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Research Article

H_∞ Networked Cascade Control System Design for Turboshaft Engines with Random Packet Dropouts

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The distributed control architecture becomes more and more important in future gas turbine engine control systems, in which the sensors and actuators will be connected to the controllers via a network. Therefore, the control problem of network-enabled high-performance distributed engine control (DEC) has come to play an important role in modern gas turbine control systems, while, due to the properties of the network, the packet dropouts must be considered. This study introduces a distributed control system architecture based on a networked cascade control system (NCCS). Typical turboshaft engine distributed controllers are designed based on the NCCS framework with H_∞ state feedback under random packet dropouts. The sufficient robust stable conditions are derived via the Lyapunov stability theory and linear matrix inequality approach. Simulations illustrate the effectiveness of the presented method.

1. Introduction

The distributed control system (DCS) is a control system wherein control elements are distributed throughout the system. This is in contrast to the centralized ones, which use a single controller at a central location. In a DCS, a hierarchy of controllers is connected by communication networks for information/data transmission. The advantages of the DCS architecture, such as reduction of system weight, higher reliability, modularity, and less low life cost, merit increasing attention from industrial companies and engineers.

Conventional gas turbine engine control systems are designed as a centralized architecture (which called as Full Authority Digital Engine Control, FADEC) to protect the control elements from the extreme environment [1]. While, with the increasingly development of sophisticated electronics with higher reliability in high temperature environment, the requirements of increased performance, more convenient operation, reduction of design, and maintenance cost make the control system to use a more effective architecture. Thus, the distributed engine control (DEC) architecture came into being [2, 3].

Due to the distributed architecture, the sensors and controllers are connected by the communication networks, as well as between the controllers and the actuators. DEC seeks to advance the state of the art in gas turbine engine control systems by using a digital communication network with a more robust network. This will lead to the development of gas turbine engine control systems with greater extensibility and higher capacity for upgrades. DEC is extensively studied in [2, 4–7] and the references therein.

The DEC architecture can be viewed as an NCCS. For example, the GE T700 turboshaft engine is a two-spool gas turbine engine consisting of a gas generator and a free power turbine [8, 9], and the power turbine is connected to the rotor system by a shaft and a gear box. Conventionally, the power turbine can be considered as a part of the rotor system [10]. The input of the rotor system is the gas generator's output and shaft torque; therefore, the whole turboshaft engine system combined with control systems can be reviewed as a cascade control system (CCS) [11].

As for the DEC using the communication networks to close the control loop, there are fundamental factors to affect the DEC system. They include network-induced time delay, packet dropouts, and bandwidth constraints [12, 13]. Therefore, to guarantee the desired performance and to ensure stability, the control system should be robust to these factors. The network-induced time delay in NCCSs occurs when the sensors, controllers and actuators transfer information/data through the networks, and it can degrade the performance of the control systems and even can destabilize the system [14]. Since the network-induced time delay is unavoidable in the NCCSs, the existing literature, such as [15–18] and the references therein, has discussed the time delay, and many useful approaches have been proposed and even applied to the industrial systems see [19–21] and the references therein.

However, few papers have discussed the DEC robust control in gas turbine engine control systems. For example, Belapurkar et al. [22] analyzed the stability of set-point controller for partially DEC systems with time delays by using LQR method. Yedavalli et al. [13] discussed the stability of DEC systems under communication packet dropouts. Merrill et al. [2] provided a DEC design approach based on quadratic invariance optimal control theory to the control performance of various types of decentralized network configurations.

This paper is concerned with the problem of H_∞ controller design for gas turbine engine distributed control by using state feedback control in the form of NCCSs with packet dropouts. The rest of the paper is organized as follows. In Section 2, the architecture of distributed engine control system is thoroughly described, and the state feedback control problem is formed. H_∞ state feedback controllers are designed based on Lyapunov stability theory and LMI approach in Section 3. A numerical simulation example is presented in Section 4 to illustrate the effectiveness of the approach. Conclusion will be found in Section 5.

Notation. Throughout the paper, the superscripts "T" and "(-1)" represent matrix transposition and matrix inverse, respectively. $\|\cdot\|$ refers to the Euclidean vector norm, and $E[\cdot]$ stands for the mathematical expectation operator with respect to the given probability measure $\mathscr{P}$. In symmetric block matrices or long matrix expressions, an asterisk $(*)$ represented a term that is induced by symmetry. diag$\{\cdot\}$ stands for a block-diagonal matrix.

2. Problem Formulation

2.1. DEC System Architecture Description. This study utilized a GE T700 turboshaft engine. Figure 1 shows the simplified diagram. The inputs to the gas generator were the power turbine speed set value, N_P, and the fuel flow rate, W_F. The outputs were the gas generator speed, N_G, engine torque transmitted by the power turbine shaft, Q_S, compressor static discharge pressure, P_{S3}, and power turbine inlet temperature, T_{45}. The controller design process begins with a linearized, state-space model of the system. Figure 2 shows the simplified model in this case.

Control laws essentially work to maintain N_P, constant at the set point by modulating W_F. The control accomplishes this by scheduling a nominal N_G speed as a function of XCPC, T_1, and P_1. The control trims this

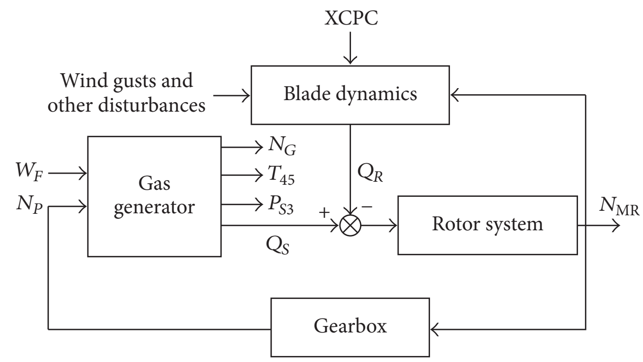

Figure 1: Block diagram of the open-loop gas generator/rotor system.

N_G demand to isochronously adjust N_P to N_P set input. PLA position limits the maximum permissible N_G, while the control further limits the maximum T_{45}. The control limits the N_G acceleration/deceleration rate as a function of N_G scheduled W_F/P_{S3} limit. The DEC discussed herein has one network, which is inserted in the gas generator controller and the gas generator. Figure 3 shows the architecture. The abovementioned description illustrates that the GE T700 control structure is a cascade control structure, wherein the desired primary process output can only be controlled by controlling the secondary control process output.

