Research Article

Design Methodology and Performance Evaluation of New Generation Sounding Rockets

Marco Pallone, Mauro Pontani, Paolo Teofilatto, and Angelo Minotti

University of Rome “La Sapienza”, Via Salaria 851/831, 00138 Rome, Italy

Correspondence should be addressed to Marco Pallone; pallone.1420138@studenti.uniroma1.it

Received 24 March 2017; Accepted 25 September 2017; Published 6 February 2018

Copyright © 2018 Marco Pallone et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Sounding rockets are currently deployed for the purpose of providing experimental data of the upper atmosphere, as well as for microgravity experiments. This work provides a methodology in order to design, model, and evaluate the performance of new sounding rockets. A general configuration composed of a rocket with four canards and four tail wings is sized and optimized, assuming different payload masses and microgravity durations. The aerodynamic forces are modeled with high fidelity using the interpolation of available data. Three different guidance algorithms are used for the trajectory integration: constant attitude, near radial, and sun-pointing. The sun-pointing guidance is used to obtain the best microgravity performance while maintaining a specified attitude with respect to the sun, allowing for experiments which are temperature sensitive. Near radial guidance has instead the main purpose of reaching high altitudes, thus maximizing the microgravity duration. The results prove that the methodology at hand is straightforward to implement and capable of providing satisfactory performance in term of microgravity duration.

1. Introduction

Sounding rockets are specialized missiles generally used to investigate the region of space between 50 and 700 km where it is difficult to enter with the traditional atmospheric balloon or low orbit satellites. They can be used in several missions, such as detection of the solar activity and anomalies, analysis of the constituents of the upper atmosphere, thermal analysis on new materials, and generally, measurements of the space surrounding the Earth [1–3]. The most important characteristic of a sounding rocket is the capability of achieving microgravity conditions for the payload, without the need of a manned mission in the ISS to perform experiments and then reducing the costs of the mission. The quality of microgravity depends on the absence of drag and other gyroscopic forces, so the guidance of the rocket assumes the utmost importance. Another good characteristic of the sounding rocket in contrast to the satellite missions is the sounding rocket’s ability to retrieve the payload with the help of a heat shield and a parachute. For all of these reasons, a vast number of sounding rocket families were developed, such as the American “Black Brant” and the European “Maxus.” The Black Brant family includes sounding rockets that range from one through three stages. The Black Brant VC [4] (Figure 1) can be launched from a steerable launch tower with different kick angles to achieve different apogee altitudes. It is controlled via spin-up motors and canard wings, which provide a fast response at the cost of small instability. Four rear tail surfaces guarantee an adequate static margin. The Maxus [4, 5] is a very powerful European single-stage sounding rocket launched in Kiruna (Sweden) and can reach a maximum altitude of 700 km, while the range does not exceed 100 km. It has a microgravity duration of about 12 min, and the microgravity level $\Delta g$ is near $10^{-4}$m/sec$^2$. TVC control system is highly sophisticated in order to guarantee strict margins for the range and to avoid damages to both people and goods. Table 1 portrays some useful data to compare the performance of the two rockets.

Both of these sounding rockets are propelled with a solid state motor, due to their simplicity and light weights with respect to liquid propellant. A crucial point is that a sounding rocket has to be cheaper than a high-tech rocket, and in order to achieve this objective, new design methodologies are needed. Many authors (e.g., Chowdhury et al. [6, 7] and...
Casalino and Pastrone [8, 9] are interested in designing new sounding rockets to explore the potentiality of new hybrid motors for hypersonic flight. Nonaka et al. [10] and Woo et al. [11] investigated the reusability of the sounding rocket, and Cremaschi et al. [12] investigated the optimization of a trajectory with the use of a commercial trajectory simulation tool (ASTOS). However, a very limited number of studies are concerned with fast sizing and aerodynamic losses during the ascent path. The purpose of this work is to find a novel and simple methodology to size and design for future sounding rockets, considering accurate aerodynamics and three guidance algorithms. In particular, the technique explained in this work is applied to a single-stage rocket whose three-dimensional trajectory is simulated using the following guidance schemes:

(i) Constant attitude guidance
(ii) Near radial guidance
(iii) Sun-pointing guidance

This configuration is chosen with the purpose of creating an algorithm as general as possible. Incidentally, this geometry is adopted by a lot of sounding rockets in the market, like the “Black Brant VC” in Figure 1.

