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Research Article

Decentralized Adaptive Control for Quasi-Consensus in Heterogeneous Nonlinear Multiagent Systems

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This paper proposes some novel decentralized adaptive control protocols to settle the quasi-consensus problem of multiagent systems with heterogeneous nonlinear dynamics. Based on local communication with the leader and between the followers, some innovative control protocols are put forward to adapt the control gains and coupling weights simultaneously and to steer the consensus errors to some bounded areas. In particular, two new inequalities are proposed to establish the Lyapunov-based adaptive controller design approach for quasi-consensus. Some quasi-consensus criteria are derived by utilizing the designed controllers, in which the error bound can be modulated on the basis of the adaptive controller parameters. Numerical tests are conducted to show the feasibility of the theoretical derivation. Our findings highlight quasi-consensus in heterogeneous multiagent systems without adding some additional complex nonlinear control terms to cancel the dynamical differences between agents.

1. Introduction

Collaborative control of multiagent systems (MASs) has become a research hotspot of distributed artificial intelligence because of its wide application in intelligent energy, multirobot formation, intelligent transportation, multi-unmanned system collaboration, and other engineering systems [1–5].

Among them, consensus or synchronization is a key common scientific issue of cooperative control of MASs, which has aroused great concern of multidisciplinary scholars. In brief, a core issue is to design appropriate control protocols to reach an agreement between agents. So far, many significant results have been acquired, regarding the consensus patterns, models, control algorithms, etc (please see [6–14] and some other results).

However, the aforesaid works [1–14] on the consensus of MASs mainly concentrate upon the case in which an agent has the same dynamics as all the neighbors or the leader. In some real scenes, the mismatched parameters or dynamical differences among agents may be almost inevitable, which will thus result in heterogeneous (or nonidentical) multi-agent systems (HMASs) [15–17]. It is remarkably that, due to heterogeneity, it is even impossible for HMASs to reach the complete consensus just by state feedback control when the coupling weights are constant. Up to now, there are few thorough research studies on complete consensus in HMASs because one has to add some additional complex nonlinear control terms or design compensators to cancel the dynamical differences [18–26]. Another alternative technique to deal with the heterogeneity is to transform HMASs to homogeneous ones [27, 28]. Unfortunately, all of the aforementioned approaches are complex and nonintuitive, which are thus not suitable for engineering applications. Nevertheless, in many practical HMASs, the consensus error may be bounded, even small enough, which is, namely, the so-called quasi-consensus (QC) [29].

Instead, an immediate and natural question, then, is how to design a simple controller to reach QC in HMASs. Similar to it, work to date has considered the quasi-synchronization (QS) in heterogeneous complex dynamical networks (HCDN) [30–34]. For example, in [33, 34], some QS criteria for fractional HCDN are derived via state feedback control and impulsive control, respectively. By contrast, there are few research studies about QC of HMASs [29, 35–38]. The definition of QC for MASs was first proposed in [38], and then, the definition of QC was further broadened in [29]. After that, the QC of HMASs has been further studied. The QC problem of nonlinear HMASs is studied via sampled-data control in [35]. In [36], sufficient conditions for the QC in switched HMASs are given, considering cooperation and competition interactions simultaneously. Ye and Shao [37] attempted to prove that QC in HMASs under DOS attacks can be realized by impulsive control. It should be noted, however, that, for large scale HCDN or HMASs, the computational complexity and conservativeness of QS or QC conditions in [29–38] impede their applications. In particular, it is difficult, even impossible, to check the LMI-based conditions without adaptive schemes for large-scale HCDN or HMASs. Obviously, it is an interesting and open problem to improve the QC conditions for HMASs by exploring some new control algorithms.

Motivated by the applications of decentralized adaptive control for synchronization in integer-order and fractional-order complex dynamical networks [39–43], this paper aims to design some decentralized adaptive protocols to reach QC in HMASs. Different from some previous studies [37, 44, 45], in our controller, the coupling and feedback values between agents change adaptively. The main contributions in this paper are as follows. First, two new inequalities are proposed to establish the Lyapunov-based adaptive controller design approach for QC in HMASs. Second, some innovative control protocols are introduced to accommodate the control gains and coupling weights adaptively, to steer the consensus errors to some bounded areas. Third, some QC criteria under the designed controllers are derived by the Lyapunov function method and the new inequalities.

2. Preliminaries and Problem Formation

2.1. Graph Theory. To carry out later research, we present some important concepts about graph theory in this section.

$\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ is defined as a weighted undirected graph with the network topology of N agents, where $\mathcal{V} = \{0, 1, \ldots, N\}$ and $\mathcal{E} = \{e_{ij}(i, j)\} \subseteq \mathcal{V} \times \mathcal{V}$ are the separate sets of nodes and undirected edges. $\mathcal{A} = (a_{ij})_{N\times N}$ is the weighted adjacency matrix of which the elements are nonnegative. The ith agent and the leader are modeled as the node $i \in \mathcal{V}, \mathcal{V} = \{0, 1, \ldots, N\}$ and node 0, respectively. As a rule, the undirected edge $(i, j) \in \mathcal{E}$ in the weighted undirected graph $\mathcal{G}$ denoted by an ordered pair $(\mathcal{V}_i, \mathcal{V}_j)$ represents that agent i and agent j become a pair of neighbors which can get their information from each other. There is an undirected spanning tree in an undirected graph if one agent exits an undirected path to every other distinct node.

There are two matrices that are considered as the network topology, i.e., the weighted adjacency matrix $\mathcal{A} = (a_{ij})_{N\times N}$ with $a_{ij} = a_{ji} > 0$ if $e_{ij} \in \mathcal{E}$, else $a_{ij} = 0$ if $e_{ij} \notin \mathcal{E}$, and the Laplacian matrix $L = (L_{ij})_{N\times N}$ which is defined as $L_{ii} = \sum_{j=1, j\neq i}^{N} a_{ij} = \deg(i)$ and $L_{ij} = -a_{ij}$, $i \neq j$, noted that all of them are asymmetric for the undirected graph.

Lemma 1 (see [46]). *The Laplacian matrix L is constructed from the undirected network. There are several properties in the following.*

(1) Eigenvalues of L satisfy $0 = \lambda_1(L) < \lambda_2(L) \leq \cdots \leq \lambda_N(L)$ and the smallest positive eigenvalue $\lambda_2(L) = \min_{x^T 1_N = 0, x\neq 0}(x^T Lx/x^T x)$ if and only if the network is connected.