Primary Plant. The state-space representation of the rotor system is provided by the following equation [9, 23]:

$$\underbrace{\begin{bmatrix} \dot{N}_P \\ \dot{N}_{MR} \\ \dot{Q}_{MR} \end{bmatrix}}_{\dot{x}_1(t)} = \underbrace{\begin{bmatrix} 0 & 0 & -\dfrac{1}{J_T} \\ 0 & -\dfrac{\mathrm{DAM}}{J_{MR}} & \dfrac{1}{J_{MR}} \\ \mathrm{KMR} & \dfrac{\mathrm{DMR}\cdot\mathrm{DAM}}{J_{MR}} - \mathrm{KMR} & -\dfrac{\mathrm{DMR}}{J_T} - \dfrac{\mathrm{DMR}}{J_{MR}} \end{bmatrix}}_{\mathscr{A}_1} \underbrace{\begin{bmatrix} N_P \\ N_{MR} \\ Q_{MR} \end{bmatrix}}_{x_1(t)} + \underbrace{\begin{bmatrix} \dfrac{2}{J_T} \\ 0 \\ \dfrac{2\cdot\mathrm{DMR}}{J_T} \end{bmatrix}}_{\mathscr{B}_1} \underbrace{Q_S}_{y_2(t)},$$

$$\underbrace{N_P}_{y_1(t)} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{C_1} \begin{bmatrix} N_P \\ N_{MR} \\ Q_{MR} \end{bmatrix}. \tag{1}$$

Secondary Plant. The continuous-time linear model of the gas generator is shown as follows:

$$\underbrace{\begin{bmatrix}\dot{N}_G\\ \dot{Q}_S\\ \dot{T}_{45}\\ \dot{P}_{S3}\\ \dot{N}_P\end{bmatrix}}_{\dot{x}_2(t)} = \underbrace{\begin{bmatrix}\dfrac{1}{J_G}\cdot\dfrac{\delta Q_G}{\delta N_G} & 0 & 0 & 0 & 0\\ \dfrac{2\cdot \mathrm{DMR}}{J_T}\cdot\dfrac{\delta Q_P}{\delta N_G} & 0 & 0 & 0 & \dfrac{2\cdot \mathrm{DMR}}{J_T}\cdot\dfrac{\delta Q_P}{\delta N_P}\\ \dfrac{\delta T_{45}}{\delta N_G} & 0 & 0 & 0 & 0\\ \dfrac{\delta P_{S3}}{\delta N_G} & 0 & 0 & 0 & 0\\ \dfrac{2}{J_T}\cdot\dfrac{\delta Q_P}{\delta N_G} & -\dfrac{1}{J_T} & 0 & 0 & \dfrac{2}{J_T}\cdot\dfrac{\delta Q_P}{\delta N_P}\end{bmatrix}}_{\mathscr{A}_2}\underbrace{\begin{bmatrix}N_G\\ Q_S\\ T_{45}\\ P_{S3}\\ N_P\end{bmatrix}}_{x_2(t)} + \underbrace{\begin{bmatrix}\dfrac{1}{J_G}\cdot\dfrac{\delta Q_G}{\delta W_F}\\ \dfrac{2\cdot \mathrm{DMR}}{J_T}\cdot\dfrac{\delta Q_P}{\delta W_F}\\ \dfrac{\delta T_{45}}{\delta W_F}\\ \dfrac{\delta P_{S3}}{\delta W_F}\\ \dfrac{2}{J_T}\cdot\dfrac{\delta Q_P}{\delta W_F}\end{bmatrix}}_{\mathscr{B}_2}\underbrace{W_F}_{u(t)} + \mathscr{B}_3 w(t)$$

$$\underbrace{Q_S}_{y_2(t)} = \underbrace{\begin{bmatrix}0 & 1 & 0 & 0 & 0\end{bmatrix}}_{C_2}\begin{bmatrix}N_G\\ Q_S\\ T_{45}\\ P_{S3}\\ N_P\end{bmatrix}, \tag{2}$$

where $w(t)$ is exogenous process white noise signal belonging to $l_2[0,\infty)$ and the noise parameters matrix $\mathscr{B}_3$ should be used as design parameters to achieve desirable system frequency response characteristics [9].

The following assumptions are partially taken from [24, 25]:

(a) The controllers are event-driven. The primary controller computes the values and sends them to the secondary controller after obtaining the latest samples of the primary plant outputs. The secondary controller then computes the control command and sends it to the actuator as soon as it receives the latest samples of the secondary plant and the control output of the primary plant controller through a common network.

(b) The actuator is time-driven. In other words, the actuator actuates the plants once it receives the control command. The actuator will then use the previous value by zero-order-hold to precede the secondary process in case of packet loss.

(c) The sensors are time-driven; that is, they periodically sample the outputs and send them to the corresponding controllers.

(d) The data packet transmitted from the controller to the plant may be delayed. The delay is assumed to be a fixed one and less than a sampling period h (i.e., $\tau_k \in [0, h]$).

(e) The data packet is assumed to be transmitted between the primary and secondary controllers in a single packet without any loss. However, the data packet transmitted between the secondary controller and the actuator may be delayed or may meet a possible failure in a random manner.

2.2. State Feedback Control of DEC System. By considering the network-induced delay τ_k, the controllers are event-driven, the actuator is time-driven, and the engine receives the piece-wised control input is given by

$$u(t) = \begin{cases}\tilde{u}_2(k-1), & kh \le t < kh + \tau_k,\\ \tilde{u}_2(k), & kh + \tau_k \le t < (k+1)h,\end{cases} \tag{3}$$

$$\tilde{u}_2(k) = \begin{cases}\tilde{u}_2(k-1), & \text{if } u_2(k) \text{ lost during transmission,}\\ u_2(k), & \text{if } u_2(k) \text{ transmitted successfully;}\end{cases} \tag{4}$$

that is, the actuator receives the signal $u_2(k)$ if the data is transmitted successfully; otherwise, the previous value will be used in the actuator by zero-order-hold, where $u_2(k)$ is the control output of the secondary controller.

Since the actuator is time-driven, the packet loss may happen in a random manner. Then, $\tilde{u}_2(k)$ can be rewritten by [25, 26]

$$\tilde{u}_2(k) = \lambda(k)\,u_2(k) + (1-\lambda(k))\,\tilde{u}_2(k-1), \tag{5}$$

where $\lambda(k)$ is a Bernoulli distributed stochastic variable taking the value 0 or 1. $\lambda(k) = 1$ represents the successful state transmission of the delayed packet and $\lambda(k) = 0$ describes the complete packet loss. It assumed that $\lambda(k)$ satisfies the Bernoulli distribution [27]:

$$\begin{aligned}\operatorname{Prob}\{\lambda(k) = 1\} &= \alpha,\\ \operatorname{Prob}\{\lambda(k) = 0\} &= 1-\alpha,\end{aligned} \tag{6}$$

where α is a positive scalar, $E[\lambda(k) - \alpha] = 0$, and $E[(\lambda(k) - \alpha)^2] = \alpha(1-\alpha)$.

