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Research Article

Existence and Uniqueness of Positive Solutions for a New System of Fractional Differential Equations

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By virtue of a recent existing fixed point theorem of increasing $\varphi-(h,e)$-concave operator by Zhai and Wang, we consider the existence and uniqueness of positive solutions for a new system of Caputo-type fractional differential equations with Riemann–Stieltjes integral boundary conditions.

1. Introduction

In this paper, we consider the following nonlinear Caputo-type fractional system:

$$\begin{cases} {}^cD_{0+}^{\theta_1}x(t)+f_1(t,x(t),y(t))=a_1(t), & t\in[0,1],\\ {}^cD_{0+}^{\theta_2}y(t)+f_2(t,x(t),y(t))=a_2(t), & t\in[0,1],\\ x(0)=x''(0)=0,\\ x(1)=\int_0^1 x(t)\mathrm{d}A_1(t),\\ y(0)=y''(0)=0,\\ y(1)=\int_0^1 y(t)\mathrm{d}A_2(t), \end{cases} \tag{1}$$

where $\theta_1\in(2,3)$, $\theta_2\in(2,3)$; ${}^cD_{0+}^{\theta_1}$ and ${}^cD_{0+}^{\theta_2}$ are the Caputo fractional derivative; f_1 and f_2: $[0,1]\times(-\infty,+\infty)\times(-\infty,+\infty)\longrightarrow(-\infty,+\infty)$ are continuous; a_1,a_2: $[0,1]\longrightarrow[0,+\infty)$ are continuous; and A_1 and A_2 are bounded variation functions with positive measures with $B_1=\int_0^1 t\mathrm{d}A_1(t)<1$, $B_2=\int_0^1 t\mathrm{d}A_2(t)<1$.

In recent decades, fractional-order calculus has been widely used in engineering, biology, physics, and so on. Based on it, many scholars have been interested in the study of the existence of nontrivial solutions for various fractional boundary value problems. For some recent works, we can refer to [1–30] and the references therein. For example, in [10], by virtue of Guo–Krasnosel'skii fixed point theorem, Ma and Cui studied the following fractional boundary value problem:

$$\begin{cases} {}^cD_{0+}^{\theta}p(y)+\mu f(y,p(y))=0, & y\in[0,1],\\ p(0)=p''(0)=0,\\ p(1)=\int_0^1 p(y)\mathrm{d}A(y), \end{cases} \tag{2}$$

where $f\in C([0,1]\times[0,+\infty),(0,+\infty))$, ${}^cD_{0+}^{\theta}$ is the Caputo fractional derivative, $\theta\in(2,3)$, and $\mu>0$ is a parameter. In the case that the parameter μ satisfied some conditions, the existence of positive solutions for the boundary value problem (2) was proved. Meanwhile, many scholars considered some various fractional systems, such as [16–30] and the reference therein. For instance, in [16], the authors obtained the existence of two positive solutions for a nonlinear Caputo-type fractional system by virtue of fixed point index theory. In [17], by using monotone iterative approach, the authors investigated the iterative positive solutions of a system of fractional Riemann–Liouville-type equations with four-point boundary conditions. In [19], by using Banach's contraction principle, the authors studied the uniqueness of solution for a system of Hadamard-type fractional differential equations with integral boundary

conditions. In [21], by using Guo–Krasnosel'skii fixed point theorem, the authors studied the existence of positive solutions for an infinite system of fractional Caputo-type differential equations.

In [22], Zhai and Wang introduced a new concept of $\varphi-(h,e)$-concave operator and obtained fixed point theorems of increasing $\varphi-(h,e)$-concave operator. In recent years, by using the fixed point theorems of $\varphi-(h,e)$-concave operator, Zhai and Jiang investigated the uniqueness of positive solutions for a Riemann–Liouville-type fractional system with integral boundary conditions in [23]; Zhai and Wang considered the uniqueness of positive solutions for a system of Hadamard fractional differential equations with integral equations in [24]; Zhai and Zhu considered the uniqueness of positive solutions for a system of Riemann–Liouville fractional differential equations in [25].

Inspired by [10, 22–25], we introduce a new system of nonlinear Caputo-type fractional differential equations (1). There are few papers about the application of $\varphi-(h,e)$-concave operator in nonlinear Caputo-type fractional boundary value problems. So, in this paper, we use the recent fixed point theorems of $\varphi-(h,e)$-concave operator by Zhai and Wang to study system (1). The result of the existence and uniqueness of positive solutions for system (1) is obtained.

2. Preliminaries

In this section, we briefly introduce Caputo's fractional derivative and the fixed point theorem of $\varphi-(h,e)$-concave operator. For details, we can refer to the literature [1, 22]. And we give some lemmas about the relevant Green's functions.

Definition 1 (see [1]). For a function $x \in C^n[0,+\infty)$, we define Caputo's fractional derivative of order $\theta > 0$ as follows:

$$ {}^{c}D_{0+}^{\theta}x(t) = \frac{1}{\Gamma(n-\theta)}\int_0^t (t-s)^{n-\theta-1}x^{(n)}(s)\mathrm{d}s, \quad n-1<\theta<n, \tag{3} $$

where n is the smallest integer greater than or equal to θ.

By [10], the following lemmas are listed.

Lemma 1 (see [10]). *Let $u \in C[0,1]$ and $\theta_1, \theta_2 \in (2,3)$. Then, the linear Caputo fractional differential equation*

$$ \begin{cases} {}^{c}D_{0+}^{\theta_i}p(t) + u(t) = 0, \quad t \in [0,1], \\ p(0) = p''(0) = 0, \\ p(1) = \int_0^1 p(t)\mathrm{d}A_i(t) \end{cases} \tag{4} $$

has a unique solution

$$ p(t) = \int_0^1 K_i(t,s)u(s)\mathrm{d}s, \tag{5} $$

where

$$ K_i(t,s) = \frac{1}{\Gamma(\theta_i)}\begin{cases} \dfrac{t}{1-B_i}\left[(1-s)^{\theta_i-1} - \displaystyle\int_s^1 (t-s)^{\theta_i-1}\mathrm{d}A_i(t)\right] - (t-s)^{\theta_i-1}, & 0 \le s \le t \le 1, \\ \dfrac{t}{1-B_i}\left[(1-s)^{\theta_i-1} - \displaystyle\int_s^1 (t-s)^{\theta_i-1}\mathrm{d}A_i(t)\right], & 0 \le t \le s \le 1, \end{cases} \tag{6} $$

and $B_i = \int_0^1 t\,dA_i(t) < 1$, $i = 1,2$.

Lemma 2. *The above Green's function $K_i(t,s)\,(i=1,2)$ has the following properties:*

(i) $K_i(t,s) \ge 0$ and $K_i(t,s)$ is continuous on $[0,1]\times[0,1]$.

(ii) $((t(1-s)^{\theta_i-1})/(\Gamma(\theta_i)(1-B_i)))\int_0^1 (t-t^{\theta_i-1})\ dA_i(t) \le K_i(t,s) \le ((t(1-s)^{\theta_i-1})/(\Gamma(\theta_i)(1-B_i)))$, $t,s \in [0,1]$.

Proof. By (6), we know that $K_i(t,s) \ge 0$ and $K_i(t,s)$ is continuous on $[0,1]\times[0,1]$.

