Research Article

Precise Orbit Determination of BDS MEO Satellites Based on Satellite TT&C Stations

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A novel method, which is based on the triple-frequency combination and Space-Based Telemetry, Tracking, and Command (STT&C) stations, is proposed in this paper. Considering BeiDou Navigation Satellite System (BDS) Geostationary Orbit (GEO) and Inclined Geostationary Orbit (IGSO) satellites as the STT&C facilities, firstly, we presented the BDS Medium Earth Orbit (MEO) satellites' precise orbit determination scheme based on triple-frequency combination. Then, we gave the sufficient and necessary conditions about the visibility and the coverage rate calculation model of STT&C to BDS MEO satellite. And then we deduced the model of BDS MEO satellites precise orbit determination based on triple-frequency combination observations. At last, we designed the simulation calculation. The simulation results show that orbit determination of BDS MEO satellite based on STT&C station can be realized at all times. And most of the simulation period time, under the condition of the dm level orbit determination for GEO/IGSO satellites, the position accuracy of the relative orbit determination is better than 4 m, the horizontal accuracy of the relative orbit determination is within 2.5 m, and the vertical accuracy of the relative orbit determination is less than 3.5 m.

1. Introduction

The orbital determination accuracy of Global Navigation Satellite System (GNSS) satellites is a vital factor in GNSS navigation and positioning. Due to the limitation of Chinese geographical cover area, only about thirty percent of orbital period of BDS MEO satellites can be tracked by using ground TT&C stations or ground differential reference stations of the Chinese's own [1, 2]. Thus it is very difficult to realize the precise orbit determination for BDS MEO satellites. But fortunately, BDS GEO/IGSO satellites can be determined by the Chinese's own ground stations. The testing broadcast ephemeris error of BDS is better than 1.5 m currently [3]. Steigenberger et al. analyzed the orbit accuracy on the several-decimeter level for the GEO and the few-decimeter level for IGSO satellites of BDS could be achieved [4]. Additionally, it is a novel way to get the high and fast accuracy positioning results by using multifrequency carrier phase combination technology. But as we know, carrier phase ambiguity must be firstly resolved in GNSS positioning. According to ambiguity

resolution (AR) problems, many scholars have done a lot of research. Vollath et al. [5] described the three-carrier ambiguity resolution (TCAR) method, and de Jonge et al. [6] and Hatch et al. [7] proposed the Cascade Integer Resolution (CIR) method. Both early TCAR and CIR use essentially the same geometry-free bootstrapping procedure. Odijk and Teunissen [8] proposed an analytical closed-form expression for the multifrequency ambiguity dilution precision. Feng [9, 10] outlined a general modeling strategy for improved AR and positioning estimation using three or more phase and code ranging signals. Paziewski and Wielgosz [11] presented a method by using frequencies L1/E1 and L5/E5a combination to account for Galileo/GPS intersystem biases in precise satellite positioning. In theory, using real tracking data, combined orbit and clock correction precision, the accuracy of the GEO satellite orbit determination can meet cm level by using multifrequency carrier phase technology [12]. On the basis of these research results, we proposed a tentative idea, which takes BDS GEO/IGSO satellites as STT&C stations and can solve the problem of BDS MEO satellite precision

FIGURE 1: The schematic flow diagram of the precise orbit determination of BDS MEO satellites based on STT&C stations.

orbit determination using multifrequency combination technology.

2. The Precise Orbit Determination Scheme of BDS MEO Satellites Based on STT&C

The precise orbit determination scheme of BDS MEO satellites based on multifrequency combination can be described by the schematic flow diagram in Figure 1.

The scheme of the specific ideas is as follows. (1) All BDS GEO/IGSO satellites can be observed, commanded, and controlled because they are within the range of the ground TT&C (GTT&C) stations or differential stations of the Chinese's own. (2) On the basis of the precise orbital determination of BDS GEO/IGSO satellites, referring to the relative navigation way in Low Earth Orbiting (LEO) by using carrier phase differential GPS [13], we can install the receivers on BDS MEO satellites, which can receive the signal from BDS GEO/IGSO satellites by using omnidirectional antenna. Thus we consider BDS GEO/IGSO satellites as STT&C stations and can use the method of multifrequency combination technology to realize the precise orbit determination of BDS MEO satellites ulteriorly.

