(t) + g(t)z(t)) + \frac{p-q}{p}K_1^{q/p}b(t) + c(t) + \frac{r}{p} \\ &\quad \cdot K_2^{(r-p)/p}z(t)m(t) \int_{\theta(t_0)}^{\theta(t)} n(\xi)g(\xi)\Delta\xi + m(t) \\ &\quad \cdot \int_{\theta(t_0)}^{\theta(t)} n(\xi) \left[\frac{r}{p}K_2^{(r-p)/p}f(\xi) + \frac{p-r}{p}K_2^{r/p} \right] \Delta\xi \\ &\quad + z(\sigma(t)) \left(\frac{l}{g} \right)^\Delta(t) \int_{t_0}^t H(s)g(\sigma(s))\Delta s + \left(\frac{l}{g} \right)^\Delta(t) \\ &\quad \cdot (t) \int_{t_0}^t H(s)f(\sigma(s))\Delta s + z(\sigma(t)) \frac{l(\sigma(t))}{g(\sigma(t))}H(t) \\ &\quad \cdot g(\sigma(t)) + \frac{l(\sigma(t))}{g(\sigma(t))}H(t)f(\sigma(t)) = L(t)z(t) \\ &\quad + F(t)z(\sigma(t)) + M(t), \end{aligned} \quad (22)$$

where $F(t)$, $L(t)$, and $M(t)$ are defined as in (12), (14), and (15). From (13), we get

$$F(t) = \frac{G(t)}{1 + \mu(t)G(t)}, \quad (23)$$

and by (22), we obtain

$$\begin{aligned} z^\Delta(t) &\leq L(t)z(t) + \frac{G(t)}{1 + \mu(t)G(t)}z(\sigma(t)) + M(t) \\ &= L(t)z(t) \\ &\quad + \frac{G(t)}{1 + \mu(t)G(t)}[z(t) + \mu(t)z^\Delta(t)] \\ &\quad + M(t), \quad t \in \mathbb{T}^\kappa. \end{aligned} \quad (24)$$

Hence

$$\frac{1}{1 + \mu(t)G(t)}z^\Delta(t) \leq \left[\frac{G(t)}{1 + \mu(t)G(t)} + L(t) \right] z(t) + M(t), \quad t \in \mathbb{T}^\kappa, \quad (25)$$

equivalently

$$\begin{aligned} z^\Delta(t) &\leq [G(t) + (1 + \mu(t)G(t))L(t)]z(t) \\ &\quad + (1 + \mu(t)G(t))M(t) \\ &= (G \oplus L)(t)z(t) + (1 + \mu(t)G(t))M(t) \\ &= (G \oplus L)(t)z(t) + A(t), \quad t \in \mathbb{T}^\kappa, \end{aligned} \quad (26)$$

where $A(t)$ is defined as in (17). Note that z is rd-continuous and $G \oplus L \in \mathcal{R}^+$; from Lemma 4 and (26), we have that

$$z(t) \leq \int_{t_0}^t e_{G \oplus L}(t, \sigma(s)) A(s) \Delta s, \quad t \in \mathbb{T}^\kappa. \quad (27)$$

This and (20) imply the desired inequality (11). This completes the proof. $\square$

If we let $g(t) = l(t)$, $m(t) = 1$, and $\theta(t) = t$ in Theorem 5, then we obtain the following corollary.