(2) For any vector $\eta = (\eta_1, \eta_2, \ldots, \eta_N)^T \in \mathbb{R}^N$, the equation satisfies

$$\eta^T L\eta = \frac{1}{2}\sum_{i=1}\sum_{j=1} G_{ij}\left(\eta_i - \eta_j\right)^2. \tag{1}$$

2.2. Problem Formation. In this paper, we consider that there are N follower multiagent systems, which can be described by

$$\dot{\omega}_i(t) = A_i\omega_i(t) + B_i f(\omega_i(t), t) + u_i(t), \tag{2}$$

where $\omega_i(t) \in R^n$ can be regarded as the position vector of the agent i. $f\colon R^n \times R^n \times R^+ \longrightarrow R^n$ represents a continuous nonlinear vector function. A_i, $B_i \in \mathbb{R}^{n\times n}$ represent the system matrices of the agent i, respectively. $u_i(t) \in R^n$ denotes the control protocol to be designed.

The leader agent can be described as follows:

$$\dot{\omega}_0(t) = A\omega_0(t) + Bf(\omega_0(t), t), \tag{3}$$

where $\omega_0(t) \in R^n$ represents the leader's state vector and A, $B \in \mathbb{R}^{n\times n}$ represent the leader's system matrices, respectively.

For the above heterogeneous leader-follower multiagent system, we give the following assumptions.

Assumption 1. Suppose there is a normal quantity l so that the vector function f for any vector $\lambda, \nu \in R^n$ satisfies

$$\|f(\lambda, t) - f(\nu, t)\| \leq l\|(\lambda - \nu)\|. \tag{4}$$

Lemma 2 (see [47]). *For any vector $x, y \in R^n$, the following holds:*

$$x^T y \leq \frac{1}{2}x^T x + \frac{1}{2}y^T y. \tag{5}$$

Lemma 3. *Any two continuous functions satisfy*

$$\dot{v}(t) + \dot{w}(t) \leq -\gamma v(t), \tag{6}$$

where $\gamma > 0$. Then, there is a $t \geq 0$ so that the following holds:

$$v(t) \leq (v(0) + w(0))e^{-\gamma t} - w(t) + \gamma e^{-\gamma t} * w(t), \quad t \geq 0, \tag{7}$$

where $*$ is the convolution.

Proof. Since $v(t) + \dot{w}(t) \leq -\gamma v(t)$, the existence of $z(t) \geq 0$ makes the following equation:

$$\dot{v}(t) + \dot{w}(t) + z(t) = -\gamma v(t). \quad (8)$$

We calculate the Laplace transform of (8) to obtain

$$sV(s) + sW(s) - (v(0) + w(0)) + Z(s) = -\gamma V(s). \quad (9)$$

Then, we can obtain

$$V(s) = (v(0) + w(0))\frac{1}{s+\gamma} - \frac{s}{s+\gamma}W(s) - \frac{1}{s+\gamma}Z(s). \quad (10)$$

The inverse Laplace transform of (10) can be obtained as

$$v(t) = (v(0) + w(0))e^{-\gamma t} - w(t) + \gamma e^{-\gamma t} * w(t) - e^{-\gamma t} * z(t). \quad (11)$$

Through (10), we can obtain

$$v(t) \leq (v(0) + w(0))e^{-\gamma t} - w(t) + \gamma e^{-\gamma t} * w(t). \quad (12)$$

Lemma 4. *Any two continuous functions satisfy*

$$\dot{v}(t) + \dot{w}(t) \leq -\gamma v(t) + \varepsilon, \quad (13)$$

where $\gamma > 0$ and $\varepsilon > 0$. Then, there is a $t \geq 0$ so that the following formula holds:

$$v(t) < (v(0) + w(0))e^{-\gamma t} - w(t) + \gamma e^{-\gamma t} * w(t) + \frac{\varepsilon}{\gamma}, \quad t \geq 0. \quad (14)$$

Proof. Since $\dot{v}(t) + \dot{w}(t) \leq -\gamma v(t) + \varepsilon$, then $(v(t) - \dot{(\varepsilon/\gamma)}) + \dot{w}(t) \leq -\gamma(v(t) - (\varepsilon/\gamma))$.

By Lemma 3, we can obtain

$$v(t) - \frac{\varepsilon}{\gamma} \leq \left(v(0) - \frac{\varepsilon}{\gamma} + w(0)\right)e^{-\gamma t} - w(t) + \gamma e^{-\gamma t} * w(t). \quad (15)$$

We can further obtain

$$v(t) \leq (v(0) + w(0))e^{-\gamma t} - w(t) + \gamma e^{-\gamma t} * w(t) + \frac{\varepsilon}{\gamma}. \quad (16)$$

Lemma 5 (see [47]). *If A, B, C, and D represent four different matrices, respectively, and the matrix products AC and $B\ D$ makes sense, the Kronecker product $\otimes$ satisfies*

$$\begin{aligned} &(1)\ A \otimes (B + C) = A \otimes B + A \otimes C, \\ &(2)\ (A \otimes B)(C \otimes D) = (AC) \otimes (B\ D). \end{aligned} \quad (17)$$

Definition 1 (see [29]). The leader-follower HMASs are decided to reach QC if

$$\lim_{t \longrightarrow \infty} \|\omega_i(t) - \omega_0(t)\| \leq \xi, \quad i = 1, 2, \ldots, N, \quad (18)$$

where ξ is a nonnegative constant.

Assumption 2. The network topology between agents is undirected and connected, and each agent can acquire the status information of the agent that has a connection relationship with it and the leader agent at any time.

2.3. Our Controller. To obtain QC between HMASs (2) and (3), we design the control input for all follower agents as

$$u_i(t) = -c\sum_{j=1}^{N} L_{ij}(t)\omega_j(t) - r_i(t)(\omega_i(t) - \omega_0(t)), \quad (19)$$

where c is a positive constant.

The adaptive law for the control gains is described as

$$\dot{r}_i(t) = \mu\left(\omega_i(t) - \omega_j(t)\right)^T(\omega_i(t) - \omega_0(t)), \quad (20)$$

where μ is a positive constant to be selected.

The adaptive law for the coupling weights is described as

$$\begin{aligned} &\dot{L}_{ij}(t) = -\alpha_{ij}\left(\omega_i(t) - \omega_j(t)\right)^T\left(\omega_i(t) - \omega_j(t)\right), \\ &L_{ij}(0) = L_{ji}(0) > 0, \quad (i, j) \in \mathscr{E}, \end{aligned} \quad (21)$$

where $\alpha_{ij} = \alpha_{ji}$ are the positive constants to be selected.