3. The Visibility and Coverage Rate Calculation Model of STT&C to BDS MEO Satellite

Using STT&C to determine the orbit of BDS MEO satellite, the visibility condition of STT&C to BDS MEO satellite must be met. The sufficient and necessary conditions about the visibility are deduced. And the coverage rate calculation model about it is given as follows.

3.1. The Description about the Visibility Condition of STT&C to BDS MEO Satellite. For the BDS MEO satellite is under the GEO/IGSO constellation, the pitch angle E of STT&C is a minus (see Figure 2).

FIGURE 2: The visibility chart of STT&C to MEO satellite.

In Figure 2, some relative quantities about the pitch angle E are given as follows: the plane α is a vertical plane about the line, which passes through GEO/IGSO satellite and the center of the Earth; h is the altitude between GEO/IGSO satellite and the surface of the Earth; θ is the antenna beam angle of GEO/IGSO satellite to MEO satellite; R_E is the Earth radius; h_{MEO} is the distance between MEO satellite and apogee point; the pitch angle E is also the line angle between the plane α and the line passing through GEO/IGSO satellite and MEO satellite. The pitch angle E can be described as

$$|E| = \frac{\pi}{2} - \frac{\theta}{2} = \frac{\pi}{2} - \arcsin \frac{R_E + h_{MEO}}{R_E + h}. \quad (1)$$

3.2. The Sufficient and Necessary Conditions about the Visibility and the Coverage Rate Calculation Model. From Figure 2, we can infer that the coverage range projection of GEO/IGSO satellite to MEO satellite on the Earth is a circle, which is the GEO/IGSO satellite projection center on the Earth. And the radius of the circle can be described as $(R_E + h_{MEO}) \sin B_S$. Here B_S is the latitude of the circle center. It is one of the coverage conditions that the track of subsatellite point of MEO satellite is within this circle.

The sufficient and necessary visibility conditions of STT&C to BDS MEO satellite must be met with these two conditions as follows:

(a) The pitch angle E must be met:

$$|E| > 90^\circ - \frac{\theta_r}{2}, \quad (2)$$

where θ_r is the valid beam angle and $\theta_r < \theta$.

(b) GEO/IGSO satellite and MEO satellite can not be shielded by the Earth. In order to avoid the effect of

the atmosphere to the received signals, we added 1000 Km to the Earth radius. Thus in the shadow area of Figure 2, STT&C and BDS MEO satellite can not see each other.

According to the two sufficient and necessary visibility conditions above, there are two visibility areas of STT&C to BDS MEO satellite. The sufficient and necessary conditions of the first visibility area are

$$\frac{\pi}{2} - \frac{\delta}{2} \geq |E| > \frac{\pi}{2} - \frac{\theta_r}{2}, \quad (3)$$

$$h_M > R \geq [(R_E + h)^2 - R_{Ex}^2]^{1/2},$$

where $\delta/2 = \arcsin(R_{Ex}/(R_E + h))$; $R_{Ex} = 1000$ Km; R is the distance between GEO/IGSO satellite and MEO satellite;

$$h_M = (h + R_E) \cos\left(\frac{\delta}{2}\right) + \sqrt{(h + R_E)^2 \cos^2\left(\frac{\delta}{2}\right) - (h + R_E)^2 + (h_{MEO} + R_E)^2}. \quad (4)$$

The sufficient and necessary conditions of the second visibility area are

$$\frac{\pi}{2} \geq |E| > \frac{\pi}{2} - \frac{\sigma}{2}, \quad (5)$$

$$[(R_E + h)^2 - R_{Ex}^2]^{1/2} > R \geq (h - h_{MEO}).$$

On the basis of sufficient and necessary conditions, the total time $\sum t_i$ tracked by STT&C will be calculated when the MEO satellite is flying. Let the total flying time of the MEO satellite be T ; the coverage rate calculation model can be described as

