

## Research Article

# $H_\infty$ Filter Design for Networked Control Systems: A Markovian Jump System Approach

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This paper puts forward a method to design the  $H_\infty$  filter for networked control systems (NCSs) with time delay and data packet loss. Based on the properties of Markovian jump system, the packet loss is treated as a constant probability independent and identically distributed Bernoulli random process. Thus, the stochastic stability condition can be acquired for the filtering error system, which meets an  $H_\infty$  performance index level  $\gamma$ . It is shown that, by introducing a special structure of the relaxation matrix, a linear representation of the filter meeting an  $H_\infty$  performance index level for NCSs with time delay and packet loss can be obtained, which uses linear matrix inequalities (LMIs). Finally, numerical simulation examples demonstrate the effectiveness of the proposed method.

## 1. Introduction

As a new generation of control systems, NCSs [1–10] have attracted more and more researchers' attention because of their extensive application. Compared with the traditional control system, NCSs have some advantages, such as easy wiring, installation, maintenance, expansion, reliability, and flexibility, so that resource sharing is achieved in such systems. However, the network also brought some new problems, in which time delay and packet dropout are two main aspects, which will not only make a negative impact on system but also may even lead to the instability of system. Recently, the problem of NCSs with time delay and packet dropout phenomenon has become a hot topic in the control field [11–13].

The Markovian jump system (MJS) refers to a stochastic system with multiple model states and the system transitions between modes in accordance with the properties of the Markov chain due to the multimode transition characteristics of the Markovian jump system in the actual engineering. It can be used to simulate many systems with abrupt characteristics, such as manufacturing systems and fault-tolerant systems [14–24]. In [25], the exponential  $L_2$ - $L_\infty$

filter problem of the linear system is explored, and the system has both distributed delay, Markovian jump parameter, and norm bounded parameter uncertainty. In [26], the  $H_\infty$  filtering design about a continuous MJS in distributed sampled-data asynchronous is involved; in addition, the system's mode jumping instants and filter are asynchronous. In [27], the design of a sampled system  $H_\infty$  filter is studied. However, many conclusions only consider time delay or packet loss separately, which is not very consistent with the actual situation of network application. In addition, the  $H_\infty$  filter design of NCSs with time delay and packet dropout has not yet been considered widely.

Based on this, this paper studies the stability of networked control systems considering both time delay and packet loss. Although the analysis process is more complicated than considering time delay or packet loss alone, however the conclusion is more general and universal, and then the existence conditions of system filters are given. The effectiveness of the proposed method is verified by simulation, and the relevant conclusions are more practical. In Section 2, NCSs with time delay are present by a Markov model. The two probabilities of packet dropout in the Bernoulli random process is designed. In Section 3,

according to Lyapunov's stability theorem, the  $H_\infty$  performance of system is proven. In Section 4, a  $H_\infty$  filter for NCSs with time delay and packet loss is designed. In Section 5, numerical simulation examples are given to verify the result of this paper. In Section 6, we make a conclusion.

Notation: here are some of the symbols in the paper. The superscript "T" means the matrix transposition, and  $R^{n_x}$  shows the  $n_x$ -dimensional Euclidean space.  $I$  denotes the unit matrix of adaptive dimension, and 0 refers to the zero matrix of adaptive dimension.  $P > 0$  indicates  $P$  is real symmetric and positive matrix, and the notation stands for the norm of matrix  $\|A\|$  that is defined according to  $\|A\| = \sqrt{\text{tr}(A^T A)}$ , where "tr" denotes the trace operator.  $\|\cdot\|_2$  indicates the usual Euclidean vector norm. Prob  $\{\cdot\}$  stands for the occurrence probability of the event " $\cdot$ ".  $E\{x\}$  and  $E\{x|y\}$  represent the expectation of event  $x$  and the expectation of  $x$  conditional on  $y$ , respectively.

## 2. Problem Formulation

First, consider the following kind of discrete-time linear systems [28] with time delay as

$$\begin{cases} x(k+1) = A(r(k))x(k) + B(r(k))w(k), \\ z(k) = L(r(k))x(k) + D(r(k))w(k), \\ y(k) = C(r(k))x(k). \end{cases} \quad (1)$$

where  $x(k) \in R^n$  refers to the state vector in the plant;  $w(k) \in R^h$  belongs to  $l_2[0, \infty)$ , which indicates the measured output;  $z(k) \in R^m$  shows the estimating intended signal; and  $A(r(k))$  indicates system parameters depend on  $r(k)$ .  $B(r(k))$ ,  $C(r(k))$ ,  $D(r(k))$ , and  $L(r(k))$  have a similar situation.  $R(k)$  is assumed to a discrete Markov chain, and the values of it are in a finite set  $h \triangleq \{1, \dots, N\}$  with a transition probabilities matrix  $\bar{\pi} = (\pi_{ij})$ ; set  $\bar{r}_k = i$  and  $\bar{r}_{k+1} = j$ , such that system mode variable  $r(k)$  satisfies  $P_r(\bar{r}_{k+1} = j | \bar{r}_k = i) = \pi_{ij}$ , where  $\pi_{ij} \geq 0$  and  $\sum_{j=1}^N \pi_{ij} = 1$ ;  $h$  has  $N$  Markov modes, and for  $\bar{r}_k = i$ ,  $A(r(k))$  are denoted by  $A_i$  with appropriate dimensions.  $B_i$ ,  $C_i$ ,  $D_i$ , and  $L_i$  also have the same situation.  $\beta \in R$  shows a Bernoulli-distributed sequence with relationship as follows:

$$\begin{cases} P_{\text{rob}}\{\beta = 1\} = E\{\beta\} \triangleq \bar{\beta}, \\ P_{\text{rob}}\{\beta = 0\} = 1 - E\{\beta\} \triangleq 1 - \bar{\beta}. \end{cases} \quad (2)$$