Corollary 6. Assume that $x, a, b, c, f, g, n, h, k, p, q, r, \alpha, \beta$ and H are defined the same as in Theorem 5 and $\mu(t)F(t) < 1$ for $t \in \mathbb{T}^\kappa$. Suppose that u satisfies

$$\begin{aligned} x^p(t) &\leq f(t) + g(t) \int_{t_0}^t \left[a(s)x^p(s) + b(s)x^q(s) \right. \\ &\quad \left. + c(s) + \int_{t_0}^s n(\xi)x^r(\xi)\Delta\xi + h(s)x^\alpha(\sigma(s)) \right. \\ &\quad \left. - k(s)x^\beta(\sigma(s)) \right] \Delta s, \quad t \in \mathbb{T}^\kappa, \end{aligned} \quad (28)$$

then

$$\begin{aligned} x(t) &\leq \left(f(t) + g(t) \int_{t_0}^t e_{G \oplus L}(t, \sigma(s)) A(s) \Delta s \right)^{1/p}, \\ &\quad \text{for any } K_1 > 0, K_2 > 0, t \in \mathbb{T}^\kappa, \end{aligned} \quad (29)$$

where

$$\begin{aligned} F(t) &:= H(t)g(\sigma(t)), \\ G(t) &:= \frac{F(t)}{1 - \mu(t)F(t)}, \end{aligned} \quad (30)$$

$$\begin{aligned} L(t) &:= a(t)g(t) + \frac{q}{p}K_1^{(q-p)/p}b(t)g(t) \\ &\quad + \frac{r}{p}K_2^{(r-p)/p} \int_{t_0}^t n(\xi)g(\xi)\Delta\xi, \end{aligned} \quad (31)$$

$$\begin{aligned} M(t) &:= a(t)f(t) + \frac{q}{p}K_1^{(q-p)/p}b(t)f(t) \\ &\quad + \frac{p-q}{p}K_1^{q/p}b(t) + c(t) \\ &\quad + \int_{t_0}^t n(\xi) \left[\frac{r}{p}K_2^{(r-p)/p}f(\xi) + \frac{p-r}{p}K_2^{r/p} \right] \Delta\xi \\ &\quad + H(t)f(\sigma(t)), \end{aligned} \quad (32)$$

$$A(t) := (1 + \mu(t)G(t))M(t). \quad (33)$$

Remark 7. Corollary 6 extends Theorem 2.1 in [37] to the cases $p < \alpha < \beta$ and $0 < \beta < \alpha < p$.

Remark 8. If $l(t) = 0$, $c(t) = 0$, and $m(t) = 0$, then Theorem 5 reduces to Theorem 3.2 in [28]. If $l(t) = 0$, $a(t) = 0$, and $m(t) = 0$, then Theorem 5 reduces to Theorem 2.4 in [29].

Theorem 9. Assume that $x, a, b, c, f, g, l, m, n, \theta, h, k, p, q, r, \alpha, \beta, H$, and L are defined the same as in Theorem 5, and $\mu(t)\tilde{F}(t) < 1$ for $t \in \mathbb{T}^\kappa$. Let $w(t, s)$ and $v(t, s)$ be defined as in Lemma 3 such that $w_t^\Delta(t, s) \geq 0$ and $v_t^\Delta(t, s) \geq 0$ for $t \geq s$ and (5) holds. Suppose that x satisfies

$$\begin{aligned} x^p(t) &\leq f(t) + g(t) \int_{t_0}^t w(t, s) \left[a(s)x^p(s) \right. \\ &\quad \left. + b(s)x^q(s) + c(s) \right. \\ &\quad \left. + m(s) \int_{\theta(t_0)}^{\theta(s)} n(\xi)x^r(\xi)\Delta\xi \right] \Delta s + l(t) \\ &\quad \cdot \int_{t_0}^t v(t, s) \left[h(s)x^\alpha(\sigma(s)) - k(s)x^\beta(\sigma(s)) \right] \Delta s, \\ &\quad t \in \mathbb{T}^\kappa, \end{aligned} \quad (34)$$

then

$$\begin{aligned} x(t) &\leq \left(f(t) + g(t) \int_{t_0}^t e_{\tilde{G} \oplus \tilde{L}}(t, \sigma(s)) \tilde{A}(s) \Delta s \right)^{1/p}, \\ &\quad \text{for any } K_1 > 0, K_2 > 0, t \in \mathbb{T}^\kappa, \end{aligned} \quad (35)$$

