

Research Article

Defending against Online Social Network Rumors through Optimal Control Approach

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Rumors have been widely spread in online social networks and they become a major concern in modern society. This paper is devoted to the design of a cost-effective rumor-containing scheme in online social networks through an optimal control approach. First, a new individual-based rumor spreading model is proposed, and the model considers the influence of the external environment on rumor spreading for the first time. Second, the cost-effectiveness is recommended to balance the loss caused by rumors against the cost of a rumor-containing scheme. On this basis, we reduce the original problem to an optimal control model. Next, we prove that this model is solvable, and we present the optimality system for the model. Finally, we show that the resulting rumor-containing scheme is cost-effective through extensive computer experiments.

1. Introduction

Nowadays, with the rapid development of Internet technology, online social network (OSN), ranging from Facebook and Twitter to YouTube and LinkedIn, has become a popular platform for people to communicate [1, 2]. On the negative side, harmful rumors can spread rapidly over OSNs, leading to huge losses [3, 4]. In 2013, a false tweet claimed that Barack Obama was injured in an explosion which resulted in a loss of 130 billion US dollars in stock value [5]. In 2015, the rumor of shootouts and kidnappings by drug gangs happening near schools in Veracruz caused severe chaos in the city [6]. The outbreak of rumors has brought many problems, some of which pose threats to our society. Therefore, how to effectively restrain the propagation of rumors over OSNs has been a research hotspot in the field of cybersecurity.

In order to protect the security of cyberspace, it is urgent to propose some effective measures to control the spread of

rumors. In recent years, researchers have suggested many measures to mitigate the impact of rumors, such as blocking or isolating some OSN users to prevent them from further spreading rumors to OSN and releasing convincing rumor-containing messages to OSN users. These measures have been proven to be effective in controlling the spread of rumors [3, 7]. In practice, almost all rumor-containing measures will consume resources (such as money and manpower) during the implementation process, resulting in a certain cost. The cost is generally borne by the OSN platform and government. Since the budget of the OSN platform and government for controlling rumor is limited, it is necessary to study the rumor-containing problem from an economic perspective. However, most of the related research works only discussed the effectiveness (or performance) of rumor-containing measures, and they all ignored the cost of implementing the measures. Different from previous research perspectives, this paper studies rumor-containing problem from an economic perspective. This work not only

considers the losses caused by the spread of rumors but also considers the cost of implementing rumor-contain measures. In this study, we use the total loss caused by rumor to characterize cost-effectiveness. If the total loss is smaller, the cost-effectiveness is better; otherwise, the cost-effectiveness is worse. Based on the definition of cost-effectiveness, cost-effectiveness can be maximized only when the total cost is minimized. This paper will try to find such a rumor-containing scheme that maximizes cost-effectiveness.

From the above discussion, it is noticeable that the rumor-containing agency faces the following challenging problem:

Rumor-containing problem: supposing that a rumor is spreading over an OSN, design a cost-effective rumor-containing scheme.

Designing a cost-effective rumor-containing scheme is a valuable research problem. There are new rumors appearing in OSNs all the time. However, due to the fact that there is no one approach that can completely control the spread of rumors, the rumor-containing problem is essentially a management problem that requires continuous investment of resources. In practice, the budget of the rumor-containing agency is limited. If the economic costs incurred in the process of controlling rumors cannot be managed well, it will be difficult to continuously implement measures to control the spread of rumors.

In this paper, we propose a novel individual-based rumor spreading model, where the effect of the external environment on the spread of the rumor is accounted for. Thereby, we estimate the cost-effectiveness of a rumor-containing scheme. On this basis, we reduce the rumor-containing problem to an optimal control model, where each control stands for a rumor-containing scheme, and the objective functional stands for the cost-effectiveness of a rumor-containing scheme. We prove that this model admits an optimal control, guaranteeing the solvability of the model. We derive the optimality system for the model, which can be used to solve the model. Through extensive comparative experiments, we show that the rumor-containing scheme so obtained is cost-effective.

This paper makes a theoretical study on rumor-containing problem, and the research results can provide some theoretical guidance for taking measures to suppress the spread of rumors. In addition, the research method proposed in this paper can be applied to analyze the cost management of the rumor containment process, and the new model proposed in this paper can be used to study the influence of some parameters on the spread of rumors. Finally, new research ideas can also be extended to other cyberspace security problems, such as malware propagation [8].

The subsequent materials are organized in this fashion: Section 2 reviews the related work. Section 3 establishes an optimal control model of the rumor-containing problem. Sections 4 and 5 solve the model. This work is summarized in Section 6.

2. Related Work

This section is devoted to reviewing the previous work that is related to the present paper. First, rumor-containing problem is introduced. Second, some rumor spreading

models are discussed. Third, the optimal control approach used to deal with rumor-containing problem is introduced.

Rumor-containing problem is devoted to finding effective strategies to control or limit the propagation of rumor in a network, so that the losses caused by rumor can be reduced. In recent years, rumor-containing problem has received significant attention of researchers. Toward this direction, there exist two main types of rumor-containing strategies [3, 9], that is, (a) preventing most influential users or community bridges from being affected by the rumor and (b) spreading convincing messages to clarify the rumor. In the past few years, there has been quite a lot of research on the first type of strategies (also called rumor blocking strategies) [10–15]. The idea of this type of strategies is to find a small set of users or community bridges in the OSN, such that isolating or protecting them will minimize the impact of rumor propagation. However, the rumor-containing strategy ultimately boils down to solving a NP-hard problem and therefore an exact solution is infeasible for large-scale OSNs. Although many heuristic algorithms have been proposed to deal with the problem, they are still too costly for large-scale OSNs. In addition, some isolating or protecting measures may violate human rights. Recently, the second type of rumor-containing strategy has attracted a lot of attention [3, 9]. The essence of the strategy is to model rumor-containing problem as a competitive propagation problem between anti-rumor information and rumor, and this type of strategies has been shown to be effective means of restraining rumor in OSNs [3, 9, 16, 17]. In practice, the above two types of rumor-containing strategies are both effective. However, researchers only studied their effectiveness in controlling rumors but did not consider the cost issues involved in the implementation.

Modeling the spreading process of rumor lays a theoretical basis for studying rumor-containing problem. Existing rumor spreading models can be classified into three categories: *compartmental* models, *network-degree* models, and *individual-based* models. Compartmental rumor spreading models are only suited to homogeneous rumor spreading networks [18–22], and network-degree rumor spreading models only apply to some special types of networks such as scale-free networks [16, 23–26]. In contrast, individual-based rumor spreading models are applicable to all rumor spreading networks [17, 27–30]. The rumor spreading models proposed in [16–22] are all based on the assumption that a rumor can only be received through OSNs. However, in practice, OSN users can also receive rumors from the external environment such as TV programs or tabloid reports [31]. Therefore, previous work may underestimate the propagation ability of rumors. Hence, it is necessary to introduce a rumor spreading model in which the effect of the external environment is accounted for. For our purpose, in the present paper, we aim to establish such an individual-based rumor spreading model.

In recent years, optimal control theory has been applied to deal with rumor-containing problem. Optimal control theory is devoted to finding a control scheme for a dynamical system so that a certain optimality criterion is met [32]. Optimal control has been applied to a variety of areas

such as malware containment [33, 34] and cybersecurity [35]. Based on network-degree rumor spreading models, the rumor-containing problem has been dealt with through optimal control approach [36–39]. Recently, this methodology has been extended to individual-based rumor spreading models. The authors of [30] suggested an isolation-conversion mechanism of restraining rumors. Owing to violation of human rights, the mechanism in [30] may be impracticable. The authors of [17] introduced a rumor-containing message-pushing mechanism, which has two defects: (1) the effect of the external environment on the spread of the rumor was neglected at all and (2) the message-pushing rate function was regarded as a rumor-containing scheme. In practice, this function may not be under direct control of the rumor-containing agency.

