Research Article

Improvement of Carrier Phase Tracking Based on a Joint Vector Architecture

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Carrier phase measurements are essential to high precision positioning. Usually, the carrier phase measurements are generated from the phase lock loop in a conventional Global Navigation Satellite System (GNSS) receiver. However there is a dilemma problem to the design of the loop parameters in a conventional tracking loop. To address this problem and improve the carrier phase tracking sensitivity, a carrier phase tracking method based on a joint vector architecture is proposed. The joint vector architecture contains a common loop based on extended Kalman filter to track the common dynamics of the different channels and the individual loops for each channel to track the satellite specific dynamics. The transfer function model of the proposed architecture is derived. The proposed method and the conventional scalar carrier phase tracking are tested with a high quality simulator. The test results indicate that carrier phase measurements of satellites start to show cycle slips using the proposed method when carrier noise ratio is equal to and below 15 dB-Hz instead of 21 dB-Hz with using the conventional phase tracking loop. Since the joint vector based tracking loops jointly process the signals of all available satellites, the potential interchannel influence between different satellites is also investigated.

1. Introduction

Carrier phase is tracked by the phase lock loop (PLL) in a conventional GNSS receiver. Compared to frequency lock loop (FLL) and code lock loop (DLL), PLL is the weakest component which is sensitive to the noise and dynamics. However, to have usable carrier phase measurements, carrier phase tracking must be maintained. There are three error sources in carrier phase tracking [1]. One is the thermal noise. The second one is the oscillator noise which includes the vibration-induced noise, Allan Deviation oscillator phase noise, and g -sensitivity stress error. The third one is the dynamic stress error. In this paper we will focus on the analysis of the carrier tracking loop performance with oscillator noise and the thermal noise. To design a tracking loop, the loop bandwidth is a key parameter. There is also a dilemma situation to deal with in the bandwidth design [2]. The bandwidth, for instance, should be narrowed in order to decrease the thermal noise error but narrowing the bandwidth will increase the oscillator noise error and dynamic stress error.

With those challenges in mind, this paper will design a tracking architecture to address the problems.

Except the thermal noise, the oscillator and dynamics stress are dominated by the receiver clock and dynamics, and the satellite dynamics will be compensated for using ephemeris. The receiver oscillator and dynamic stress are common for different channels. If the common error can be estimated before the correlator for individual channels, the bandwidth of channel loop can be reduced without increasing the common error. One idea to handle these common errors is to use vector tracking. Leimer and Kohli [3] proposed a phase noise mitigation method which first used all available satellites to estimate the oscillator phase noise and then remove this oscillator error before the correlator. However they did not consider the common dynamics influence. Zhodzishsky et al. [4] proposed a Co-Op architecture for phase tracking based on the least square (LS) method. The Co-Op tracking architecture contains a common high-bandwidth loop besides the conventional individual loop. However LS method is usually based on fixed bandwidth

The TF model of the common loop based on EKF can be written as follows [9]:

$$\vec{f} = \left(I_{n \times n} + \frac{G_{n \times 4} K_{4 \times n}}{s} \right)^{-1} G_{n \times 4} K_{4 \times n} (\vec{\varphi} + \vec{n} - \vec{\theta}). \quad (1)$$

For all channels, the TF model is

$$\vec{\theta} = [F(s) (\vec{\varphi} + \vec{n} - \vec{\theta}) + \vec{f}] \frac{1}{s} + \theta_c I_{n \times 1}. \quad (2)$$

Substitute (1) to (2) and get the TF model for all channels:

$$\vec{\theta} = \left(F(s) I_{n \times n} + \left(I_{n \times n} + \frac{G_{n \times 4} K_{4 \times n}}{s} \right)^{-1} G_{n \times 4} K_{4 \times n} \right) \cdot (\vec{\varphi} + \vec{n} - \vec{\theta}) \frac{1}{s} + \theta_c I_{n \times 1}. \quad (3)$$

For an individual channel, the TF model is

$$\theta_i = [F(s) (\varphi_i + n_i - \theta_i) + f_i] \frac{1}{s} + \theta_c, \quad (4)$$

where f_i is the output from the common filter for the i th channel which is calculated based on all available channels in the EKF as described in (1).