Remark 1. It should be noted that, in controller (19), adaptive laws (20) and (21), the coupling weights $L_{ij}(t)$ and the control gains $r_i(t)$ are adjusted adaptively based on local communication with the leader and between the followers. Combining adaptation of the coupling weights and control gains, adaptive law (20) ensures the QC of the follower agents, while adaptive law (21) drives the follower agents to the leader agent.

Let the QC error vector be $e_i(t) = \omega_i(t) - \omega_0(t)$. Then, the error model with the controller iscom

$$\begin{aligned} \dot{e}_i(t) = {} & A_i e_i(t) + B_i\tilde{f}(e_i(t), t) + h_i(\omega_0(t), t) \\ & - c\sum_{j=1}^{N} L_{ij}(t)e_j(t) - r_i(t)e_i(t), \end{aligned} \quad (22)$$

where $\tilde{f}(e_i(t), t) = f(\omega_i(t), t) - f(\omega_0(t), t)$ and $h_i(\omega_0(t), t) = (A_i - A)\omega_0(t) + (B_i - B)f(\omega_0(t), t)$.

$h_i(\omega_0(t), t)$ represents the difference between different agents. It can be obtained by Assumption 1:

$$\begin{aligned} \|h_i(\omega_0(t), t)\| &= \|(A_i - A)\omega_0(t) + (B_i - B)f(\omega_0(t), t)\| \\ &\leq \|A_i - A\|\|\omega_0(t)\| + \|B_i - B\|\|f(\omega_0(t), t)\| \\ &\leq \|A_i - A\|\|\omega_0(t)\| + \|B_i - B\|l_{\max}\|\omega_0(t)\|. \end{aligned} \quad (23)$$

Since $\omega_0(t)$ is bounded, it can be obtained that $h_i(\omega_0(t), t)$ is bounded.

3. Main Results

In this section, we present some theorems for achieving QC in HMASs.

3.1. Adaptive Control Protocol

Theorem 1. *It is assumed that $f(\omega_i(t), t)$ satisfies Assumption 1 and the follower system (2) satisfies Assumption 2. Under the action of controller (19) and adaptive laws (20) and (21), HMASs (2) and (3) can achieve QC.*

The Lyapunov function we constructed is

$$V_1(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)e_i(t) + \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} \frac{c}{4\alpha_{ij}}\left(L_{ij}(t) + k_{ij}\right)^2 + \sum_{i=1}^{N}\frac{1}{2\mu}\left(r_i(t) - d_i^*\right)^2, \tag{24}$$

where $k_{ij} = k_{ji}\,(i \neq j)$ is a nonnegative constant if and only if $L_{ij}(t) = 0$ is $k_{ij} = 0$. d_i^* is the normal constant waiting for the value.

We take the derivative of $V_1(t)$ along (22) together with controller (19) and adaptive laws (20) and (21), and we can obtain

$$\begin{aligned}
\dot{V}_1(t) &= \sum_{i=1}^{N} e_i^T(t)\dot{e}_i(t) + \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N}\frac{c}{2\alpha_{ij}}\left(L_{ij}(t) + k_{ij}\right)\dot{L}_{ij}(t) + \frac{1}{\mu}\sum_{i=1}^{N}\left(r_i(t) - d_i^*\right)\dot{r}_i(t)\\
&= \sum_{i=1}^{N} e_i^T(t)\left(A_i e_i(t) + B_i\widetilde{f}(e_i(t), t) + h_i(\omega_0(t), t) - c\sum_{j=1}^{N} L_{ij}(t)e_j(t) - r_i(t)e_i(t)\right)\\
&\quad - \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N}\frac{c}{2}\left(L_{ij}(t) + k_{ij}\right)\left(\omega_i(t) - \omega_j(t)\right)^T\left(\omega_i(t) - \omega_j(t)\right)\\
&\quad + \sum_{i=1}^{N}\left(r_i(t) - d_i^*\right)e_i^T(t)e_i(t)\\
&= \sum_{i=1}^{N} e_i^T(t)A_i e_i(t) + \sum_{i=1}^{N} e_i^T(t)B_i\widetilde{f}(e_i(t), t) + \sum_{i=1}^{N} e_i^T(t)h_i(\omega_0(t), t)\\
&\quad - \sum_{i=1}^{N} e_i^T(t)c\sum_{j=1}^{N} L_{ij}(t)e_j(t) - \sum_{i=1}^{N} d_i^* e_i^T(t)e_i(t)\\
&\quad - \frac{c}{2}\sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N}\left(L_{ij}(t) + k_{ij}\right)\left(\omega_i(t) - \omega_j(t)\right)^T\left(\omega_i(t) - \omega_j(t)\right).
\end{aligned} \tag{25}$$

Through Assumption 1 and Lemma 2, one has

$$\begin{aligned}
\sum_{i=1}^{N} e_i^T(t)B_i\widetilde{f}(e_i(t), t) &= \sum_{i=1}^{N} e_i^T(t)B_i\left(f(\omega_i(t), t) - f(\omega_0(t), t)\right)\\
&\leq \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)B_iB_i^T e_i(t) + \frac{1}{2}\sum_{i=1}^{N}\left\|f(\omega_i(t), t) - f(\omega_0(t), t)\right\|^2\\
&\leq \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)\left(B_iB_i^T + l^2 I_n\right)e_i(t),
\end{aligned} \tag{26}$$

$$\sum_{i=1}^{N} e_i^T(t)h_i(\omega_0(t), t) \leq \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)e_i(t) + \frac{1}{2}\sum_{i=1}^{N}\left\|h_i(\omega_0(t), t)\right\|^2. \tag{27}$$

From (25)–(27), we can obtain

$$\begin{aligned}\dot{V}_1(t) \le{}& \sum_{i=1}^{N} e_i^T(t)A_i e_i(t) + \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)e_i(t) + \frac{1}{2}\sum_{i=1}^{N}\left\|h_i(\omega_0(t),t)\right\|^2 \\ &- \sum_{i=1}^{N} e_i^T(t)c\sum_{j=1}^{N} L_{ij}(t)e_j(t) + \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)\left(B_iB_i^T + l^2 I_n\right)e_i(t) - \sum_{i=1}^{N} d_i^* e_i^T(t)e_i(t) \\ &- \frac{c}{2}\sum_{i=1}^{N}\sum_{j=1,j\ne i}^{N}\left(L_{ij}(t)+k_{ij}\right)\left(\omega_i(t)-\omega_j(t)\right)^T\left(\omega_i(t)-\omega_j(t)\right).\end{aligned} \tag{28}$$