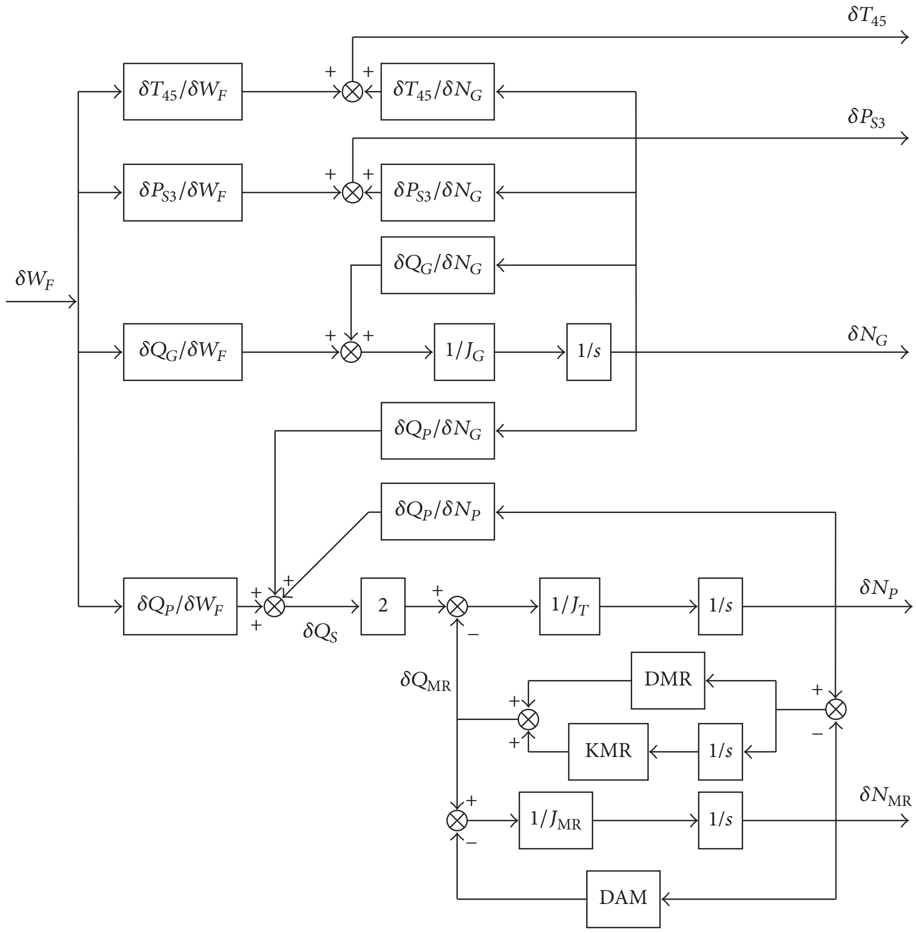

Figure 2: Block diagram of the simplified linearized gas generator and rotor system.

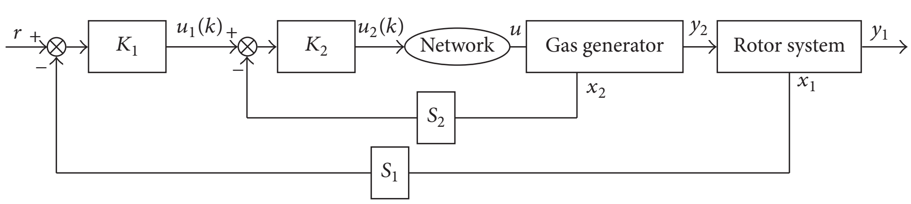

Figure 3: Block diagram of the NCCS model.

Considering the system reference input $N_{\mathrm{Pr}} = 0$, the static state feedback controller is utilized by a discrete-time form:

$$u_2(k) = u_1(k) + K_2(\alpha)\, x_2(k), \tag{7}$$

where $K_2(\alpha)$ is the probability-dependent (packet dropouts probability) state feedback gain matrix given by $K_2(\alpha) = K_{21} + \alpha K_{22}$ and K_1 is

$$u_1(k) = K_1 x_1(k), \tag{8}$$

where $x_1(k)$ is the state vector of rotor system in discrete-time form and K_1 is the state feedback gain.

By using (3), the rotor system and engine with sampling period, $[kh, (k+1)h]$, are discretized to

$$\begin{aligned} x_1(k+1) &= A_1 x_1(k) + B_1 y_2(k), \\ y_1(k) &= C_1 x_1(k), \end{aligned} \tag{9}$$

where $A_1 = e^{\mathscr{A}_1 h}$, $B_1 = \int_0^h e^{\mathscr{A}_1 s} ds \mathscr{B}_1$, and

$$\begin{aligned} x_2(k+1) &= A_2 x_2(k) + B_{21}\tilde{u}_2(k-1) + B_{22}\tilde{u}_2(k) \\ &\quad + B_3 w(k), \\ y_2(k) &= C_2 x_2(k), \end{aligned} \tag{10}$$

where $A_2 = e^{\mathscr{A}_2 h}$, $B_{21} = \int_{h-\tau_k}^{h} e^{\mathscr{A}_2 s} ds \mathscr{B}_2$, $B_{22} = \int_0^{h-\tau_k} e^{\mathscr{A}_2 s} ds \mathscr{B}_2$, and $B_3 = \int_0^h e^{\mathscr{A}_2 s} ds \mathscr{B}_3$. Then, considering the random packet loss, combined (5) and (7) with (10), (10) becomes

$$\begin{aligned} x_2(k+1) = {} & A_2 x_2(k) \\ & + \left(B_{21} - (1-\lambda(k)) B_{22}\right) \tilde{u}_2(k-1) \\ & + \lambda(k) B_{22} \left(K_1 x_1(k) + K_2(\alpha) x_2(k)\right) \\ & + B_3 w(k), \\ y_2(k) = {} & C_2(k) x_2(k). \end{aligned} \tag{11}$$

Thus, the discretized system can be further expanded as

$$\begin{aligned} & x_2(k+1) \\ & \quad = \left(A_2 + \alpha B_{22} K_2(\alpha)\right) x_2(k) \\ & \qquad + (\lambda(k) - \alpha) B_{22} \left(K_1 x_1(k) + K_2(\alpha) x_2(k)\right) \\ & \qquad + \alpha B_{22} K_1 x_1(k) + \left(B_{21} + \left(1 - \alpha B_{22}\right)\right) \tilde{u}_2(k-1) \\ & \qquad + (\alpha - \lambda(k)) B_{22} \tilde{u}_2(k-1) + B_3 w(k), \\ & y_2(k) = C_2(k) x_2(k). \end{aligned} \tag{12}$$

Since the goal of this paper is to design the state controllers to regulate the power turbine speed in presence of disturbances, the output of the closed-loop is determined by $y_1(k)$, and the input is exogenous disturbance $w(k)$. Observing (9) and (12), $x_1(k)$, $x_2(k)$, and $\tilde{u}_2(k-1)$ are chosen as the closed-loop state vectors. Therefore, the closed-loop state-space form is given by

$$\begin{aligned} x(k+1) = {} & (A(\alpha) + (\lambda(k) - \alpha) B(\alpha)) x(k) \\ & + D w(k), \\ y(k) = {} & C x(k), \end{aligned} \tag{13}$$

where $A(\alpha)$, $B(\alpha)$, C, and D can be seen in

$$\begin{aligned} & A(\alpha) \\ & = \begin{bmatrix} A_1 & B_1 C_2 & 0 \\ \alpha B_{22} K_1 & A_2 + \alpha B_{22} K_2(\alpha) & B_{21} + (1-\alpha) B_{22} \\ \alpha K_1 & \alpha K_2(\alpha) & (1-\alpha) I \end{bmatrix}, \\ & B(\alpha) = \begin{bmatrix} 0 & 0 & 0 \\ B_{22} K_1 & B_{22} K_2(\alpha) & -B_{22} \\ K_1 & K_2(\alpha) & -I \end{bmatrix}, \\ & C = \begin{bmatrix} C_1^T \\ 0 \\ 0 \end{bmatrix}^T, \\ & D = \begin{bmatrix} 0 \\ B_3 \\ 0 \end{bmatrix}. \end{aligned} \tag{14}$$