where

$$\begin{aligned} \tilde{F}(t) := & \left(\frac{l}{g} \right)^\Delta (t) \int_{t_0}^t v(t, s) H(s) g(\sigma(s)) \Delta s \\ & + \frac{l(\sigma(t))}{g(\sigma(t))} \left[\int_{t_0}^t v_t^\Delta(t, s) H(s) g(\sigma(s)) \Delta s \right. \\ & \left. + v(\sigma(t), t) H(t) g(\sigma(t)) \right], \end{aligned} \quad (36)$$

$$\tilde{G}(t) := \frac{\tilde{F}(t)}{1 - \mu(t) \tilde{F}(t)}, \quad (37)$$

$$\tilde{L}(t) := \int_{t_0}^t w_t^\Delta(t, s) L(s) \Delta s + w(\sigma(t), t) L(t), \quad (38)$$

$$\begin{aligned} \tilde{M}(t) := & \int_{t_0}^t w_t^\Delta(t, s) \left\{ a(s) f(s) + b(s) \left[\frac{q}{p} \right. \right. \\ & \cdot K_1^{(q-p)/p} f(s) + \frac{p-q}{p} K_1^{q/p} \left. \right] + c(s) + m(s) \\ & \cdot \int_{\theta(t_0)}^{\theta(s)} n(\xi) \\ & \cdot \left[\frac{r}{p} K_2^{(r-p)/p} f(\xi) + \frac{p-r}{p} K_2^{r/p} \right] \Delta \xi \left. \right\} \Delta s \\ & + w(\sigma(t), t) \left\{ a(t) f(t) + b(t) \left[\frac{q}{p} K_1^{(q-p)/p} f(t) \right. \right. \\ & + \frac{p-q}{p} K_1^{q/p} \left. \right] + c(t) + m(t) \int_{\theta(t_0)}^{\theta(t)} n(\xi) \left[\frac{r}{p} \right. \\ & \cdot K_2^{(r-p)/p} f(\xi) + \frac{p-r}{p} K_2^{r/p} \left. \right] \Delta \xi \left. \right\} + \left(\frac{l}{g} \right)^\Delta \\ & \cdot (t) \int_{t_0}^t v(t, s) H(s) f(\sigma(s)) \Delta s \\ & + \frac{l(\sigma(t))}{g(\sigma(t))} \left[\int_{t_0}^t v_t^\Delta(t, s) H(s) f(\sigma(s)) \Delta s + v(\sigma(t), t) \right. \\ & \left. \cdot H(t) f(\sigma(t)) \right], \end{aligned} \quad (39)$$

$$\tilde{A}(t) := (1 + \mu(t) \tilde{G}(t)) \tilde{M}(t), \quad (41)$$

and $L(t)$ is defined as in (14).

Proof. From Lemma 1 and (34), we get that

$$\begin{aligned} x^p(t) \leq & f(t) + g(t) \int_{t_0}^t w(t, s) \left[a(s) x^p(s) \right. \\ & + b(s) x^q(s) + c(s) \\ & \left. + m(s) \int_{\theta(t_0)}^{\theta(s)} n(\xi) x^r(\xi) \Delta \xi \right] \Delta s + l(t) \end{aligned}$$

$$\begin{aligned} & \cdot \int_{t_0}^t v(t, s) [h(s) x^\alpha(\sigma(s)) - k(s) x^\beta(\sigma(s))] \Delta s \\ & \leq f(t) + g(t) \int_{t_0}^t w(t, s) \left[a(s) x^p(s) + b(s) x^q(s) \right. \\ & \left. + c(s) + m(s) \int_{\theta(t_0)}^{\theta(s)} n(\xi) x^r(\xi) \Delta \xi \right] \Delta s + l(t) \\ & \cdot \int_{t_0}^t v(t, s) H(s) x^p(\sigma(s)) \Delta s, \quad t \in \mathbb{T}^\kappa, \end{aligned} \quad (42)$$