In the present paper, we deal with the rumor-containing problem through optimal control approach but from a more practical perspective. First, we consider the influence of external environment on rumor spreading and propose a new rumor spreading model. Second, we regard the growth rate function of the rumor-containing cost as a rumor-containing scheme, and we define the cost-effectiveness of a rumor-containing scheme. Finally, we modeled the rumor-containing problem as the problem of finding the most cost-effective rumor-containing scheme, and we solve the problem by applying the optimal control theory.

3. The Modeling of the Rumor-Containing Problem

This section is devoted to the modeling of the rumor-containing problem. Based on a novel individual-based rumor spreading model, we measure the cost-effectiveness of a rumor-containing scheme. On this basis, we reduce the rumor-containing problem to an optimal control model.

3.1. A Rumor Spreading Model. Consider an OSN of N users denoted u_1 through u_N . Let $G_{\text{net}} = (U, E)$ denote the topological structure of the OSN, i.e., the node set $U = \{u_1, \dots, u_N\}$, and each edge $(u_i, u_j) \in E$ stands for the fact that the users u_i and u_j are mutual OSN friends. Let $\mathbf{A} = (a_{ij})_{N \times N}$ denote the adjacency matrix of G_{net} ; that is, $a_{ij} = 1$ if $(u_i, u_j) \in E$, and $a_{ij} = 0$ otherwise.

Suppose a rumor is spreading over the network G_{net} . In order to mitigate the impact of the rumor, a rumor-containing agency must collect rumor-containing evidence through continuous investment in a prescribed time horizon $[0, T]$. For $0 \leq t \leq T$, let $C(t)$ denote the cumulative rumor-containing cost in the time horizon $[0, t]$. Then $(dC(t)/dt)$ stands for the growth rate of the rumor-containing cost at time t . We refer to the function G defined by $G(t) = (dC(t)/dt)$, $t \in [0, T]$, as a *rumor-containing scheme*. Obviously, this scheme is under control of the rumor-containing agency.

For ease in realization, we assume that all feasible rumor-containing schemes are Lebesgue integrable [40]. Additionally, based on sociological evidence, it can be concluded that $G(t)$ is bounded. Let \bar{G} be the supremum of $G(t)$,

$0 \leq t \leq T$. By combining the above discussions, we get that the set of all feasible rumor-containing schemes is

$$G = \{G(t) \in L[0, T]: G(t) \leq \bar{G}, \quad 0 \leq t \leq T\}, \quad (1)$$

where $L[0, T]$ denotes the set of all Lebesgue integrable functions defined on the interval $[0, T]$.

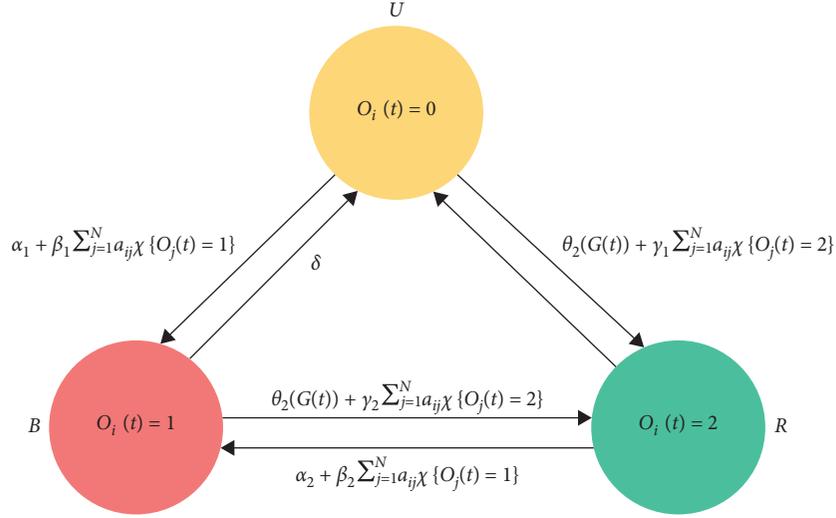
Combined with sociological evidence and rational analysis, we can know that at any time $t \in [0, T]$ each network user is either rumor-uncertain, rumor-believing, or rumor-refusing. Rumor-uncertain means that a user's attitude toward rumor is uncertain. Rumor-believing means that a user believes in a rumor. Rumor-refusing means that a user does not believe in a rumor. Let $O_i(t) = 0, 1,$ and 2 stand for the fact that the user u_i is rumor-uncertain, rumor-believing, and rumor-refusing at time t , respectively. We refer to $O_i(t)$ as the state of the user u_i at time t , and the vector

$$O(t) = (O_1(t), \dots, O_N(t)), \quad (2)$$

as the state of the network at time t . The network state evolves over time. In order to describe the evolutionary process of the network state, let us introduce the following notations:

- (i) β_1 (resp., β_2): the probability with which, owing to the influence of a rumor-believing OSN friend, a rumor-uncertain (resp., rumor-refusing) user becomes rumor-believing at any time, and $\beta_1, \beta_2 > 0$.
- (ii) α_1 (resp., α_2): the probability with which, owing to the influence of the external environment, a rumor-uncertain (resp., rumor-refusing) user becomes rumor-believing at any time, and $\alpha_1, \alpha_2 \geq 0$.
- (iii) γ_1 (resp., γ_2): the probability with which, owing to the influence of a rumor-refusing OSN friend, a rumor-uncertain (resp., rumor-believing) user becomes rumor-refusing at any time, and $\gamma_1, \gamma_2 > 0$.
- (iv) δ : the probability with which, owing to the limited memory, a rumor-believing or rumor-refusing user becomes rumor-uncertain at any time, and $\delta > 0$.
- (v) $\theta_1(\bar{G})$ (resp., $\theta_2(\bar{G})$), $\bar{G} \in [0, \infty)$: the probability with which, owing to the growth rate \bar{G} of the rumor-containing cost, a rumor-uncertain (resp., rumor-believing) user becomes rumor-refusing. Obviously, $\theta_1(0) = \theta_2(0) = 0$, θ_1 and θ_2 are strictly increasing.

In practice, the first seven parameters can be estimated through online questionnaire survey, and the last two functions can be approximated through regression based on historical data. In particular, we introduce the parameter α_1 to characterize the effect of the external environment on the spread of rumors, and $\alpha_1 = 0$ refers to the scenario that does not consider the influence of external environment; this scenario has been studied by many researchers [16–30]. In this paper, we will consider a more general case, i.e., $\alpha_1 > 0$. Let $\chi_{\{O_i(t)=1\}}$ and $\chi_{\{O_i(t)=2\}}$ denote the indicator function of the event $O_i(t) = 1$ and the event $O_i(t) = 2$, respectively. Based on the theory of continuous-time Markov chain [41], Figure 1 exhibits the diagram of state transition of the user u_i .

FIGURE 1: The diagram of state transition of the user u_i .