We can define the Laplacian matrix $\Omega = (\tau_{ij})_{N\times N}$, where $\tau_{ij} = k_{ij}, i \ne j$ and $\tau_{ii} = -\sum_{j=1, j\ne i}^{N} \tau_{ij}$. By Lemma 1, we can obtain

$$\begin{aligned}&\frac{c}{2}\sum_{i=1}^{N}\sum_{j=1,j\ne i}^{N}\left(L_{ij}(t)+k_{ij}\right)\left(e_i(t)-e_j(t)\right)^T\left(e_i(t)-e_j(t)\right) \\ &\quad = -c\sum_{i=1}^{N}\sum_{j=1}^{N} L_{ij}(t)e_i^T(t)e_j(t) + c\sum_{i=1}^{N}\sum_{j=1}^{N}\tau_{ij}e_i^T(t)e_j(t).\end{aligned} \tag{29}$$

Combining with (28) and (29), we have

$$\begin{aligned}\dot{V}_1(t) \le{}& \sum_{i=1}^{N} e_i^T(t)A_i e_i(t) + \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)e_i(t) + \frac{1}{2}\sum_{i=1}^{N}\left\|h_i(\omega_0(t),t)\right\|^2 \\ &- \sum_{i=1}^{N} e_i^T(t)c\sum_{i=1}^{N} L_{ij}(t)e_j(t) + \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)\left(B_iB_i^T + l^2 I_n\right)e_i(t) \\ &+ c\sum_{i=1}^{N}\sum_{i=1}^{N} L_{ij}(t)e_i^T(t)e_j(t) - c\sum_{i=1}^{N}\sum_{i=1}^{N}\tau_{ij}e_i^T(t)e_j(t) - \sum_{i=1}^{N} d_i^* e_i^T(t)e_i(t) \\ ={}& \sum_{i=1}^{N} e_i^T(t)A_i e_i(t) + \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)\left(B_iB_i^T + \left(l^2+1\right)I_n - 2d_i^*\cdot I_n\right)e_i(t) \\ &+ \frac{1}{2}\sum_{i=1}^{N}\left\|h_i(\omega_0(t),t)|\right\|^2 - c\sum_{i=1}^{N}\sum_{i=1}^{N}\tau_{ij}e_i^T(t)e_j(t).\end{aligned} \tag{30}$$

Then, there is a unitary matrix $U = (u_1, \ldots, u_N)$ so that $U^T\Omega U = \Delta$, $m(t) = (U^T \otimes I_n)e(t)$,

$$\begin{aligned}\dot{V}_1(t) \le{}& e^T(t)\left[A + \frac{1}{2}\left(BB^T + I_N\otimes\left(l^2+1\right)I_n - 2\left(D^*\otimes I_n\right)\right) - c\left(\Omega\otimes I_n\right)\right]e(t) + \frac{1}{2}\left\|h\left(\omega_0(t),t\right)\right\|^2 \\ ={}& e^T(t)\left[A + \frac{1}{2}\left(BB^T + I_N\otimes\left(l^2+1\right)I_n - 2\left(D^*\otimes I_n\right)\right)\right] \\ &-c\left(U\otimes I_n\right)\left(\Delta\otimes I_n\right)\left(U^T\otimes I_n\right)\big)\big]e(t) + \frac{1}{2}\left\|h\left(\omega_0(t),t\right)\right\|^2 \\ ={}& e^T(t)\left[A + \frac{1}{2}\left(BB^T + I_N\otimes\left(l^2+1\right)I_n - 2\left(D^*\otimes I_n\right)\right)\right]e(t) - cm^T(t)\left(\Delta\otimes I_n\right)m(t) \\ &+ \frac{1}{2}\left\|h\left(\omega_0(t),t\right)\right\|^2,\end{aligned} \tag{31}$$

where $D^* = \mathrm{diag}(d_1^*, d_2^*, \ldots, d_N^*)$, $B = \mathrm{diag}(B_1, B_2, \ldots, B_N)$, and $A = \mathrm{diag}(A_1, A_2, \ldots, A_N)$.

Since I_n is positive definite, by Lemma 1, we can get the following:

$$m^T(t)(\Delta \otimes I_n)m(t) \geq \lambda_2(\Omega)m^T(t)(I_N \otimes I_n)m(t). \quad (32)$$

Then, we can obtain

$$\begin{aligned}\dot{V}_1(t) &\leq e^T(t)\left[A + \frac{1}{2}\left(BB^T + I_N \otimes \left(l^2+1\right)I_n - 2\left(D^* \otimes I_n\right)\right)\right]e(t) + \frac{1}{2}\left\|h\left(\omega_0(t), t\right)\right\|^2 \\ &\quad - c\lambda_2(\Omega)m^T(t)\left(I_N \otimes I_n\right)m(t) \\ &= e^T(t)\left[A + \frac{1}{2}\left(BB^T + I_N \otimes \left(l^2+1\right)I_n - 2\left(D^* \otimes I_n\right)\right)\right]e(t) + \frac{1}{2}\left\|h\left(\omega_0(t), t\right)\right\|^2 \\ &\quad - c\lambda_2(\Omega)e^T(t)\left(P \otimes I_n\right)\left(I_N \otimes I_n\right)\left(P^T \otimes I_n\right)e(t) \\ &= e^T(t)\left[A + \frac{1}{2}\left(BB^T + I_N \otimes \left(l^2+1\right)I_n - 2\left(D^* \otimes I_n\right)\right) - c\lambda_2(\Omega)\left(I_N \otimes I_n\right)\right]e(t) \\ &\quad + \frac{1}{2}\left\|h\left(\omega_0(t), t\right)\right\|^2.\end{aligned} \quad (33)$$

We can choose large enough k_{ij} and d_i^* so that

$$\lambda_{\max}\left(A + \frac{1}{2}\left(BB^T + I_N \otimes \left(l^2+1\right)I_n - 2\left(D^* \otimes I_n\right)\right) - c\lambda_2(\Omega)\left(I_N \otimes I_n\right)\right) + \theta \leq 0, \quad (34)$$

where $\theta > 0$ is a positive constant.

$$\dot{V}_1(t) \leq -\theta e^T(t)e(t) + \frac{1}{2}\left\|h\left(\omega_0(t), t\right)\right\|^2, \quad (35)$$

where $e(t) = (e_1^T, e_2^T, e_3^T, \ldots, e_N^T)^T$.