After calculations, another important expectation can be shown as follows:

$$\text{var}\{E(\beta - \bar{\beta})^2\} = (1 - \bar{\beta})\bar{\beta} = \sigma. \quad (3)$$

*Remark 1.* For network packet loss, both Bernoulli distribution and Poisson distribution have been considered. According to the network protocols adopted in actual systems, such as industrial Ethernet and profibus, it is more practical to model network packet loss with Bernoulli distribution in this paper.

Mathematical model description of filtering is in the following formula:

$$\begin{cases} \hat{x}(k+1) = A_{fi}\hat{x}(k) + B_{fi}\hat{y}(k), \\ \hat{z}(k) = C_{fi}\hat{x}(k) + D_{fi}\hat{y}(k), \\ \hat{y}(k) = \beta y(k) + (1 - \beta)y(k+1), \end{cases} \quad (4)$$

where  $\hat{x}(k) \in R^n$  and  $\hat{y}(k) \in R^m$  are the state vector of filter mode estimator and  $\hat{z}(k) \in R^p$  indicates the output vector of the estimator.  $A_{fi}$ ,  $B_{fi}$ ,  $C_{fi}$ , and  $D_{fi}$  are real matrices to be determined with compatible dimensions. Combining (4) and (1), a filter error system with Markov chain can be shown as

$$\begin{cases} \tilde{x}(k+1) = \tilde{A}_i\tilde{x}(k) + \tilde{A}_i\tilde{x}(k-1) + \tilde{B}_i\omega(k) \\ \tilde{z}(k) = \tilde{C}_i\tilde{x}(k) + \tilde{C}_i\tilde{x}(k-1) + \tilde{D}_i\omega(k) \end{cases}, \quad (5)$$

where

$$\begin{aligned} \tilde{A}_i &= \tilde{A}_{1i} + (\beta - \bar{\beta})\tilde{A}_{2i} = \begin{bmatrix} A_i & 0 \\ \beta B_{fi}C_i & A_{fi} \end{bmatrix} = \begin{bmatrix} A_i & 0 \\ \bar{\beta} B_{fi}C_i & A_{fi} \end{bmatrix} \\ &\quad + (\beta - \bar{\beta}) \begin{bmatrix} 0 & 0 \\ B_{fi}C_i & 0 \end{bmatrix}, \\ \hat{A}_i &= \hat{A}_{1i} - (\beta - \bar{\beta})\hat{A}_{2i} = \begin{bmatrix} 0 & 0 \\ (1 - \beta)B_{fi}C_i & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ (1 - \bar{\beta})B_{fi}C_i & 0 \end{bmatrix} - (\beta - \bar{\beta}) \begin{bmatrix} 0 & 0 \\ B_{fi}C_i & 0 \end{bmatrix}, \\ \tilde{C}_i &= \tilde{C}_{1i} - (\beta - \bar{\beta})\tilde{C}_{2i} = [L_i - \beta D_{fi}C_i - C_{fi}] \\ &= [L_i - \bar{\beta} D_{fi}C_i - C_{fi}] - (\beta - \bar{\beta}) [D_{fi}C_i \ 0], \\ \hat{C}_i &= \hat{C}_{1i} - (\beta - \bar{\beta})\hat{C}_{2i} = [-(1 - \beta)D_{fi}C_i \ 0] \\ &= [-(1 - \bar{\beta})D_{fi}C_i \ 0] + (\beta - \bar{\beta}) [D_{fi}C_i \ 0], \\ \tilde{B}_i &= \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \\ \tilde{D}_i &= D_i. \end{aligned} \quad (6)$$

Obviously, a filter error system (5) is a Markovian jump system with packet loss. We will use some important definitions in the following for essential later steps.

*Definition 1* (see [29]). A filter error system (5) is stochastically stable when  $\omega(k) = 0$ . There  $x_0 \in R$  and  $r_0 \in I$ , and the following inequality exists:

$$E \left\{ \sum_{k=0}^{\infty} \|\tilde{x}(k)\| \mid \tilde{x}_0, r_0 \right\} < \infty. \quad (7)$$

*Definition 2* (see [29]). Give a scalar  $\gamma > 0$ , assume that system (5) is stochastically stable, and (5) also can meet an  $H_\infty$  performance index level  $\gamma$  under zero conditions for all nonzero  $\omega(k) \in L_2[0, \infty)$  if it satisfies

$$E \left\{ \sum_{k=0}^{\infty} \|\tilde{z}(k)\|^2 \right\} < \gamma^2 \|\omega(k)\|^2. \quad (8)$$

### 3. Main Results and Proofs

Based on the previous known conditions, a stochastically stable condition meets an  $H_\infty$  performance index level  $\gamma$  for NCSs with the development of time delay and packet loss.