From (3), we can get $\vec{\theta}$ as

$$\begin{aligned} \vec{\theta} = & \left[I_{n \times n} + \frac{1}{s} \left(F(s) I_{n \times n} \right. \right. \\ & + \left. \left. \left(I_{n \times n} + \frac{G_{n \times 4} K_{4 \times n}}{s} \right)^{-1} G_{n \times 4} K_{4 \times n} \right) \right]^{-1} \left\{ \left[F(s) I_{n \times n} \right. \right. \\ & + \left. \left. \left(I_{n \times n} + \frac{G_{n \times 4} K_{4 \times n}}{s} \right)^{-1} G_{n \times 4} K_{4 \times n} \right] (\vec{\varphi} + \vec{n}) \right. \\ & \left. + \theta_c I_{n \times 1} \right\}, \end{aligned} \quad (5)$$

where $\vec{\varphi}$ is the input signal carrier phase vector including all channels: $\vec{\varphi} = [\varphi_1 \ \varphi_2 \ \cdots \ \varphi_n]^T$; $\vec{\theta}$ is a replica of the local signal carrier phase vector including all channels: $\vec{\theta} = [\theta_1 \ \theta_2 \ \cdots \ \theta_n]^T$; $\vec{n}$ is the noise vector for all channels: $\vec{n} = [n_1 \ n_2 \ \cdots \ n_n]^T$; $\vec{f}$ is the common loop output vector for all channels: $\vec{f} = [f_1 \ f_2 \ \cdots \ f_n]^T$; $I_{n \times n}$ is an $n \times n$ identity matrix; $K_{4 \times n}$ is the Kalman gain; $G_{n \times 4}$ is the geometry matrix; θ_c is the local oscillator phase noise; n_i is the thermal noise in the i th channel; $F(s)$ is the individual local filter transfer function.

For an individual channel in (4), the output of the NCO, θ_i is not only related to the channel's inputs such as φ_i and n_i , but also related to f_i which is related to all available channels in the EKF. In (5), there are two components, namely, the input signal and thermal noise ($\vec{\varphi} + \vec{n}$) and the oscillator phase

noise θ_c . We therefore can separate (5) into the following two equations:

$$\begin{aligned} \vec{\theta}_s = & \left[I_{n \times n} + \frac{1}{s} \left(F(s) I_{n \times n} \right. \right. \\ & + \left. \left. \left(I_{n \times n} + \frac{G_{n \times 4} K_{4 \times n}}{s} \right)^{-1} G_{n \times 4} K_{4 \times n} \right) \right]^{-1} \left[F(s) I_{n \times n} \right. \\ & + \left. \left. \left(I_{n \times n} + \frac{G_{n \times 4} K_{4 \times n}}{s} \right)^{-1} G_{n \times 4} K_{4 \times n} \right] (\vec{\varphi} + \vec{n}), \end{aligned} \quad (6)$$

$$\begin{aligned} \vec{\theta}_o = & \left[I_{n \times n} + \frac{1}{s} \left(F(s) I_{n \times n} \right. \right. \\ & + \left. \left. \left(I_{n \times n} + \frac{G_{n \times 4} K_{4 \times n}}{s} \right)^{-1} G_{n \times 4} K_{4 \times n} \right) \right]^{-1} \theta_c I_{n \times 1}, \end{aligned} \quad (7)$$

where $\vec{\theta}_s$ is the NCO output related to the input signal including the thermal noise; $\vec{\theta}_o$ is the NCO output related to the oscillator noise.