By Lemma 4, it yields

$$\begin{aligned}e^T(t)e(t) &\leq \left(\sum_{i=1}^{N} e_i^T(0)e_i(0) + \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} \frac{c}{2\alpha_{ij}}\left(L_{ij}(0) - k_{ij}\right)^2\right. \\ &\quad \left. + \sum_{i=1}^{N}\frac{1}{\mu}\left(r_i(0) - d_i^*\right)^2\right)e^{-2\theta t} + 2\theta e^{-2\theta t} * \left(\sum\sum_{i=1}^{N}\frac{1}{\mu}\left(r_i(0) - d_i^*\right)^2\right. \\ &\quad \left. + \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N}\frac{c}{2\alpha_{ij}}\left(L_{ij}(0) - k_{ij}\right)^2\right) + \frac{\left\|h\left(\omega_0(t), t\right)\right\|^2}{2\theta}.\end{aligned} \quad (36)$$

The error will eventually converge to finite region $\Xi = \left\{e(t) \in R^n \middle| \|(t)\| \leq \sqrt{\left(\|h(\omega_0(t), t)\|^2/2\theta\right)}\right\}$ as $t \longrightarrow +\infty$. This completes the proof.

3.2. Adaptive Pinning Control Protocol. In the previous section, an adaptive controller (19) and an adaptive laws (20) and (21) for all follower agents are designed. However, it is neither realistic nor economical to control all follower agents in engineering. In view of this, the adaptive pinning control schemes are considered, where a fraction of the control gains and the coupling weights are adapted.

Suppose that $\tilde{\varepsilon}$ is a subset of $\mathscr{E}$ and HMASs (2) and (3) are connected by the undirected edge. $\tilde{\varepsilon}$ is connected. The pinning adaptive protocol is written as follows:

$$u_i(t) = -c\sum_{j=1}^{N} L_{ij}(t)\omega_j(t) - \vartheta_i r_i(t)\left(\omega_i(t) - \omega_0(t)\right), \quad (37)$$

where $\vartheta_i = \begin{cases} 1, & \text{for } i = 1, 2, \ldots, N_s, \\ 0, & \text{for } i = N_s + 1, \ldots, N \end{cases}$.

The adaptive law for $r_i(t)$ is the same as (20).

The adaptive law for $L_{ij}(t)$ is described as

$$\dot{L}_{ij}(t) = -\alpha_{ij}\big(\omega_i(t) - \omega_j(t)\big)^T\big(\omega_i(t) - \omega_j(t)\big), \tag{38}$$

where $(i, j) \in \tilde{\varepsilon}$ and $L_{ij}(0) = L_{ji}(0) > 0$.

Theorem 2. *Suppose that Assumption 1 is valid and the follower system (2) satisfies Assumption 2. The HMASs (2) and (3) can achieve QC under the pinning adaptive protocol combined with (20), (21), and (37).*

Proof. Take into account the Lyapunov function candidate:

$$V_2(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)e_i(t) + \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} \frac{c}{4\alpha_{ij}}\big(L_{ij}(t) + k_{ij}\big)^2 + \sum_{i=1}^{N_s}\frac{1}{2\mu}\big(r_i(t) - d_i^*\big)^2, \tag{39}$$

where $c_{ij} = c_{ji}\,(i \neq j)$ is a nonnegative quantity if and only if $L_{ij}(t) = 0$ and $k_{ij} = 0$. d_i^* is the normal constant waiting for the value if $i = 1, 2, \ldots, N_s, N_s \geq 1$, or $d_i^* = 0$.

The derivative of $V_2(t)$ along (22), controller (37), and the decentralized adaptive pinning laws (20) and (21) gives

$$\begin{aligned}
\dot{V}_2(t) &= \sum_{i=1}^{N} e_i^T(t)\dot{e}_i(t) + \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} \frac{c}{2\alpha_{ij}}\big(L_{ij}(t) + k_{ij}\big)\dot{L}_{ij}(t) + \sum_{i=1}^{N_s}\frac{1}{\mu}\big(r_i(t) - d_i^*\big)\dot{d}_i(t) \\
&\leq \sum_{i=1}^{N} e_i^T(t)A_i e_i(t) + \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)\big(B_iB_i^T + (l^2+1)I_n - 2d_i^* \otimes I_n\big)e_i(t) \\
&\quad + \frac{1}{2}\sum_{i=1}^{N}\big\|h_i(\omega_0(t), t)\big\|^2 - c\sum_{i=1}^{N}\sum_{j=1}^{N}\tau_{ij}e_i^T(t)e_j(t) \\
&\leq e^T(t)\Big[A + \frac{1}{2}\big(BB^T + I_N \otimes (l^2+1)I_n - 2\big(\tilde{D}^* \otimes I_n\big)\big) - c\lambda_2(\Omega)\big(I_N \otimes I_n\big)\Big]e(t) \\
&\quad + \frac{1}{2}\big\|h(\omega_0(t), t)\big\|^2,
\end{aligned} \tag{40}$$

where $\tilde{D}^* = \operatorname{diag}(d_1^*, d_2^*, \ldots, d_{N_s}^*, 0, \ldots, 0)$.

The proof can be completed by using the similar analysis method in Theorem 1.

Theorem 3. *Suppose that Assumptions 1 and 2 hold. The HMASs (2) and (3) can achieve QC under the pinning adaptive protocol combined with (19), (20), and (38).*

Proof. Construct the Lyapunov functional as

$$V_3(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)e_i(t) + \sum_{i=1}^{N}\sum_{(i,j)\varepsilon\tilde{\varepsilon}} \frac{c}{4\alpha_{ij}}\big(L_{ij}(t) + \tilde{k}_{ij}\big)^2 + \sum_{i=1}^{N}\frac{1}{2\mu}\big(r_i(t) - d_i^*\big)^2, \tag{41}$$

where $\tilde{k}_{ij} = \tilde{k}_{ji} > 0$, $(i, j)\varepsilon\tilde{\varepsilon}$, and $\tilde{k}_{ij} = 0\,(i \neq j)$, else.