Let $U_i(t)$, $B_i(t)$, and $R_i(t)$ denote the probability that the user u_i is rumor-uncertain, rumor-believing, and rumor-refusing at time t , respectively:

$$\begin{aligned} U_i(t) &= \Pr\{O_i(t) = 0\}, \\ B_i(t) &= \Pr\{O_i(t) = 1\}, \\ R_i(t) &= \Pr\{O_i(t) = 2\}. \end{aligned} \quad (3)$$

Since $U_i(t) = 1 - B_i(t) - R_i(t)$, we refer to the vector

$$E(t) = (B_1(t), \dots, B_N(t), R_1(t), \dots, R_N(t)), \quad (4)$$

as the expected state of the network at time t . Let $E_0 = E(0)$ denote the initial expected network state.

Theorem 1. *The evolutionary process of the expected network state is described by the following system of ordinary differential equations:*

$$\begin{cases} \frac{dB_i(t)}{dt} = \left[\alpha_1 + \beta_1 \sum_{j=1}^N a_{ij} B_j(t) \right] [1 - B_i(t) - R_i(t)] + \left[\alpha_2 + \beta_2 \sum_{j=1}^N a_{ij} B_j(t) \right] R_i(t) - \left[\theta_2(G(t)) + \gamma_2 \sum_{j=1}^N a_{ij} R_j(t) + \delta \right] B_i(t), & 0 \leq t \leq T, i = 1, \dots, N, \\ \frac{dR_i(t)}{dt} = \left[\theta_1(G(t)) + \gamma_1 \sum_{j=1}^N a_{ij} R_j(t) \right] [1 - B_i(t) - R_i(t)] + \left[\theta_2(G(t)) + \gamma_2 \sum_{j=1}^N a_{ij} R_j(t) \right] B_i(t) - \left[\alpha_2 + \beta_2 \sum_{j=1}^N a_{ij} B_j(t) + \delta \right] R_i(t), & 0 \leq t \leq T, i = 1, \dots, N, \\ E(0) = E_0. \end{cases} \quad (5)$$

Proof. Let $\mathbb{E}(\cdot)$ denote the mathematical expectation of a random variable. Then, the rumor-uncertain user u_i becomes rumor-believing at time t at the expected rate

$$\mathbb{E} \left(\alpha_1 + \beta_1 \sum_{j=1}^N a_{ij} \chi_{\{O_j(t)=1\}} \right) = \alpha_1 + \beta_1 \sum_{j=1}^N a_{ij} B_j(t), \quad (6)$$

the rumor-refusing user u_i becomes rumor-believing at time t at the expected rate

$$\mathbb{E} \left(\alpha_2 + \beta_2 \sum_{j=1}^N a_{ij} \chi_{\{O_j(t)=1\}} \right) = \alpha_2 + \beta_2 \sum_{j=1}^N a_{ij} B_j(t), \quad (7)$$

the rumor-believing user u_i becomes rumor-refusing at time t at the expected rate

$$\mathbb{E} \left(\theta_2(G(t)) + \gamma_2 \sum_{j=1}^N a_{ij} \chi_{\{O_j(t)=2\}} \right) = \theta_2(G(t)) + \gamma_2 \sum_{j=1}^N a_{ij} R_j(t), \quad (8)$$

and the rumor-believing user u_i becomes rumor-uncertain at time t at the expected rate δ . The first N equations in system (5) follow. The last N equations in the system can be derived analogously.

System (5) is a novel individual-based rumor spreading model, in which the effect of the external environment is accounted for. For $1 \leq i \leq N$ and $0 \leq t \leq T$, let

$$\begin{aligned} f_i(\mathbf{E}(t), G(t)) &= \left[\alpha_1 + \beta_1 \sum_{j=1}^N a_{ij} B_j(t) \right] [1 - B_i(t) - R_i(t)] + \left[\alpha_2 + \beta_2 \sum_{j=1}^N a_{ij} B_j(t) \right] R_i(t) - \left[\theta_2(G(t)) + \gamma_2 \sum_{j=1}^N a_{ij} R_j(t) + \delta \right] B_i(t), \\ f_{N+i}(\mathbf{E}(t), G(t)) &= \left[\theta_1(G(t)) + \gamma_1 \sum_{j=1}^N a_{ij} R_j(t) \right] [1 - B_i(t) - R_i(t)] + \left[\theta_2(G(t)) + \gamma_2 \sum_{j=1}^N a_{ij} R_j(t) \right] B_i(t) - \left[\alpha_2 + \beta_2 \sum_{j=1}^N a_{ij} B_j(t) + \delta \right] R_i(t). \end{aligned} \tag{9}$$

For $0 \leq t \leq T$, let

$$f(\mathbf{E}(t), G(t)) = (f_1(\mathbf{E}(t), G(t)), \dots, f_{2N}(\mathbf{E}(t), G(t))). \tag{10}$$

Then, model (5) is abbreviated as

$$\begin{cases} \frac{d\mathbf{E}(t)}{dt} = f(\mathbf{E}(t), G(t)), & 0 \leq t \leq T, \\ \mathbf{E}(0) = \mathbf{E}_0. \end{cases} \tag{11}$$

Next, we show that model (5) is positively invariant.

Theorem 2. *Let \mathbf{E} be the solution to model (5). Then, $\mathbf{E}(t) \in [0, 1]^{2N}$, $0 \leq t \leq T$.*

Proof. Obviously, $\mathbf{E}(0) \in [0, 1]^{2N}$. For $0 \leq t \leq T$, we proceed by distinguishing among three possibilities.

Case 1. $B_i(t) = 1$. Then, $R_i(t) = 0$. Hence, $(dB_i(t)/dt) = -[\theta_2(G(t)) + \gamma_2 \sum_{j=1}^N a_{ij} R_j(t) + \delta] < 0$.

Case 2. $R_i(t) = 0$. Then, $B_i(t) = 0$. Hence, $(dR_i(t)/dt) = -[\alpha_2 + \beta_2 \sum_{j=1}^N a_{ij} B_j(t) + \delta] < 0$.

Case 3. $0 \leq B_i(t) \leq 1, 0 \leq R_i(t) \leq 1$, and $B_i(t) + R_i(t) = 1$. Then, $(d[B_i(t) + R_i(t)]/dt) = -\delta < 0$.

Hence, $\mathbf{E}(t) \in [0, 1]^{2N}$, $0 \leq t \leq T$.

In what follows, let

$$\begin{aligned} \mathbf{E}(t, G(t)) &= (B_1(t, G(t)), \dots, B_N(t, G(t)), R_1(t, G(t)), \\ &\quad \dots, R_N(t, G(t))), \end{aligned} \tag{12}$$

stand for the solution to model (5).

3.2. The Optimal Control Modeling of the Rumor-Containing Problem. For our modeling purpose, we need to estimate the total loss caused by rumor, and then we will define cost-effectiveness. To this end, let w denote the average loss per unit time of a rumor-believing user. In practice, w can be estimated by assessing the potential consequence of the rumor under consideration.

Theorem 3. *The expected total loss of all network users in the time range $[0, T]$ is*

$$J_1(G) = w \int_0^T \sum_{i=1}^N B_i(t, G(t)) dt. \tag{13}$$

Proof. Let $dt > 0$ denote an infinitesimal. The average loss of the user u_i in the time range $[t, t + dt]$ is $w dt$ or zero according to whether he is rumor-believing at time t or not. Hence, his expected loss in the time range $[t, t + dt]$ is $w B_i(t, G(t)) dt$. Equation (13) follows.