By (6), we easily know that

$$ K_i(t,s) \le \frac{t(1-s)^{\theta_i-1}}{\Gamma(\theta_i)(1-B_i)}. \tag{7} $$

From (6), when $0 \le s \le t \le 1$, we have

$$ \begin{aligned} &\frac{t}{1-B_i}\left[(1-s)^{\theta_i-1} - \int_s^1 (t-s)^{\theta_i-1}\mathrm{d}A_i(t)\right] - (t-s)^{\theta_i-1} \\ &\ge \frac{t}{1-B_i}\left[(1-s)^{\theta_i-1} - \int_s^1 (t-ts)^{\theta_i-1}\mathrm{d}A_i(t)\right] - (t-ts)^{\theta_i-1} \end{aligned} $$

$$
\begin{aligned}
&= \frac{t(1-B_i+B_i)}{1-B_i}(1-s)^{\theta_i-1} - \frac{t}{1-B_i}\int_s^1 (t-ts)^{\theta_i-1}\mathrm{d}A_i(t) - t^{\theta_i-1}(1-s)^{\theta_i-1} \\
&= (1-s)^{\theta_i-1}\left(t-t^{\theta_i-1}\right) + \frac{t}{1-B_i}\left((1-s)^{\theta_i-1}\int_0^1 t\mathrm{d}A_i(t) - \int_s^1 (t-ts)^{\theta_i-1}\mathrm{d}A_i(t)\right) \\
&\geq (1-s)^{\theta_i-1}\left(t-t^{\theta_i-1}\right) + \frac{t}{1-B_i}(1-s)^{\theta_i-1}\int_0^1\left(t-t^{\theta_i-1}\right)\mathrm{d}A_i(t) \\
&\geq \frac{t}{1-B_i}(1-s)^{\theta_i-1}\int_0^1\left(t-t^{\theta_i-1}\right)\mathrm{d}A_i(t).
\end{aligned} \tag{8}
$$

When $t \leq s$, we have

$$
\begin{aligned}
&\frac{t}{1-B_i}\left[(1-s)^{\theta_i-1} - \int_s^1 (t-s)^{\theta_i-1}\mathrm{d}A_i(t)\right] \\
&\quad\geq \frac{t}{1-B_i}\left[(1-s)^{\theta_i-1} - \int_s^1 (t-ts)^{\theta_i-1}\mathrm{d}A_i(t)\right] \\
&\quad\geq \frac{t}{1-B_i}(1-s)^{\theta_i-1}\left[1-\int_s^1 t^{\theta_i-1}\mathrm{d}A_i(t)\right] \\
&\quad\geq \frac{t}{1-B_i}(1-s)^{\theta_i-1}\left[1-\int_0^1 t^{\theta_i-1}\mathrm{d}A_i(t)\right] \\
&\quad\geq \frac{t}{1-B_i}(1-s)^{\theta_i-1}\int_0^1\left(t-t^{\theta_i-1}\right)\mathrm{d}A_i(t).
\end{aligned} \tag{9}
$$

By (8) and (9), we have

$$
K_i(t,s) \geq \left(\frac{t(1-s)^{\theta_i-1}}{\Gamma(\theta_i)(1-B_i)}\right)\int_0^1\left(t-t^{\theta_i-1}\right)\mathrm{d}A_i(t). \tag{10}
$$

Let E be a real Banach space and $P \subset E$ be a cone. A partial order on E is induced by P. For any $x, y \in E$, the notation $x \sim y$ denotes that there are $\lambda > 0$ and $\mu > 0$ such that $\lambda x \leq y \leq \mu x$. Take $h > \theta$ (i.e., $h \geq \theta$ and $h \neq \theta$); let $P_h = \{x \in E \mid x \sim h\}$; then obviously, $P_h \subset P$. Choose $e \in P$ with $\theta \leq e \leq h$, and let $P_{h,e} = \{x \in E \mid x + e \in P_h\}$. □

Definition 2 (see [22]). Let T: $P_{h,e} \longrightarrow E$ be an operator. If for any $x \in P_{h,e}$ and $\eta \in (0,1)$, there exists $\varphi(\eta) > \eta$ such that

$$
T(\eta x + (\eta - 1)e) \geq \varphi(\eta)Tx + (\varphi(\eta) - 1)e. \tag{11}
$$

Then, T is called a $\varphi - (h,e)$-concave operator.

Lemma 3 (see [22]). *Let P be a normal cone. Suppose that T is an increasing $\varphi - (h,e)$-concave operator and $Th \in P_{h,e}$; then, T has a unique fixed point $u \in P_{h,e}$. For any $u_0 \in P_{h,e}$, the sequence $u_n = Tu_{n-1}, n = 1, 2, \ldots$, and then $\|u_n - u\| \longrightarrow 0$ as $n \longrightarrow \infty$.*

3. Main Results

Let $\overline{E} = C[0,1]$ with the norm $\|x\| = \max_{0\leq t\leq 1}|x(t)|$. Let $E = \overline{E} \times \overline{E}$ with the norm $\|(x,y)\|_E = \max\{\|x\|, \|y\|\}$. Let $\overline{P} = \{x \in \overline{E}: x(t) \geq 0, t \in [0,1]\}$. Then, $\overline{P}$ is a normal cone. Let $P = \overline{P} \times \overline{P}$. It is obvious that P is a normal cone of E. We define the following partial order on space E: $(x_1, y_1) \leq (x_2, y_2) \Longleftrightarrow x_1(t) \leq x_2(t), y_1(t) \leq y_2(t), t \in [0,1]$. For the detailed knowledge about the cone, we can refer to [31].

Define the following operators T_1: $E \longrightarrow \overline{E}$, T_2: $E \longrightarrow \overline{E}$ and T: $E \longrightarrow E$:

$$
\begin{aligned}
T_1(x,y)(t) &= \int_0^1 K_1(t,s)f_1(s,x(s),y(s))\mathrm{d}s - \int_0^1 K_1(t,s)a_1(s)\mathrm{d}s, \quad t \in [0,1], \\
T_2(x,y)(t) &= \int_0^1 K_2(t,s)f_2(s,x(s),y(s))\mathrm{d}s - \int_0^1 K_2(t,s)a_2(s)\mathrm{d}s, \quad t \in [0,1], \\
T(x,y)(t) &= \left(T_1(x,y), T_2(x,y)\right)(t), \quad (x,y) \in E, t \in [0,1],
\end{aligned} \tag{12}
$$

where $K_i(t,s)\,(i = 1,2)$ is defined by (6).

By [10], we easily know that fixed points of the operator T are solutions of system (1).

