A Terminal Guidance Law Based on Motion Camouflage Strategy of Air-to-Ground Missiles

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A guidance law for attacking ground target based on motion camouflage strategy is proposed in this paper. According to the relative position between missile and target, the dual second-order dynamics model is derived. The missile guidance condition is given by analyzing the characteristic of motion camouflage strategy. Then, the terminal guidance law is derived by using the relative motion of missile and target and the guidance condition. In the process of derivation, the three-dimensional guidance law could be designed in a two-dimensional plane and the difficulty of guidance law design is reduced. A two-dimensional guidance law for three-dimensional space is derived by bringing the estimation for target maneuver. Finally, simulation for the proposed guidance law is taken and compared with pure proportional navigation. The simulation results demonstrate that the proposed guidance law can be applied to air-to-ground missiles.

1. Introduction

Proportional navigation (PN), which has a simple form and is implemented easily, is widely used in the missile interception field. Through decades of development, proportional navigation law has been improved to different forms, including true proportional navigation (TPN), pure proportional navigation (PPN), augmented proportional navigation (APN), and bias-proportional navigation (BPN) [1–3]. The ultimate goal of these guidance laws is to make the line of sight (LOS) angle rate converge to zero as much as possible. However, the LOS angle rate convergence to zero is difficult for high maneuvering target. This comes with some of the inherent problems of PN guidance, such as lateral acceleration singularity at the end time when range-to-go or time-to-go approaches zero [4]. Traditional proportional navigation law requires the normal acceleration and the LOS rate is proportional to the ratio; the bias guidance law is to make the normal line of sight angular rate and acceleration give a small deviation term. The modified bias-proportional navigation deals with angle constraint by increasing two time-varying terms, but it requires a time-to-go estimation and the velocity of the missile to be constant [5, 6].

In recent years, the optimal guidance law is investigated intensively based on optimal control theory [7]. The different forms of guidance law can be achieved by different performance indexes, such as the minimum miss distance, the minimum consumption, and the minimum time. In [8, 9], optimal control laws, for a missile with arbitrary order dynamics trying to attack a stationary target, were proposed with a similar cost function and an LOS fixed the coordinate system. The proposed law was implemented for lag-free and first-order lag missile systems. In the optimal guidance law, the time-to-go has significant effects on the guidance commands and even performance index. Therefore, the key issue is how to accurately estimate the remaining time so that we can improve the performance of guidance law. Hexner et al. [10] derived an optimal guidance law by analyzing an intercept scenario in the framework of a linear quadratic Gaussian terminal control problem with bounded acceleration command. Ratnoo and Ghose [11] introduced a tracking filter to estimate the relative motion for obtaining estimates of the time-to-go. In the ideal case, the optimal guidance law can get a good trajectory, but the ballistic performance maybe gets poor in uncertainty [12].
This paper proposes a dimension-reduction guidance law for attacking ground target based on motion camouflage strategy, which has compensation for target maneuvering. First, the interception condition of the missile is derived from motion camouflage characteristics which is obtained by the theory of motion camouflage. Then, the two-dimensional guidance law based on the condition is designed to the three-dimensional space model. This dimension reduction method not only simplifies the design steps of guidance law but also reduces the design difficulty. Finally, some simulations are carried out. The simulation results show the effectiveness of the proposed guidance law.

2. Dynamics Model

The relative relationships of the missile and the target are shown in Figure 1.

The relative displacement vector from the missile to the target is given by

\[ \mathbf{r} = \mathbf{r}_t - \mathbf{r}_m = r \mathbf{e}_r, \]

where \( \mathbf{r} \) is the relative distance between missile and target. \( \mathbf{e}_r \) is a fixed unit vector along the line of sight. Differentiating \( \mathbf{e}_r \) with respect to time yields

\[ \dot{\mathbf{e}}_r = \mathbf{\omega} \times \mathbf{e}_r = \omega \mathbf{e}_\omega \times \mathbf{e}_r. \]

The vector \( \mathbf{e}_\theta \) is defined as

\[ \mathbf{e}_\theta = \mathbf{e}_\omega \times \mathbf{e}_r. \]