**Theorem 1.** *If there exists symmetric matrices  $P_i$  and  $R_i > 0$ , consider NCSs with filter in (5); when  $\omega(k) = 0$ , system (5) will be stochastically stable such that*

$$\begin{bmatrix} \bar{Q}_i - P_i & * & * & * \\ 0 & -Q_i & * & * \\ \tilde{A}_{1i} & \hat{A}_{1i} & -\tilde{P}_i^{-1} & * \\ \delta\tilde{A}_{2i} & \delta\hat{A}_{2i} & 0 & -\delta\tilde{P}_i^{-1} \end{bmatrix} < 0, \quad (9)$$

where  $\bar{Q}_i \triangleq \sum_{j=1}^N \pi_{ij} Q_j$  and  $\bar{P}_i \triangleq \sum_{j=1}^N \pi_{ij} P_j$ .

*Proof.* Consider a Lyapunov functional candidate as follows:

$$\tilde{\Lambda} = \begin{bmatrix} \tilde{A}_{1i}^T \tilde{P}_i \tilde{A}_{1i} + \delta\tilde{A}_{2i}^T \tilde{P}_i \tilde{A}_{2i} + \tilde{Q}_i - P_i & \tilde{A}_{1i}^T \tilde{P}_i \hat{A}_{1i} - \delta\tilde{A}_{2i}^T \tilde{P}_i \hat{A}_{2i} \\ \hat{A}_{1i}^T \tilde{P}_i \tilde{A}_{1i} - \delta\hat{A}_{2i}^T \tilde{P}_i \tilde{A}_{2i} & \hat{A}_{1i}^T \tilde{P}_i \hat{A}_{1i} + \delta\hat{A}_{2i}^T \tilde{P}_i \hat{A}_{2i} - Q_i \end{bmatrix}. \quad (12)$$

Applying Schur complement theorem to the above  $\tilde{\Lambda}$ ,

$$\Lambda = \begin{bmatrix} \bar{Q}_i - P_i & * & * & * \\ 0 & -Q_i & * & * \\ \tilde{A}_{1i} & \hat{A}_{1i} & -\tilde{P}_i^{-1} & * \\ \delta\tilde{A}_{2i} & -\delta\hat{A}_{2i} & 0 & -\delta\tilde{P}_i^{-1} \end{bmatrix}. \quad (13)$$

From Theorem 3, we can get

$$\begin{aligned} E\{\Delta V(k)\} < 0 &\Rightarrow E\{V(\tilde{x}(k+1), r(k+1)) | \tilde{x}(k), r(k+1)\} \\ &\quad - V(\tilde{x}(k), r(k)) \\ &\leq -\lambda_{\min}(-\Lambda) \leq -\varepsilon \tilde{x}^T(k) \tilde{x}(k), \end{aligned} \quad (14)$$

where  $\lambda_{\min}(-\Lambda)$  indicates the minimum value of the eigenvalue for  $-\Lambda$  and  $\varepsilon = \inf\{\lambda_{\min}(-\Lambda)\}$  is the lower bound of  $\lambda_{\min}(-\Lambda)$ ; for any  $M \geq 1$ , we can obtain

$$\begin{aligned} E\{V(\tilde{x}(M+1), r(M+1))\} - E\{V(\tilde{x}(0), r(0))\} \\ \leq -\varepsilon \sum_{k=0}^M E\{\tilde{x}^T(k) \tilde{x}(k)\}. \end{aligned} \quad (15)$$

Thus, we can get

$$\sum_{k=0}^M E\{\tilde{x}^T(k) \tilde{x}(k)\} \leq \frac{1}{\varepsilon} E\{V(\tilde{x}(0), r(0))\} < \infty. \quad (16)$$

As a result, it can be proved that system (5) will be stochastically stable. The proof is completed.  $\square$

$$V(k) = \tilde{x}^T(k) P_i \tilde{x}(k) + \tilde{x}^T(k-1) Q_i \tilde{x}(k-1), \quad (10)$$

and next

$$\begin{aligned} E\{\Delta V(k)\} &= E\{\tilde{x}^T(k+1) P_i \tilde{x}(k+1)\} + \tilde{x}^T(k) \bar{Q}_i \tilde{x}(k) \\ &\quad - \tilde{x}^T(k) P_i \tilde{x}(k) - \tilde{x}^T(k-1) Q_i \tilde{x}(k-1) \\ &= E\left\{ \left[ \tilde{x}^T(k) \tilde{A}_{1i}^T + (\beta - \bar{\beta}) \tilde{x}^T(k) \tilde{A}_{2i}^T + \tilde{x}^T(k-1) \tilde{A}_{1i}^T \right. \right. \\ &\quad \left. \left. - (\beta - \bar{\beta}) \tilde{x}^T(k-1) \tilde{A}_{2i}^T \right]^T P_i [\tilde{A}_{1i} \tilde{x}(k) + (\beta - \bar{\beta}) \tilde{A}_{2i} \tilde{x}(k) \right. \right. \\ &\quad \left. \left. + \hat{A}_{1i} \tilde{x}(k-1) - (\beta - \bar{\beta}) \hat{A}_{2i} \tilde{x}(k-1) \right] \right\} \\ &\quad + \tilde{x}^T(k) \bar{Q}_i \tilde{x}(k) - \tilde{x}^T(k) P_i \tilde{x}(k) \\ &\quad - \tilde{x}^T(k-1) Q_i \tilde{x}(k-1) \\ &= \eta^T(k) \tilde{\Lambda} \eta(k), \end{aligned} \quad (11)$$