Theorem 4. *The rumor-containing cost in the time range $[0, T]$ is*

$$J_2(G) = \int_0^T G(t) dt. \tag{14}$$

Proof. Let $dt > 0$ denote an infinitesimal. The rumor-containing cost in the time range $[t, t + dt]$ is $G(t) dt$. Equation (14) follows.

Based on Theorems 3 and 4, for a rumor-containing scheme G , the total loss caused by a rumor can be measured by the quantity

$$J(G) = J_1(G) + J_2(G) = \int_0^T \left[w \sum_{i=1}^N B_i(t, G(t)) + G(t) \right] dt. \tag{15}$$

In this paper, we use the quantity $J(G)$ to characterize cost-effectiveness, and the smaller the quantity $J(G)$ is, the more cost-effective the rumor-containing scheme will be. In practice, we hope to achieve maximum cost-effectiveness by minimizing the quantity $J(G)$.

Based on the above discussions, we model the rumor-containing problem as the following optimal control problem:

$$\min_{G \in \mathcal{G}} J(G) = \int_0^T F(\mathbf{E}(t), G(t)) dt,$$

$$\text{subject to } \begin{cases} \frac{d\mathbf{E}(t)}{dt} = f(\mathbf{E}(t), G(t)), & 0 \leq t \leq T, \\ \mathbf{E}(0) = \mathbf{E}_0. \end{cases} \tag{16}$$

Here, $F(\mathbf{E}(t), G(t)) = w \sum_{i=1}^N B_i(t, G(t)) + G(t)$.

We refer to the optimal control problem as the rumor-containing model. Each instance of the model is given by the 14-tuple as follows:

$$M = (G, \beta_1, \beta_2, \alpha_1, \alpha_2, \gamma_1, \gamma_2, \delta, \bar{G}, w, \theta_1, \theta_2, T, \mathbf{E}_0). \quad (17)$$

4. Dealing with the Rumor-Containing Model

This section aims to deal with the rumor-containing model (16). First, we show that the model is solvable. Second, we derive the optimality system for the model.

4.1. Solvability of the Rumor-Containing Model. The following lemma is a direct corollary of a theorem in [32].

Lemma 1. *The rumor-containing model (16) admits an optimal control if the following six conditions hold simultaneously:*

- (1) \mathcal{G} is closed.
- (2) \mathcal{G} is convex.
- (3) There exists $G \in \mathcal{G}$ such that model (5) is solvable.
- (4) $\mathbf{f}(\mathbf{E}, G)$ is bounded by a linear function in \mathbf{E} .
- (5) $F(\mathbf{E}, G)$ is convex on \mathcal{G} .

(6) There exist $\rho > 1, d_1 > 0$, and d_2 such that $F(\mathbf{E}, G) \geq d_1 G^\rho + d_2$.

We are ready to show the solvability of the rumor-containing model.

Theorem 5. *The rumor-containing model (16) admits an optimal control.*

Proof. First, let G^* be an accumulation point of the set \mathcal{G} . Then, there is a sequence of points, G_1, G_2, \dots , in \mathcal{G} that approaches G^* . On one hand, $G^* \in L[0, T]$ follows from the completeness of $L[0, T]$. On the other hand, $G^* = \lim_{n \rightarrow \infty} G_n(t) \leq \bar{G}, 0 \leq t \leq T$. Hence, $G^* \in \mathcal{G}$. The closeness of \mathcal{G} is proven. Second, let $G_1, G_2 \in \mathcal{G}, 0 < \sigma < 1$, and $G^* = (1 - \sigma)G_1 + \sigma G_2$. On one hand, $G^* \in L[0, T]$ follows from the fact that $L[0, T]$ is a vector space. On the other hand, it is obvious that $G^*(t) \leq \bar{G}, 0 \leq t \leq T$. The convexity of \mathcal{G} is proven. Thirdly, the solvability of the system $(d\mathbf{E}(t)/dt) = \mathbf{f}(\mathbf{E}(t), G), 0 \leq t \leq T$, follows from the continuous differentiability of the function $\mathbf{f}(\mathbf{E}, G)$. Fourthly, for $1 \leq i \leq N$, we have

$$\begin{aligned} -\left[\theta_2(\bar{G}) + \gamma_2 \sum_{j=1}^N a_{ij} + \delta\right] B_i \leq f_i(\mathbf{E}, G) \leq \alpha_1 + \alpha_2 + (\beta_1 + \beta_2) \sum_{j=1}^N a_{ij} B_j, \\ -\left[\alpha_2 + \beta_2 \sum_{j=1}^N a_{ij} + \delta\right] R_i \leq f_{N+i}(\mathbf{E}, G) \leq \theta_1(\bar{G}) + \theta_2(\bar{G}) + (\gamma_1 + \gamma_2) \sum_{j=1}^N a_{ij} R_j. \end{aligned} \quad (18)$$

Next, the convexity of $F(\mathbf{E}, G)$ on \mathcal{G} follows from its linearity on \mathcal{G} . Finally, we have $F(\mathbf{E}, G) \geq 0 \geq G^2 - \bar{G}^2$. Hence, the claim follows from Lemma 1.

4.2. A Necessary Condition for the Optimal Control. The Hamiltonian of the rumor-containing model (16) is

$$\begin{aligned} H(\mathbf{E}, R, \lambda, \mu) = w \sum_{i=1}^N B_i + G + \sum_{i=1}^N \lambda_i \left[\left(\alpha_1 + \beta_1 \sum_{j=1}^N a_{ij} B_j \right) (1 - B_i - R_i) + \left(\alpha_2 + \beta_2 \sum_{j=1}^N a_{ij} B_j \right) R_i - \left(\theta_2(G) + \gamma_2 \sum_{j=1}^N a_{ij} R_j + \delta \right) B_i \right] \\ + \sum_{i=1}^N \mu_i \left[\left(\theta_1(G) + \gamma_1 \sum_{j=1}^N a_{ij} R_j \right) (1 - B_i - R_i) + \left(\theta_2(G) + \gamma_2 \sum_{j=1}^N a_{ij} R_j \right) B_i - \left(\alpha_2 + \beta_2 \sum_{j=1}^N a_{ij} B_j + \delta \right) R_i \right], \end{aligned} \quad (19)$$

where $\lambda = (\lambda_1, \dots, \lambda_N)$ and $\mu = (\mu_1, \dots, \mu_N)$. λ and μ constitute the adjoint of H .

We are ready to present a necessary condition for the optimal control of model (16).