Let

$$
\begin{aligned}
e_1(t) &= \int_0^1 K_1(t,s)a_1(s)\mathrm{d}s, \\
e_2(t) &= \int_0^1 K_2(t,s)a_2(s)\mathrm{d}s, \quad t \in [0,1], \\
h_1(t) &= C_1 t, \\
h_2(t) &= C_2 t, \quad t \in [0,1],
\end{aligned} \tag{13}
$$

where

$$C_1 > \frac{1}{\Gamma(\theta_1)(1-B_1)} \int_0^1 (1-s)^{\theta_1-1} a_1(s)\mathrm{d}s,$$
$$C_2 > \frac{1}{\Gamma(\theta_2)(1-B_2)} \int_0^1 (1-s)^{\theta_2-1} a_2(s)\mathrm{d}s. \tag{14}$$

Let

$$\begin{aligned} \overline{P}_{h_1} &= \{x \in \overline{E}\colon x \sim h_1\}, \\ \overline{P}_{h_2} &= \{x \in \overline{E}\colon x \sim h_2\}, \\ \overline{P}_{h_1,e_1} &= \{x \in \overline{E}\colon x + e_1 \sim h_1\}, \\ \overline{P}_{h_2,e_2} &= \{x \in \overline{E}\colon x + e_2 \sim h_2\}. \end{aligned} \tag{15}$$

Set $h(t) = (h_1(t), h_2(t))$ and $e(t) = (e_1(t), e_2(t))$. By [29], we have

$$\begin{aligned} P_h &= \overline{P}_{h_1} \times \overline{P}_{h_2}, \\ P_{h,e} &= \overline{P}_{h_1,e_1} \times \overline{P}_{h_2,e_2}. \end{aligned} \tag{16}$$

Let $M_1 = \max_{t\in[0,1]} e_1(t)$, $M_2 = \max_{t\in[0,1]} e_2(t)$.

For convenience, the following conditions are given.

(i) (S_1) $f_1\colon [0,1] \times [-M_1, +\infty) \times [-M_2, +\infty) \longrightarrow (-\infty, +\infty)$ is increasing about the second and third variables; $f_2\colon [0,1] \times [-M_1, +\infty) \times [-M_2, +\infty) \longrightarrow (-\infty, +\infty)$ is increasing about the second and third variables.

(ii) (S_2) For $\eta \in (0,1)$, there exists $\varphi(\eta) > \eta$ such that

$$\begin{aligned} f_1(t, \eta u_1 + (\eta-1)v_1, \eta u_2 + (\eta-1)v_2) &\geq \varphi(\eta) f_1(t, u_1, u_2), \\ f_2(t, \eta u_1 + (\eta-1)v_1, \eta u_2 + (\eta-1)v_2) &\geq \varphi(\eta) f_2(t, u_1, u_2), \end{aligned} \tag{17}$$

where

$$t \in [0,1], u_1, u_2 \in (-\infty, +\infty), v_1 \in [0, M_1], v_2 \in [0, M_2]. \tag{18}$$

(iii) (S_3) $f_1(t,0,0) \geq 0$, $f_2(t,0,0) \geq 0$ and $f_1(t,0,0) \equiv 0$, $f_2(t,0,0) \equiv 0$, $\forall t \in [0,1]$.

Theorem 1. *Suppose that conditions (S_1), (S_2), and (S_3) are satisfied. Then, system (1) has a unique solution $(x^*, y^*) \in P_{h,e}$, and for any given $(x_0, y_0) \in P_{h,e}$, we have $x_{n+1}(t) \longrightarrow x^*(t)\,(n \longrightarrow \infty)$, $y_{n+1}(t) \longrightarrow y^*(t)\,(n \longrightarrow \infty)$, where the sequences:*

$$\begin{aligned} x_{n+1}(t) &= \int_0^1 K_1(t,s) f_1(s, x_n(s), y_n(s))\mathrm{d}s \\ &\quad - \int_0^1 K_1(t,s) a_1(s)\mathrm{d}s, \\ y_{n+1}(t) &= \int_0^1 K_2(t,s) f_2(s, x_n(s), y_n(s))\mathrm{d}s \\ &\quad - \int_0^1 K_2(t,s) a_2(s)\mathrm{d}s, \end{aligned} \tag{19}$$

where $n = 0, 1, 2, \ldots$.

Proof. By Lemma 2, we know that $K_i(t,s) \geq 0$, $i = 1, 2$. So,

$$\begin{aligned} e_1(t) &= \int_0^1 K_1(t,s) a_1(s)\mathrm{d}s \geq 0, \quad t \in [0,1], \\ e_2(t) &= \int_0^1 K_2(t,s) a_2(s)\mathrm{d}s \geq 0, \quad t \in [0,1]. \end{aligned} \tag{20}$$

By Lemma 2, we have

$$\begin{aligned} &\frac{t(1-s)^{\theta_i-1}}{\Gamma(\theta_i)(1-B_i)} \int_0^1 \left(t - t^{\theta_i-1}\right)\mathrm{d}A_i(t) \leq K_i(t,s) \\ &\quad \leq \frac{t(1-s)^{\theta_i-1}}{\Gamma(\theta_i)(1-B_i)}, \quad t, s \in [0,1], i = 1, 2. \end{aligned} \tag{21}$$

From (21), for $t \in [0,1]$, we have

$$\begin{aligned} e_1(t) &= \int_0^1 K_1(t,s) a_1(s)\mathrm{d}s \leq \frac{t}{\Gamma(\theta_1)(1-B_1)} \int_0^1 (1-s)^{\theta_1-1} a_1(s)\mathrm{d}s \leq C_1 t = h_1(t), \\ e_2(t) &= \int_0^1 K_2(t,s) a_2(s)\mathrm{d}s \leq \frac{t}{\Gamma(\theta_2)(1-B_2)} \int_0^1 (1-s)^{\theta_2-1} a_2(s)\mathrm{d}s \leq C_2 t = h_2(t). \end{aligned} \tag{22}$$

Thus, we obtain that

$$\begin{aligned} 0 &\leq e_1 \leq h_1, \\ 0 &\leq e_2 \leq h_2. \end{aligned} \tag{23}$$

In the following, we divide three parts to prove this theorem. Firstly, we prove that $T\colon P_{h,e} \longrightarrow E$ is a $\varphi - (h,e)$ concave operator. By (S_2), for $\eta \in (0,1)$ and $\eta \in (0,1)$, we have

$$\begin{aligned}
&T_1(\eta x+(\eta-1)e_1,\eta y+(\eta-1)e_2)(t)\\
&\quad=\int_0^1 K_1(t,s)f_1(s,\eta x(s)+(\eta-1)e_1(s),\eta y(s)+(\eta-1)e_2(s))\mathrm{d}s-\int_0^1 K_1(t,s)a_1(s)\mathrm{d}s\\
&\quad\geq\int_0^1 K_1(t,s)\varphi(\eta)f_1(s,x(s),y(s))\mathrm{d}s-\int_0^1 K_1(t,s)a_1(s)\mathrm{d}s\\
&\quad=\varphi(\eta)\int_0^1 K_1(t,s)f_1(s,x(s),y(s))\mathrm{d}s-e_1(t)\\
&\quad=\varphi(\eta)\left[\int_0^1 K_1(t,s)f_1(s,x(s),y(s))\mathrm{d}s-\int_0^1 K_1(t,s)a_1(s)\mathrm{d}s\right]+(\varphi(\eta)-1)e_1(t)\\
&\quad=\varphi(\eta)T_1(x,y)(t)+(\varphi(\eta)-1)e_1(t).
\end{aligned}\tag{24}$$