where $H(t)$ is defined as in (16). Denote

$$\begin{aligned} z(t) = & \int_{t_0}^t w(t, s) \left[a(s) x^p(s) + b(s) x^q(s) + c(s) \right. \\ & \left. + m(s) \int_{\theta(t_0)}^{\theta(s)} n(\xi) x^r(\xi) \Delta \xi \right] \Delta s + \frac{l(t)}{g(t)} \\ & \cdot \int_{t_0}^t v(t, s) H(s) x^p(\sigma(s)) \Delta s, \quad t \in \mathbb{T}^\kappa. \end{aligned} \quad (43)$$

From the assumptions on $x, w, v, a, b, c, m, n, \theta, g, l, H$, (42) and (43), we obtain that z is nondecreasing and

$$x(t) \leq (f(t) + g(t) z(t))^{1/p}, \quad t \in \mathbb{T}^\kappa. \quad (44)$$

Combining (43) and (44), we have that

$$\begin{aligned} z^\Delta(t) = & \int_{t_0}^t w_t^\Delta(t, s) \left[a(s) x^p(s) + b(s) x^q(s) + c(s) \right. \\ & \left. + m(s) \int_{\theta(t_0)}^{\theta(s)} n(\xi) x^r(\xi) \Delta \xi \right] \Delta s + w(\sigma(t), t) \\ & \cdot \left[a(t) x^p(t) + b(t) x^q(t) + c(t) + m(t) \right. \\ & \cdot \int_{\theta(t_0)}^{\theta(t)} n(\xi) x^r(\xi) \Delta \xi \left. \right] + \left(\frac{l}{g} \right)^\Delta (t) \int_{t_0}^t v(t, s) \\ & \cdot H(s) x^p(\sigma(s)) \Delta s + \frac{l(\sigma(t))}{g(\sigma(t))} \left[\int_{t_0}^t v_t^\Delta(t, s) \right. \\ & \cdot H(s) x^p(\sigma(s)) \Delta s + v(\sigma(t), t) H(t) x^p(\sigma(t)) \left. \right] \\ & \leq \int_{t_0}^t w_t^\Delta(t, s) \left[a(s) (f(s) + g(s) z(s)) + b(s) \right. \\ & \left. \cdot (f(s) + g(s) z(s))^{q/p} + c(s) + m(s) \right] \Delta s \end{aligned}$$

$$\begin{aligned}
& \cdot \int_{\theta(t_0)}^{\theta(s)} n(\xi) (f(\xi) + g(\xi) z(\xi))^{r/p} \Delta \xi \Big] \Delta s \\
& + w(\sigma(t), t) \left[a(t) (f(t) + g(t) z(t)) + b(t) (f(t) \right. \\
& + g(t) z(t))^{q/p} + c(t) + m(t) \int_{\theta(t_0)}^{\theta(t)} n(\xi) \\
& \cdot (f(\xi) + g(\xi) z(\xi))^{r/p} \Delta \xi + \left(\frac{l}{g} \right)^\Delta (t) \\
& \cdot \int_{t_0}^t v(t, s) H(s) \\
& \cdot (f(\sigma(s)) + g(\sigma(s)) z(\sigma(s))) \Delta s \\
& + \frac{l(\sigma(t))}{g(\sigma(t))} \left[\int_{t_0}^t v_t^\Delta(t, s) H(s) \right. \\
& \cdot (f(\sigma(s)) + g(\sigma(s)) z(\sigma(s))) \Delta s + v(\sigma(t), t) \\
& \cdot H(t) (f(\sigma(t)) + g(\sigma(t)) z(\sigma(t))) \Big] , \quad t \in \mathbb{T}^\kappa.
\end{aligned} \tag{45}$$