$$\zeta = \frac{\sum t_i}{T}. \quad (6)$$

4. The Precise Orbit Determination Model of BDS MEO Satellites Based on the Multifrequency Combination

As measured using periodic event, in most cases will appear ambiguity problem. Moreover, for the wavelength of carrier signal is generally just about 20 cm, it is really difficult to determine the phase ambiguity about the carrier signal, from tens of thousands of kilometers. The longer the carrier wavelength is, the easier it is to calculate its phase ambiguity. Here take BDS signals as an example; their service carrier frequencies broadcasted are as follows: B1 1561.098 Mhz, B2 1207.14 Mhz, and B3 1268.52 Mhz [14]. Currently BDS clock precision can be superior to 6 ns [3]. Moreover, Yang et al. achieved 6 h prediction accuracy better than 2 ns by using the AR model [15]. As we know, measured error due to the multipath of carrier phase observation is less than a quarter of the wavelength. Additionally referring to the literature [4], let us suppose the GEO/IGSO orbit consistency is on the 2 dm level. Each carrier signal wavelength and its measurement precision of BDS are listed in Table 1.

TABLE 1: BDS wavelength and the observation precision of each carrier signal.

Carrier signal	B1	B2	B3
Wavelength	19.2 cm	24.8 cm	23.6 cm
Phase noise	0.192 cm	0.248 cm	0.236 cm
Multipath error	4.8 cm	6.2 cm	5.9 cm
GEO/IGSO orbit error	2 dm		
GEO/IGSO clock error	2 ns		

4.1. The Triple-Frequency Combination Model of BDS. When the receivers, which are installed on BDS MEO satellites, receive signal from GEO/IGSO satellites, the measurement errors do not contain ionosphere and troposphere propagation delay errors because the area of signal propagation path is located in more than 20,000 kilometers above ground, especially due to sufficient and necessary conditions of (3) and (5). In the moment of t epoch, considering the impact of satellite orbital error, satellite clock error, multipath error, and phase measurement noise, carrier phase observation equations of BDS B1, B2, and B3 signals can be written as

$$\begin{aligned} \bar{\rho}_{k,B1}^S(t) &= \rho_k^S(t) - \lambda_{B1} \cdot N_{k,B1}^S(t_0) + d_{orb}^S(t) + c \\ &\quad \cdot \delta t_k(t) - c \cdot \delta t^S(t) + m_{k,L-B1}^S(t) \\ &\quad + \varepsilon_{k,L-B1}^S(t), \\ \bar{\rho}_{k,B2}^S(t) &= \rho_k^S(t) - \lambda_{B2} \cdot N_{k,B2}^S(t_0) + d_{orb}^S(t) + c \\ &\quad \cdot \delta t_k(t) - c \cdot \delta t^S(t) + m_{k,L-B2}^S(t) \\ &\quad + \varepsilon_{k,L-B2}^S(t), \\ \bar{\rho}_{k,B3}^S(t) &= \rho_k^S(t) - \lambda_{B3} \cdot N_{k,B3}^S(t_0) + d_{orb}^S(t) + c \\ &\quad \cdot \delta t_k(t) - c \cdot \delta t^S(t) + m_{k,L-B3}^S(t) \\ &\quad + \varepsilon_{k,L-B3}^S(t), \end{aligned} \quad (7)$$

where $\bar{\rho}_{k,B1}^S(t)$, $\bar{\rho}_{k,B2}^S(t)$, and $\bar{\rho}_{k,B3}^S(t)$ are the phase measurements in meters in the moment of t epoch; $\rho_k^S(t)$ is the ideal geometrical distance between the receiver of number k MEO satellite and number S GEO/IGSO satellite in meters in the moment of t epoch; λ_{B1} , λ_{B2} , and λ_{B3} separately stand for BDS B1, B2, and B3 carrier signal wavelength; $N_{k,B1}^S(t_0)$, $N_{k,B2}^S(t_0)$, and $N_{k,B3}^S(t_0)$, respectively, stand for the corresponding integer ambiguity of BDS B1, B2, and B3 carrier phase measurements; $d_{orb}^S(t)$ is the relative orbit equivalent distance error between number k MEO satellite and number S GEO/IGSO satellite; $\delta t_k(t)$, $\delta t^S(t)$ separately stand for the corresponding receiver clock biases and satellite clock biases; $m_{k,L-B1}^S(t)$, $m_{k,L-B2}^S(t)$, and $m_{k,L-B3}^S(t)$, respectively, stand for the multipath error; $\varepsilon_{k,L-B1}^S(t)$, $\varepsilon_{k,L-B2}^S(t)$, and $\varepsilon_{k,L-B3}^S(t)$, respectively, stand for each phase measurement noise which corresponded to carrier measurements.