Similarly, by (S_2), for $\eta\in(0,1)$ and $\eta\in(0,1)$, we obtain

$$\begin{aligned}
&T_2(\eta x+(\eta-1)e_1,\eta y+(\eta-1)e_2)(t)\\
&\quad=\int_0^1 K_2(t,s)f_2(s,\eta x(s)+(\eta-1)e_1(s),\eta y(s)+(\eta-1)e_2(s))\mathrm{d}s-\int_0^1 K_2(t,s)a_2(s)\mathrm{d}s\\
&\quad\geq\int_0^1 K_2(t,s)\varphi(\eta)f_2(s,x(s),y(s))\mathrm{d}s-\int_0^1 K_2(t,s)a_2(s)\mathrm{d}s\\
&\quad=\varphi(\eta)\int_0^1 K_2(t,s)f_2(s,x(s),y(s))\mathrm{d}s-e_2(t)\\
&\quad=\varphi(\eta)\left[\int_0^1 K_2(t,s)f_2(s,x(s),y(s))\mathrm{d}s-\int_0^1 K_2(t,s)a_2(s)\mathrm{d}s\right]+(\varphi(\eta)-1)e_2(t)\\
&\quad=\varphi(\eta)T_2(x,y)(t)+(\varphi(\eta)-1)e_2(t).
\end{aligned}\tag{25}$$

By (24) and (25), for $(x,y)\in P_{h,e}$, $\eta\in(0,1)$, and $t\in[0,1]$, we have

$$\begin{aligned}
&T(\eta(x,y)+(\eta-1)e)(t)\\
&\quad=T(\eta(x,y)+(\eta-1)(e_1,e_2))(t)\\
&\quad=T(\eta x+(\eta-1)e_1,\eta y+(\eta-1)e_2)(t)\\
&\quad=(T_1(\eta x+(\eta-1)e_1,\eta y+(\eta-1)e_2)(t),T_2(\eta x+(\eta-1)e_1,\eta y+(\eta-1)e_2)(t))\\
&\quad\geq(\varphi(\eta)T_1(x,y)(t)+(\varphi(\eta)-1)e_1(t),\varphi(\eta)T_2(x,y)(t)+(\varphi(\eta)-1)e_2(t))\\
&\quad=\varphi(\eta)(T_1(x,y)(t),T_2(x,y)(t))+(\varphi(\eta)-1)(e_1(t),e_2(t))\\
&\quad=\varphi(\eta)T(x,y)(t)+(\varphi(\eta)-1)e(t).
\end{aligned}\tag{26}$$

Namely, for $(x,y)\in P_{h,e}$ and $\eta\in(0,1)$, we obtain that

$$T(\eta(x,y)+(\eta-1)e)(t)\geq\varphi(\eta)T(x,y)+(\varphi(\eta)-1)e.\tag{27}$$

Secondly, we show that $T\colon P_{h,e}\longrightarrow E$ is an increasing operator. By the definition of $P_{h,e}$, for $(x,y)\in P_{h,e}$, we get that $(x,y)+e\in P_h$, i.e., $(x+e_1,y+e_2)\in\overline{P}_{h_1}\times\overline{P}_{h_2}$. By the definitions of $\overline{P}_{h_1}$ and $\overline{P}_{h_2}$, we obtain that there exist $\eta_1>0$ and $\eta_2>0$ such that

$$\begin{aligned}
&x(t)+e_1(t)\geq\eta_1h_1(t),\\
&y(t)+e_2(t)\geq\eta_2h_2(t),\quad t\in[0,1].
\end{aligned}\tag{28}$$

So,

$$\begin{aligned}
&x(t)\geq\eta_1h_1(t)-e_1(t)\geq-e_1(t)\geq-M_1,\\
&y(t)\geq\eta_2h_2(t)-e_2(t)\geq-e_2(t)\geq-M_2,\quad t\in[0,1].
\end{aligned}\tag{29}$$

From (S_1), we easily know that the operators T_1 and T_2 are increasing, so it is obvious that $T: P_{h,e} \longrightarrow E$ is increasing.

In the end, we prove $Th \in P_{h,e}$. By [22], we know that $P_{h,e} = \overline{P}_{h_1,e_1} \times \overline{P}_{h_2,e_2}$. Since $(Th)(t) = T(h_1,h_2)(t) = (T_1(h_1,h_2)(t), T_2(h_1,h_2)(t))$, we need to prove $T_1(h_1,h_2) \in \overline{P}_{h_1,e_1}, T_2(h_1,h_2) \in \overline{P}_{h_2,e_2}$. In the following, by the definitions of $\overline{P}_{h_1,e_1}$ and $\overline{P}_{h_2,e_2}$, we prove $T_1(h_1,h_2) + e_1 \in \overline{P}_{h_1}, T_2(h_1,h_2) + e_2 \in \overline{P}_{h_2}$, respectively. From (S_1) and (S_3), combine (21), and we have

$$\begin{aligned}
&T_1(h_1,h_2)(t) + e_1(t) \\
&\quad = \int_0^1 K_1(t,s) f_1(s,h_1(s),h_2(s))\mathrm{d}s \\
&\quad \le \frac{t}{\Gamma(\theta_1)(1-B_1)} \int_0^1 (1-s)^{\theta_1-1} f_1(s,h_1(s),h_2(s))\mathrm{d}s \\
&\quad \le \frac{t}{\Gamma(\theta_1)(1-B_1)} \int_0^1 (1-s)^{\theta_1-1} f_1(s,C_1,C_2)\mathrm{d}s \\
&\quad = \frac{h_1(t)}{C_1\Gamma(\theta_1)(1-B_1)} \int_0^1 (1-s)^{\theta_1-1} f_1(s,C_1,C_2)\mathrm{d}s, \\
&T_1(h_1,h_2)(t) + e_1(t) \\
&\quad = \int_0^1 K_1(t,s) f_1(s,h_1(s),h_2(s))\mathrm{d}s \\
&\quad \ge \frac{t}{\Gamma(\theta_1)(1-B_1)} \int_0^1 (1-s)^{\theta_1-1} f_1(s,h_1(s),h_2(s))\mathrm{d}s \int_0^1 \left(t - t^{\theta_1-1}\right)\mathrm{d}A_1(t) \\
&\quad \ge \frac{t}{\Gamma(\theta_1)(1-B_1)} \int_0^1 (1-s)^{\theta_1-1} f_1(s,0,0)\mathrm{d}s \int_0^1 \left(t - t^{\theta_1-1}\right)\mathrm{d}A_1(t) \\
&\quad = \frac{h_1(t)}{C_1\Gamma(\theta_1)(1-B_1)} \int_0^1 (1-s)^{\theta_1-1} f_1(s,0,0)\mathrm{d}s \int_0^1 \left(t - t^{\theta_1-1}\right)\mathrm{d}A_1(t).
\end{aligned} \tag{30}$$

Let

$$\begin{aligned}
\lambda_1 &= \frac{1}{C_1\Gamma(\theta_1)(1-B_1)} \int_0^1 (1-s)^{\theta_1-1} f_1(s,0,0)\mathrm{d}s \\
&\quad \cdot \int_0^1 \left(t - t^{\theta_1-1}\right)\mathrm{d}A_1(t), \\
\lambda_2 &= \frac{1}{C_1\Gamma(\theta_1)(1-B_1)} \int_0^1 (1-s)^{\theta_1-1} f_1(s,C_1,C_2)\mathrm{d}s.
\end{aligned} \tag{31}$$

Obviously, $\lambda_1 > 0, \lambda_2 > 0$. So, we have $\lambda_1 h_1(t) \le T_1(h_1,h_2)(t) + e_1(t) \le \lambda_2 h_2(t)$. And $T_1(h_1,h_2) + e_1 \in \overline{P}_{h_1}$. Similarly, we have $T_2(h_1,h_2) + e_2 \in \overline{P}_{h_2}$. So, $Th \in P_{h,e}$ is proved.