Every rocket configuration in this work is defined by the total length \( l \) and the maximum diameter \( d \), while all the other parameters like the propulsive ones are obtained through scaling factors. The grid used for the computation is

(i) \( l \) from 3 m to 15 m with step of 1 m
(ii) \( d \) from 0.2 m to 1.5 m with step of 0.05 m.

Thus, a total of 243 different configurations were analyzed. The design methodology that is being presented relates geometry, weight, aerodynamics, and propulsive performances.

2.2. Main Body. For each configuration, the length of the ogive is proportional to the total length of the sounding rocket

\[ l_{\text{ogive}} = C_{\text{ogive}} l, \]

where \( C_{\text{ogive}} = 0.17 \) is a dimensionless coefficient. The dimensional ogive point distribution is then obtained by multiplying the nondimensional points in the \((\xi, \eta)\) frame in Figure 2 with \( l_{\text{ogive}} \) and \( d \). The main body, which contains the propulsive system and the avionics, is cylindrical with diameter \( d \), while its length is found through

\[ l_{\text{body}} = l - l_{\text{ogive}}, \]

2.3. Aerodynamic Surfaces. The aerodynamic surfaces are sized according to the following ratios:

\[ A_P = (1 - l_{\text{ogive}}) d, \]
\[ A_C = \frac{1}{45} A_P, \]
\[ A_T = \frac{1}{12} A_P. \]

Then the following quantities are computed as follows:

\[ h_T = \sqrt{\frac{4}{3} A_C}, \]
\[ h_C = \sqrt{2 A_T}. \]
while the distances with respect to the nose tip are as follows:

\[ d_C = l_{ogive} + 0.4 \text{m}, \]
\[ d_T = l - h_T. \]  \hspace{1cm} (5)

The main concept under this procedure is to maintain the shape of the wings, while selecting their size in relation to the overall size of the main body. This is in order to avoid wings with nonconventional shapes that would result in undesired aerodynamic effects. Figure 2 portrays the wing geometry with the profile adopted for both wing systems. The nonvariable distance in (5) 0.4 m is meant to provide a fixed volume needed for the canard avionic and the recovery system. Figure 2 portrays an example of a configuration with the disposition of the leading edges along the longitudinal axis of the rocket.

### Table 2: Auxiliary mass \( m_{aux} \) of the ogive.

<table>
<thead>
<tr>
<th>Auxiliary Mass (kg)</th>
<th>( m_{aux} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parachute mass (kg)</td>
<td>10</td>
</tr>
<tr>
<td>Heat shield mass (kg)</td>
<td>35</td>
</tr>
<tr>
<td>Metal case mass (kg)</td>
<td>5</td>
</tr>
<tr>
<td>Total ( m_{aux} ) (kg)</td>
<td>50</td>
</tr>
</tbody>
</table>

2.4. Mass Scaling. In order to describe properly a real sounding rocket mission, a fixed mass including parachute, heat shield, and metal case is used for every configuration. Table 2 portrays in detail these auxiliary masses, which form \( m_{aux} \).