From (7), it can be seen that the oscillator phase noise is not only processed by the local filter but also by the EKF. If the EKF can estimate the oscillator error which is common to all channels, the design of the local filter $F(s)$ is then not limited by the oscillator noise. Similar to the oscillator noise shown in (6), the input signal and thermal noise are processed jointly by the EKF. The dynamic stress error can be estimated jointly by the joint VPLL. There are four states in the EKF. If the measurements of the EKF are more than four, then the variance of the dynamic stress error and oscillator noise error will be decreased which is similar to a vector delay lock loop (VDLL) [10, 11]. If the error of the dynamic stress and oscillator noise can be estimated more precisely, the bandwidth of the local filter can be decreased which subsequently decreases the thermal noise error. In the optimal state, the dynamic stress error and the oscillator phase noise error could be estimated perfectly by the EKF. This allows the local filter $F(s)$ to be optimized according to the thermal noise which contains mainly channel-based error.

Although the dynamic stress error and oscillator phase noise error can be decreased using the joint VPLL, the joint tracking method will make the errors in one channel spread to other channels. Usually the thermal noise error is derived from the single channel in a conventional receiver. However for the joint VPLL, except the thermal noise error from the channel itself, the thermal noise error in this channel should include the errors generated from the common loop.

As we see in (6) and (7), the transfer function includes a matrix inverse which is related to all tracked satellites; it is very hard to derive the equivalent bandwidth in theory. So we use a simulator to assess the performance of the proposed VPLL. The focus will be on stationary receivers, since this will imply that thermal noise and oscillator phase noise are the main error sources. The dynamics induced by the satellite is calculated and compensated using ephemeris and user

location. In the following section, the implementation of the joint VPLL is described in detail.

2.3. Implementation of the Joint VPLL. Based on the software receiver [12], the joint VPLL architecture is implemented. The individual channel of the joint VPLL is the same with the conventional PLL, although the loop parameters are different. For the EKF of the joint VPLL, the states, system model, and measurement model are all described below.

The state vector of the EKF is $x = [\delta x \ \delta y \ \delta z \ \delta t]^T$ which includes the errors in prediction of the change of the receiver position in the ECEF (Earth-Centered, Earth-Fixed) coordinate systems and the clock bias.

The system model is

$$x_k = Ax_{k-1} + w_{k-1}, \quad (8)$$

where A is the transition matrix: $A = I_{4 \times 4}$ and $I_{4 \times 4}$ is an identity matrix; $w_k = [w_x \ w_y \ w_z \ w_t]^T$ represent the process noises in the three different coordinate directions and the oscillator induced noise, respectively. For the oscillator phase noise, here we consider the Allan deviation oscillator phase noise since the front-end is static. So the variance of the oscillator phase noise with Allan variance parameters can be calculated as follows [13]:

$$\sigma_t^2 = \frac{h_0}{2} \Delta t + \frac{2}{3} \pi^2 h_{-2} \Delta t^3, \quad (9)$$

where h_0 is white frequency noise and h_{-2} is random walk frequency noise; $h_0 = 2 \times 10^{-19}$ and $h_{-2} = 2 \times 10^{-20}$ are for low quality temperature compensated crystal oscillator (TCXO) [13, 14]. Then $\sigma_t^2 = 1.8 \times 10^{-4}$ in units of square meter for coherent integration time as 20 ms. However, the precise Allan variance coefficients of the oscillator used in the test are unknown; we have test different values of σ_t^2 and set $\sigma_t^2 = 10 \times 10^{-4}$ in the following test. For the covariance matrix of the dynamic noise, a value of $1 \times 10^{-12} \text{ m}^2$ for σ_x^2 , σ_y^2 , and σ_z^2 is used in the following test.