Referring to Cocard et al. [16], the triple-frequency combination observation equation of BDS can be given as follows:

$$\Phi_{k,TC}^S(t) = \alpha \cdot \bar{\rho}_{k,B1}^S(t) + \beta \cdot \bar{\rho}_{k,B2}^S(t) + \gamma \cdot \bar{\rho}_{k,B3}^S(t), \quad (8)$$

where $\Phi_{k,TC}^S(t)$ is the virtual phase measurement of triple-frequency combination in the moment of t epoch; subscript TC stands for the abbreviation of triple-frequency combination; α , β , and γ are combination coefficient.

By substituting (7) in (8), the expression for the triple-frequency combination observation equation of BDS becomes

$$\begin{aligned} \Phi_{k,TC}^S(t) = & \rho_k^S(t) \cdot (\alpha + \beta + \gamma) - [\alpha \cdot \lambda_{B1} \cdot N_{k,B1}^S(t_0) \\ & + \beta \cdot \lambda_{B2} \cdot N_{k,B2}^S(t_0) + \gamma \cdot \lambda_{B3} \cdot N_{k,B3}^S(t_0)] \\ & + d_{orb}^S(t) \cdot (\alpha + \beta + \gamma) + c \cdot \delta t_k(t) \cdot (\alpha + \beta + \gamma) - c \\ & \cdot \delta t^S(t) \cdot (\alpha + \beta + \gamma) + [\alpha \cdot m_{k,L-B1}^S(t) + \beta \\ & \cdot m_{k,L-B2}^S(t) + \gamma \cdot m_{k,L-B3}^S(t)] + [\alpha \cdot \varepsilon_{k,L-B1}^S(t) + \beta \\ & \cdot \varepsilon_{k,L-B2}^S(t) + \gamma \cdot \varepsilon_{k,L-B3}^S(t)], \end{aligned} \quad (9)$$

where $\rho_k^S(t) \cdot (\alpha + \beta + \gamma)$ is the virtual geometrical distance between the receiver of number k MEO satellite and number S GEO/IGSO satellite in the moment of t epoch. Of course, when $\alpha + \beta + \gamma = 1$, it is the real geometrical distance between the receiver of number k MEO satellite and number S GEO/IGSO satellite.

Let $\lambda_{TC} \cdot N_{k,TC}^S(t_0) = \alpha \cdot \lambda_{B1} \cdot N_{k,B1}^S(t_0) + \beta \cdot \lambda_{B2} \cdot N_{k,B2}^S(t_0) + \gamma \cdot \lambda_{B3} \cdot N_{k,B3}^S(t_0)$, where λ_{TC} stands for the virtual wavelength of the triple-frequency combination carrier phase measurement. Hence, the corresponding integer ambiguity $N_{k,TC}^S(t_0)$ can be expressed by

$$\begin{aligned} N_{k,TC}^S(t_0) = & \alpha \cdot \frac{\lambda_{B1}}{\lambda_{TC}} \cdot N_{k,B1}^S(t_0) + \beta \cdot \frac{\lambda_{B2}}{\lambda_{TC}} \cdot N_{k,B2}^S(t_0) \\ & + \gamma \cdot \frac{\lambda_{B3}}{\lambda_{TC}} \cdot N_{k,B3}^S(t_0). \end{aligned} \quad (10)$$

Let

$$\begin{aligned} l = & \alpha \cdot \frac{\lambda_{B1}}{\lambda_{TC}}, \\ m = & \beta \cdot \frac{\lambda_{B2}}{\lambda_{TC}}, \\ n = & \gamma \cdot \frac{\lambda_{B3}}{\lambda_{TC}}. \end{aligned} \quad (11)$$

By substituting (11) in (10), one obtains an explicit type:

$$\begin{aligned} N_{k,TC}^S(t_0) = & l \cdot N_{k,B1}^S(t_0) + m \cdot N_{k,B2}^S(t_0) + n \\ & \cdot N_{k,B3}^S(t_0), \end{aligned} \quad (12)$$

where l , m , and n are triple-frequency combination coefficient; they are arbitrary integers in order to ensure cycle properties of $N_{k,TC}^S(t_0)$.