where  $\eta(k) = [\tilde{x}(k)^T \tilde{x}(k-1)^T]^T$  and

**Theorem 2.** *For NCSs (5), give a scalar  $\gamma > 0$ . For all  $\omega(k) \neq 0$ , if there are symmetric matrices  $P_i, R_i > 0$  satisfied the following matrix inequalities, and the system (5) will be stochastically stable, which meets the  $H_\infty$  norm performance level  $\gamma$ :*

$$\begin{bmatrix} \bar{Q}_i - P_i & * & * & * & * & * & * \\ 0 & -Q_i & * & * & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * & * \\ \tilde{A}_{1i} & \hat{A}_{1i} & \tilde{B}_i & -\tilde{P}_i^{-1} & * & * & * \\ \delta\tilde{A}_{2i} & -\delta\hat{A}_{2i} & 0 & 0 & -\delta\tilde{P}_i^{-1} & * & * \\ \tilde{C}_{1i} & \tilde{C}_{1i} & \tilde{D}_i & 0 & 0 & -I & * \\ \delta\tilde{C}_{2i} & -\delta\tilde{C}_{2i} & \delta\tilde{D}_i & 0 & 0 & 0 & -\delta I \end{bmatrix} < 0. \quad (17)$$

*Proof.* When  $\omega(k) \neq 0$  is similar to Theorem 3 proof process, we obtain the following equation:

$$E\{\Delta V(k)\} + E\{\tilde{z}^T(k) \tilde{z}(k)\} - \gamma^2 E\{\omega^T(k) \omega(k)\} = \tilde{\eta}^T \Lambda_1 \tilde{\eta}, \quad (18)$$

where  $\tilde{\eta} = (\tilde{x}^T(k) \tilde{x}^T(k-1) \omega^T(k))^T$ ,

$$\Lambda_1 = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \tilde{A}_{1i}^T \tilde{P}_i \tilde{B}_i + \tilde{C}_{1i}^T \tilde{D}_i \\ \Lambda_{21} & \Lambda_{13} & \hat{A}_{1i}^T \tilde{P}_i \tilde{B}_i + \tilde{C}_{1i}^T \tilde{D}_i \\ \tilde{B}_i^T \tilde{P}_i \tilde{A}_{1i} + \tilde{D}_i^T \tilde{C}_{1i} & \tilde{B}_i^T \tilde{P}_i \hat{A}_{1i} + \tilde{D}_i^T \tilde{C}_{1i} & \tilde{B}_i^T \tilde{P}_i \tilde{B}_i + \tilde{D}_i^T \tilde{D}_i \end{bmatrix}, \quad (19)$$

in which

$$\begin{aligned}
\Lambda_{11} &= \tilde{A}_{1i}^T \tilde{P}_i \tilde{A}_{1i} + \delta \tilde{A}_{2i}^T \tilde{P}_i \tilde{A}_{2i} + \bar{Q}_i - P_i + \tilde{C}_{1i}^T \tilde{C}_{1i} + \delta \tilde{C}_{2i}^T \tilde{C}_{2i}, \\
\Lambda_{21} &= A_{1i}^T \tilde{P}_i \tilde{A}_{1i} - \delta \tilde{A}_{2i}^T \tilde{P}_i \tilde{A}_{2i} + \tilde{C}_{1i}^T \tilde{C}_{1i} - \delta \tilde{C}_{2i}^T \tilde{C}_{2i}, \\
\Lambda_{12} &= \tilde{A}_{1i}^T \tilde{P}_i \hat{A}_{1i} - \delta \tilde{A}_{2i}^T \tilde{P}_i \hat{A}_{2i} + \tilde{C}_{1i}^T \tilde{C}_{1i} - \delta \tilde{C}_{2i}^T \tilde{C}_{2i}, \\
\Lambda_{11} &= \hat{A}_{1i}^T \tilde{P}_i \hat{A}_{1i} + \delta \hat{A}_{2i}^T \tilde{P}_i \hat{A}_{2i} - \bar{Q}_i + \tilde{C}_{1i}^T \tilde{C}_{1i} + \delta \tilde{C}_{2i}^T \tilde{C}_{2i}.
\end{aligned} \tag{20}$$