The measurement model is

$$z_k = Gx_k + v_k, \quad (10)$$

where $z = [\Delta\theta_1 \ \cdots \ \Delta\theta_N]^T$ is a $N \times 1$ vector which includes N satellite phase error outputs from the phase discriminators; G is the design matrix composed of slowly changing unit vectors dependent on the relative positions of the receiver and the satellites; $v = [v_1 \ \cdots \ v_N]^T$ represent the measurement noise which is assumed to be uncorrelated between different channels. The design matrix G is

$$G = \begin{bmatrix} \alpha_x^1 & \alpha_y^1 & \alpha_z^1 & -1 \\ \vdots & \vdots & \vdots & \vdots \\ \alpha_x^i & \alpha_y^i & \alpha_z^i & -1 \\ \vdots & \vdots & \vdots & \vdots \\ \alpha_x^n & \alpha_y^n & \alpha_z^n & -1 \end{bmatrix}, \quad (11)$$

where α_x^i , α_y^i , and α_z^i are the line of sight unit vectors from the receiver's estimated position to the i th satellite's position. The Kalman gain matrix K in (1) can be calculated as

$$K = P^- G^T (G P^- G^T + R)^{-1}, \quad (12)$$

where P^- is the covariance matrix of the predicted estimation error and R is the covariance of the measurements noise. Since the measurements of different channels are uncorrelated, R is a diagonal matrix shown as follows:

$$R = \begin{bmatrix} \sigma_1^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_n^2 \end{bmatrix}, \quad (13)$$

where σ_i^2 is the carrier phase discriminator output noise variance of the i th satellite. An arctangent is used as discriminator. However the gain of an arctangent discriminator will be quickly decreased with decreasing SNR after the SNR is below 10 dB. The process gain of the discriminator can be compensated as described in Curran [15], although the variance of the discriminator output will be increased when the process gain is adjusted. The variance of the discriminator output can be calculated according to Curran [15] as

$$\sigma_i^2 \approx \frac{(e^{-c_1 \sqrt{\text{SNR}}} + 1) \text{erf}(c_2 \text{SNR})}{\text{SNR}}, \quad (14)$$

where $c_1 = 0.8046$ and $c_2 = 0.3977$; SNR is the signal to noise ratio which can be derived from the CNR. CNR is calculated using the moments method [16].

3. Test and Performance Comparison

3.1. Test Description. A Spirent 8000 simulator is used to test the performance of the joint VPLL and compare to the performance using the conventional PLL method. In this research, we focus on the sensitivity improvement using the joint vector based tracking architecture. To isolate other error sources, the test antenna is put in a static mode during the experiments and the dynamics induced by satellite is to be estimated and compensated using the satellite ephemeris. As a result, the carrier phase error mostly would come from the oscillator and thermal noise. IF data for about 6 minutes are collected from the simulator with a CNR of 47 dB-Hz for all satellite channels. In the experiments, Gaussian white noises are added to the last 5 minutes of the data to adjust the CNR for the purpose of assessing the performance using different tracking methods. Except the original data set with 47 dB-Hz, we generate 17 data sets which show that CNR are from 45 dB-Hz down to 13 dB-Hz with 2 dB intervals.

The geometry of satellites during the test period is shown in Figure 3, with 11 tracked satellites. Since no multipath and atmosphere influence are simulated, the satellites with lower elevation angle can also be used to analyze the performance using different tracking methods.

In the test, the coherent integration time is set as 20 ms to improve the tracking sensitivity. The dynamics of the satellite

FIGURE 3: Geometry of satellites in the test.

FIGURE 4: PLL filter output in the conventional GNSS receiver.

is compensated using the ephemeris and the receiver location information. Figure 4 shows the PLL filter output when the satellite dynamics is compensated for the original data with 47 dB-Hz.

In Figure 4, it is obvious that the PLL filter output does not change much after the satellite dynamics are compensated at the time of 40th second. After the compensation of satellite dynamics, the oscillator phase noise and thermal noise become the main factors that affect the carrier phase tracking.