Additionally, (11) can also be described as

$$\begin{aligned} \alpha = & l \cdot \frac{\lambda_{TC}}{\lambda_{B1}}, \\ \beta = & m \cdot \frac{\lambda_{TC}}{\lambda_{B2}}, \\ \gamma = & n \cdot \frac{\lambda_{TC}}{\lambda_{B3}}. \end{aligned} \quad (13)$$

When $\alpha + \beta + \gamma = 1$, the wavelength λ_{TC} of combination observation is

$$\lambda_{TC} = \frac{\lambda_{B1} \cdot \lambda_{B2} \cdot \lambda_{B3}}{\alpha \cdot \lambda_{B2} \cdot \lambda_{B3} + \beta \cdot \lambda_{B1} \cdot \lambda_{B3} + \gamma \cdot \lambda_{B1} \cdot \lambda_{B2}}. \quad (14)$$

For $\lambda = c/f$, where c is the speed of light in vacuum and f is the frequency of carrier phase, hence, λ_{TC} can be given by

$$\lambda_{TC} = \frac{c}{(l \cdot f_{B1} + m \cdot f_{B2} + n \cdot f_{B3})}. \quad (15)$$

And the frequency f_{TC} of combination observation is given as

$$f_{TC} = l \cdot f_{B1} + m \cdot f_{B2} + n \cdot f_{B3}. \quad (16)$$

When $\alpha + \beta + \gamma = 1$, (9) can be simplified as

$$\begin{aligned} \Phi_{k,TC}^S(t) = & \rho_k^S(t) - \lambda_{TC} \cdot N_{k,TC}^S(t_0) + d_{orb}^S(t) + c \\ & \cdot \delta t_k(t) - c \cdot \delta t^S(t) \\ & + [\alpha \cdot m_{k,L-B1}^S(t) + \beta \cdot m_{k,L-B2}^S(t) + \gamma \cdot m_{k,L-B3}^S(t)] \\ & + [\alpha \cdot \varepsilon_{k,L-B1}^S(t) + \beta \cdot \varepsilon_{k,L-B2}^S(t) + \gamma \cdot \varepsilon_{k,L-B3}^S(t)]. \end{aligned} \quad (17)$$

4.2. The Precision Orbit Determination Model of Triple-Frequency Combination of BDS. Carrier phase measurement noise error is usually one percent of its wavelength. According to error propagation law, carrier measurement noise δ_{TC} of BDS triple-frequency combination can be expressed as $\delta_{TC} = \sqrt{\alpha^2 \cdot \delta_{B1}^2 + \beta^2 \cdot \delta_{B2}^2 + \gamma^2 \cdot \delta_{B3}^2}$. On the basis of triple-frequency combination model of BDS above, according to the selection criteria and constraint conditions of combination coefficients [14, 17], considering the properties of (7) and ensuring the magnitude of error δ_{TC} is of cm level, the combination coefficients and their other properties of some wide-lane (WL) and extra-wide-lane (EWL) of BDS triple-frequency combination are listed in Table 2.

TABLE 2: BDS multifrequency combination observation and their properties.

Carrier signal	Combination	Combination coefficient						Combination frequency/MHz	Combination wavelength/m	Combination phase noise/m
		l	m	n	α	β	γ			
B2-B3	EWL	0	-1	1	0	-19.69	20.69	61.38	4.884	0.069
B1-B2-B3	WL	1	1	-2	6.75	5.23	-10.99	231.198	1.297	0.032
B1-B3	WL	1	0	-1	5.34	0	-4.34	292.578	1.025	0.014
B1-B2-B3	WL	1	-2	1	3.76	-5.82	3.06	415.338	0.722	0.018
B1-B2-B3	WL	1	-3	2	3.28	-7.62	5.32	476.718	0.629	0.023
B1-B2-B3	WL	2	1	-3	5.97	2.31	-7.28	523.776	0.573	0.021