Therefore, by Lemma 3, the operator T has a unique fixed point $(x^*, y^*) \in P_{h,e}$. For any given $(x_0, y_0) \in P_{h,e}$, define the sequences:

$$\begin{aligned}
x_{n+1}(t) &= \int_0^1 K_1(t,s) f_1(s,x_n(s),y_n(s))\mathrm{d}s \\
&\quad - \int_0^1 K_1(t,s) a_1(s)\mathrm{d}s, \\
y_{n+1}(t) &= \int_0^1 K_2(t,s) f_2(s,x_n(s),y_n(s))\mathrm{d}s \\
&\quad - \int_0^1 K_2(t,s) a_2(s)\mathrm{d}s,
\end{aligned} \tag{32}$$

where $n = 0, 1, 2, \ldots$. □

4. Application

Example 1. We study the following fractional system with integral boundary conditions:

$$\begin{cases} {}^{c}D_{0+}^{(5/2)}x(t)+\left(\dfrac{2835\sqrt{\pi}}{944}x+1\right)^{(1/5)}\left[\dfrac{16}{105\sqrt{\pi}}\left(t^{(5/2)}+\dfrac{32}{27}\right)\right]^{(1/5)}t^{(1/5)} \\ +\left(\dfrac{2835\sqrt{\pi}}{1888}y+1\right)^{(1/5)}\left[\dfrac{32}{105\sqrt{\pi}}\left(t^{(5/2)}+\dfrac{32}{27}\right)\right]^{(1/5)}t^{(1/5)}=t, \quad t\in[0,1], \\ {}^{c}D_{0+}^{(5/2)}y(t)+\left(\dfrac{2835\sqrt{\pi}}{944}x+1\right)^{(1/3)}\left[\dfrac{16}{105\sqrt{\pi}}\left(t^{(5/2)}+\dfrac{32}{27}\right)\right]^{(1/3)}t^{(1/3)} \\ +\left(\dfrac{2835\sqrt{\pi}}{1888}y+1\right)^{(1/3)}\left[\dfrac{32}{105\sqrt{\pi}}\left(t^{(5/2)}+\dfrac{32}{27}\right)\right]^{(1/3)}t^{(1/3)}=2t, \quad t\in[0,1], \\ x(0)=x''(0)=0, \\ x(1)=\dfrac{1}{2}\displaystyle\int_0^1 x(t)\mathrm{d}t, \\ y(0)=y''(0)=0, \\ y(1)=\dfrac{1}{2}\displaystyle\int_0^1 y(t)\mathrm{d}t, \end{cases} \tag{33}$$

where $\theta_1=\theta_2=(5/2)$; $A_1(t)=A_2(t)=(1/2)t$; $B_1=B_2=(1/4)$; $a_1(t)=t$; $a_2(t)=2t$; and

$$f_1(t,x,y)=\left(\frac{2835\sqrt{\pi}}{944}x+1\right)^{(1/5)}\left[\frac{16}{105\sqrt{\pi}}\left(t^{(5/2)}+\frac{32}{27}\right)\right]^{(1/5)}t^{(1/5)}+\left(\frac{2835\sqrt{\pi}}{1888}y+1\right)^{(1/5)}\left[\frac{32}{105\sqrt{\pi}}\left(t^{(5/2)}+\frac{32}{27}\right)\right]^{(1/5)}t^{(1/5)}, \tag{34}$$

$$f_2(t,x,y)=\left(\frac{2835\sqrt{\pi}}{944}x+1\right)^{(1/3)}\left[\frac{16}{105\sqrt{\pi}}\left(t^{(5/2)}+\frac{32}{27}\right)\right]^{(1/3)}t^{(1/3)}+\left(\frac{2835\sqrt{\pi}}{1888}y+1\right)^{(1/3)}\left[\frac{32}{105\sqrt{\pi}}\left(t^{(5/2)}+\frac{32}{27}\right)\right]^{(1/3)}t^{(1/3)}. \tag{35}$$

Obviously,

$$f_1(t,0,0)=\left[\frac{16}{105\sqrt{\pi}}\left(t^{(5/2)}+\frac{32}{27}\right)\right]^{(1/5)}t^{(1/5)}+\left[\frac{32}{105\sqrt{\pi}}\left(t^{(5/2)}+\frac{32}{27}\right)\right]^{(1/5)}t^{(1/5)},$$
$$f_2(t,0,0)=\left[\frac{16}{105\sqrt{\pi}}\left(t^{(5/2)}+\frac{32}{27}\right)\right]^{(1/3)}t^{(1/3)}+\left[\frac{32}{105\sqrt{\pi}}\left(t^{(5/2)}+\frac{32}{27}\right)\right]^{(1/3)}t^{(1/3)}, \tag{36}$$

and $f_1(t,0,0)\equiv 0, f_2(t,0,0)\geq 0,$ $f_1(t,0,0)\geq 0,$ $f_2(t,0,0)\geq 0.$ Green's functions are as follows:

$$K_1(t-s)=K_2(t-s)=K(t-s)=\frac{1}{\Gamma(5/2)}\begin{cases}\frac{4t}{3}\left[(1-s)^{(3/2)}-\frac{1}{5}(1-s)^{(5/2)}\right]-(t-s)^{(3/2)}, & 0\leq s\leq t\leq 1,\\ \frac{4t}{3}\left[(1-s)^{(3/2)}-\frac{1}{5}(1-s)^{(5/2)}\right], & 0\leq s\leq t\leq 1.\end{cases} \tag{37}$$

Then, we have

$$\begin{aligned}
e_1(t)&=\int_0^1 K_1(t,s)a_1(s)\mathrm{d}s\\
&=\frac{4t}{3\Gamma(5/2)}\left[\int_0^t(1-s)^{(3/2)}s\mathrm{d}s-\frac{1}{5}\int_0^t(1-s)^{(5/2)}s\mathrm{d}s\right]-\frac{1}{\Gamma(5/2)}\int_0^t(t-s)^{(3/2)}s\mathrm{d}s\\
&\quad+\frac{4t}{3\Gamma(5/2)}\left[\int_t^1(1-s)^{(3/2)}s\mathrm{d}s-\frac{1}{5}\int_t^1(1-s)^{(5/2)}s\mathrm{d}s\right]\\
&=\frac{4t}{3\Gamma(5/2)}\left[-\frac{2}{5}t(1-t)^{(5/2)}-\frac{4}{35}(1-t)^{(7/2)}+\frac{4}{35}+\frac{2}{35}t(1-t)^{(7/2)}+\frac{4}{315}(1-t)^{(9/2)}-\frac{4}{315}\right]+\frac{1}{\Gamma(5/2)}\frac{4}{35}t^{(7/2)}\\
&\quad+\frac{4t}{3\Gamma(5/2)}\left[\frac{2}{5}t(1-t)^{(5/2)}+\frac{4}{35}(1-t)^{(7/2)}-\frac{2}{35}t(1-t)^{(7/2)}-\frac{4}{315}(1-t)^{(9/2)}\right]\\
&=\frac{16t}{105\sqrt{\pi}}\left(\frac{32}{27}+t^{(5/2)}\right),
\end{aligned}$$