Applying Lemma 2 on the right side of (45), we have that

$$\begin{aligned}
z^\Delta(t) & \leq \int_{t_0}^t w_t^\Delta(t, s) \left\{ a(s) (f(s) + g(s) z(s)) + b(s) \right. \\
& \cdot \left[\frac{q}{p} K_1^{(q-p)/p} (f(s) + g(s) z(s)) + \frac{p-q}{p} K_1^{q/p} \right] \\
& + c(s) + m(s) \int_{\theta(t_0)}^{\theta(s)} n(\xi) \\
& \cdot \left[\frac{r}{p} K_2^{(r-p)/p} (f(\xi) + g(\xi) z(\xi)) \right. \\
& + \left. \frac{p-r}{p} K_2^{r/p} \right] \Delta \xi \Big\} \Delta s + w(\sigma(t), t) \left\{ a(t) \right. \\
& \cdot (f(t) + g(t) z(t)) + b(t) \left[\frac{q}{p} K_1^{(q-p)/p} (f(t) \right. \\
& + g(t) z(t)) + \frac{p-q}{p} K_1^{q/p} \Big] + c(t) + m(t) \\
& \cdot \int_{\theta(t_0)}^{\theta(t)} n(\xi) \left[\frac{r}{p} K_2^{(r-p)/p} (f(\xi) + g(\xi) z(\xi)) \right. \\
& + \left. \frac{p-r}{p} K_2^{r/p} \right] \Delta \xi \Big\} + \left(\frac{l}{g} \right)^\Delta (t) \int_{t_0}^t v(t, s) H(s) \\
& \cdot (f(\sigma(s)) + g(\sigma(s)) z(\sigma(s))) \Delta s \\
& + \frac{l(\sigma(t))}{g(\sigma(t))} \left[\int_{t_0}^t v_t^\Delta(t, s) H(s) (f(\sigma(s)) + g(\sigma(s)) \right. \\
& \cdot z(\sigma(s))) \Delta s + v(\sigma(t), t) H(t) (f(\sigma(t))
\end{aligned}$$

$$\begin{aligned}
& + g(\sigma(t)) z(\sigma(t))) \Big] \leq z(t) \int_{t_0}^t w_t^\Delta(t, s) \left[a(s) g(s) \right. \\
& + b(s) \frac{q}{p} K_1^{(q-p)/p} g(s) + m(s) \int_{\theta(t_0)}^{\theta(s)} n(\xi) \\
& \cdot \frac{r}{p} K_2^{(r-p)/p} g(\xi) \Delta \xi \Big] \Delta s + \int_{t_0}^t w_t^\Delta(t, s) \\
& \cdot \left\{ a(s) f(s) + b(s) \left[\frac{q}{p} K_1^{(q-p)/p} f(s) + \frac{p-q}{p} \right. \right. \\
& \cdot K_1^{q/p} \Big] + c(s) + m(s) \int_{\theta(t_0)}^{\theta(s)} n(\xi) \left[\frac{r}{p} K_2^{(r-p)/p} f(\xi) \right. \\
& + \left. \frac{p-r}{p} K_2^{r/p} \right] \Delta \xi \Big\} \Delta s + z(t) w(\sigma(t), t) \\
& \cdot \left\{ a(t) g(t) + b(t) \left[\frac{q}{p} K_1^{(q-p)/p} g(t) \right] + m(t) \right. \\
& \cdot \int_{\theta(t_0)}^{\theta(t)} n(\xi) \frac{r}{p} K_2^{(r-p)/p} g(\xi) \Delta \xi \Big\} + w(\sigma(t), t) \\
& \cdot \left\{ a(t) f(t) + b(t) \left[\frac{q}{p} K_1^{(q-p)/p} f(t) + \frac{p-q}{p} \right. \right. \\
& \cdot K_1^{q/p} \Big] + c(t) + m(t) \int_{\theta(t_0)}^{\theta(t)} n(\xi) \left[\frac{r}{p} \right. \\
& \cdot K_2^{q/p(r-p)/p} f(\xi) + \frac{p-r}{p} K_2^{r/p} \Big] \Delta \xi \Big\} \\
& + z(\sigma(t)) \left\{ \left(\frac{l}{g} \right)^\Delta (t) \int_{t_0}^t v(t, s) H(s) g(\sigma(s)) \Delta s \right. \\
& + \frac{l(\sigma(t))}{g(\sigma(t))} \left[\int_{t_0}^t v_t^\Delta(t, s) H(s) g(\sigma(s)) \Delta s \right. \\
& + v(\sigma(t), t) H(t) g(\sigma(t)) \Big] \Big\} + \left(\frac{l}{g} \right)^\Delta (t) \\
& \cdot \int_{t_0}^t v(t, s) H(s) f(\sigma(s)) \Delta s \\
& + \frac{l(\sigma(t))}{g(\sigma(t))} \left[\int_{t_0}^t v_t^\Delta(t, s) H(s) f(\sigma(s)) \Delta s \right. \\
& + v(\sigma(t), t) H(t) f(\sigma(t)) \Big] \leq \left(\int_{t_0}^t w_t^\Delta(t, s) \right. \\
& \cdot L(s) \Delta s + w(\sigma(t), t) L(t) \Big) z(t) + \left\{ \left(\frac{l}{g} \right)^\Delta \right. \\
& \cdot (t) \int_{t_0}^t v(t, s) H(s) g(\sigma(s)) \Delta s \\
& + \frac{l(\sigma(t))}{g(\sigma(t))} \left[\int_{t_0}^t v_t^\Delta(t, s) H(s) g(\sigma(s)) \Delta s
\end{aligned}$$