3. Main Results

3.1. System Performance Requirement. In this paper, the goal is to design controllers (7) and (8) for the turboshaft engine NCCS, such that, in the presence of random packet losses, the closed-loop system (13) is stable, and the H_∞ performance constraint is satisfied [28]

$$\sum_{k=0}^{\infty} E\left[\|y(k)\|^2\right] < \gamma^2 \sum_{k=0}^{\infty} E\left[\|w(k)\|^2\right] \tag{15}$$

for all nonzero $w(k)$, where $\gamma > 0$ is a prescribed scalar.

3.2. Controller Design

Lemma 1 (Schur complement). *Given constant matrices Ω_1, Ω_2, and Ω_3, where $\Omega_1 = \Omega_1^T$ and $\Omega_2 = \Omega_2^T > 0$, then $\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0$ if and only if*

$$\begin{gathered} \begin{bmatrix} \Omega_1 & \Omega_3^T \\ \Omega_3 & -\Omega_2 \end{bmatrix} < 0, \\ \text{or} \begin{bmatrix} -\Omega_2 & \Omega_3 \\ \Omega_3^T & \Omega_1 \end{bmatrix} < 0. \end{gathered} \tag{16}$$

Theorem 2. *Let the positive scalar α be given. The closed-loop system (13) is stable with an H_∞ performance index γ, if there exist symmetric positive definite matrices P and matrices R_1, R_2, R_3, $\mathscr{K}_1$, $\mathscr{K}_{21}$, and $\mathscr{K}_{22}$ such that the following LMI holds:*

$$\begin{bmatrix} O_1 & 0 & O_2 & O_3 & O_4 \\ * & -\gamma^2 I & O_5 & 0 & 0 \\ * & * & O_6 & 0 & 0 \\ * & * & * & O_6 & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \tag{17}$$

where

$$O_1 = P - \operatorname{diag}\left\{R_1 + R_1^T, R_2 + R_2^T, R_3 + R_3^T\right\},$$

$$O_2 = \begin{bmatrix} R_1 A_1^T & \alpha \mathscr{K}_1^T B_{22}^T & \alpha \mathscr{K}_1^T \\ R_2^T C_2^T B_1^T & R_2^T A_2^T + \alpha \mathscr{K}_2^T(\alpha) B_{22}^T & \mathscr{K}_2^T(\alpha) \\ 0 & R_3^T B_{21}^T + (1-\alpha) R_3^T B_{22}^T & (1-\alpha) R_3^T \end{bmatrix},$$

$$O_3 = \begin{bmatrix} 0 & \mathscr{K}_1^T B_{22}^T & \mathscr{K}_1^T \\ 0 & \mathscr{K}_2^T(\alpha) B_{22}^T & \mathscr{K}_2^T(\alpha) \\ 0 & -R_3^T B_{22}^T & -R_3^T \end{bmatrix},$$

$$O_4 = \begin{bmatrix} R_1^T C_1^T \\ 0 \\ 0 \end{bmatrix},$$

$$O_5 = \begin{bmatrix} 0 \\ B_3 \\ 0 \end{bmatrix}^T, \tag{18}$$

$$O_6 = -P,$$

$$\mathscr{K}_2(\alpha) = \mathscr{K}_{21} + \alpha \mathscr{K}_{22},$$

and the state feedback gain matrices can be gained by

$$K_1 = \mathscr{K}_1 R_1^{-1},$$
$$K_{21} = \mathscr{K}_{21} R_2^{-1}, \tag{19}$$
$$K_{22} = \mathscr{K}_{22} R_3^{-1}.$$

Proof. In order to conclude the controller design conditions, the following Lyapunov function can be defined:

$$V(k) = x^T(k) P^{-1} x(k). \tag{20}$$

Now, for any nonzero $w(k)$,

$$\begin{aligned} &E[V(k+1)] - E[V(k)] + E\left[y(k)^T y(k)\right] \\ &\quad - \gamma^2 E\left[w(k)^T w(k)\right] = E\left[(A(\alpha)\right. \\ &\quad + (\lambda(k) - \alpha) B(\alpha) x(k+1) + Dw(k))^T P^{-1} (A(\alpha) \\ &\quad + (\lambda(k) - \alpha) B(\alpha) x(k+1) + Dw(k)) - x(k)^T \\ &\quad \left.\cdot P^{-1} x(k) + y(k)^T y(k) - \gamma^2 w(k)^T w(k)\right] \\ &\quad \le E\left[(A(\alpha x(k))^T P^{-1} (A(\alpha x(k)) + \alpha(1-\alpha)\right. \\ &\quad \cdot x^T(k) B^T(\alpha) P^{-1} B(\alpha) x(k) - x(k)^T P^{-1} x(k) \\ &\quad \left. + y(k)^T y(k) - \gamma^2 w(k)^T w(k)\right] \\ &\quad = E\left[\begin{bmatrix} x(k) \\ w(k) \end{bmatrix}^T \cdot O(\alpha) \cdot \begin{bmatrix} x(k) \\ w(k) \end{bmatrix}\right], \end{aligned} \tag{21}$$

where $O(\alpha)$ can be seen in

$$O(\alpha) = \begin{bmatrix} A(\alpha)^T P^{-1} A(\alpha) + \alpha(1-\alpha) B(\alpha)^T P^{-1} B(\alpha) + C^T C - P^{-1} & A(\alpha)^T P^{-1} D \\ D^T P^{-1} A(\alpha) & D^T P^{-1} D - \gamma^2 I \end{bmatrix}. \tag{22}$$

and $O(\alpha)$ can be rewritten in

$$\begin{aligned} O(\alpha) = &\left(\begin{bmatrix} -P^{-1} & 0 \\ * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} A(\alpha)^T \\ D^T \end{bmatrix} \cdot P^{-1}\right. \\ &\cdot \begin{bmatrix} A(\alpha) & D \end{bmatrix} + \alpha(1-\alpha) \begin{bmatrix} B(\alpha)^T \\ 0 \end{bmatrix} \cdot P^{-1} \\ &\left.\cdot \begin{bmatrix} B(\alpha) & 0 \end{bmatrix} + \begin{bmatrix} C^T \\ 0 \end{bmatrix} \cdot \begin{bmatrix} C & 0 \end{bmatrix}\right). \end{aligned} \tag{23}$$