The set of unit vectors \( \{ \mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_\omega \} \) constitutes a reference frame. This frame is a rotating coordinate system and the origin is the mass center of the missile. Differentiating (1) yields

\[ \dot{\mathbf{r}} = \dot{r} \mathbf{e}_r + r \dot{\mathbf{e}}_r = \dot{r} \mathbf{e}_r + r \mathbf{\omega} \mathbf{e}_\omega. \]

Obviously, the relative velocity vector is constituted by the radial velocity and the normal velocity. Let \( \mathbf{a}_r \) and \( \mathbf{a}_\theta \) be the maneuvering acceleration of missile and target; they are expressed in the rotating coordinate system as

\[ \mathbf{a}_m = a_{mr} \mathbf{e}_r + a_{m\theta} \mathbf{e}_\theta + a_{m\omega} \mathbf{e}_\omega, \]

\[ \mathbf{a}_t = a_{tr} \mathbf{e}_r + a_{t\theta} \mathbf{e}_\theta + a_{t\omega} \mathbf{e}_\omega. \]

Therefore, the relative acceleration of the missile and the target can be expressed as

\[ \dot{\mathbf{r}} = \mathbf{a}_t - \mathbf{a}_m = (a_{tr} - a_{mr}) \mathbf{e}_r + (a_{t\theta} - a_{m\theta}) \mathbf{e}_\theta + (a_{t\omega} - a_{m\omega}) \mathbf{e}_\omega. \]

From the above equation, we can derive a second-order dynamic equation of relative movement as

\[ \ddot{\mathbf{r}} = r \ddot{\mathbf{e}}_r - a_{mr} + a_{tr}, \]

\[ \ddot{\mathbf{e}}_r = (a_{t\theta} - a_{m\theta}) \frac{\mathbf{e}_\theta}{r} + (a_{t\omega} - a_{m\omega}) \frac{\mathbf{e}_\omega}{r} - \frac{\mathbf{r}}{r^2} \mathbf{e}_r - \frac{2\dot{r}}{r} \mathbf{e}_r. \]

Therefore, the guidance problem can be described as finding the acceleration of the missile \( a_{mr}, a_{m\theta}, \) and \( a_{m\omega} \) to let \( r \) converge to zero in finite time.
3. Guidance Law Implementation

3.1. Motion Camouflage Theory. Motion camouflage strategy is a new form of stealth strategy which describes the relative motion relationship of the pursuer, target, and reference point: the movement of them as shown in Figure 2.

The pursuer’s path is controlled by the path control parameter (PCP) \( c(t) \) as

\[
\mathbf{x}_p = \mathbf{x}_r + c(t) \mathbf{x}_r,
\]

where \( \mathbf{x}_r = \mathbf{x}_t - \mathbf{x}_m \) are the relative distance vector from the reference point to the target. The selected PCP and reference point determine the speed and curvature of the trajectory in the constructed subspace.

If the position of the reference point is a fixed camouflage background, motion camouflage strategy is similar to the three-point guidance law. And if the reference point is chosen at the infinity, it is similar to the constant-bearing navigation. Therefore, motion camouflage strategy has both features of the three-point guidance law and constant-bearing navigation.

3.2. Guidance Law Based on Motion Camouflage. Let the pursuer and target be the missile and the ground target, respectively. And setting the reference point as infinity yields

\[
\mathbf{r} = \mathbf{r}_m - \mathbf{r}_m = c(t) \mathbf{e}_r.
\]

The component of the missile velocity transverse to the baseline is

\[
\hat{\mathbf{r}}_{m\perp} = \hat{\mathbf{r}}_m - (\mathbf{e}_r \cdot \hat{\mathbf{r}}_m) \mathbf{e}_r,
\]

and, similarly, that of the target is

\[
\hat{\mathbf{r}}_{t\perp} = \hat{\mathbf{r}}_t - (\mathbf{e}_r \cdot \hat{\mathbf{r}}_t) \mathbf{e}_r.
\]

The relative transverse component is

\[
\lambda = (\hat{\mathbf{r}}_m - \hat{\mathbf{r}}_m) - (\mathbf{e}_r \cdot (\hat{\mathbf{r}}_m - \hat{\mathbf{r}}_m)) \mathbf{e}_r = \hat{\mathbf{r}} - (\mathbf{e}_r \cdot \hat{\mathbf{r}}) \mathbf{e}_r.
\]

The missile-target system is in a state of motion camouflage without collision on an interval iff \( \lambda = 0 \) on that interval.