In the test, we use the original 47 dB-Hz data to generate the carrier phase measurements $\theta_{47\text{ dB-Hz}}$ as reference. Then the other data sets with different CNR are processed to generate the carrier phase measurements θ_{CNR} . The indication of the performance to compare the two methods is the single difference of carrier phase measurements between $\theta_{47\text{ dB-Hz}}$ and θ_{CNR} which is shown as follows:

$$\theta_{\text{diff}} = \theta_{\text{CNR}} - \theta_{47\text{ dB-Hz}} \quad (15)$$

θ_{diff} should show white noise since we only add Gaussian white noises to the original data set. However since we use different bandwidths to process the data sets, θ_{diff} will not show white noise and the standard deviation (STD) of θ_{diff} will not be able to be used as performance indicator since the

FIGURE 5: Mean of PLI under different bandwidth for the data set with 47 dB-Hz data set.

STD depends on the bandwidth. In this paper, we check the cycle slips of θ_{diff} to indicate the performance of the tracking loop.

Firstly, to establish a reference for the test, we have tried different bandwidths to process the original data set to find the optimal bandwidth and the corresponding phase lock indicator (PLI) [2] under different bandwidths are shown in Figure 5.

In Figure 5, it can be seen that the PLI is greater than 0.96 when the bandwidth is above 4 Hz. Since the product of bandwidth and integration time should be much less than 1 in order to ensure the stability of the loop [17], it is better to use a smaller bandwidth. However it is obvious that PLI values are significantly reduced when the bandwidth is reduced to 1 Hz. To verify the influence of the oscillator, the PLI for different satellites with 1 Hz bandwidth is shown in Figure 6.

As shown in Figure 6, after the bandwidth is set to 1 Hz which starts at 50 seconds, the PLI values for all satellites are reduced because the oscillator has caused the same influence on all satellites. Considering all above influences, a bandwidth of 5 Hz will be applied to generate the reference carrier phase measurements.

3.2. Performance Comparison. An analysis on the single difference of carrier phase measurements indicates that cycle slips will be present when the CNR is equal to or less than 21 dB-Hz. Therefore, the data sets with 21 dB-Hz and less CNR will be used in the following performance comparison between the conventional scalar tracking and the joint VPLL.

Given in Figure 7 are the single differences of carrier phase measurements using the conventional PLL method. Four satellites, namely, PRN 4, PRN 8, PRN 9, and PRN 15, have experienced cycle slips in their carrier phase measurements. Since the optimal bandwidth 3 Hz (obtained through testing different bandwidths) has been used, the result is considered to be the best performance using the conventional

FIGURE 6: PLI for different satellites with 1 Hz bandwidth.

FIGURE 7: Single difference of carrier phase between 47 dB-Hz and 21 dB-Hz data sets with PLL.

FIGURE 8: Single difference of carrier phase between 47 dB-Hz and 21 dB-Hz data sets with the joint VPLL.

FIGURE 9: Single difference of carrier phase between 47 dB-Hz and 19 dB-Hz data sets with PLL.

PLL method. More cycle slips however would be expected when a less desired bandwidth is applied.

Shown in Figure 8 are the single differences of carrier phase measurements using the proposed joint VPLL method. The results indicate that no cycle slips are present for all satellites although the standard deviation of the carrier phase measurements is large. A comparison between Figure 8 and Figure 7 confirms that the proposed tracking method can provide more robust performance than the conventional PLL architecture.

Shown in Figures 9 and 10 are the results when the CNR is 19 dB-Hz, using the conventional PLL and the proposed joint VPLL methods, respectively. We see that all carrier phase measurements are experienced with cycle slips with the

conventional PLL method. However, there are no cycle slips for all satellites when using the proposed joint VPLL method.

Similar results are gotten when the CNR is 17 dB-Hz and no cycle slips are observed using the joint VPLL but all satellites experience cycle slips with conventional PLL. However, it is worth mentioning that all satellite carrier phase measurements would experience cycle slips when the CNR of the data set is equal to or below 15 dB-Hz, even using the proposed method as shown in Figures 11 and 12. Nevertheless, it is still obvious that the proposed method shows that it would provide more robust performance compared to the conventional PLL method.