By Schur complement, (19) is equivalent to the following formula:

$$\begin{bmatrix}
\bar{Q}_i - P_i & * & * & * & * & * & * \\
0 & -Q_i & * & * & * & * & * \\
0 & 0 & -\gamma^2 I & * & * & * & * \\
\tilde{A}_{1i} & \hat{A}_{1i} & \tilde{B}_i & -\tilde{P}_i^{-1} & * & * & * \\
\delta \tilde{A}_{2i} & -\delta \hat{A}_{2i} & 0 & 0 & -\delta \tilde{P}_i^{-1} & * & * \\
\tilde{C}_{1i} & \tilde{C}_{1i} & \tilde{D}_i & 0 & 0 & -I & * \\
\delta \tilde{C}_{2i} & -\delta \tilde{C}_{2i} & \delta \tilde{D}_i & 0 & 0 & 0 & -\delta I
\end{bmatrix} < 0, \tag{21}$$

where  $\bar{\eta}^T \Lambda \bar{\eta} < 0$ , and it can obtain that

$$\begin{aligned}
E\{\Delta V(k)\} + E\{\bar{z}^T(k)\bar{z}(k)\} - \gamma^2 E\{\omega^T(k)\omega(k)\} &< 0, \\
\sum_{k=0}^{\infty} E\{\|\bar{z}(k)\|^2\} &< \gamma^2 E\{\|\omega(k)\|^2\} + E\{V(0)\} - E\{V(\infty)\},
\end{aligned} \tag{22}$$

initial conditions  $V(0)=0$  and  $E\{V(\infty)\} \geq 0$ ; system (5) meets  $H_\infty$  norm performance level  $\gamma$ , and it can clearly see that

$$E\left\{\sum_{k=0}^{\infty} \|\bar{z}(k)\|^2\right\} < \gamma^2 \|\omega(k)\|^2. \tag{23}$$

The proof is over.  $\square$

#### 4. Filter Design

Here, we will go to solve the system filter.

**Theorem 3.** Consider NCSs (5) with a scalar  $\gamma > 0$ . If symmetric matrix  $P_{1i} > 0$ ,  $P_{3i} > 0$ ,  $Q_{1i} > 0$ ,  $Q_{3i} > 0$ ,  $X_i > 0$ , and  $Y_i > 0$  and  $P_{2i}$ ,  $Q_{2i}$ ,  $Z_i$ ,  $A_{Fi}$ ,  $B_{Fi}$ ,  $C_{Fi}$  and  $D_{Fi}$  satisfied the following matrix inequalities, then

$$\begin{bmatrix}
\psi_{11} & * & * & * & * & * \\
0 & \psi_{22} & * & * & * & * \\
0 & 0 & -\gamma^2 I & * & * & * \\
\psi_{41} & \psi_{42} & \psi_{43} & \psi_{44} & * & * \\
\psi_{51} & \psi_{52} & 0 & 0 & \psi_{55} & * \\
\psi_{61} & \psi_{62} & \psi_{63} & 0 & 0 & \psi_{66}
\end{bmatrix} < 0, \tag{24}$$

in which

$$\begin{aligned}
\psi_{11} &= \begin{bmatrix} \bar{Q}_{1i} - P_{1i} & * \\ \bar{Q}_{2i} - P_{2i} & \bar{Q}_{3i} - P_{3i} \end{bmatrix}, \\
\psi_{22} &= \begin{bmatrix} -Q_{1i} & * \\ -Q_{2i} & -Q_{3i} \end{bmatrix}, \\
\psi_{41} &= \begin{bmatrix} X_i^T A_i + \bar{\beta} B_{Fi} C_{Fi} & A_{Fi} \\ Z_i^T A_i + \bar{\beta} B_{Fi} C_{Fi} & A_{Fi} \end{bmatrix}, \\
\psi_{42} &= \begin{bmatrix} (1 - \bar{\beta}) B_{Fi} C_{Fi} & 0 \\ (1 - \bar{\beta}) B_{Fi} C_{Fi} & 0 \end{bmatrix}, \\
\psi_{43} &= \begin{bmatrix} X_i B_i \\ Z_i B_i \end{bmatrix}, \\
\psi_{44} &= \begin{bmatrix} P_{1i} - X_i^T - X_i & * \\ P_{2i} - Y_i^T - Z_i & P_{3i} - Y_i^T - Y_i \end{bmatrix}, \\
\psi_{51} &= \begin{bmatrix} \delta B_{Fi} C_{Fi} & 0 \\ \delta B_{Fi} C_{Fi} & 0 \end{bmatrix}, \\
\psi_{52} &= \begin{bmatrix} -\delta B_{Fi} C_{Fi} & 0 \\ -\delta B_{Fi} C_{Fi} & 0 \end{bmatrix}, \\
\psi_{61} &= \begin{bmatrix} L_i - \bar{\beta} D_{Fi} C_{Fi} & -C_{Fi} \\ \delta D_{Fi} C_{Fi} & 0 \end{bmatrix}, \\
\psi_{55} &= \begin{bmatrix} \delta P_{1i} - \delta X_i^T - \delta X_i & * \\ \delta P_{2i} - \delta Y_i^T - \delta Z_i & \delta P_{3i} - \delta Y_i^T - \delta Y_i \end{bmatrix}, \\
\psi_{62} &= \begin{bmatrix} -(1 - \bar{\beta}) D_{Fi} C_{Fi} & 0 \\ -\delta D_{Fi} C_{Fi} & 0 \end{bmatrix}, \\
\psi_{63} &= \begin{bmatrix} D_{Fi} \\ 0 \end{bmatrix}, \\
\psi_{66} &= \begin{bmatrix} -I & * \\ 0 & -\delta I \end{bmatrix}.
\end{aligned} \tag{25}$$