Theorem 6. *Let G be an optimal control for model (16), let \mathbf{E} be the resulting solution to model (5). Then, there exists an adjoint (λ, μ) such that*

$$\begin{aligned}
\frac{d\lambda_i(t)}{dt} &= -\omega + \left[\alpha_1 + \theta_2(G(t)) + \delta + \sum_{j=1}^N a_{ij}(\beta_1 B_j(t) + \gamma_2 R_j(t)) \right] \lambda_i(t) + \left[\theta_1(G(t)) - \theta_2(G(t)) + (\gamma_1 - \gamma_2) \sum_{j=1}^N a_{ij} R_j(t) \right] \mu_i(t) \\
&\quad + \sum_{j=1}^N \beta_2 a_{ij} R_j(t) \mu_j(t), \\
&\quad - \sum_{j=1}^N a_{ij} [\beta_1(1 - B_j(t) - R_j(t)) + \beta_2 R_j(t)] \lambda_j(t), \quad 0 \leq t \leq T, i = 1, \dots, N, \\
\frac{d\mu_i(t)}{dt} &= \left[(\alpha_1 - \alpha_2) + \sum_{j=1}^N (\beta_1 - \beta_2) a_{ij} B_j(t) \right] \lambda_i(t) + \sum_{j=1}^N \gamma_2 a_{ij} B_j(t) \lambda_j(t) + \left[\theta_1(G(t)) + \alpha_2 + \delta + \sum_{j=1}^N a_{ij} (\gamma_1 R_j(t) + \beta_2 B_j(t)) \right] \mu_i(t) \\
&\quad - \sum_{j=1}^N a_{ij} [\gamma_1(1 - B_j(t) - R_j(t)) + \gamma_2 B_j(t)] \mu_j(t), \quad 0 \leq t \leq T, i = 1, \dots, N.
\end{aligned} \tag{20}$$

Moreover, $\lambda(T) = \mu(T) = 0$, and

$$G(t) \in \arg \min_{\tilde{G} \in [0, \bar{G}]} \left\{ \tilde{G} - \sum_{i=1}^N \lambda_i(t) B_i(t) \theta_2(\tilde{G}) + \sum_{i=1}^N \mu_i(t) [(1 - B_i(t) - R_i(t)) \theta_1(\tilde{G}) + B_i(t) \theta_2(\tilde{G})] \right\}, \quad 0 \leq t \leq T. \tag{21}$$

Proof. It follows from Pontryagin's Minimum Principle [32] that there exists (λ, μ) such that

$$\begin{cases} \frac{d\lambda_i(t)}{dt} = \frac{\partial H(\mathbf{E}(t), G(t), \lambda(t), \mu(t))}{\partial B_i}, & 0 \leq t \leq T, i = 1, \dots, N, \\ \frac{d\mu_i(t)}{dt} = \frac{\partial H(\mathbf{E}(t), G(t), \lambda(t), \mu(t))}{\partial R_i}, & 0 \leq t \leq T, i = 1, \dots, N. \end{cases} \tag{22}$$

System (20) follows by direct calculations. As the terminal cost is unspecified and the final state is free, we have $\lambda(T) = \mu(T) = 0$. Again, by Pontryagin's Minimum Principle, we get

$$G(t) \in \arg \min_{G \in \mathcal{E}} H(\mathbf{E}(t), \tilde{G}, \lambda(t), \mu(t)), \quad 0 \leq t \leq T. \tag{23}$$

System (21) follows from direct calculations.

Based on the previous discussions, we get that the optimality system for model (16) comprises system (5), system (20), system (21), and $\lambda(T) = \mu(T) = 0$. The optimality system can be solved by invoking the well-known Forward-Backward Euler Method [42]. We refer to the control obtained in this way as a *promising rumor-containing scheme* for model (16). This is because the scheme may be optimal in terms of cost-effectiveness.

5. The Cost-Effectiveness of the Promising Rumor-Containing Scheme

At the end of the previous section, we proposed the notion of promising rumor-containing scheme. In this section, we

assess the cost-effectiveness of this scheme through comparative experiments.

5.1. Experiment Design. In each of the following experiments, we conduct the following operations: (1) generate an instance of the rumor-containing model (16), (2) obtain a promising rumor-containing scheme for the instance by invoking Forward-Backward Euler Method, and (3) compare this scheme with a set of static rumor-containing schemes in terms of the cost-effectiveness. All the following experiments are carried out on a PC with Inter® Core™ i5-7500 CPU @ 3.40 GHz and 8 GB RAM.

Studies show that some social platforms such as Facebook [43], Twitter [44], and YouTube [45] have provided a way for rumors to generate and spread. To simulate the environment in which rumors spread, we choose three real-world OSNs. First, consider the Facebook network and the Twitter network provided by SNAP, the well-known network library [46]. Due to memory limitation, we randomly choose a subnet of the original network without loss of generality. Choose a 100-node subnet of the original Facebook network (dataset name: ego-Facebook), denoted by G_F , and a 100-node subset of the original Twitter network (dataset name: ego-Twitter), denoted by G_T , respectively. Second, consider the YouTube network in Network Repository [47]. Choose a 100-node subnet of the YouTube network, denoted by G_Y . Figure 2 displays these three networks.

The cost-effectiveness is the focus in this section. Common OSN platforms generally have information security agencies. When rumors break out on OSN, the agency

is responsible for collecting and disseminating truths to dispel rumors, for example, when President Trump declared on Twitter that mail voting would lead to a “rigged election.” In order to control the spread of the rumor, Twitter tagged Trump’s tweets with the label “Getting the facts about mailing votes” and redirected users to a fact-checking page to provide comprehensive investigation information for the misleading article. In practice, the agency’s budget is limited. In order to control the spread of rumors, the agency needs to find a cost-effective rumor control scheme; that is, the scheme can minimize the total loss caused by rumors.

In all the following experiments, let $\bar{G} = 1$, $w = 0.1$, $T = 10$, and $\mathbf{E}_0 = (0.1, \dots, 0.1)$. Let $G^P(t) (0 \leq t \leq T)$ denote the promising rumor-containing scheme, and $G_a(t) = a (0 \leq t \leq T)$ a static rumor-containing scheme. Let

$$\mathbb{M}_i = (G_{\text{net}}^i, \beta_1, \beta_2, \alpha_1, \alpha_2, \gamma_1, \gamma_2, \delta, \bar{G}, w, \theta_1(x), \theta_2(x), T, \mathbf{E}_0), \quad i = 1, 2, 3, \dots, \quad (25)$$

where $G_{\text{net}}^1 = G_F$, $G_{\text{net}}^2 = G_T$, $G_{\text{net}}^3 = G_Y$, $\beta_1 = 0.2$, $\beta_2 = 0.1$, $\alpha_1 = 0$, $\alpha_2 = 0.15$, $\gamma_1 = 0.2$, $\gamma_2 = 0.12$, $\delta = 0.1$, $\bar{G} = 1.0$, $w = 0.1$, $\theta_1(x) = 0.3$, and $\theta_2(x) = 0.2$. In order to show the influence of external environment on the propagation of rumors, we only change the value of α_1 and let $\alpha_1 = 0, 0.1, 0.2, 0.3, 0.4, 0.5$, respectively. Let $B(t)$ denote the expected probability of rumor-believing users in the OSN, and $B(t)$ is given by $B(t) = (1/N) \sum_{i=1}^N B_i(t)$. As shown in Figure 3, compared with the situation that does not consider the effect of external environment (i.e., $\alpha_1 = 0$), increasing α_1 will cause the value of $B(t)$ to increase, especially when t is relatively small. The finding indicates that external environment has a great effect on the expected probability of rumor-believing users in OSNs, especially in the early stages of the spread of rumors, and if we ignore the effect of external environment, the propagation ability of rumors will be seriously underestimated.

$$\mathbb{M} = (G, \beta_1, \beta_2, \alpha_1, \alpha_2, \gamma_1, \gamma_2, \delta, \bar{G}, w, \theta_1, \theta_2, T, \mathbf{E}_0). \quad (26)$$

In practice, the closed-form formula for the functions θ_1 and θ_2 can be approximated through regression based on historical data, and both θ_1 and θ_2 are monotonically increasing functions. Supposing that there is a rumor spreading on the network G_F , we consider three different forms of θ_1 and θ_2 , and the experimental settings are as follows.