With regard to the initial mass at launch, a common initial density is used in order to find a reasonable and scaled mass distribution for every rocket configuration. This initial density is \( C_{m_0} = 1092.3 \text{kg/m}^3 \), which is taken from the Black...
Brant VC and is considered to have a reasonable value. This value includes propellant, structural, and auxiliary masses. So the initial mass for every configuration without the ogive mass is

\[ m_{0\text{body}} = C_{m_0} V_{\text{body}}. \]  

For every configuration, 50, 100, and 200 kg of scientific payload are used, so the total initial mass at launch is

\[ m_0 = m_{0\text{body}} + m_U + m_{\text{aux}}. \]  

The propulsive technology of this single-stage rocket is a solid propellant one, allowing for the following structural coefficient:

\[ \epsilon = \frac{m_S}{m_U + m_S} = 0.11. \]  

The structural and propellant masses are obtained through the following expressions:

\[ u = \frac{m_U + \epsilon m_0 - \epsilon m_U}{m_0}, \]

\[ m_p = \frac{m_0}{1 - u}, \]

\[ m_S = \frac{\epsilon m_0 (1 - u)}{1 - \epsilon}. \]  

In the end, (6), (7), (8), and (9) yield the overall rocket mass at launch, portrayed in Figure 3 as a function of \( l \) and \( d \), together with the corresponding propellant mass.

2.5. Propulsion System Scaling. To scale the thrust for every sounding rocket size, the initial launch acceleration and the specific impulse are maintained constant:

\[ T = n_0 g m_0, \]

where \( n_0 = 3g \), while the propellant mass consumption is

\[ \dot{m} = \frac{T}{I_{sp} g}. \]  

The value for \( n_0 \) is chosen in order to constrain the rocket dynamical stress at launch. Equation (11) leads to the burnout time:

\[ t_B = \frac{m_p}{m}. \]  

Figure 4 portrays the thrust and the burnout time scaling with the rocket size.

3. Rocket Aerodynamic Modeling

Aerodynamic forces affect the rocket motion, which partially occurs in the atmosphere. Thus, the rocket aerodynamics is to be modeled appropriately. In this work, four quantities are assumed to determine the aerodynamic forces: Mach number \( M \), Reynolds number \( \text{Re} \), angle of attack \( \alpha \), and sideslip angle \( \beta \). To provide an accurate model for each configuration, two fundamental steps are needed:

1. Derivation of the aerodynamic coefficients \( C_D \), \( C_L \), and \( C_Q \) from DATCOM [13] at a relevant number of Mach, Reynolds, angles of attack, and sideslip angles

2. Cubic interpolation of the aerodynamic coefficients during the integration of the dynamics equations

Following the approach presented by Mangiacasale [14], Pallone et al. [15], and Pontani [16], the aerodynamics of the rocket was modeled through the Missile DATCOM software [13]. This software provides an accurate esteem of the aerodynamic forces without using a lot of computational resources; thus, it can provide a complete aerodynamic database for each configuration in a short time. The following discrete values of \( M \), \( \text{Re} \), \( \alpha \), and \( \beta \) are used:

\[ (i) \quad M = [0.2, 0.5, 0.6, 0.8, 0.9, 1.1, 1.2, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6, 6.5, 7, 7.5, 8, 10] \]

\[ (ii) \quad \text{Re} = [0.5, 1, 1.5, 2] \times 10^5 \]

\[ (iii) \quad \alpha = [0, 1, 2, 3, 4, 5] \text{ deg} \]
This means that DATCOM was run 3024 times for each \( l, d \) combination. The aerodynamic surfaces used in the computation are the cross surfaces of each configuration and correspond to the circular section, that is, 

\[
S_i = \pi \left( \frac{d}{2} \right)^2.
\]  

Figure 5 portrays an example of the aerodynamic coefficients provided by the DATCOM software for the configuration \( l = 6 \text{ m} \) and \( d = 0.8 \text{ m} \).

4. Rocket Dynamics

The problem investigated in this work consists in finding the sounding rocket with the minimum total volume that can guarantee \( \Delta T_{mg} \) of 10 and 15 min, fixing the propulsive technology \( (I_{sp}, \epsilon) \) and the geometrical ratios between its parts \( (C_{m_i}, C_{ogive}, A_C/A_P, A_T/A_P) \).