Let $\tilde{K} = (\tilde{k}_{ij})_{N\times N}, \tilde{k}_{ii} = -\sum_{i=1, i\neq j}^{N}\tilde{k}_{ij}$; then,

$$G_{ij} = \begin{cases} L_{ij}(0), & (i, j) \in E - \tilde{\varepsilon}, \\ -\displaystyle\sum_{j=1, j\neq i}^{N} L_{ij}(0), & i = j, \\ 0, & \text{other.} \end{cases} \tag{42}$$

After the derivative of $V_3(t)$ along (22), controller (19), and the decentralized adaptive pinning laws (20) and (38), the following holds:

$$\begin{aligned}
\dot{V}_3(t) &= \sum_{i=1}^{N} e_i^T(t)\dot{e}_i(t) + \sum_{i=1}^{N}\sum_{(i,j)\varepsilon\varepsilon} \frac{c}{2\alpha_{ij}}\left(L_{ij}(t)+\widetilde{k}_{ij}\right)\dot{L}_{ij}(t) + \frac{1}{\mu}\sum_{i=1}^{N}\left(r_i(t)-d_i^*\right)\dot{d}_i(t) \\
&= \sum_{i=1}^{N} e_i^T(t)A_i e_i(t) + \sum_{i=1}^{N} e_i^T(t)B_i\widetilde{f}\left(e_i(t),t\right) + \sum_{i=1}^{N} e_i^T(t)h_i\left(\omega_0(t),t\right) \\
&\quad - \sum_{i=1}^{N} e_i^T(t)c\sum_{i=1}^{N} L_{ij}(t)e_j(t) - \sum_{i=1}^{N} d_i^* e_i^T(t)e_i(t) \\
&\quad - \frac{c}{2}\sum_{i=1}^{N}\sum_{(i,j)\varepsilon\varepsilon}\left(L_{ij}(t)+\widetilde{k}_{ij}\right)\left(\omega_i(t)-\omega_j(t)\right)^T\left(\omega_i(t)-\omega_j(t)\right) \\
&\leq \sum_{i=1}^{N} e_i^T(t)A_i e_i(t) + \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)e_i(t) + \frac{1}{2}\sum_{i=1}^{N}\left\|h_i\left(\omega_0(t),t\right)\right\|^2 \\
&\quad + \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)\left(B_iB_i^T + l^2 I_n\right)e_i(t) - c\sum_{i=1}^{N}\sum_{j=1}^{N} G_{ij}(t)e_i^T(t)e_j(t) \\
&\quad - c\sum_{i=1}^{N}\sum_{j=1}^{N}\widetilde{\tau}_{ij}e_i^T(t)e_j(t) - \sum_{i=1}^{N} d_i^* e_i^T(t)e_i(t) \\
&= e^T(t)\left[A + c\left(G\otimes I_n\right) + \frac{1}{2}\left(I_N\otimes\left(BB^T + \left(l^2+1\right)I_n\right)\right)\right] \\
&\quad \left(D^*\otimes I_n\right) - c\lambda_2\left(\Omega^*\right)\left(I_N\otimes I_n\right)]e(t) + \frac{1}{2}\left\|h\left(\omega_0(t),t\right)\right\|^2,
\end{aligned} \tag{43}$$

where $G = (G_{ij})_{N\times N}$, $\Omega^* = (\widetilde{\tau}_{ij})_{N\times N}$, $\widetilde{\tau}_{ij} = -\widetilde{k}_{ij}, i \neq j$, and $\widetilde{\tau}_{ii} = -\sum_{\substack{j=1 \\ j\neq i}}^{N}\widetilde{\tau}_{ij}$.

The following proof is similar to the previous derivation in Theorem 1; thus, we will omit this part here.

Theorem 4. *Suppose that Assumptions 1 and 2 hold. The HMASs (2) and (3) can achieve QC under the pinning adaptive protocol combined with (20), (37), and (38).*

Proof. The Lyapunov functional is considered as follows:

$$V_4(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)e_i(t) + \sum_{i=1}^{N}\sum_{(i,j)\widetilde{\varepsilon\varepsilon}}\frac{c}{4\alpha_{ij}}\left(L_{ij}(t)+\widetilde{k}_{ij}\right)^2 + \sum_{i=1}^{N_s}\frac{1}{2\mu}\left(r_i(t)-d_i^*\right)^2. \tag{44}$$

The derivative of $V_4(t)$ along the error system (22), controller (37), and decentralized adaptive pinning laws (20) and (38) gives

$$\begin{aligned}
\dot{V}_4(t) &= \sum_{i=1}^{N} e_i^T(t)\dot{e}_i(t) + \sum_{i=1}^{N}\sum_{(i,j)\varepsilon\varepsilon}^{N} \frac{c}{2\alpha_{ij}}\left(L_{ij}(t)+\widetilde{k}_{ij}\right)\dot{L}_{ij}(t) + \frac{1}{\mu}\sum_{i=1}^{N_s}\left(r_i(t)-d_i^*\right)\dot{d}_i(t) \\
&\le \sum_{i=1}^{N} e_i^T(t)A_i e_i(t) + \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)e_i(t) + \frac{1}{2}\sum_{i=1}^{N}\left\|h_i(\omega_0(t),t)\right\|^2 \\
&\quad + \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)\left(B_iB_i^T + l^2 I_n\right)e_i(t) + c\sum_{i=1}^{N}\sum_{j=1}^{N} G_{ij}(t)e_i^T(t)e_j(t) \\
&\quad - c\sum_{i=1}^{N}\sum_{j=1}^{N}\widetilde{\tau}_{ij}e_i^T(t)e_j(t) - \sum_{i=1}^{N_s} d_i^* e_i^T(t)e_i(t) \\
&= e^T(t)\Big[A + c\left(G\otimes I_n\right) + \frac{1}{2}\left(I_N\otimes\left(BB^T + \left(l^2+1\right)I_n\right)\right) - \left(\widetilde{D}^*\otimes I_n\right)\Big] \\
&\quad - c\lambda_2\left(\Omega^*\right)\left(I_N\otimes I_n\right)]e(t) + \frac{1}{2}\left\|h(\omega_0(t),t)\right\|^2,
\end{aligned} \tag{45}$$

where $G=(G_{ij})_{N\times N}$, $\Omega^*=(\widetilde{\tau}_{ij})_{N\times N}$, $\widetilde{\tau}_{ij}=-\widetilde{k}_{ij}, i\neq j$, $\widetilde{\tau}_{ii}=-\sum_{j=1, j\neq i}^{N}\widetilde{\tau}_{ij}$, and $\widetilde{D}^*=\mathrm{diag}(d_1^*, d_2^*, \ldots, d_{N_s}^*, 0, \ldots, 0)$.

The rest of the proof is similar to Theorem 1. To save space, it is thus omitted here.

Remark 2. In Theorems 2–4, some sufficient conditions are given to realize QC of HMASs (2) and (3) by using the adaptive pinning control schemes. Actually, if the follower agents (3) are connected, one can randomly choose a small fraction of coupling weights and/or the control gains to adapt. In particular, it is possible to obtain the QC by pinning one follower agent.

4. Numerical Examples

In this section, we will confirm the theory proposed in the paper by using digital simulation experiments.