Thus, by applying Lemma 1, (24) can be obtained

$$O(\alpha) = \begin{bmatrix} -P^{-1} & 0 & A(\alpha)^T & \alpha(1-\alpha) B(\alpha)^T & C^T \\ * & -\gamma^2 I & D^T & 0 & 0 \\ * & * & -P & 0 & 0 \\ * & * & * & -P & 0 \\ * & * & * & * & -I \end{bmatrix}. \tag{24}$$

Now, the goal is to prove $O(\alpha) < 0$. By means of the partition matrices $R = \operatorname{diag}\{R_1, R_2, R_3\}$, P, the values of $A(\alpha)$,

$B(\alpha)$, C, D, and $\mathcal{K}_1$, $\mathcal{K}_{21}$, and $\mathcal{K}_{22}$ in LMI (17), the following inequality (25) can be gotten:

$$\begin{bmatrix} P - R + R^T & 0 & R^T A(\alpha)^T & \alpha(1-\alpha) R^T B(\alpha)^T & R^T C^T \\ * & -\gamma^2 I & D^T & 0 & 0 \\ * & * & -P & 0 & 0 \\ * & * & * & -P & 0 \\ * & * & * & * & -I \end{bmatrix} < 0. \tag{25}$$

If (25) holds, then by using $P - 2R \geq -R^T P^{-1} R$, the following inequality (26) can be gained:

$$\begin{bmatrix} -R^T P^{-1} R & 0 & R^T A(\alpha)^T & \alpha(1-\alpha) R^T B(\alpha)^T & R^T C^T \\ * & -\gamma^2 I & D^T & 0 & 0 \\ * & * & -P & 0 & 0 \\ * & * & * & -P & 0 \\ * & * & * & * & -I \end{bmatrix} < 0. \tag{26}$$

Equation (17) can then be obtained by pre- and postmultiplying (26) by $\mathrm{diag}(R^{-1}, I, I, I, I)$. Therefore, for zero to ∞ with respect to k, it yields:

$$\begin{aligned} \sum_{k=0}^{\infty} E\left[\|y(k)\|^2\right] &< \gamma^2 \sum_{k=0}^{\infty} E\left[\|w(k)\|^2\right] + E[V(0)] \\ &\quad - E[V(\infty)]. \end{aligned} \tag{27}$$

Since $\left[\begin{smallmatrix} x(0) \\ w(0) \end{smallmatrix}\right] = \left[\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right]$, the closed-loop system (13) is stable, and it satisfies (15). □

Algorithm for the Controllers Design

(a) The continuous closed-loop system parameters are derived based on Figure 2.

(b) The continuous system parameters are discretized.

(c) The convex optimization problem (17) is solved to obtain the feasible solutions in terms of positive definite matrices P, nonsingular slack matrices R_i, ($i = 1, 2, 3$), and matrices $\mathcal{K}_1$, $\mathcal{K}_{21}$, $\mathcal{K}_{22}$, and γ.

(d) The controller parameters K_1, K_{21}, and K_{22} are derived based on Theorem 2.

(e) Stop.

4. Simulation Examples

This section presents the effectiveness evaluation of the proposed method under simulations in the GE T700 turboshaft gas turbine engine DEC control systems. The model of the engine is based on partial derivatives calculated from an accurate nonlinear model [1]. The rotor system and the gas generator models in continuous time form are provided:

$$\begin{aligned}
\begin{bmatrix} \dot{N}_P \\ \dot{N}_{MR} \\ \dot{Q}_{MR} \end{bmatrix} &= \begin{bmatrix} 0 & 0 & -285.7143 \\ 0 & -0.4533 & 9.0662 \\ 5.2650 & -5.2131 & -42.5958 \end{bmatrix} \begin{bmatrix} N_P \\ N_{MR} \\ Q_{MR} \end{bmatrix} + \begin{bmatrix} 571.4286 \\ 0 \\ 82.5714 \end{bmatrix} Q_S, \\
N_P &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} N_P \\ N_{MR} \\ Q_{MR} \end{bmatrix}, \\
\begin{bmatrix} \dot{N}_G \\ \dot{Q}_S \\ \dot{T}_{45} \\ \dot{P}_{S3} \\ \dot{N}_P \end{bmatrix} &= \begin{bmatrix} -126.8 & 27.04 & 12.36 & 22.17 & 16.72 \\ 54.67 & 57.21 & -77.02 & -76.21 & 50.81 \\ -336.6 & 223.3 & -130.7 & -83.32 & 172.1 \\ 161.2 & 2.459 & -21.8 & -63.09 & 1.799 \\ 62.42 & -73.55 & -104.2 & -91.44 & -102.3 \end{bmatrix} \begin{bmatrix} N_G \\ Q_S \\ T_{45} \\ P_{S3} \\ N_P \end{bmatrix} \\
&\quad + \begin{bmatrix} -11.7 \\ 44.24 \\ 53.56 \\ 17.45 \\ 59.35 \end{bmatrix} W_F + \begin{bmatrix} 0.02 \\ 0.02 \\ 0.02 \\ 0.02 \\ 0.02 \end{bmatrix} w, \\
Q_S &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} N_G \\ Q_S \\ T_{45} \\ P_{S3} \\ N_P \end{bmatrix}.
\end{aligned} \tag{28}$$

The coefficients after the discretization are provided as follows:

$$A_1 = \begin{bmatrix} 0.9352 & 0.0640 & -2.2670 \\ 0.0021 & 0.9934 & 0.0718 \\ 0.0418 & -0.0413 & 0.5952 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 4.5703 \\ 0.0362 \\ 0.7847 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.3927 & 0.1572 & -0.0524 & 0.0010 & 0.0772 \\ 0.6985 & 1.0228 & -0.4391 & -0.4080 & 0.0411 \\ -0.5933 & 0.5866 & -0.1759 & -0.8311 & 0.3484 \\ 0.8005 & 0.0546 & -0.0665 & 0.6367 & 0.0187 \\ 0.1142 & -0.8350 & 0.0242 & 0.0769 & 0.0953 \end{bmatrix},$$

$$B_{21} = \begin{bmatrix} -0.0153 \\ 0.1908 \\ 0.3524 \\ 0.0502 \\ 0.1099 \end{bmatrix},$$

$$B_{22} = \begin{bmatrix} 0.0206 \\ 0.0920 \\ 0.2553 \\ 0.0138 \\ -0.1128 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 0.1213 \\ 0.0715 \\ -0.1214 \\ 0.1019 \\ 0.0328 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix}. \tag{29}$$