Based on the test results described above, the proposed joint VPLL architecture can improve the carrier phase

FIGURE 10: Single difference of carrier phase between 47 dB-Hz and 19 dB-Hz data sets with the joint VPLL.

FIGURE 11: Single difference of carrier phase between 47 dB-Hz and 15 dB-Hz data sets with PLL.

tracking performance by about 6 dB when compared to the conventional PLL architecture. However, the interchannel influences between different satellites have to be accounted for using the vector based PLL especially when the channel with lower CNR affects the channels with higher CNR. This will be described in the next section.

4. Interchannel Influence of Different Satellites in the Joint Vector Tracking

To assess the interchannel influence, another data set is collected which is the same as the data described in the previous section except with a different CNR. In this data set, the CNR for satellite PRN27 is set to be 27 dB-Hz while

FIGURE 12: Single difference of carrier phase between 47 dB-Hz and 15 dB-Hz data sets with the joint VPLL.

other satellites are with 47 dB-Hz. We call this data set data-set A in the sequel. Gaussian white noise is added to data-set A to adjust the CNR and generate data-set B which shows 12 dB lower compared to data-set A. So in data-set B, CNR of satellite PRN 27 is 15 dB-Hz and other satellites show 35 dB-Hz.

The carrier phase measurement of data-set A is set as reference. We use the same analysis method described in previous section to process data-set A and data-set B. However to explore the influence of the lower CNR satellites, two strategies are used to process the data-set B.

Strategy One. Using the joint VPLL method: set measurements variances for different satellites in the R matrix are the same which means all the measurements variances are calculated based on 35 dB-Hz.

Strategy Two. Using the joint VPLL method: calculate measurement noise variance of satellite PRN 27 using 15 dB-Hz and other satellites using 35 dB-Hz.

The result of single difference of carrier phase measurements between data-set A and data-set B with two strategies for PRN 27 is shown in Figure 13.

In Figure 13, the second strategy shows less cycle slips and better performance compared to the first strategy. The influence of PRN 27 for the joint VPLL is reduced in the second strategy since the covariance of PRN measurements is calculated using 15 dB-Hz rather than 35 dB-Hz compared to the first strategy. The single difference of carrier phase measurements for PRN 2 is shown in Figure 14.

With strategy two, satellite PRN 2 shows better performance than that using strategy one in Figure 14. Standard deviation of residuals for PRN 2 using strategy two is 0.0129; however strategy one shows 0.0165. The lower CNR satellite can impose more noise on the higher CNR satellites due to

FIGURE 13: Residuals of satellite PRN 27 using two processing strategies.

FIGURE 14: Residuals of satellite PRN 2 using two processing strategies.

the common loop. In the joint VPLL tracking, since R matrix of EKF is calculated based on the CNR, the influence of lower CNR satellite can be reduced which is similar to strategy two.

5. Conclusions

In this paper, the influence of the oscillator in the GNSS receiver is analyzed. To address the dilemma problem of loop bandwidth design, a carrier phase tracking method based on a joint vector architecture is proposed. The transfer function model of the proposed method is derived and analyzed. Based on the same oscillator, we test different CNR through adding white noise on the IF data. With the IF data collected from simulator and the data sets with more white noise, the proposed method is proved to improve the carrier phase tracking sensitivity by around 6 dB when compared to the conventional scalar tracking which uses the optimal bandwidth during the test. Interchannel influence of the proposed method is also analyzed. Due to the joint tracking in the joint VPLL, the lower CNR satellite imposes more noise on the higher CNR satellite. However, as R matrix of EKF in

the joint VPLL is calculated based on CNR, the influence of the satellite with lower CNR can be reduced.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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