$$\begin{aligned}
e_2(t)&=\int_0^1 K_2(t,s)a_2(s)\mathrm{d}s=\frac{8t}{3\Gamma(5/2)}\left[\int_0^t(1-s)^{(3/2)}s\mathrm{d}s-\frac{1}{5}\int_0^t(1-s)^{(5/2)}s\mathrm{d}s\right]-\frac{2}{\Gamma(5/2)}\int_0^t(t-s)^{(3/2)}s\mathrm{d}s\\
&\quad+\frac{8t}{3\Gamma(5/2)}\left[\int_t^1(1-s)^{(3/2)}s\mathrm{d}s-\frac{1}{5}\int_t^1(1-s)^{(5/2)}s\mathrm{d}s\right]
\end{aligned}$$

$$= \frac{8t}{3\Gamma(5/2)}\left[-\frac{2}{5}t(1-t)^{(5/2)} - \frac{4}{35}(1-t)^{(7/2)} + \frac{4}{35} + \frac{2}{35}t(1-t)^{(7/2)} + \frac{4}{315}(1-t)^{(9/2)} - \frac{4}{315}\right] + \frac{2}{\Gamma(5/2)}\frac{4}{35}t^{(7/2)}$$

$$+ \frac{8t}{3\Gamma(5/2)}\left[\frac{2}{5}t(1-t)^{(5/2)} + \frac{4}{35}(1-t)^{(7/2)} - \frac{2}{35}t(1-t)^{(7/2)} - \frac{4}{315}(1-t)^{(9/2)}\right]$$

$$= \frac{32t}{105\sqrt{\pi}}\left(\frac{32}{27} + t^{(5/2)}\right), \tag{38}$$

$$M_1 = \max_{t\in[0,1]} e_1(t) = \max_{t\in[0,1]} \frac{16t}{105\sqrt{\pi}}\left(\frac{32}{27} + t^{(5/2)}\right) = \frac{944}{2835\sqrt{\pi}},$$

$$M_2 = \max_{t\in[0,1]} e_2(t) = \max_{t\in[0,1]} \frac{32t}{105\sqrt{\pi}}\left(\frac{32}{27} + t^{(5/2)}\right) = \frac{1888}{2835\sqrt{\pi}}.$$

Let $h_1(t) = C_1 t, h_2(t) = C_2 t$, where

$$C_1 > \frac{4}{3\Gamma(5/2)}\int_0^1 (1-s)^{(3/2)} s ds = \frac{4}{3\Gamma(5/2)}\frac{4}{35} = \frac{16}{105\Gamma(5/2)},$$

$$C_2 > \frac{8}{3\Gamma(5/2)}\int_0^1 (1-s)^{(3/2)} s ds = \frac{8}{3\Gamma(5/2)}\frac{4}{35} = \frac{32}{105\Gamma(5/2)}. \tag{39}$$

We have

$$e_1(t) = \frac{16t}{105\sqrt{\pi}}\left(\frac{32}{27} + t^{(5/2)}\right) \le \frac{4t}{3\Gamma(5/2)}\int_0^1 (1-s)^{(3/2)} s ds$$

$$= \frac{16t}{105\Gamma(5/2)} \le C_1 t = h_1(t),$$

$$e_2(t) = \frac{32t}{105\sqrt{\pi}}\left(\frac{32}{27} + t^{(5/2)}\right) \le \frac{8t}{3\Gamma(5/2)}\int_0^1 (1-s)^{(3/2)} s ds$$

$$= \frac{32t}{105\Gamma(5/2)} \le C_2 t = h_2(t). \tag{40}$$

By (34) and (35), we have

$$f_1(t,x,y) = \left(\frac{2835\sqrt{\pi}}{944}x + 1\right)^{(1/5)}\left[\frac{16}{105\sqrt{\pi}}\left(t^{(5/2)} + \frac{32}{27}\right)\right]^{(1/5)} t^{(1/5)} + \left(\frac{2835\sqrt{\pi}}{1888}y + 1\right)^{(1/5)}\left[\frac{32}{105\sqrt{\pi}}\left(t^{(5/2)} + \frac{32}{27}\right)\right]^{(1/5)} t^{(1/5)}$$

$$= \left(\frac{2835\sqrt{\pi}}{944}x + 1\right)^{(1/5)} (e_1(t))^{(1/5)} + \left(\frac{2835\sqrt{\pi}}{1888}y + 1\right)^{(1/5)} (e_2(t))^{(1/5)}$$

$$= \left(\frac{2835\sqrt{\pi}}{944}xe_1(t) + e_1(t)\right)^{(1/5)} + \left(\frac{2835\sqrt{\pi}}{1888}ye_2(t) + e_2(t)\right)^{(1/5)},$$

$$\begin{aligned} f_2(t,x,y) &= \left(\frac{2835\sqrt{\pi}}{944}x+1\right)^{(1/3)}\left[\frac{16}{105\sqrt{\pi}}\left(t^{(5/2)}+\frac{32}{27}\right)\right]^{(1/3)}t^{(1/3)}+\left(\frac{2835\sqrt{\pi}}{1888}y+1\right)^{(1/3)}\left[\frac{32}{105\sqrt{\pi}}\left(t^{(5/2)}+\frac{32}{27}\right)\right]^{(1/3)}t^{(1/3)} \\ &= \left(\frac{2835\sqrt{\pi}}{944}x+1\right)^{(1/3)}(e_1(t))^{(1/3)}+\left(\frac{2835\sqrt{\pi}}{1888}y+1\right)^{(1/3)}(e_2(t))^{(1/3)} \\ &= \left(\frac{2835\sqrt{\pi}}{944}xe_1(t)+e_1(t)\right)^{(1/3)}+\left(\frac{2835\sqrt{\pi}}{1888}ye_2(t)+e_2(t)\right)^{(1/3)}. \end{aligned} \tag{41}$$