Experiment 2. Consider three instances of the rumor-containing model:

$$\mathbb{M}_F^i = (G_F, \beta_1, \beta_2, \alpha_1, \dots, \theta_2^i(x), T, \mathbf{E}_0), \quad i = 1, 2, 3, \dots, \quad (27)$$

where $\beta_1 = 0.2$, $\beta_2 = 0.1$, $\alpha_1 = 0.2$, $\alpha_2 = 0.15$, $\gamma_1 = 0.2$, $\gamma_2 = 0.12$, $\delta = 0.1$, $\bar{G} = 1.0$, $w = 0.1$, $\theta_1^1(x) = 0.3x$, $\theta_2^1(x) = 0.2x$, $\theta_1^2(x) = 0.3x^{(1/2)}$, $\theta_2^2(x) = 0.2x^{(1/2)}$, $\theta_1^3(x) = (1.5x/4 + x)$, and $\theta_2^3(x) = (x/4 + x)$. Let G^P denote the promising rumor-containing scheme. By applying the

$$A = \{G_a: a = 0, 0.1, 0.2, \dots, 1.0\}. \quad (24)$$

In particular, the static rumor-containing scheme $G_a(t) = 0$ stands for taking no rumor-containing scheme; that is, let the rumors spread freely.

5.2. Experimental Results

Experiment 1. For the rumor-containing model $\mathbb{M} = (G, \beta_1, \beta_2, \alpha_1, \dots, \theta_2, T, \mathbf{E}_0)$, $\alpha_1 = 0$ stands for ignoring the influence of external environment on the propagation of a rumor. Consider three instances as follows:

approach introduced in Section 4.2, we get a promising control G^P , which is shown in Figure 4. It is seen that the promising rumor-containing scheme G^P of the three instances first stays at the maximum allowable rate and then drops to the zero rates.

Furthermore, we compare the cost-effectiveness between the promising control strategy G^P and a group of static control strategies $A = \{G_a: 0, 0.1, 0.2, \dots, 1.0\}$. The comparison result can be found in Figure 5. It is seen that $J(G^P) < J(G_a)$, $G_a \in A$. The result shows that our proposed rumor-containing scheme G^P obtains the highest cost-effectiveness; hence, it performs much better than all the static rumor-containing schemes.

Similarly, supposing that there is a rumor spreading on the network G_T , we consider three different forms of θ_1 and θ_2 , and the experimental settings are as follows.

Experiment 3. Consider three instances of the rumor-containing model:

$$\mathbb{M}_T^i = (G_T, \beta_1, \beta_2, \alpha_1, \alpha_2, \gamma_1, \gamma_2, \delta, \bar{G}, w, \theta_1^i(x), \theta_2^i(x), T, \mathbf{E}_0), \quad i = 1, 2, 3, \dots, \quad (28)$$

where $\beta_1 = 0.15$, $\beta_2 = 0.1$, $\alpha_1 = 0.15$, $\alpha_2 = 0.12$, $\gamma_1 = 0.2$, $\gamma_2 = 0.1$, $\delta = 0.1$, $\bar{G} = 1.0$, $w = 0.1$, $\theta_1^1(x) = 0.4x$, $\theta_2^1(x) = 0.3x$, $\theta_1^2(x) = 0.4x^{(1/2)}$, $\theta_2^2(x) = 0.3x^{(1/2)}$, $\theta_1^3(x) = (0.8x/1 + x)$, and $\theta_2^3(x) = (0.6x/1 + x)$. Let G^P denote the promising rumor-containing scheme. By applying the approach introduced in Section 4.2, we get a promising control G^P , which is shown in Figure 6. It is seen that the promising rumor-containing scheme G^P of the three instances first stays at the maximum allowable rate and then drops to the zero rates.

Furthermore, we compare the cost-effectiveness between the promising control strategy G^P and a group of static control strategies $A = \{G_a: 0, 0.1, 0.2, \dots, 1.0\}$. The comparison result can be found in Figure 7. It is seen that $J(G^P) < J(G_a)$, $G_a \in A$. The result shows that our proposed

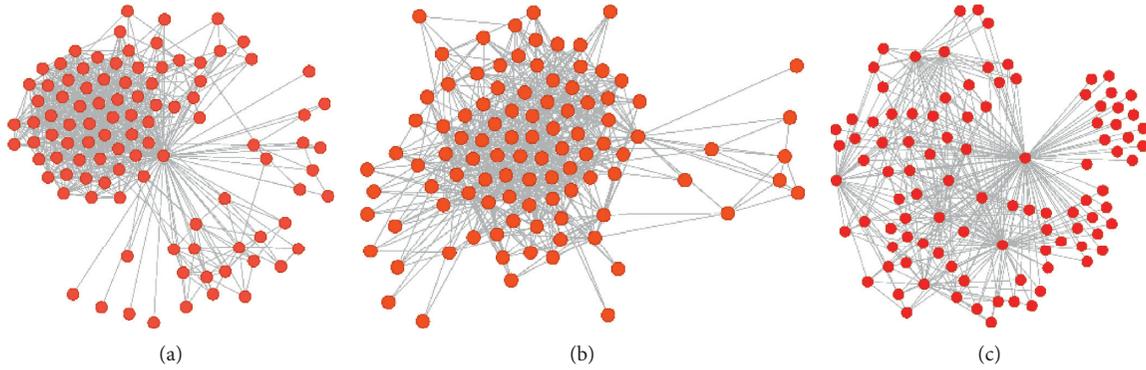


FIGURE 2: Three real-world OSNs: (a) G_F , (b) G_T , and (c) G_Y .

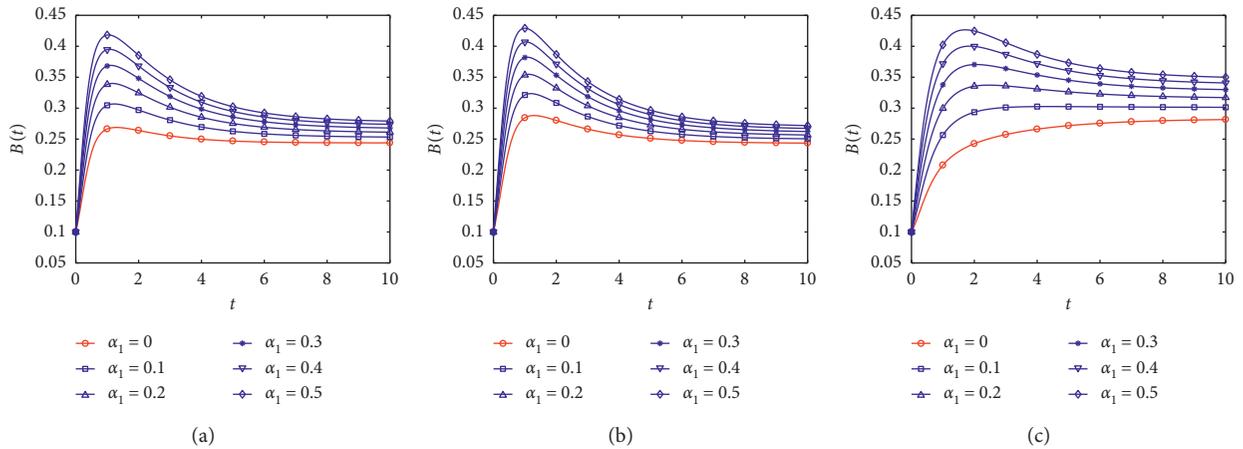


FIGURE 3: The effect of α_1 on $B(t)$ for the three OSNs: (a) Facebook network G_F , (b) Twitter network G_T , and (c) YouTube network G_Y .