Simulation 1. Let the packet loss probability value $\alpha = 0.3$, and given the initial conditions as $x_1(0) = \begin{bmatrix} 1 & 0.2 & 0.2 \end{bmatrix}^T$, $x_2(0) = \begin{bmatrix} 0.9000 & 0.4189 & 0.7843 & 0.6498 & 1.0000 \end{bmatrix}^T$, the simulation time is $T = 20$ s, sampling time is $h = 0.01$ s, and assuming that the two network-induced delays are both equivalent to τ_k, which is not longer than the sampling period, $\tau_k = 0.005$ s. The goal of this simulation is to design the controller gains such that the closed-loop system is robustly stable with a disturbance attenuation level $\gamma > 0$. The optimized solution of (17) can be calculated by using the LMI toolbox in MATLAB:

$$P = 10^{-7} \times \begin{bmatrix} -0.4640 & -0.0037 & 0.0128 & -0.0398 & 0.0684 & 0.0015 & -0.0110 & -0.0418 & -0.0011 \\ -0.0037 & -0.5633 & -0.0005 & 0.0001 & 0.0003 & -0.0002 & -0.0006 & -0.0005 & 0.0001 \\ 0.0128 & -0.0005 & -0.6465 & -0.0180 & 0.0233 & 0.0038 & 0.0120 & -0.0035 & -0.0015 \\ -0.0398 & 0.0001 & -0.0180 & -0.6070 & -0.0263 & 0.0130 & 0.0106 & -0.0424 & -0.0142 \\ 0.0684 & 0.0003 & 0.0233 & -0.0263 & -0.7761 & -0.0993 & 0.0161 & 0.1228 & -0.0320 \\ 0.0015 & -0.0002 & 0.0038 & 0.0130 & -0.0993 & -0.5875 & -0.0724 & 0.0404 & -0.0347 \\ -0.0110 & -0.0006 & 0.0120 & 0.0106 & 0.0161 & -0.0724 & -0.5874 & 0.0041 & 0.0364 \\ -0.0418 & -0.0005 & -0.0035 & -0.0424 & 0.1228 & 0.0404 & 0.0041 & -0.5022 & -0.0119 \\ -0.0011 & 0.0001 & -0.0015 & -0.0142 & -0.0320 & -0.0347 & 0.0364 & -0.0119 & -0.5462 \end{bmatrix},$$

$$R_1 = 10^{-5} \times \begin{bmatrix} 0.2236 & -0.0052 & 0.0866 \\ -0.0052 & 0.0172 & -0.0017 \\ 0.0866 & -0.0017 & 0.0363 \end{bmatrix},$$

$$R_2 = 10^{-4} \times \begin{bmatrix} 0.0188 & 0.0001 & 0.0485 & -0.0217 & 0.0007 \\ 0.0001 & 0.0001 & 0.0004 & -0.0000 & 0.0001 \\ 0.0485 & 0.0004 & 0.1304 & -0.0584 & 0.0030 \\ -0.0217 & -0.0000 & -0.0584 & 0.0278 & 0.0040 \\ 0.0007 & 0.0001 & 0.0030 & 0.0040 & 0.0237 \end{bmatrix},$$

Table 1: Optimized γ_{opt} and control parameters for various values of α.

α	K_1	K_{21} & K_{22}	γ_{opt}
0	$[0.0009\ \ -0.0000\ \ -0.0016]$	$[0.0421\ \ 0.2644\ \ -0.0754\ \ -0.1307\ \ 0.0297]$ $[0\ \ 0\ \ 0\ \ 0\ \ 0]$	0.2999
0.3	$[-0.0016\ \ -0.0001\ \ 0.0035]$	$[-0.0380\ \ -0.2780\ \ 0.0635\ \ 0.1037\ \ -0.0242]$ $[0.0287\ \ -0.0008\ \ 0.0847\ \ -0.0323\ \ -0.0080]$	0.3035
0.7	$[-0.0047\ \ -0.0003\ \ 0.0102]$	$[-0.1013\ \ -0.6667\ \ 0.1705\ \ 0.2835\ \ -0.0654]$ $[0.0741\ \ -0.0013\ \ 0.2336\ \ -0.0698\ \ -0.0287]$	0.2970
1.0	$[-0.0075\ \ -0.0004\ \ 0.0162]$	$[-0.1611\ \ -0.9143\ \ 0.2504\ \ 0.4084\ \ -0.0926]$ $[0.0972\ \ -0.0003\ \ 0.3161\ \ -0.0869\ \ -0.0415]$	0.3014

$$\begin{aligned}
R_3 &= 10^{-8} \times 6.8719, \\
K_1 &= [-0.0016\ \ -0.0001\ \ 0.0035], \\
K_{21} &= [-0.0380\ \ -0.2780\ \ 0.0635\ \ 0.1037\ \ -0.0242], \\
K_{22} &= [0.0287\ \ -0.0008\ \ 0.0847\ \ -0.0323\ \ -0.0080], \\
\gamma_{\text{opt}} &= 0.3035.
\end{aligned} \tag{30}$$

Figures 4 and 5 show the responses of the state variables in the closed-loop system under packet dropouts, and the system states converge to zero. Meanwhile, Figure 6 illustrates that the gas generator control loop (inner loop) is much faster than the rotor system control loop (outer loop). Therefore, by Theorem 2, the closed-loop system (13) is robust stable with H_∞ disturbance-rejection-attenuation level γ, and it is noted that the obtained controllers make sure the fast response of inner loop to eliminate disturbances.

Simulation 2. In order to show the effectiveness evaluation of the proposed method under different values of packet loss probability α, the obtained state feedback controller parameters and disturbance attenuation level γ are presented in Table 1. In Table 1, the case $\alpha = 0$ represents the successful transmission, $\alpha \neq 1$ representing the packet dropout, and $\alpha = 1$ denotes the complete loss of transmission case.

Figures 7 and 8 show that the closed-loop system (13) can be stabilized with or without packet dropouts. Figure 9 illustrates that the dynamical behavior of the closed-loop system takes longer to converge to zero. Figures 10 and 11 show the network-induced packet loss responses. Thus, the designed controllers are well suited for the considered turboshaft engine model and work well over the network-induced imperfections and input disturbances.

5. Conclusions

This study considered the novel robust H_∞ distributed engine control problem to guarantee the engine performance with random packet dropouts and disturbances. A distributed control system architecture of a typical turboshaft engine was also described accordingly. This distributed architecture can be transformed into a networked cascade control system. The state feedback controllers were designed to robustly stabilize the closed-loop system under packet loss and disturbances. The sufficient conditions for stability were derived based on the Lyapunov stability and the LMI approach. The controller design problem under consideration is solvable if the LMI was feasible. Simulation examples were provided to show the effectiveness of the approach.

Notations

PLA: Power lever angle (throttle)
N_G: Gas generator speed
N_P: Power turbine speed
N_{MR}: Main rotor blade velocity
Q_{MR}: Rotor torque state
Q_S: Engine shaft torque
XCPC: Collective pitch
P_1: Inlet pressure

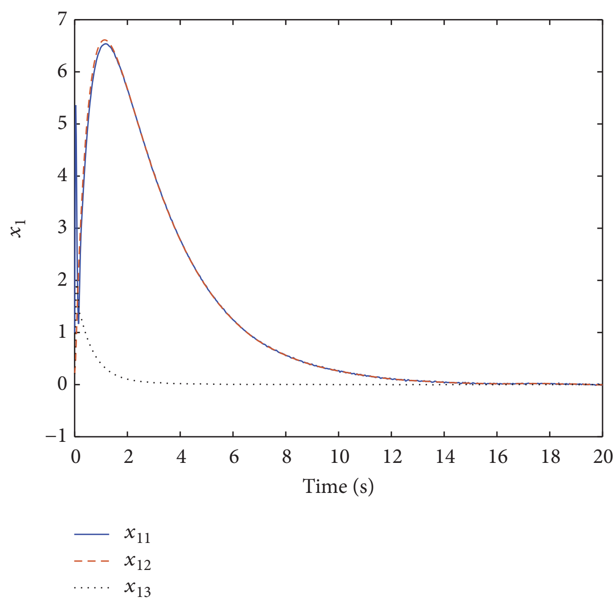

FIGURE 4: Rotor system state response x_1.