We use five following agents and one leader agent, and the initial communication Laplacian matrix for follower agents is

$$L=\begin{pmatrix} 1.5 & -0.3 & -0.3 & -0.4 & -0.5 \\ -0.3 & 0.9 & -0.6 & 0 & 0 \\ -0.3 & -0.6 & 0.9 & 0 & 0 \\ -0.4 & 0 & 0 & 0.7 & -0.3 \\ -0.5 & 0 & 0 & -0.3 & 0.8 \end{pmatrix}. \tag{46}$$

The leader's system matrices can be described as $A=\begin{pmatrix} -2.5 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -18 & 0 \end{pmatrix}$ and $B=\begin{pmatrix} 35/6 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, and the nonlinear function of the entire system can be assumed to be $f(t,x)=\begin{pmatrix} 0.5\left(|x_1(t)+1|-|x_1(t)-1|\right) \\ 0 \\ 0 \end{pmatrix}$. The following system matrices are, respectively, assumed to be $A_i=\begin{pmatrix} -2.5+0.1*i & 10+0.2*i & 0 \\ 1+0.2*i & -1+0.2*i & 1+0.2*i \\ 0 & -18+0.2*i & 0 \end{pmatrix}$ and $B_i=\begin{pmatrix} 35/6 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $(i=1,2,\ldots,5)$. We arbitrarily choose the initial value as $x_0(0)=(3.3,\ 0.66,\ 0.1)^T$ and $x_i(0)=(11.5+1.8*i,\ 4.2\ +1.1*i,\ 4.9+1.5*i)^T$ $(i=1,2,\ldots,5)$. Figure 1 shows the change of $\|e_i(t)\|_2$ $(i=1,2,\ldots,5)$ for HMASs without the controller.

4.1. Example 1. In this case, the parameters of controller (19) and adaptive laws (20) and (21) are chosen according to Theorem 1. All agent system parameters are in accordance with A_i and B_i. We choose $c=2.6$, $\mu=0.05$, $\alpha_{12}=\alpha_{21}=2.7$, $\alpha_{13}=\alpha_{31}=2.4$, $\alpha_{14}=\alpha_{41}=2.1$, $\alpha_{15}=\alpha_{51}=1.8$, $\alpha_{23}=\alpha_{32}=2.1$, and $\alpha_{45}=\alpha_{54}=2.2$. We arbitrarily choose the initial value as $r(0)=(2.3,\ 2.1,\ 1.8,\ 1.8,\ 2.1)^T$, $x_0(0)=(3.3,\ 0.66,\ 0.1)^T$, and $x_i(0)=(11.5+1.8*i,\ 4.2+1.1*i,\ 4.9+1.5*i)^T$ $(i=1,2,\ldots,5)$. Figure 2 shows

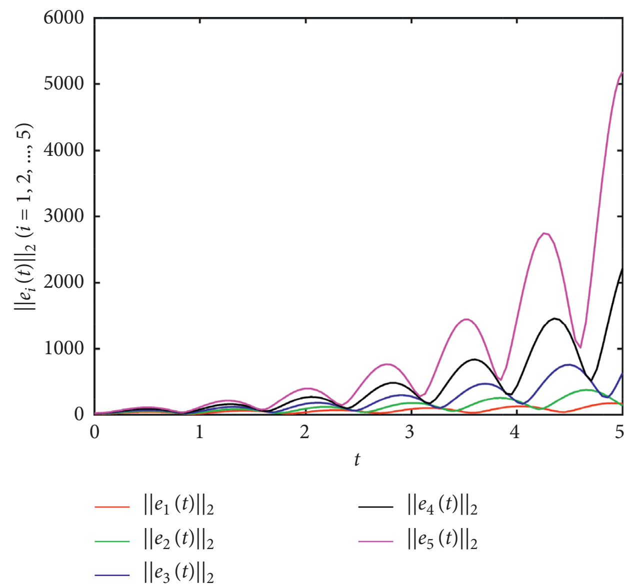

FIGURE 1: The changes of $\|e_i(t)\|_2 (i = 1, 2, \ldots, 5)$ for the system without thw controller.

the change in the state of the system under the increment controller. It visibly shows that, under the action of the adaptive controller and adaptive law, the error of the leader and follower finally converges to a finite region.

4.2. Example 2. In this example, we only use the coupling between followers and leaders 1 and 2 to achieve QC of the entire system. All agent system parameters are in accordance with A_i and B_i. We choose $c = 2.6$, $\mu = 0.01$, $\alpha_{12} = \alpha_{21} = 8.1$, $\alpha_{13} = \alpha_{31} = 7.2$, $\alpha_{14} = \alpha_{41} = 6.3$, $\alpha_{15} = \alpha_{51} = 5.4$, $\alpha_{23} = \alpha_{32} = 6.3$, and $\alpha_{45} = \alpha_{54} = 7.2$. We arbitrarily choose the 1 and 2 agents, and their initial values are selected as $r(0) = (2.3, 2.1)^T$. The initial value of each agent is selected as $x_0(0) = (3.3, 0.66, 0.1)^T$ and $x_i(0) = (-6 - 1.6 * i, 5 + 1.1 * i, 9.4 + 2 * i)^T$ $(i = 1, 2, \ldots, 5)$. Figure 3 shows $e_i(t)_2 (i = 1, 2, \ldots, 5)$ eventually tends to a finite region.

4.3. Example 3. In this example, we use a control strategy that pinning the coupling between followers to QC of HMASs (2) and (3). All agent system parameters are in accordance with A_i and B_i. We choose $c = 1.2$. The parameters in the adaptive law are $\mu = 0.01$, $\alpha_{14} = \alpha_{41} = 7.2$, $\alpha_{15} = \alpha_{51} = 6.3$, and $\alpha_{23} = \alpha_{32} = 5.4$. We arbitrarily choose the initial value as $r(0) = (2.1, 2.1, 2, 1.9, 2.1)^T$, $x_0(0) = (3.3, 0.66, 0.1)^T$ and $x_i(0) = (-5.3 - 0.5 * i, 3 + 1.6 * i, 2.7 + 1.4 * i)^T$ $(i = 1, 2, \ldots, 5)$. From Figure 4, we can distinctly see that, under the control strategy designed by Theorem 3, $e_i(t)_2 (i = 1, 2, \ldots, 5)$ of HMASs (2) and (3) are concentrated in a limited area.