System (5) meets  $H_\infty$  norm performance level  $\gamma$ , which is stochastically stable. Then, the filter which is achieved by the desired  $\gamma$  is calculated by  $A_{fi} = Y_i^{-1} A_{Fi}$ ,  $B_{fi} = Y_i^{-1} B_{Fi}$ ,  $C_{fi} = C_{Fi}$ , and  $D_{fi} = D_{Fi}$ .

*Proof.* Slack matrix approach can be used for (17) by setting

$$\begin{aligned}
P_i &= \begin{bmatrix} P_{1i} & P_{2i} \\ * & P_{3i} \end{bmatrix}, \\
Q_i &= \begin{bmatrix} Q_{1i} & Q_{2i} \\ * & Q_{3i} \end{bmatrix}, \\
R_i &= \begin{bmatrix} X_i & Z_i \\ Y_i & Y_i \end{bmatrix} > 0.
\end{aligned} \tag{26}$$

Then, we have following equation using (26):

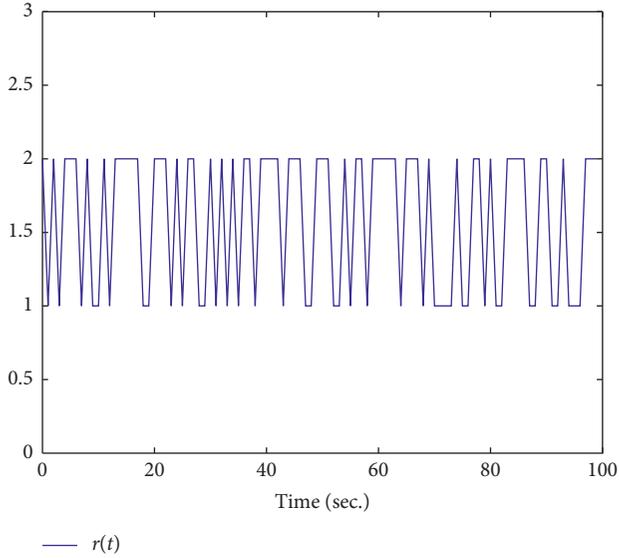
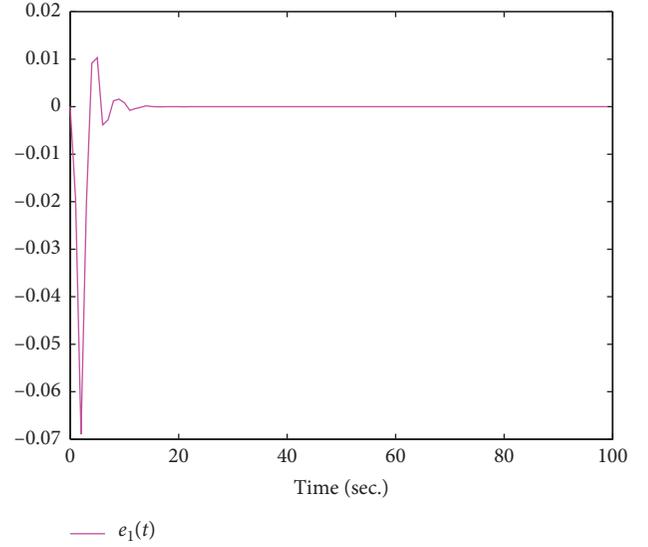
$$\begin{aligned}
 \bar{Q}_i - P_i &= \begin{bmatrix} \bar{Q}_{1i} - P_{1i} & * \\ \bar{Q}_{2i} - P_{2i} & \bar{Q}_{3i} - P_{3i} \end{bmatrix}, \\
 R_i^T \hat{A}_{1i} &= \begin{bmatrix} (1 - \bar{\beta})Y_i B_{fi} C_i & 0 \\ (1 - \bar{\beta})Y_i B_{fi} C_i & 0 \end{bmatrix}, \\
 R_i^T \hat{A}_{1i} &= \begin{bmatrix} Y_i B_{fi} C_i & 0 \\ Y_i B_{fi} C_i & 0 \end{bmatrix}, \\
 R_i^T \tilde{A}_{1i} &= \begin{bmatrix} X_i^T A_i + \bar{\beta} Y_i B_{fi} C_i & Y_i A_{fi} \\ Z_i^T A_i + \bar{\beta} Y_i B_{fi} C_i & Y_i A_{fi} \end{bmatrix}, \\
 R_i^T \tilde{B}_{1i} &= \begin{bmatrix} X_i^T B_i \\ Z_i^T B_i \end{bmatrix}, \\
 \bar{P}_i - R_i^T - R_i &= \begin{bmatrix} \bar{P}_{1i} - R_{1i}^T - R_i & * \\ \bar{P}_{2i} - R_{2i}^T - R_i & \bar{P}_{3i} - R_{3i}^T - R_i \end{bmatrix}, \\
 R_i^T \tilde{A}_{1i} &= \begin{bmatrix} Y_i B_{fi} C_i & 0 \\ Y_i B_{fi} C_i & 0 \end{bmatrix}.
 \end{aligned} \tag{27}$$