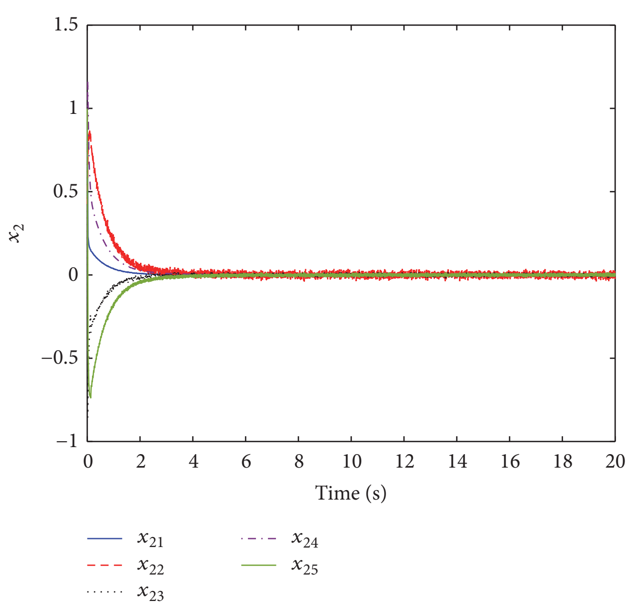

FIGURE 5: Gas generator state response x_2.

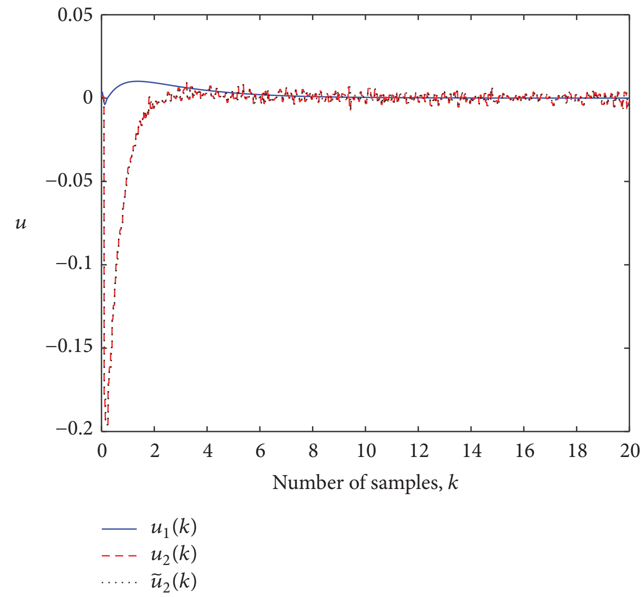

FIGURE 6: Rotor system controller output u.

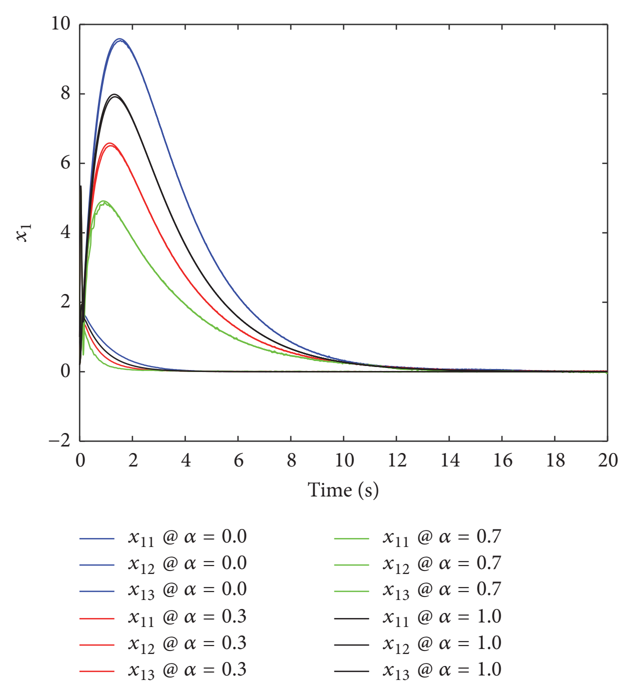

FIGURE 7: Rotor system state x_1 under different α.

P_{S3}: Static pressure at Station 3
T_1: Inlet temperature
T_{45}: Interturbine gas temperature
W_F: Fuel flow
J_G: Power turbine inertia
J_T: Lumped power turbine/dynamometer inertia
J_{MR}: Main rotor blade inertia
KMR: Stiffness of the centrifugal restoring springs
DMR: Lag hinge damping
DAM: Aero damping
r: Reference input
x: Model state vector
y: Model output vector
u: Model input vector
δ: Partial derivatives
ω: Process white noise.

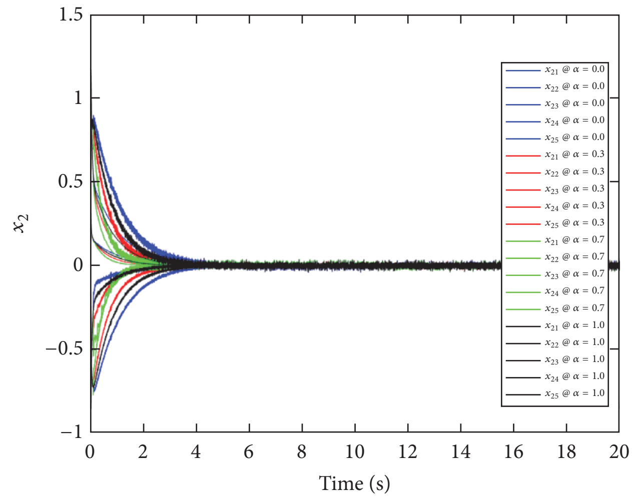

Figure 8: Gas generator state x_2 under different α.