$$\begin{aligned}
& + v(\sigma(t), t) H(t) g(\sigma(t)) \Big] \Big\} z(\sigma(t)) + \int_{t_0}^t w_t^\Delta(t, s) \\
& \cdot \left\{ a(s) f(s) + b(s) \left[\frac{q}{p} K_1^{(q-p)/p} f(s) + \frac{p-q}{p} \right. \right. \\
& \cdot K_1^{q/p} \Big] + c(s) + m(s) \int_{\theta(t_0)}^{\theta(s)} n(\xi) \left[\frac{r}{p} K_2^{(r-p)/p} f(\xi) \right. \\
& \cdot \frac{p-r}{p} K_2^{r/p} \Big] \Delta \xi \Big\} \Delta s + w(\sigma(t), t) \left\{ a(t) \right. \\
& \cdot f(t) + b(t) \left[\frac{q}{p} K_1^{(q-p)/p} f(t) + \frac{p-q}{p} K_1^{q/p} \right] \\
& + c(t) + m(t) \int_{\theta(t_0)}^{\theta(t)} n(\xi) \left[\frac{r}{p} K_2^{(r-p)/p} f(\xi) + \frac{p-r}{p} \right. \\
& \cdot K_2^{r/p} \Big] \Delta \xi \Big\} + \left(\frac{l}{g} \right)^\Delta(t) \int_{t_0}^t v(t, s) H(s) \\
& \cdot f(\sigma(s)) \Delta s + \frac{l(\sigma(t))}{g(\sigma(t))} \left[\int_{t_0}^t v_t^\Delta(t, s) H(s) \right. \\
& \cdot f(\sigma(s)) \Delta s + v(\sigma(t), t) H(t) f(\sigma(t)) \Big] \\
& = \tilde{L}(t) z(t) + \tilde{F}(t) z(\sigma(t)) + \tilde{M}(t),
\end{aligned} \tag{46}$$

where $\tilde{F}(t)$, $\tilde{L}(t)$, and $\tilde{M}(t)$ are defined as in (36), (38), and (40). By a similar argument with Theorem 5 in the remaining proof of theorem, one can prove (35). This completes the proof. $\square$

If we let $g(t) = l(t)$, $m(t) = 1$, $w(t, s) = v(t, s)$, and $\theta(t) = t$ in Theorem 9, then we obtain the following corollary.