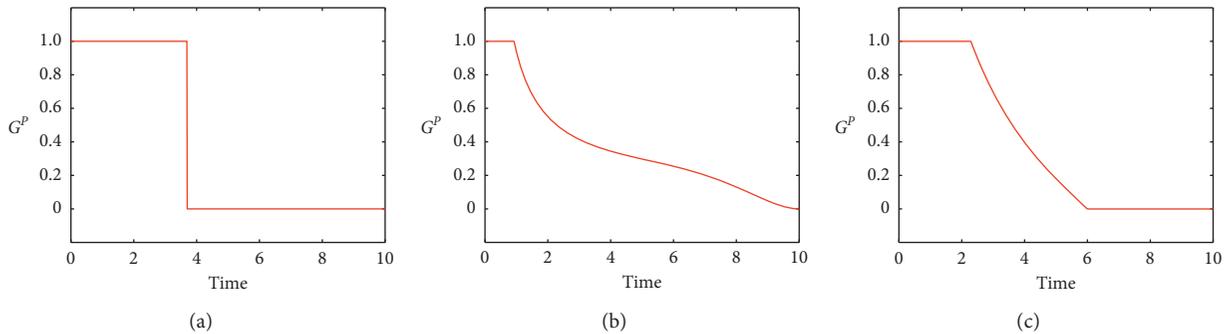


FIGURE 4: The promising rumor-containing scheme G^P for three pairs of $(\theta_1(x), \theta_2(x))$ functions: (a) $\theta_1(x) = \theta_1^1(x)$ and $\theta_2(x) = \theta_2^1(x)$, (b) $\theta_1(x) = \theta_1^2(x)$ and $\theta_2(x) = \theta_2^2(x)$, and (c) $\theta_1(x) = \theta_1^3(x)$ and $\theta_2(x) = \theta_2^3(x)$.

rumor-containing scheme G^P obtains the highest cost-effectiveness; hence, it performs much better than all the static rumor-containing schemes.

Again, supposing that there is a rumor spreading on the network G_Y , we consider three different forms of θ_1 and θ_2 , and the experimental settings are as follows.

Experiment 4. Consider three instances of the rumor-containing model:

$$M_Y^i = (G_Y, \beta_1, \beta_2, \alpha_1, \alpha_2, \gamma_1, \gamma_2, \delta, \bar{G}, w, \theta_1^i(x), \theta_2^i(x), T, E_0), \quad i = 1, 2, 3, \dots, \quad (29)$$

where $\beta_1 = 0.15$, $\beta_2 = 0.13$, $\alpha_1 = 0.15$, $\alpha_2 = 0.16$, $\gamma_1 = 0.2$, $\gamma_2 = 0.1$, $\delta = 0.1$, $\bar{G} = 1.0$, $w = 0.1$, $\theta_1^1(x) = 0.3x$, $\theta_1^2(x) = 0.2x$, $\theta_1^3(x) = (1.5x/4 + x)$, $\theta_2^1(x) = 0.3x^{(1/4)}$, $\theta_2^2(x) = 0.2x^{(1/4)}$, and $\theta_2^3(x) = (x/4 + x)$. Let G^P denote the promising rumor-containing scheme. By applying the approach introduced in Section 4.2, we get a promising control

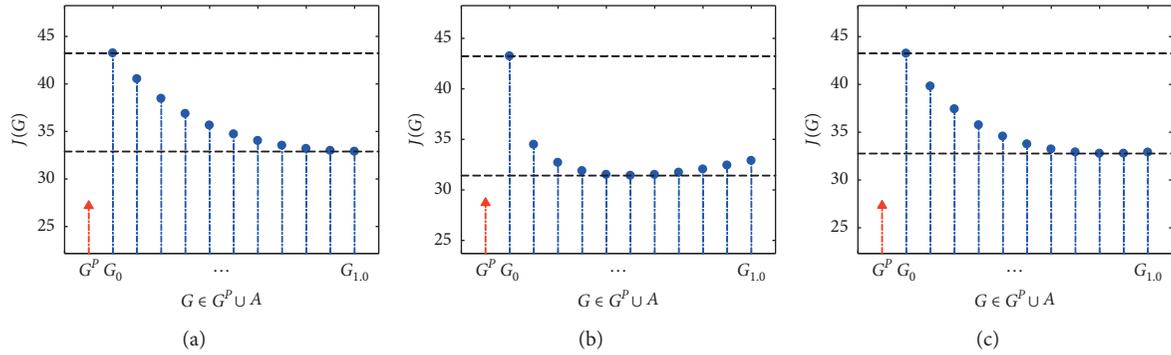


FIGURE 5: A comparison between the three promising controls G^P in Figure 2 and the set of static controls A in terms of total cost.

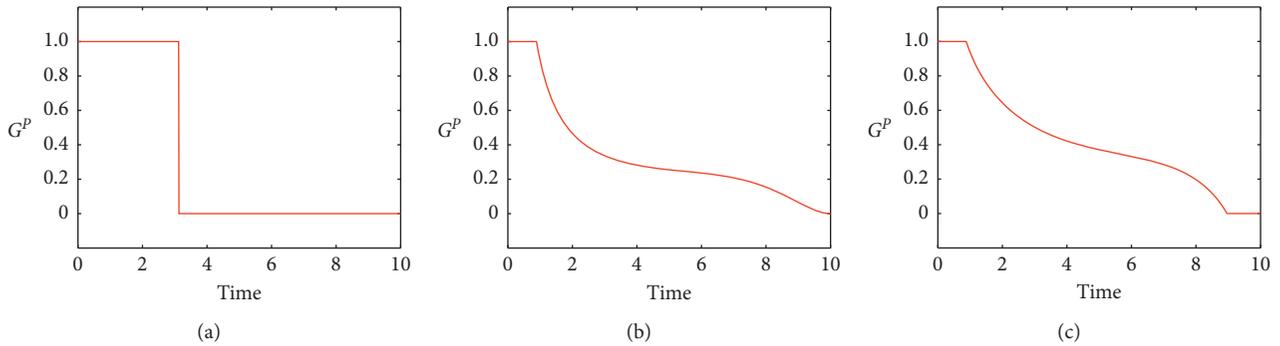


FIGURE 6: The promising rumor-containing scheme G^P for three pairs of $(\theta_1(x), \theta_2(x))$ functions: (a) $\theta_1(x) = \theta_1^1(x)$ and $\theta_2(x) = \theta_2^1(x)$, (b) $\theta_1(x) = \theta_1^2(x)$ and $\theta_2(x) = \theta_2^2(x)$, and (c) $\theta_1(x) = \theta_1^3(x)$ and $\theta_2(x) = \theta_2^3(x)$.

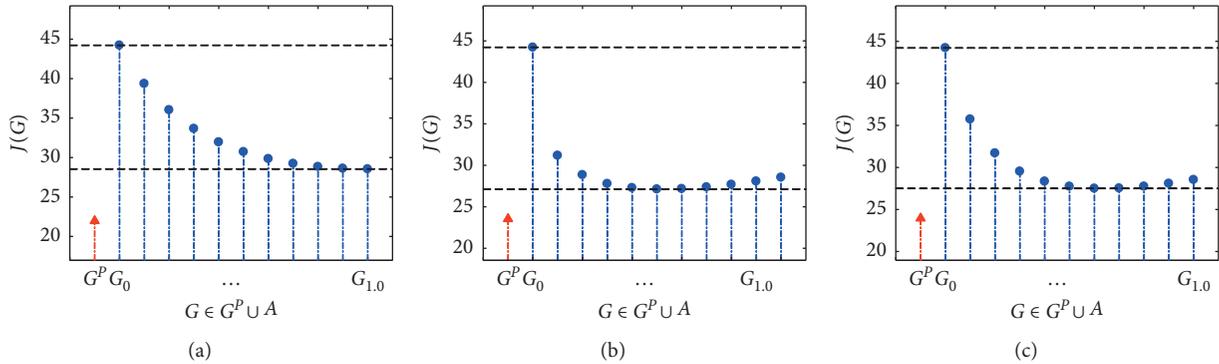


FIGURE 7: A comparison between the three promising controls G^P in Figure 4 and the set of static controls A in terms of total cost.