4.1. Reference Frames. A description of the reference frames used in this work is useful to understand the guidance implementation. In the following, the notation \( R_j(\mp \xi) \) \((j=1, 2, 3)\) denotes the elementary rotation about axis \( j \) by angle \( \xi \) while the \( \mp \) sign means, respectively, clockwise and counterclockwise.

1. The Earth-centered frame (ECI) \( (\hat{\mathbf{r}}, \hat{\mathbf{u}}, \hat{\mathbf{i}}) \) is a Cartesian inertial reference frame defined as follows. Its origin \( O \) is the center of the Earth. The unit vector \( \hat{\mathbf{u}} \) is aligned with the Earth axis of rotation and is positive northward, whereas \( \hat{\mathbf{i}} \) is aligned with the vernal axis, which corresponds to the intersection of the Earth equatorial plane with the ecliptic plane.

2. The Earth-centered Earth-fixed frame (ECEF) \( (\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}) \) represents a reference system that is rigidly attached to the Earth. Under the assumption that
the planet rotates with a constant angular rate \( \omega_R \), also the ECEF frame rotates with angular rate \( \omega_E \). The origin of the coordinate system is the center of the Earth, while the unit vector \( \hat{\mathbf{I}} \) intersects the Greenwich reference meridian at all times, and \( \hat{\mathbf{K}} \) is aligned with the planet axis of rotation and is positive northward. \( \omega_E \hat{\mathbf{K}} \) is the vector rotation rate of the ECEF frame with respect to the ECI frame. The reference meridian is the Greenwich side-real-time \( \theta_G \), given by \( \theta_G(t) = \theta_G(\tilde{T}) + \omega_E (t - \tilde{T}) \) where \( \tilde{T} \) refers to a generic time instant. The ECEF frame is obtained through a counterclockwise rotation by angle \( \theta_G \) around the third axis, that is, 
\[
[\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}] = R_3(\theta_G)[\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, \hat{\mathbf{e}}_3]^T.
\]

(3) The local horizontal frame (LH) \((\hat{\mathbf{r}}, \hat{\mathbf{E}}, \hat{\mathbf{N}})\) is a reference system with the origin in the rocket center of mass. Specifically, \( \hat{\mathbf{r}} \) is the unit vector aligned with the instantaneous position \( \hat{\mathbf{r}}, \hat{\mathbf{E}} \) is directed along the local east direction, and \( \hat{\mathbf{N}} \) is aligned with the local north direction. LH frame is obtained from the ECEF frame through the following rotations: 
\[
[\hat{\mathbf{r}}, \hat{\mathbf{E}}, \hat{\mathbf{N}}]^T = R_2(\omega_L) R_3(\lambda) [\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}]^T.
\]

(4) The auxiliary orbital frame (AO) \((\hat{\mathbf{r}}, \hat{\mathbf{R}}, \hat{\mathbf{h}})\) is defined making reference to the rocket relative velocity \( v_R = v_l - \omega_R \times r \). The unit vector \( \hat{\mathbf{R}} \) is aligned with the projection of the velocity \( v \) onto the horizontal plane \((\hat{\mathbf{E}}, \hat{\mathbf{N}})\). The AO frame is obtained from the LH frame with the simple rotation 
\[
[\hat{\mathbf{r}}, \hat{\mathbf{R}}, \hat{\mathbf{h}}]^T = R_1(\gamma) [\hat{\mathbf{r}}, \hat{\mathbf{E}}, \hat{\mathbf{N}}]^T.
\]

(5) The relative velocity frame (RV) \((\hat{\mathbf{n}}_R, \hat{\mathbf{v}}_R, \hat{\mathbf{h}}_R)\) is obtained from the AO frame through the following clockwise rotation: 
\[
[\hat{\mathbf{n}}_R, \hat{\mathbf{v}}_R, \hat{\mathbf{h}}_R]^T = R_3(-\gamma_R) [\hat{\mathbf{r}}, \hat{\mathbf{R}}, \hat{\mathbf{h}}]^T.
\]