According to the fact that the final goal of guidance problem is such that the relative distance converges to zero, we consider the ratio as follows:

\[
Z = \frac{\hat{\mathbf{r}}}{|\hat{\mathbf{r}}|}.
\]

which compares the rate of change of the baseline length to the absolute rate of change of the baseline vector. If the baseline experiences pure lengthening, then the ratio assumes its maximum value, \( Z = +1 \). If the baseline experiences pure shortening, then the ratio assumes its minimum value, \( Z = -1 \).

Equation (13) can be written as

\[
Z = \frac{\mathbf{r} \cdot \hat{\mathbf{r}}}{r |\hat{\mathbf{r}}|}.
\]

Thus, \( Z \) is the dot product of two unit vectors: one in the direction of \( \mathbf{r} \) and the other in the direction of \( \hat{\mathbf{r}} \). According to (12), the magnitude squared of \( \lambda \) is

\[
|\lambda|^2 = |\hat{\mathbf{r}}|^2 - 2 (\mathbf{e}_r \cdot \hat{\mathbf{r}})^2 + (\mathbf{e}_r \cdot \hat{\mathbf{r}})^2 = |\hat{\mathbf{r}}|^2 (1 - Z^2).
\]

Obviously, the requirement of the component of the missile velocity is equal to that of the target: it could be transferred to \( Z = -1 \). Thus, our objective is to design a guidance law to guarantee \( Z = -1 \).

Differentiating \( Z \) gives

\[
\dot{Z} = \frac{(\mathbf{r} \cdot \hat{\mathbf{r}} + \mathbf{r} \cdot \hat{\mathbf{r}}) \cdot \mathbf{r} |\hat{\mathbf{r}}| - \mathbf{r} \cdot \hat{\mathbf{r}} \cdot \mathbf{r} |\hat{\mathbf{r}}|^2}{(r |\hat{\mathbf{r}}|)^2} - \frac{\mathbf{r} \cdot \hat{\mathbf{r}} \cdot \mathbf{r} \cdot \mathbf{r}}{(r |\hat{\mathbf{r}}|)^2} |\hat{\mathbf{r}}|^2
\]

\[
= (\frac{\hat{\mathbf{r}} \cdot \hat{\mathbf{r}}}{r |\hat{\mathbf{r}}|}) - (\frac{\hat{\mathbf{r}} \cdot \mathbf{r}}{|\hat{\mathbf{r}}|}) (\frac{\mathbf{r} \cdot \hat{\mathbf{r}}}{r |\hat{\mathbf{r}}|})
\]

\[
= |\mathbf{r}| \left[ 1 - \left(\frac{\hat{\mathbf{r}} \cdot \mathbf{r}}{|\hat{\mathbf{r}}|} \right)^2 \right] + \frac{1}{|\hat{\mathbf{r}}|} \left[ \mathbf{r} - (\frac{\mathbf{r} \cdot \hat{\mathbf{r}}}{|\hat{\mathbf{r}}|}) \hat{\mathbf{r}} \right] \cdot \hat{\mathbf{r}}.
\]

We define

\[
\xi = \frac{1}{|\mathbf{r}|} \left[ \frac{\mathbf{r}}{|\mathbf{r}|} - (\frac{\mathbf{r}}{|\mathbf{r}|}) \frac{\mathbf{r} \cdot \hat{\mathbf{r}}}{|\hat{\mathbf{r}}|} \right].
\]

Using the formula \( a \times (b \times c) = b(a \cdot c) - c(a \cdot b) \) and (4), we compute

\[
\xi = -\frac{1}{|\mathbf{r}|^2} \left[ \mathbf{r} \times \left( \mathbf{r} \times \frac{\mathbf{r}}{|\mathbf{r}|} \right) \right]
\]

\[
= -\frac{1}{|\mathbf{r}|^2} \left[ \mathbf{r} \left( \frac{\mathbf{r} \cdot \mathbf{r}}{|\mathbf{r}|^2} \right) - r \left( \mathbf{r} \cdot \mathbf{r} \right) \right]
\]

\[
= -\frac{1}{|\mathbf{r}|^2} \left[ \mathbf{r} \left( \mathbf{e}_r + r \omega e_\theta \right) - e_r \left( r^2 e_r + r^2 \omega^2 \right) \right]
\]