Corollary 10. Assume that $x, f, g, w, a, b, c, n, h, k, p, q, r, \alpha, \beta$, and H are defined the same as in Theorem 9 and $\mu(t)\tilde{F}(t) < 1$ for $t \in \mathbb{T}^\kappa$. Suppose that x satisfies

$$\begin{aligned}
x^p(t) & \leq f(t) + g(t) \int_{t_0}^t w(t, s) \left[a(s) x^p(s) \right. \\
& + b(s) x^q(s) + c(s) + \int_{t_0}^s n(\xi) x^r(\xi) \Delta \xi \\
& \left. + h(s) x^\alpha(\sigma(s)) - k(s) x^\beta(\sigma(s)) \right] \Delta s, \\
& t \in \mathbb{T}^\kappa,
\end{aligned} \tag{47}$$

then

$$\begin{aligned}
x(t) & \leq \left(f(t) + g(t) \int_{t_0}^t e_{\tilde{G} \oplus \tilde{L}}(t, \sigma(s)) \tilde{A}(s) \Delta s \right)^{1/p}, \\
& \text{for any } K_1 > 0, K_2 > 0, t \in \mathbb{T}^\kappa,
\end{aligned} \tag{48}$$

where

$$\begin{aligned}
\tilde{F}(t) & := \int_{t_0}^t w_t^\Delta(t, s) F(s) \Delta s + w(\sigma(t), t) F(t), \\
\tilde{G}(t) & := \frac{\tilde{F}(t)}{1 - \mu(t) \tilde{F}(t)}, \\
\tilde{L}(t) & := \int_{t_0}^t w_t^\Delta(t, s) L(s) \Delta s + w(\sigma(t), t) L(t), \\
\tilde{M}(t) & := \int_{t_0}^t w_t^\Delta(t, s) M(s) \Delta s + w(\sigma(t), t) M(t), \\
\tilde{A}(t) & = (1 + \mu(t) \tilde{G}(t)) \tilde{M}(t),
\end{aligned} \tag{49}$$

and $F(t)$, $L(t)$, and $M(t)$ are defined the same as in (30), (31), and (32).

Remark 11. Corollary 10 extends Theorem 2.2 in [37] to the cases $p < \alpha < \beta$ and $0 < \beta < \alpha < p$.

Remark 12. If $c(t) = 0$, $m(t) = 0$, and $l(t) = 0$, then Theorem 9 reduces to Theorem 3.8 in [28]. If $m(t) = 0$ and $l(t) = 0$, then Theorem 9 reduces to Theorem 2.11 in [29].

4. Applications

In this section, we apply our results to study the boundedness of the solutions of a dynamic equation on time scales.

Example 13. Consider the following dynamic equation on time scales:

$$\begin{aligned}
& (x^p(t))^\Delta \\
& = P \left(t, x(t), x(\sigma(t)), \int_{\theta(t_0)}^{\theta(t)} Q(s, x(s)) \Delta s \right),
\end{aligned} \tag{50}$$

$$x(t_0) = x_0, \quad t_0, t \in \mathbb{T}^\kappa, \tag{51}$$

where $p > 0$ and u_0 are constants, $\theta : \mathbb{T}^\kappa \rightarrow \mathbb{T}^\kappa$ is continuous function satisfying $\theta(t) \leq t$ and $\theta(t_0) = t_0$ for $t \in \mathbb{T}^\kappa$, and $P : \mathbb{T}^\kappa \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ and $Q : \mathbb{T}^\kappa \times \mathbb{R} \rightarrow \mathbb{R}$ are continuous functions.