In order to realize precise orbit determination for BDS MEO satellites, if we choose the EWL of B2-B3 carrier combination in Table 2, (17) can be recovered as follows:

$$\begin{aligned} \Phi_{k,TC}^S(t) = & \rho_k^S(t) - 4.884N_{k,TC}^S(t_0) + d_{orb}^S(t) + c \\ & \cdot \delta t_k(t) - c \cdot \delta t^S(t) - 19.69m_{k,L-B2}^S(t) \\ & + 20.69m_{k,L-B3}^S(t) - 19.69\epsilon_{k,L-B2}^S(t) \\ & + 20.69\epsilon_{k,L-B3}^S(t), \end{aligned} \quad (18)$$

where $\rho_k^S(t) = [(X^S(t) - x_k(t))^2 + (Y^S(t) - y_k(t))^2 + (Z^S(t) - z_k(t))^2]^{1/2}$; $[X^S(t) \ Y^S(t) \ Z^S(t)]^T$ is the space coordinate of number S GEO/IGSO satellite in the measurement of t epoch; $[x_k(t) \ y_k(t) \ z_k(t)]^T$ represents the space coordinate of number k MEO satellite in the measurement of t epoch.

For $\rho_k^S(t)$ is the nonlinear function about $[x_k(t) \ y_k(t) \ z_k(t)]^T$, which are space coordinates of number k MEO satellite in the moment of t epoch, it should be linearized when it is processed by computer.

Assuming that $[x_k(t)|_0 \ y_k(t)|_0 \ z_k(t)|_0]^T$ is the approximate coordinate of number k MEO satellite in the moment of t epoch, letting

$$\begin{aligned} x_k(t) &= x_k(t)|_0 + \delta x_k(t), \\ y_k(t) &= y_k(t)|_0 + \delta y_k(t), \\ z_k(t) &= z_k(t)|_0 + \delta z_k(t), \end{aligned} \quad (19)$$

we can deduce the linearization formula of (18) as follows:

$$\begin{aligned} \Phi_{k,TC}^S(t) = & (\rho_k^S(t))_0 - a_k^S(t) \cdot \delta x_k(t) - b_k^S(t) \cdot \delta y_k(t) \\ & - c_k^S(t) \cdot \delta z_k(t) - 4.884N_{k,TC}^S(t_0) \\ & + d_{orb}^S(t) + c \cdot \delta t_k(t) - c \cdot \delta t^S(t) \\ & - 19.69m_{k,L-B2}^S(t) + 20.69m_{k,L-B3}^S(t) \\ & - 19.69\epsilon_{k,L-B2}^S(t) + 20.69\epsilon_{k,L-B3}^S(t), \end{aligned} \quad (20)$$

where $a_k^S(t)$, $b_k^S(t)$, $c_k^S(t)$ are the coefficients of the linearization formula and $[a_k^S(t) \ b_k^S(t) \ c_k^S(t)]^T$ is also regarded as the direction cosines vector from number k MEO satellite to number S GEO/IGSO satellite in the moment of t epoch.

Generally, when the receiver of number k MEO satellite observes continuously for a time, the number n_Σ of the carrier observation equations would be greater than the total number n_u of the unknown parameters. When $n_\Sigma > n_u$, the error matrix equation with respect to the unknown parameters of (20) can be given as

$$\mathbf{V}_k(t) = \mathbf{B}_k(t) \delta \mathbf{G}_k(t) - \mathbf{L}_k(t), \quad (21)$$

$n_\Sigma \times 1 \quad n_\Sigma \times n_u \quad n_u \times 1 \quad n_\Sigma \times 1$

where $\mathbf{B}_k(t)$ is the coefficient matrix with respect to the unknown parameters; $\delta \mathbf{G}_k(t)$ is the matrix with respect to the unknown parameters; $\mathbf{L}_k(t)$ is the matrix of constant term.