\[
= -\frac{1}{|\mathbf{r}|^2} \left[ \left( r^2 e_r + r \omega e_\theta \right) - e_r \left( r^2 + r^2 \omega^2 \right) \right]
\]

\[
= \frac{1}{|\mathbf{r}|} \left( r^2 \omega^2 e_r - r \omega e_\theta \right).
\]
Then
\[
\begin{aligned}
\xi \cdot \ddot{r} &= \frac{1}{|r|^3} \left( r^2 \omega^2 e_r - i r \omega e_{\theta} \right) \cdot (P e_{\theta} + Q e_{\omega} + R e_r) \\
&= \frac{1}{|r|^3} \left( r^2 \omega^2 R - i r \omega P \right).
\end{aligned}
\] (19)

Substituting (19) into (16) yields
\[
\begin{aligned}
\dot{Z} &= \frac{|r|}{r} \left( 1 - Z^2 \right) - \frac{i r \omega}{|r|^3} (a_{m\theta} - a_{m\alpha}) \\
&\quad + \frac{\omega^2 r^2}{|r|^3} (a_{tr} - a_{mr}).
\end{aligned}
\] (20)

As can be seen from the above results, the acceleration term \(a_{m\alpha}\) has been eliminated. Thus, we only design the tangential acceleration and normal acceleration of missile to make \(\dot{Z} < 0\). According to the literature [3], the relative acceleration \((a_{m\alpha} - a_{m\omega})\) is high-order small quantity relative to the other direction of the acceleration and can be neglected. Thus, the final task of the designed threedimensional guidance law is to give the analytical expression of the acceleration \(a_{m\theta}\).

We give the guidance law as
\[
a_{m\theta} = \frac{v_m}{ \alpha_r} \left( \dot{r} - \frac{|r|^2}{r} \right) + a_{t\theta}
\] (21)
and substitute into (20)
\[
\dot{Z} = \left( 1 - Z^2 \right) \left( \frac{|r|}{r} - \frac{\mu v_m}{ \alpha_r} \right)
\] (22)

We assume that the upper and lower bounds \([v_{m}^{-}, v_{m}^{+}]\) and \([v_{t}, v_{t}^{-}]\) exist such that
\[
\frac{v_{t}}{v_{m}} \leq K < 1.
\] (23)

For the interception process, the relative velocity of the missile and the target should satisfy the following relationship:
\[
v_{m}^{-} (1 - K) \leq |r| \leq v_{m}^{+} (1 + K).
\] (24)

We define
\[
\mu = \frac{v_{m}^{+} (K + 1)}{v_{m}^{-}} \left( \frac{v_{m}^{-} (K + 1)}{r_o} + \sigma \right)
\] (25)
and hence
\[
\mu \geq \frac{v_{m}^{+} (K + 1)}{v_{m}^{-}} \left( \frac{v_{m}^{+} (K + 1)}{r} + \sigma \right),
\] (26)
where \(r_o > 0, \sigma > 0\). Thus, for \(r > r_o\), (22) becomes
\[
\dot{Z} \leq \left( 1 - Z^2 \right) \left[ \frac{v_{m}^{+} (1 + K)}{r} - \frac{v_{m}^{-} (1 + K)}{v_{m}^{-}} \left( \frac{v_{m}^{+} (K + 1)}{r} + \sigma \right) \right]
\] (27)
\[- \frac{v_{m}^{-} (K + 1)}{v_{m}^{-}} \left( \frac{v_{m}^{+} (K + 1)}{r} + \sigma \right)
\]
\[- \left( 1 - Z^2 \right) \sigma.
\]

Obviously, \(\dot{Z} < 0\) can be held for \(\sigma > 0\). Therefore, (21) can guarantee interception. However, the target acceleration information of guidance law is not measurable and is only estimated approximately. We assume that a constant \(W\) exists such that
\[
|a_{t\alpha}| \leq W.
\] (28)

The target acceleration \(a_{t\alpha}\) is replaced by \(W \text{ sgn}(\omega)\) and (20) is given by
\[
a_{m\delta} = \frac{v_m}{ \alpha_r} \left( \dot{r} - \frac{|r|^2}{r} \right) + W \text{ sgn} (\omega).
\] (29)