Take $\varphi(\eta)=\eta^{(1/3)}$; then, $\varphi(\eta)>\eta, \eta\in(0,1)$. For $\eta\in(0,1)$, $u_1,u_2\in(-\infty,+\infty)$, $v_1\in[0,M_1]$, and $v_2\in[0,M_2]$, we have

$$\begin{aligned} f_1(t,\eta u_1+(\eta-1)v_1,\eta u_2+(\eta-1)v_2) &= \left[\frac{2835\sqrt{\pi}}{944}e_1(t)[\eta u_1+(\eta-1)v_1]+e_1(t)\right]^{(1/5)} \\ &\quad+\left[\frac{2835\sqrt{\pi}}{1888}e_2(t)[\eta u_2+(\eta-1)v_2]+e_2(t)\right]^{(1/5)} \\ &= \eta^{(1/5)}\left[\frac{2835\sqrt{\pi}}{944}e_1(t)\left[u_1+\left(1-\frac{1}{\eta}\right)v_1\right]+\frac{1}{\eta}e_1(t)\right]^{(1/5)} \\ &\quad+\eta^{(1/5)}\left[\frac{2835\sqrt{\pi}}{1888}e_2(t)\left[u_2+\left(1-\frac{1}{\eta}\right)v_2\right]+\frac{1}{\eta}e_2(t)\right]^{(1/5)} \\ &= \eta^{(1/5)}\left[\frac{2835\sqrt{\pi}}{944}e_1(t)u_1+\frac{2835\sqrt{\pi}}{944}\left(1-\frac{1}{\eta}\right)e_1(t)v_1+\frac{1}{\eta}e_1(t)\right]^{(1/5)} \\ &\quad+\eta^{(1/5)}\left[\frac{2835\sqrt{\pi}}{1888}e_2(t)u_2+\frac{2835\sqrt{\pi}}{1888}\left(1-\frac{1}{\eta}\right)e_2(t)v_2+\frac{1}{\eta}e_2(t)\right]^{(1/5)} \\ &\geq \eta^{(1/5)}\left[\frac{2835\sqrt{\pi}}{944}e_1(t)u_1+\left(1-\frac{1}{\eta}\right)e_1(t)+\frac{1}{\eta}e_1(t)\right]^{(1/5)} \\ &\quad+\eta^{(1/5)}\left[\frac{2835\sqrt{\pi}}{1888}e_2(t)u_2+\left(1-\frac{1}{\eta}\right)e_2(t)+\frac{1}{\eta}e_2(t)\right]^{(1/5)} \\ &= \eta^{(1/5)}\left[\frac{2835\sqrt{\pi}}{944}e_1(t)u_1+e_1(t)\right]^{(1/5)}+\eta^{(1/5)}\left[\frac{2835\sqrt{\pi}}{1888}e_2(t)u_2+e_2(t)\right]^{(1/5)} \end{aligned}$$

$$
\begin{aligned}
&= \eta^{(1/5)} f_1(t, u_1, u_2) \geq \varphi(\eta) f_1(t, u_1, u_2), \\
f_2(t, \eta u_1 + (\eta-1)v_1, \eta u_2 + (\eta-1)v_2) &= \left[\frac{2835\sqrt{\pi}}{944} e_1(t)[\eta u_1 + (\eta-1)v_1] + e_1(t)\right]^{(1/3)} \\
&\quad + \left[\frac{2835\sqrt{\pi}}{1888} e_2(t)[\eta u_2 + (\eta-1)v_2] + e_2(t)\right]^{(1/3)} \\
&= \eta^{(1/3)}\left[\frac{2835\sqrt{\pi}}{944} e_1(t)\left[u_1 + \left(1-\frac{1}{\eta}\right)v_1\right] + \frac{1}{\eta} e_1(t)\right]^{(1/3)} \\
&\quad + \eta^{(1/3)}\left[\frac{2835\sqrt{\pi}}{1888} e_2(t)\left[u_2 + \left(1-\frac{1}{\eta}\right)v_2\right] + \frac{1}{\eta} e_2(t)\right]^{(1/3)} \\
&= \eta^{(1/3)}\left[\frac{2835\sqrt{\pi}}{944} e_1(t)u_1 + \frac{2835\sqrt{\pi}}{944}\left(1-\frac{1}{\eta}\right)e_1(t)v_1 + \frac{1}{\eta} e_1(t)\right]^{(1/3)} \\
&\quad + \eta^{(1/3)}\left[\frac{2835\sqrt{\pi}}{1888} e_2(t)u_2 + \frac{2835\sqrt{\pi}}{1888}\left(1-\frac{1}{\eta}\right)e_2(t)v_2 + \frac{1}{\eta} e_2(t)\right]^{(1/3)} \\
&\geq \eta^{(1/3)}\left[\frac{2835\sqrt{\pi}}{944} e_1(t)u_1 + \left(1-\frac{1}{\eta}\right)e_1(t) + \frac{1}{\eta} e_1(t)\right]^{(1/3)} \\
&\quad + \eta^{(1/3)}\left[\frac{2835\sqrt{\pi}}{1888} e_2(t)u_2 + \left(1-\frac{1}{\eta}\right)e_2(t) + \frac{1}{\eta} e_2(t)\right]^{(1/3)} \\
&= \eta^{(1/3)}\left[\frac{2835\sqrt{\pi}}{944} e_1(t)u_1 + e_1(t)\right]^{(1/3)} + \eta^{(1/3)}\left[\frac{2835\sqrt{\pi}}{1888} e_2(t)u_2 + e_2(t)\right]^{(1/3)} \\
&= \eta^{(1/3)} f_2(t, u_1, u_2) \geq \varphi(\eta) f_2(t, u_1, u_2).
\end{aligned}
\tag{42}
$$

Therefore, from Theorem 1, we know that system (33) has a unique solution $(x^*, y^*) \in P_{h,e}$, where

$$
\begin{aligned}
e(t) &= \left(\frac{16t}{105\sqrt{\pi}}\left(\frac{32}{27} + t^{(5/2)}\right), \frac{32t}{105\sqrt{\pi}}\left(\frac{32}{27} + t^{(5/2)}\right)\right), \quad t \in [0,1], \\
h(t) &= (C_1 t, C_2 t), \quad t \in [0,1].
\end{aligned}
\tag{43}
$$

For any given $(x_0, y_0) \in P_{h,e}$, we obtain the following sequences:

$$
\begin{aligned}
x_{n+1}(t) &= \int_0^1 K(t,s)\left(\left[\frac{2835\sqrt{\pi}}{944} x_n(s) + 1\right]^{(1/5)}\left[\frac{16}{105\sqrt{\pi}}\left(s^{(5/2)} + \frac{32}{27}\right)\right]^{(1/5)} s^{(1/5)}\right. \\
&\quad \left. + \left[\frac{2835\sqrt{\pi}}{1888} y_n(s) + 1\right]^{(1/5)}\left[\frac{32}{105\sqrt{\pi}}\left(s^{(5/2)} + \frac{32}{27}\right)\right]^{(1/5)} s^{(1/5)}\right) \mathrm{d}s - \frac{16t}{105\sqrt{\pi}}\left(\frac{32}{27} + t^{(5/2)}\right), \\
y_{n+1}(t) &= \int_0^1 K(t,s)\left(\left[\frac{2835\sqrt{\pi}}{944} x_n(s) + 1\right]^{(1/3)}\left[\frac{16}{105\sqrt{\pi}}\left(s^{(5/2)} + \frac{32}{27}\right)\right]^{(1/3)} s^{(1/3)}\right. \\
&\quad \left. + \left[\frac{2835\sqrt{\pi}}{1888} y_n(s) + 1\right]^{(1/3)}\left[\frac{32}{105\sqrt{\pi}}\left(s^{(5/2)} + \frac{32}{27}\right)\right]^{(1/3)} s^{(1/3)}\right) \mathrm{d}s - \frac{32t}{105\sqrt{\pi}}\left(\frac{32}{27} + t^{(5/2)}\right),
\end{aligned}
\tag{44}
$$

where $n = 0, 1, 2, \ldots$, $K(t, s)$ is defined by (37), $x_{n+1}(t) \longrightarrow x^*(n \longrightarrow \infty)$, and $y_{n+1}(t) \longrightarrow y^*(n \longrightarrow \infty)$.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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References

[1] A. A. Kilbas, H. M. Srivastava, and J. J. Trujillo, *Theory and Applications of Fractional Diferential Equations*, Elsevier, NewYork, NY, USA, 2006.