According to Gaussian least-square principle, in order to calculate the matrix $\delta \mathbf{G}_k(t)$ with respect to the unknown parameters in (21), the condition of $\mathbf{V}_k^T(t) \mathbf{V}_k(t) = \min$ must be satisfied. Referring to the mathematical function extremum method, one leads to

$$\frac{\partial \mathbf{V}_k^T(t) \mathbf{V}_k(t)}{\partial \delta \mathbf{G}_k(t)} = 2 \mathbf{V}_k^T(t) \mathbf{B}_k(t) = 0. \quad (22)$$

Transposition leads to

$$\mathbf{B}_k^T(t) \mathbf{V}_k(t) = 0. \quad (23)$$

By substituting (21) in (23), the optimal value $\delta \widehat{\mathbf{G}}_k(t)$ of the matrix $\delta \mathbf{G}_k(t)$ with respect to the unknown parameters can be calculated by

$$\begin{aligned} \delta \widehat{\mathbf{G}}_k(t) &= (\mathbf{B}_k^T(t) \mathbf{B}_k(t))^{-1} \mathbf{B}_k^T(t) \mathbf{L}_k(t) \\ &= \widehat{\mathbf{Q}}_{\delta \widehat{\mathbf{G}} \delta \widehat{\mathbf{G}}} \mathbf{B}_k^T(t) \mathbf{L}_k(t), \end{aligned} \quad (24)$$

where $\widehat{\mathbf{Q}}_{\delta \widehat{\mathbf{G}} \delta \widehat{\mathbf{G}}}$ is the corresponding cofactor matrix about $\delta \widehat{\mathbf{G}}_k(t)$.

4.3. The Orbit Determination Accuracy Evaluation Model of MEO Satellite. Referring to the error propagation law, the corresponding mean square error δ_0 with respect to combination carrier measurement can be expressed as follows:

$$\delta_0 = \pm \sqrt{\frac{\mathbf{V}_k^T(t) \cdot \mathbf{V}_k(t)}{f}}, \quad (25)$$

where $f = n_\Sigma - n_u$.

FIGURE 3: The visibility number of STT&C to BDS MEO satellite.

Thus the accuracy evaluation equation of the optimal value $\delta\hat{\mathbf{G}}_k(t)$ can be expressed as follows:

$$(m_{\delta\hat{\mathbf{G}}_k})_j = \delta_0 \sqrt{(\hat{\mathbf{Q}}_{\delta\hat{\mathbf{G}}_k\delta\hat{\mathbf{G}}_k})_{jj}}, \quad (26)$$

where $(\hat{\mathbf{Q}}_{\delta\hat{\mathbf{G}}_k\delta\hat{\mathbf{G}}_k})_{jj}$ is the main number j diagonal element of $\hat{\mathbf{Q}}_{\delta\hat{\mathbf{G}}_k\delta\hat{\mathbf{G}}_k}$.

The PDOP (Position Dilution of Precision) of the orbit determination of MEO satellite in the moment of t epoch can be described as

$$\text{PDOP} = \sqrt{Q_{\delta\hat{x}_k(t)\delta\hat{x}_k(t)} + Q_{\delta\hat{y}_k(t)\delta\hat{y}_k(t)} + Q_{\delta\hat{z}_k(t)\delta\hat{z}_k(t)}}. \quad (27)$$

Additionally the corresponding horizontal precision of the orbit determination of MEO satellite in the moment of t epoch can be also represented as

$$m_P(t) = \delta_0 \cdot \text{PDOP}. \quad (28)$$

FIGURE 4: PDOP value of BDS MEO satellite.

The HDOP (Horizontal Dilution of Precision) of the orbit determination of MEO satellite in the moment of t epoch can be described as

$$\text{HDOP} = \sqrt{Q_{\delta \hat{x}_k(t) \delta \hat{x}_k(t)} + Q_{\delta \hat{y}_k(t) \delta \hat{y}_k(t)}}. \quad (29)$$

The corresponding horizontal precision of the orbit determination of MEO satellite in the moment of t epoch can be also represented as

$$m_H(t) = \delta_0 \cdot \text{HDOP}. \quad (30)$$

The VDOP (Vertical Dilution of Precision) of the orbit determination of MEO satellite in the moment of t epoch can be described as

$$\text{VDOP} = \sqrt{Q_{\delta \hat{z}_k(t) \delta \hat{z}_k(t)}}. \quad (31)$$

The corresponding vertical precision of the orbit determination of MEO satellite in the moment of t epoch can be also represented as

$$m_V(t) = \delta_0 \cdot \text{VDOP}. \quad (32)$$

FIGURE 5: HDOP value of BDS MEO satellite.