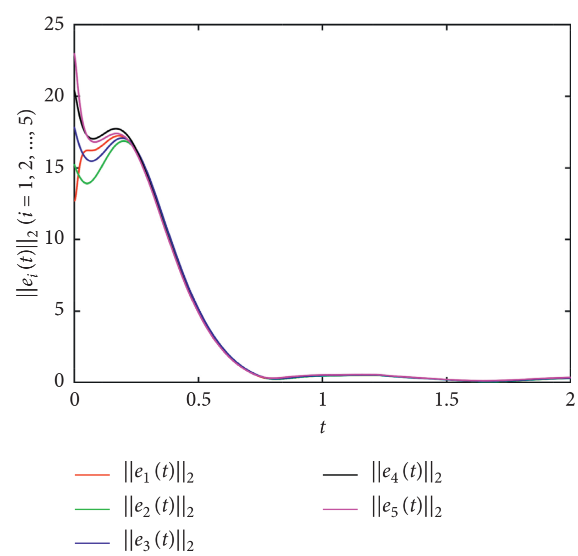

FIGURE 2: The changes of $\|e_i(t)\|_2 (i = 1, 2, \ldots, 5)$ for the system under the control strategy designed by Theorem 1.

4.4. Example 4. In this case, by controlling the coupling between part of the follower agent and the leader and the coupling between the follower agents, we realize the QC of the whole system. All agent system parameters are in accordance with A_i and B_i. We choose $c = 3$. The parameters in the adaptive law are $\mu = 0.01$, $\alpha_{14} = \alpha_{41} = 4.9$, $\alpha_{15} = \alpha_{51} = 4.2$, and $\alpha_{23} = \alpha_{32} = 4.9$. We arbitrarily choose the 1, 2, and 3 agents, and their initial values are selected as $r(0) = (1.8, 2.1, 1.8)^T$. The initial value of each agent is selected as $x_0(0) = (3.3, 0.66, 0.1)^T$ and $x_1(0) = x_2(0) = x_3(0) = x_4(0) = x_5(0) = (12.3, 5, 5.1)^T$.

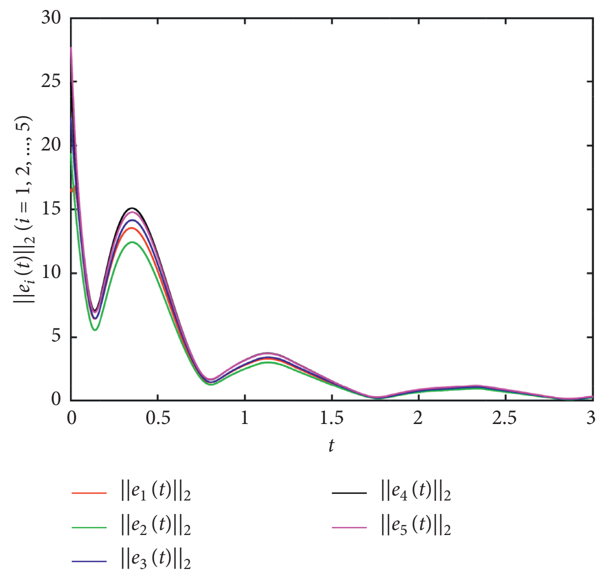

FIGURE 3: The changes of $\|e_i(t)\|_2 (i = 1, 2, \ldots, 5)$ for the system under the control strategy designed by Theorem 2.

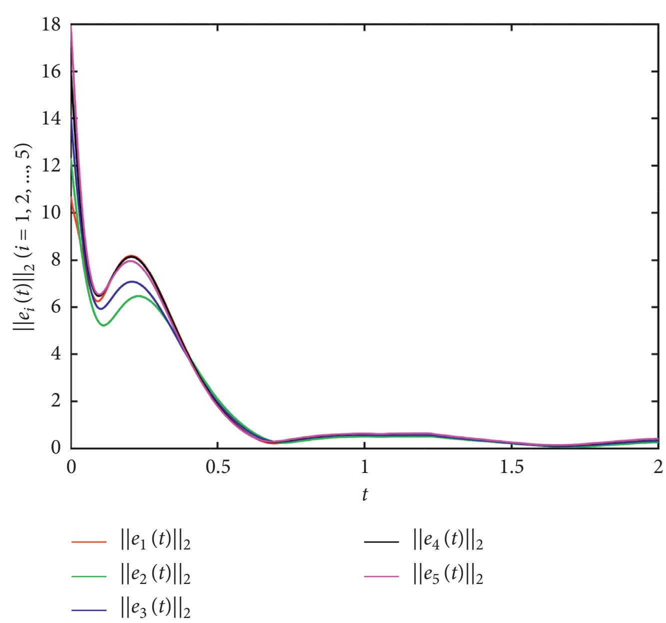

FIGURE 4: The changes of $\|e_i(t)\|_2 (i = 1, 2, \ldots, 5)$ for the system under the control strategy designed by Theorem 3.

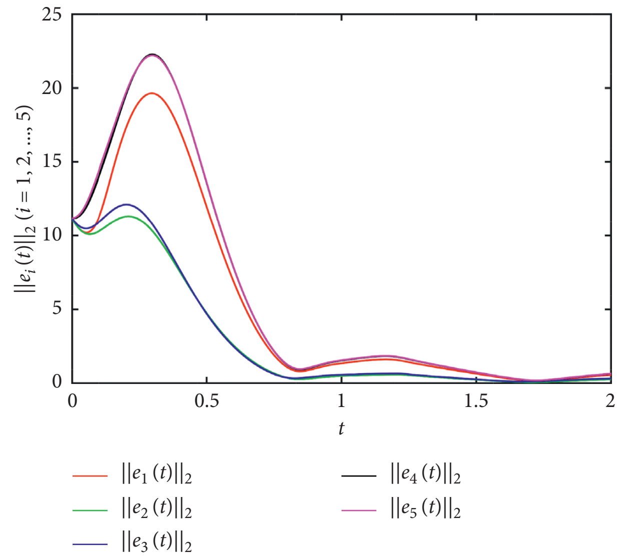

Figure 5: The changes of $\|e_i(t)\|_2 (i = 1, 2, \ldots, 5)$ for the system under the control strategy designed by Theorem 4.

Figure 5 also verifies that, under the control strategy designed by Theorem 4, the entire system gradually converges to a limited area.

5. Conclusions

The decentralized adaptive control for QC of HMASs has been studied. The combined adaptation of the coupling weights and control gains allows to drive HMASs (2) and (3) to some bounded areas. In addition, some pinning schemes have been proposed to adjust a fraction of the coupling weights and control gains. To deal with the heterogeneity, two new lemmas are proposed to derive the QC criteria. It has been shown that the QC can be obtained without requiring any global. In future works, we will extend the results to more general HMASs, such as fractional-order HMASs, HMASs with time delay, and cooperative-competitive interaction. At the same time, we will attempt to optimize control protocol and extend it to some other systems, i.e., fractional-order systems [48], memristive neural networks [49, 50], and complex-valued neural networks [51, 52].

Data Availability

The data used to support the findings of the study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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