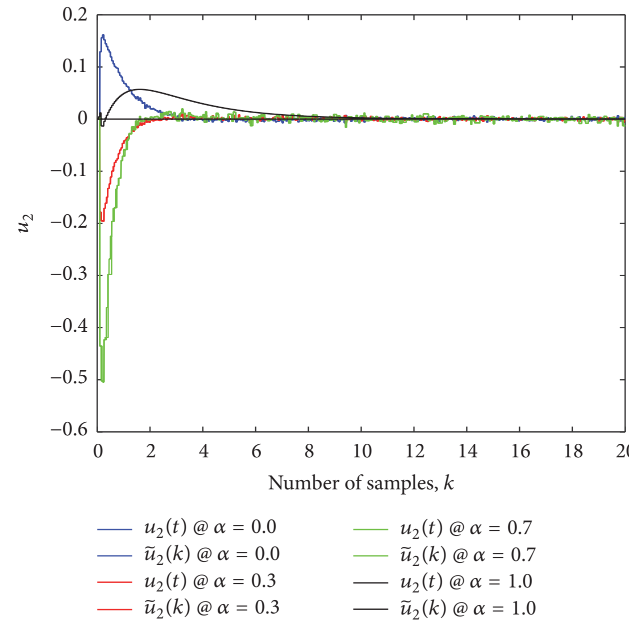

Figure 10: Gas generator controller output u_2 under different α.

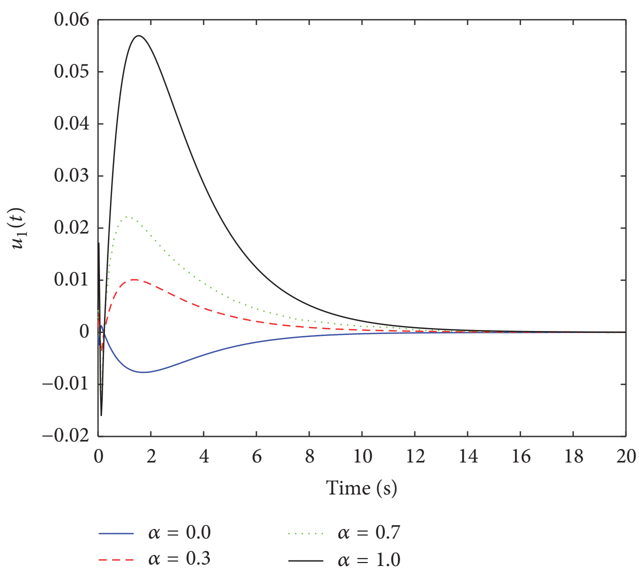

Figure 9: Rotor system controller output u_1 under different α.

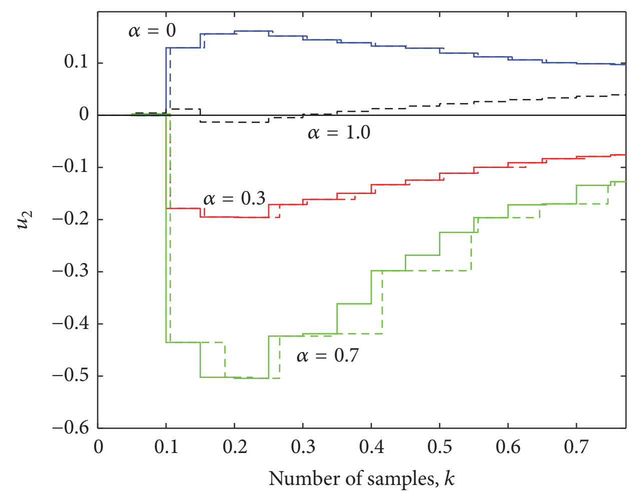

Figure 11: Gas generator controller output u_2 under different α (partial view).

Competing Interests

The authors declare that they have no competing interests.

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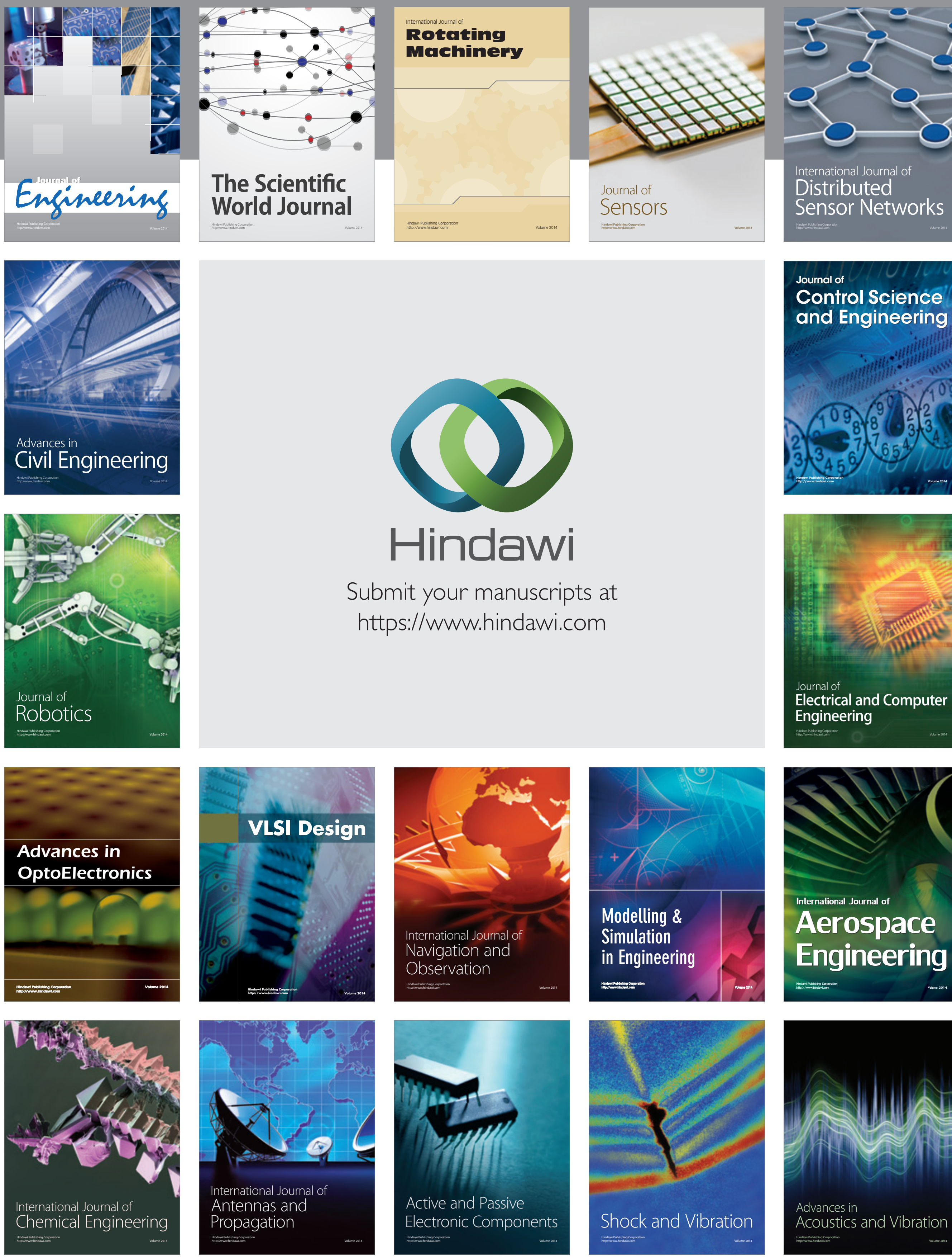
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