G^P , which is shown in Figure 8. It is seen that the promising rumor-containing scheme G^P of the three instances first stays at the maximum allowable rate and then drops to the zero rates.

Furthermore, we compare the cost-effectiveness between the promising control strategy G^P and a group of static control strategies $A = \{G_\alpha: 0, 0.1, 0.2, \dots, 1.0\}$. The comparison result can be found in Figure 9. It is seen that $J(G^P) < J(G_\alpha)$, $G_\alpha \in A$. The result shows that our proposed rumor-containing scheme G^P obtains the highest cost-effectiveness; hence, it performs much better than all the static rumor-containing schemes.

Based on the results of Experiments 2–4, we can draw some conclusions as follows: (a) if we do not take any rumor-containing scheme, the spread of rumors will cause great losses, and (b) the proposed rumor-containing scheme can greatly mitigate the impact of rumor and performs much better than all the static rumor-containing schemes in terms of the cost-effectiveness. Apart from the above three experiments, we conduct 1,000 similar experiments as well. In all these experiments, we obtain similar and consistent results. Therefore, we conclude that the promising rumor-containing scheme is cost-effective.

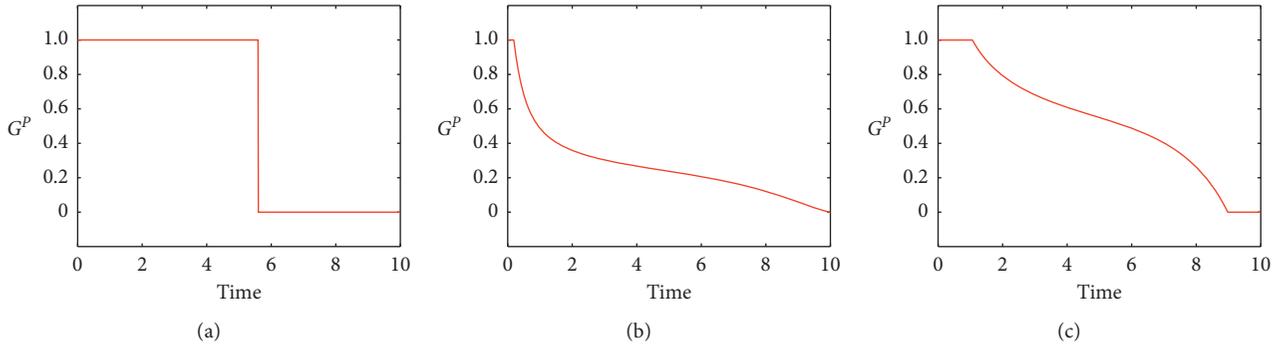


FIGURE 8: The promising rumor-containing scheme G^P for three pairs of $(\theta_1(x), \theta_2(x))$ functions: (a) $\theta_1(x) = \theta_1^1(x)$ and $\theta_2(x) = \theta_2^1(x)$, (b) $\theta_1(x) = \theta_1^2(x)$ and $\theta_2(x) = \theta_2^2(x)$, and (c) $\theta_1(x) = \theta_1^3(x)$ and $\theta_2(x) = \theta_2^3(x)$.

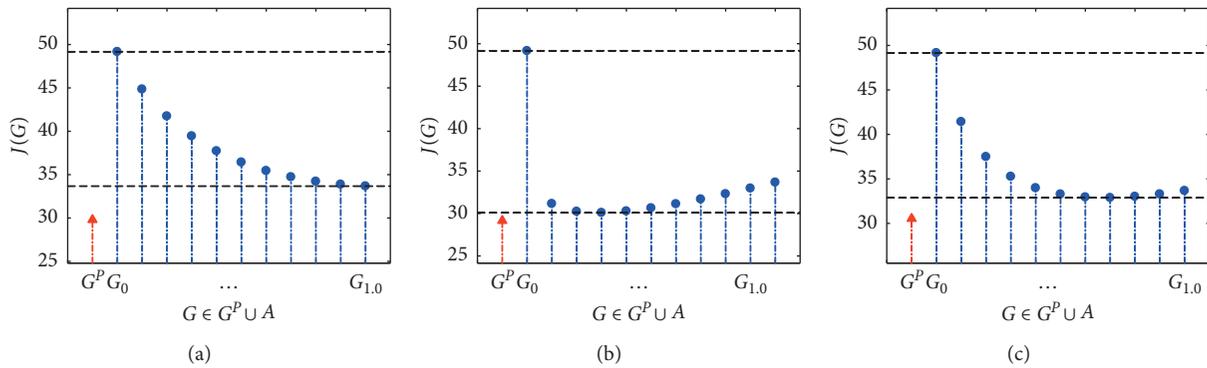


FIGURE 9: A comparison between the three promising controls G^P in Figure 6 and the set of static controls A in terms of total cost.

6. Concluding Remarks

In this study, we have studied the problem of developing a cost-effective rumor-containing scheme. Based on a node-level rumor spreading model that takes account of the effect of external environment, we have measured the impact of rumors. On this basis, we have modeled the rumor-containing problem as an optimal control problem. The optimization goal of the problem is to find a rumor-containing scheme that minimizes the total loss, and simulation results show that the proposed scheme is cost-effective. This work has studied the propagation of rumor from theoretical modeling and cost management perspectives. The research results can provide some theoretical guidance for taking measures to suppress the spread of rumors. The new model proposed in this paper can be used to study the influence of some parameters on the spread of rumors, and the research ideas can also be applied to study other cyberspace security problems.

The spread of rumors in the real world may be more complicated, and there are some open problems. First, since there is more than one OSN in the real world [48, 49], the rumor-containing problem should be extended to multiplex OSNs. Second, since realistic OSNs are varying over time [50, 51], it is necessary to study the rumor-containing problem with dynamic OSNs. Thirdly, in some application scenarios, the spread of a rumor can be captured by a system of partial differential equations [52–54]; it is worth adapting this work to these situations. Next, it is necessary to apply

our methodology to some other areas such as disease spreading [55, 56], malware propagation [57, 58], and cybersecurity [59, 60]. Finally, in this work, the rumor-monger is assumed to be nonstrategic. In practice, however, the rumormonger may well be strategic. In this situation, the rumor-containing problem should be treated in the framework of game theory [61–63].

Data Availability

The data used to support the findings of this study are available, and the sources of the datasets have been given in the paper.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

Authors' Contributions

Da-Wen Huang contributed to formal analysis, software, and roles/writing of the original draft. Lu-Xing Yang contributed to formal analysis, investigation, and validation. Xiaofan Yang contributed to supervision, methodology, and writing, reviewing, and editing of the paper. Yuan Yan Tang contributed to supervision and writing, reviewing, and editing of the paper. Jichao Bi contributed to writing, reviewing, and editing of the paper.

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