According to equations in (17) and (27), we can get

$$\begin{bmatrix}
 \bar{Q}_{1i} - P_{1i} & * & * & * & * \\
 \bar{Q}_{2i} - P_{2i} & \bar{Q}_{3i} - P_{3i} & * & * & * \\
 0 & 0 & -Q_{1i} & * & * \\
 0 & 0 & -Q_{2i} & -Q_{3i} & * \\
 0 & 0 & 0 & 0 & -\gamma^2 I \\
 X_i^T A_i + \bar{\beta} Y_i B_{fi} C_i & Y_i A_{fi} & (1 - \bar{\beta}) Y_i B_{fi} C_{fi} & 0 & X_i B_i \\
 Z_i^T A_i + \bar{\beta} Y_i B_{fi} C_i & Y_i A_{fi} & (1 - \bar{\beta}) Y_i B_{fi} C_{fi} & 0 & Z_i B_i \\
 \delta Y_i B_{fi} C_{fi} & 0 & -\delta Y_i B_{fi} C_{fi} & 0 & 0 \\
 \delta Y_i B_{fi} C_{fi} & 0 & -\delta Y_i B_{fi} C_{fi} & 0 & 0 \\
 L_i - \bar{\beta} D_{fi} C_{fi} & -C_{Fi} & -(1 - \bar{\beta}) D_{fi} C_{fi} & 0 & D_{fi} \\
 \delta D_{fi} C_{fi} & 0 & -\delta D_{fi} C_{fi} & 0 & 0 \\
 * & * & * & * & * \\
 * & * & * & * & * \\
 * & * & * & * & * \\
 * & * & * & * & * \\
 * & * & * & * & * \\
 P_{1i} - X_i^T - X_i & * & * & * & * \\
 P_{2i} - Y_i^T - Z_i & P_{3i} - Y_i^T - Y_i & * & * & * \\
 0 & 0 & \delta P_{1i} - \delta X_i^T - \delta X_i & * & * \\
 0 & 0 & \delta P_{2i} - \delta Y_i^T - \delta Z_i & \delta P_{3i} - \delta Y_i^T - \delta Y_i & * \\
 0 & 0 & 0 & 0 & -I \\
 0 & 0 & 0 & 0 & 0 \quad -\delta I
 \end{bmatrix} < 0. \tag{28}$$

The proof is over.

□

FIGURE 1: Parameters change of  $r(t)$ .FIGURE 2: Parameters change of  $e_1(t)$ .

## 5. Simulation Result

Consider system (5) with

$$\begin{aligned}
 A_1 &= \begin{bmatrix} -0.2 & -0.405 \\ 0 & -0.55 \end{bmatrix}, \\
 A_2 &= \begin{bmatrix} -0.2 & -2.673 \\ 0 & -0.230 \end{bmatrix}, \\
 B_1 &= \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \\
 B_2 &= \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \\
 C_1 &= [0.4 \quad -0.4], \\
 C_2 &= [0.5 \quad -0.2], \\
 D_1 &= [0.1 \quad -0.1], \\
 D_2 &= [0.1 \quad -0.1], \\
 L_1 &= [1 \quad 0], \\
 L_2 &= [1 \quad 0].
 \end{aligned} \tag{29}$$

Two models in the simulation are set as those data in (29). In the model, the transition probabilities are  $\pi_{ij} = \begin{bmatrix} 0.3 & 0.7 \\ 0.4 & 0.6 \end{bmatrix}$ , the initial value of  $\bar{\beta}$  is 0.5, and the initial condition is set to zero. It is easily observed that the method is feasible, and the system with time delay and packet loss becomes stochastically stable, which meets  $H_\infty$  norm performance level  $\gamma$ . Some simulation results are shown in Figures 1–5.

Figure 1 shows the time response of  $r(t)$ .

Figure 2 is the parameter change of the system error  $e_1(t)$ . It can be seen from the figure that, with the passage of time,  $e_1(t)$  changes from a large fluctuation to a stable state one.

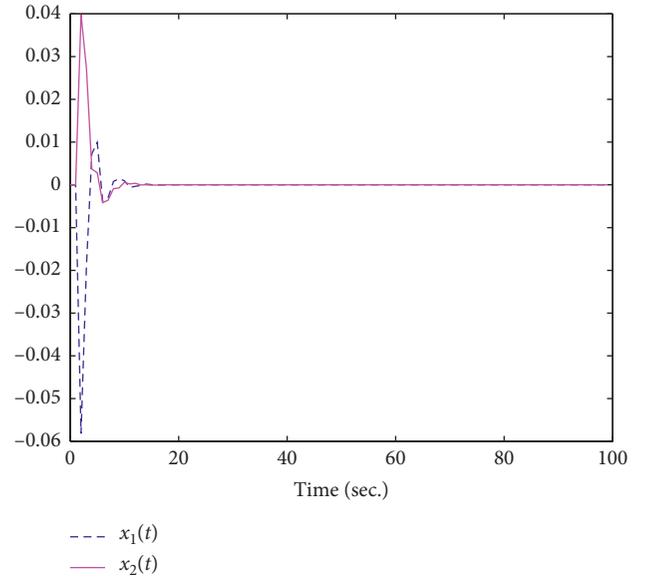
FIGURE 3: Parameters change of different  $x(t)$ .