5. Performance Analysis of the Precise Orbit Determination of BDS MEO Satellite

On the basis of the visibility and coverage rate calculation model above, the simulation calculation has been designed here. According to the literature [18], we select the simulation parameters of BDS as follows: the five GEO satellites' orbital altitudes are all 35786 km; the 27 MEO satellites' orbital altitudes are all 21528 km, and their orbital inclinations are all 55 degrees; the three IGSO satellites' orbital altitudes are

all 35786 km, and their orbital inclinations are all 55 degrees too. Referring to the literature [11], let the beam angles of GEO/IGSO satellites be 160 degrees. According to these main parameters, the simulation results of two days are given in Figures 3–9.

Figure 3 shows the visibility number of 27 BDS MEO satellite receivers to STT&C stations in the simulation period. Here MEO1, MEO2, and MEO3 stand for the orbital plane number. MEO1-1, MEO1-2, MEO1-3, ..., and MEO3-9 are the serial numbers of 27 BDS MEO satellites. As we can

FIGURE 6: VDOP value of BDS MEO satellite.

see, most of the time, the visibility number is more than six and, for only a few times, the visibility number is five. Thus orbit determination of BDS MEO satellite based on STT&C stations can be realized for all period time.

Figures 4–6 give the results of 27 BDS MEO satellite receivers' PDOP, HDOP, and VDOP with respect to STT&C stations. It can be found from Figure 4, most of the time, that PDOP value is less than six, and about 50% of simulation period time, the value is less than 4, while it is seldom greater than 10. From Figure 5, we can see, most of the time, that HDOP value is within 4, while it is seldom slightly greater

than 5. And Figure 6 shows that VDOP value is within 5 most of the period time, while it is seldom greater than 9. Combining with Figure 3, we can see that the PDOP, HDOP, and VDOP value is at accord with the visibility number of 27 BDS MEO satellite receivers to STT&C stations; that is, the greater the visibility number is, the smaller the value of PDOP, HDOP, and VDOP is.

Referring to the accuracy of Tables 1 and 2 given and according to the simulation design of the BDS constellation above, ephemeris data and observation data are simulated. Using the precise orbit determination model of BDS MEO

FIGURE 7: The simulation precision m_p of the orbit determination of BDS MEO satellite.

satellite based on multifrequency combination above, the accuracy results of the orbit determination of BDS MEO satellites are showed in Figures 7–9.

Figures 7–9 give the results of 27 BDS MEO satellite receivers' position accuracy m_p , horizontal accuracy m_H , and vertical accuracy m_V based on STT&C stations. It can be found from Figure 7, most of the time, that the position accuracy m_p is less than 4 m, and about 50% of simulation period time, the value is less than 3 m, while it is seldom greater than 6 m. From Figure 8, we can see,

most of the time, that the horizontal accuracy m_H is within 2.5 m, and about 50% of simulation period time, the value is less than 2 m, while it is seldom greater than 3 m. And Figure 9 shows that the vertical accuracy m_V is within 3.5 m most of the period time, and about 50% of simulation period time, the value is less than 3 m, while it is seldom greater than 5 m. Therefore it is one of the high and autonomous accuracy methods to determine BDS MEO satellite position by using STT&C stations for all period time.

FIGURE 8: The simulation precision m_H of the orbit determination of BDS MEO satellite.

6. Conclusion

It is difficult to realize the whole period and high precise orbit determination for BDS MEO satellites by using ground TT&C stations or ground differential reference stations of the Chinese's own. The method of precise orbit determination of BDS MEO satellites based on STT&C stations was proposed in this paper, the precise orbit determination model based on BDS EWL of B2-B3 combination was deduced, and its orbit determination accuracy was evaluated. The results show that the method and the precise orbit determination model of BDS MEO satellites based on the multifrequency

combination mentioned in this paper can solve the problem of orbit determination with respect to BDS MEO satellites. Moreover, on the basis of the dm level orbit determination for GEO/IGSO satellites, this method can meet the meter-level higher accuracy requirement and can realize precise orbit determination of BDS MEO satellites for all period time.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

FIGURE 9: The simulation precision m_V of the orbit determination of BDS MEO satellite.

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