Theorem 14. Suppose that the functions P and Q in (50) satisfy the conditions

$$\begin{aligned}
|P(t, u, v, w)| & \leq c(t) + a(t) |u|^p + b(t) |u|^q \\
& + h(t) |v|^\alpha - k(t) |v|^\beta \\
& + m(t) |w|,
\end{aligned} \tag{52}$$

$$t \in \mathbb{T}^\kappa, u, v, w \in \mathbb{R},$$

$$|Q(t, u)| \leq n(t) |u|^r, \quad t \in \mathbb{T}^\kappa, u \in \mathbb{R}, \tag{53}$$

where $a, b, c, m, n : \mathbb{T}^\kappa \rightarrow \mathbb{R}_+$, and $h, k : \mathbb{T}^\kappa \rightarrow (0, \infty)$ are rd-continuous functions and q, r, α , and β are constants

satisfying (i) $0 \leq q \leq p, 0 \leq r \leq p, p < \alpha < \beta$ or (ii) $0 \leq q \leq p, 0 \leq r \leq p, 0 < \beta < \alpha < p$.

If x is a solution of (50) and $\mu(t)F(t) < 1$ for $t \in \mathbb{T}^\kappa$, then

$$x(t) \leq \left(|x_0|^p + \int_{t_0}^t e_{G \oplus L}(t, \sigma(s)) A(s) \Delta s \right)^{1/p}, \quad (54)$$

for any $K_1 > 0, K_2 > 0, t \in \mathbb{T}^\kappa$,

where

$$F(t) := \frac{h(t)(\beta - \alpha)}{\beta - p} \left(\frac{(\beta - p)k(t)}{(\alpha - p)h(t)} \right)^{(\alpha - p)/(\alpha - \beta)}, \quad (55)$$

$$G(t) := \frac{F(t)}{1 - \mu(t)F(t)}, \quad (56)$$

$$L(t) := a(t) + \frac{q}{p} K_1^{(q-p)/p} b(t) + \frac{r}{p} K_2^{(r-p)/p} m(t) \quad (57)$$

$$\begin{aligned} & \cdot \int_{\theta(t_0)}^{\theta(t)} n(\xi) \Delta \xi, \\ M(t) &:= |x_0|^p a(t) + \frac{q}{p} K_1^{(q-p)/p} |x_0|^p b(t) + \frac{p-q}{p} \\ & \cdot K_1^{q/p} b(t) + c(t) \\ & + \left(\frac{r}{p} K_2^{(r-p)/p} |x_0|^p + \frac{p-r}{p} K_2^{r/p} \right) m(t) \end{aligned} \quad (58)$$

$$\begin{aligned} & \cdot \int_{\theta(t_0)}^{\theta(t)} n(\xi) \Delta \xi + |x_0|^p F(t), \\ A(t) &:= (1 + \mu(t)G(t))M(t). \end{aligned} \quad (59)$$

Proof. The equivalent integral equation of (50) is denoted by

$$\begin{aligned} x^p(t) &= x_0^p + \int_{t_0}^t P \\ & \cdot \left(s, x(s), x(\sigma(s)), \int_{\theta(t_0)}^{\theta(s)} Q(\xi, x(\xi)) \Delta \xi \right) \Delta s, \end{aligned} \quad (60)$$

$t \in \mathbb{T}^\kappa$.

Using the assumptions (52) and (53), we have

$$\begin{aligned} |x^p(t)| &\leq |x_0|^p + \int_{t_0}^t \left[a(s) |x(s)|^p + b(s) |x(s)|^q \right. \\ & \left. + c(s) \right] \Delta s \end{aligned}$$

$$\begin{aligned} & + m(s) \int_{\theta(t_0)}^{\theta(s)} n(\xi) |x(\xi)|^r \Delta \xi + h(s) |x(\sigma(s))|^\alpha \\ & - k(s) |x(\sigma(s))|^\beta \Big] \Delta s, \quad t \in \mathbb{T}^\kappa. \end{aligned} \quad (61)$$

Then, a suitable application of Theorem 5 to (61) yields (54). $\square$

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this article.

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