[2] P. Agarwal, D. Baleanu, Y. Q. Chen, S. Momani, and J. A. Tenreiro Machado, "Fractional calculus: ICFDA 2018," *Proceedings in Mathematics and Statistics*, Springer, Amman, Jordan, 2019.

[3] P. Agarwal, R. P. Agarwal, and M. Ruzhansky, *Special Functions and Analysis of Differential Equations*, CRC Press, Boca Raton, FL, USA, 2020.

[4] R. P. Agarwal, M. Benchohra, and S. Hamani, "A survey on existence results for boundary value problems of nonlinear fractional differential equations and inclusions," *Acta Applicandae Mathematicae*, vol. 109, no. 3, pp. 973–1033, 2010.

[5] B. Azizollah and T. Abdeljawad, "A Caputo fractional orer boundary value problem with integral boundary conditions," *Journal of Computational Analysis and Applications*, vol. 15, no. 4, pp. 753–763, 2013.

[6] A. Cabada and Z. Hamdi, "Nonlinear fractional differential equations with integral boundary value conditions," *Applied Mathematics and Computation*, vol. 228, pp. 251–257, 2014.

[7] Y. Cui, "Uniqueness of solution for boundary value problems for fractional differential equations," *Applied Mathematics Letters*, vol. 51, pp. 48–54, 2016.

[8] X. Zhang and Q. Zhong, "Uniqueness of solution for higher-order fractional differential equations with conjugate type integral conditions," *Fractional Calculus and Applied Analysis*, vol. 20, no. 6, pp. 1471–1484, 2017.

[9] X. G. Zhang, L. S. Liu, Y. H. Wu, and Y. J. Cui, "New result on the critical exponent for solution of an ordinary fractional differential problem," *Journal of Function Spaces*, vol. 2017, Article ID 3976469, 4 pages, 2017.

[10] W. J. Ma and Y. J. Cui, "The eigenvalue problem for Caputo type fractional differential equation with Riemann-Stieltjes integral boundary conditions," *Journal of Function Spaces*, vol. 2018, Article ID 2176809, 9 pages, 2018.

[11] J. Wu, X. G. Zhang, L. S. Liu, Y. H. Wu, and Y. J. Cui, "The convergence analysis and error estimation for unique solution of a p-Laplacian fractional differential equation with singular decreasing nonlinearity," *Boundary Value Problems*, vol. 2018, Article ID 82, 15 pages, 2018.

[12] Q. L. Song and Z. B. Bai, "Positive solutions of fractional differential equations involving the Riemann-Stieltjes integral boundary condition," *Advances in Difference Equations*, vol. 2018, Article ID 183, 2018.

[13] Q. Sun, H. W. Ji, and Y. J. Cui, "Positive solutions for boundary value problems of fractional differential equation with integral boundary conditions," *Journal of Function Spaces*, vol. 2018, Article ID 6461930, 6 pages, 2018.

[14] M. A. Alqudah, C. Ravichandran, T. Abdeljawad, and N. Valliammal, "New results on Caputo fractional-order neutral differential inclusions without compactness," *Advances in Difference Equations*, vol. 2019, Article ID 528, 2019.

[15] S. Y. Song and Y. J. Cui, "Existence of solutions for integral boundary value problems of mixed fractional differential equations under resonance," *Boundary Value Problems*, vol. 2020, Article ID 23, 2020.

[16] H. Y. Li and Y. Chen, "Multiple positive solutions for a system of nonlinear Caputo-type fractional differential equations," *Journal of Function Spaces*, vol. 2020, Article ID 2437530, 10 pages, 2020.

[17] C. Z. Bai and D. D. Yang, "The iterative positive solution for a system of fractional q-difference equations with four-point boundary conditions," *Discrete Dynamics in Nature and Society*, vol. 2020, Article ID 3970903, 8 pages, 2020.

[18] S. Y. Song and Y. J. Cui, "Multiplicity solutions for integral boundary value problem of fractional differential systems," *Discrete Dynamics in Nature and Society*, vol. 2020, Article ID 2651845, 10 pages, 2020.

[19] U. Riaz, A. Zada, Z. Ali, Y. J. Cui, and J. F. Xu, "Analysis of coupled systems of implicit impulsive fractional differential equations involving Hadamard derivatives," *Advances in Difference Equations*, vol. 2019, Article ID 226, 2019.

[20] W. Cheng, J. F. Xu, Y. J. Cui, and Q. Ge, "Positive solutions for a class of fractional difference systems with coupled boundary conditions," *Advances in Difference Equations*, vol. 2019, Article ID 249, 2019.

[21] F. L. Wang and Y. J. Cui, "Positive solutions for an infinite system of fractional order boundary value problems," *Advances in Difference Equations*, vol. 2019, Article ID 169, 2019.

[22] C. Zhai and L. Wang, "φ–(h, e)-concave operators and applications," *Journal of Mathematical Analysis and Applications*, vol. 454, no. 2, pp. 571–584, 2017.

[23] C. B. Zhai and R. T. Jiang, "Unique solutions for a new coupled system of fractional differential equations," *Advances in Difference Equations*, vol. 2019, Article ID 324, 2019.

[24] C. Zhai and W. Wang, "Solutions for a system of Hadamard fractional differential equations with integral conditions," *Numerical Functional Analysis and Optimization*, vol. 41, no. 2, pp. 209–229, 2020.

[25] C. B. Zhai and X. L. Zhu, "Unique solution for a new system of fractional differential equations," *Advances in Difference Equations*, vol. 2019, Article ID 394, 19 pages, 2019.

[26] H. Y. Li and J. T. Zhang, "Positive solutions for a system of fractional differential equations with two parameters," *Journal of Function Spaces*, vol. 2018, Article ID 1462505, 9 pages, 2018.

[27] X. W. Qiu, J. F. Xu, D. ORegan, and Y. J. Cui, "Positive solutions for a system of nonlinear semipositone boundary value problems with Riemann-Liouville fractional derivatives," *Journal of Function Spaces*, vol. 2018, Article ID 7351653, 10 pages, 2018.

[28] T. Qi, Y. Liu, and Y. Zou, "Existence result for a class of coupled fractional differential systems with integral boundary

value conditions," *The Journal of Nonlinear Sciences and Applications*, vol. 10, no. 7, pp. 4034–4045, 2017.

[29] C. B. Zhai, W. X. Wang, and H. Y. Li, "A uniqueness method to a new Hadamard fractional differential system with four-point boundary conditions," *Journal of Inequalities and Applications*, vol. 2018, Article ID 207, 2018.

[30] B. Ahmad and S. K. Ntouyas, "A fully hadamard fractional type integral boundary value problem of a coupled system of fractional differential equations," *Fractional Calculus and Applied Analysis*, vol. 17, pp. 348–360, 2014.

[31] D. Guo and V. Lakshmikantham, *Nonlinear Problems in Abstract Cones*, Academic Press, New York, NY, USA, 1988.