The switching term would lead to the appearance of the chattering effect on acceleration. To remove the chattering, the signum function can be smoothened, usually replacing \(\text{sgn}(x)\) with a saturation function expressed as
\[
\text{sat}(x, \delta) = \begin{cases} 
1 & x > \delta \\
\frac{x}{\delta} & |x| \leq \delta \\
-1 & x < -\delta.
\end{cases}
\] (30)

The final three-dimensional guidance law can be designed as
\[
a_m = \left( \frac{v_m}{ \alpha_r} \left( \dot{r} - \frac{|r|^2}{r} \right) + W \cdot \text{sat} (\omega, \delta) \right) \cdot e_{\theta}.
\] (31)

The guidance law only has a normal component of LOS, so the three-dimensional guidance law based on motion camouflage strategy can be converted directly into the rotation plane of LOS. If the target does not maneuver, the designed guidance law only requires the LOS rate, the relative distance, and the velocity of the missile. The proposed guidance law reduces the difficulty of detection (without obtaining pre-angle information) compared with the constant-bearing method. Also, it can ensure a smaller overload at the terminal stage than proportional navigation method, because the guidance law contains the relative motion information, although it needs more measurement information.

4. Simulation

4.1. Comparison of Different Gains. In order to verify the validity of the designed guidance law, the different coefficients \(\mu\) will be given for comparison simulation. The initial position and the initial velocity of the missile are \(\mathbf{r}_m = [131 \text{ km}, 5 \text{ km}, 0 \text{ km}]\) and \(v_m = 700 \text{ m/s}\). The initial position and the initial velocity of the target are \(\mathbf{r}_t = [142 \text{ km}, 0 \text{ km}, 2.54 \text{ km}]\) and \(v_t = 60 \text{ m/s}\). Firstly, the target moves in a straight line and the guidance coefficients are specified as 0.5 and 2. The simulation results are shown in Figures 3–5.

As can be seen from the figures, the acceleration amplitudes of motion camouflage guidance law are closely related to the guidance coefficient. The trajectory and overload are also different when different guidance coefficients are taken.
Because the coefficient $\mu = 0.5$ is small, the overload of the missile is small in the initial stage and thus cannot keep up with the target. Whereas when $\mu = 2$ the missile overload is larger in the terminal stage, the miss distance is smaller than the other. Therefore, the guidance coefficient should be selected reasonably and it can guarantee that the proper overload of missile is smooth and can also achieve a smaller miss distance.

4.2. Comparison of Different Guidance Laws. The pure proportional navigation (PPN) is chosen for comparing with the proposed guidance law. The guidance coefficients of MCPG and PPN are 0.5 and 3, respectively. The simulation conditions are not changed. The target’s maneuvering acceleration is $1\text{g}$. The simulation results are shown in Figures 6–10.

The miss distances of MCPG and PPN are 0.3254 m and 0.5851 m, respectively. In the initial stage of interception, the acceleration of MCPG is larger than that of PPN. However, the acceleration of MCPG has a faster response so that the missile can track the maneuvering of the target better. Figure 9 presents the rate of rotation of LOS and illustrates that the MCPG can restrain the rate rotation before hitting the target.

Figure 10 shows the values of $Z$. Note that the proposed guidance law trends $Z$ to $-1$ during the process of interception. The value of $Z$ always fluctuates around $-1$. Thus, the relative motion satisfies the status of motion camouflage.

According to the above analysis, the MCPG has a large overload at the initial stage, but the proposed guidance law can satisfy the demand for rapid response and maneuvering.

5. Conclusion

This paper presents a three-dimensional guidance law for intercepting the ground target, which is based on the motion camouflage theory and the proposed dynamic equations. To
improve the robustness of the guidance law, a compensation is given for the target maneuver. The designed guidance law does not require too much measurement information so that it is implemented easily. Also, its expression only has a normal component of acceleration, and it can reduce the difficulty of the design process. According to the simulation and comparison of MCPG and PPN, the results show that the three-dimensional guidance law based on the motion camouflage theory can destroy ground targets effectively. The proposed guidance law has a faster response of acceleration and a smaller acceleration at the last moment.

Competing Interests
The authors declare that they have no competing interests.

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References


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