Figure 3 shows the values of  $x_1(t)$  and  $x_2(t)$ . It can be seen from the figure that  $x_1(t)$  and  $x_2(t)$  remain stable after about 20 seconds.

Figure 4 is the parameter change of the filter state  $\hat{x}_1(t)$  and  $\hat{x}_2(t)$ . From the figure, it can be seen that the value of  $\hat{x}_1(t)$  gradually becomes equal to the value of  $\hat{x}_2(t)$  overtime and finally both values remain stable.

Figure 5 is the parameter change of  $z_1(t)$  and  $\hat{z}_1(t)$ . From the figure, it can be seen that the initial time of  $z_1(t)$  fluctuates greatly, and it tends to be consistent with  $\hat{z}_1(t)$  in the end.

*Remark 2.* As can be seen from the ordinate of the system state diagram of Figure 3 and the filter state diagram of Figure 4, the difference between the two is 100 times, so the

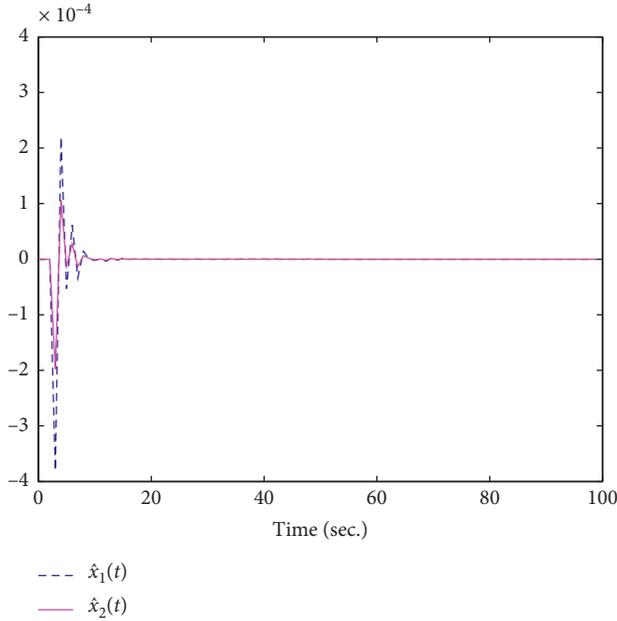


FIGURE 4: Parameters change of filter parameters.

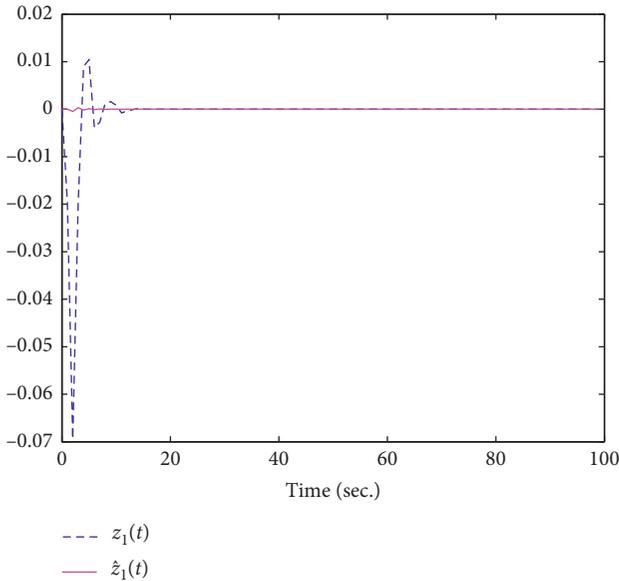


FIGURE 5: Parameters change of output parameters.

change of  $\hat{z}_1(t)$  appears to be small in Figure 5. This also reflects from one side that the filter designed in this paper has a smaller overshoot and more stable output.

## 6. Conclusion

This paper has investigated the  $H_\infty$  filtering problem about NCSs with time delay and packet dropout phenomenon. Packet dropout is treated as a constant probability independent and identically distributed Bernoulli random process. By the Lyapunov stability theory, system (5)

meeting  $H_\infty$  norm performance level  $\gamma$  is proven. By introducing a special structure of the relaxation matrix, the solution of filter which meets an  $H_\infty$  performance index level of NCSs with time delay and packet dropout is completed. Finally, a simulation result is given to prove the validity of the new design scheme.

## Data Availability

The data used to support the findings of this study are included within the article. Because it is a numerical simulation example, readers can get the same results as this article by using the LMI toolbox of Matlab and the theorem given in this article.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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