(6) The wind axis frame (WA) \((\hat{\mathbf{n}}_W, \hat{\mathbf{v}}_W, \hat{\mathbf{h}}_W)\) is defined with reference to the rocket velocity with respect to the local velocity of the atmosphere \( v_a = \omega_R \times r \) so \( v_a = v_R \) and just a counterclockwise rotation about axis 2 by the bank angle \( \sigma \): 
\[
[\hat{\mathbf{n}}_W, \hat{\mathbf{v}}_W, \hat{\mathbf{h}}_W]^T = R_2(\sigma) [\hat{\mathbf{n}}_R, \hat{\mathbf{v}}_R, \hat{\mathbf{h}}_R]^T.
\]

(7) The auxiliary body axis frame (ABA) \((\hat{\mathbf{i}}_B, \hat{\mathbf{j}}_B, \hat{\mathbf{k}}_B)\) is a reference system aligned with the rocket inertia axes. The unit vector \( \hat{\mathbf{k}}_B \) is orthogonal to the rocket plane of symmetry, while the two remaining unit vectors lie on it. The ABA frame is obtained from the WA frame through a sequence of two rotations: 
\[
[\hat{\mathbf{i}}_B, \hat{\mathbf{j}}_B, \hat{\mathbf{k}}_B]^T = R_3(-\alpha) R_1(\beta) [\hat{\mathbf{n}}_W, \hat{\mathbf{v}}_W, \hat{\mathbf{h}}_W]^T.
\]

(8) The body axis (BA) \((\hat{\mathbf{x}}_B, \hat{\mathbf{y}}_B, \hat{\mathbf{z}}_B)\) is, like the ABA frame, also attached to the rocket inertial axes only with a different orientation. Specifically, \( \hat{\mathbf{x}}_B \) is aligned with the rocket longitudinal axis \((\hat{\mathbf{x}}_B = \hat{\mathbf{j}}_B)\); \( \hat{\mathbf{z}}_B \) is directed downward so \( \hat{\mathbf{z}}_B = -\hat{\mathbf{i}}_B \) and as consequence \( \hat{\mathbf{y}}_B = -\hat{\mathbf{k}}_B \). These relations lead to the following elementary rotation:
\[
\begin{bmatrix}
\hat{x}_B \\
\hat{y}_B \\
\hat{z}_B
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 1 \\
0 & 0 & -1 \\
-1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\hat{i}_B \\
\hat{j}_B \\
\hat{k}_B
\end{bmatrix} = R_A [\hat{i}_B, \hat{j}_B, \hat{k}_B].
\tag{14}
\]

The knowledge of the instantaneous orientation of \((\hat{\mathbf{x}}_B, \hat{\mathbf{y}}_B, \hat{\mathbf{z}}_B)\) is equivalent to identifying the instantaneous attitude of the rocket.

(9) The launch inertial frame (LI) \((\hat{\mathbf{i}}_L, \hat{\mathbf{j}}_L, \hat{\mathbf{k}}_L)\) is an inertial system centered in the launch location at the launch instant. The unit vector \( \hat{\mathbf{i}}_L \) is aligned with the local north direction, \( \hat{\mathbf{j}}_L \) is directed toward the local east direction, and \( \hat{\mathbf{k}}_L \) is aligned with the downward direction. The LI frame is obtained from the ECI frame with a sequence of three rotations: the first two of them take the ECI frame to an intermediary reference frame, named auxiliary launch inertial frame (ALI) \((\hat{\mathbf{i}}_0, \hat{\mathbf{j}}_0, \hat{\mathbf{k}}_0)\) using the geographical latitude and longitude of the launch base at the launch time, while the third one relates the axes of the ALI frame to the LI frame. The complete sequence is as follows:
\[
\begin{bmatrix}
\hat{\mathbf{i}}_0 \\
\hat{\mathbf{j}}_0 \\
\hat{\mathbf{k}}_0
\end{bmatrix} = R_2(-L_L) R_3(\lambda_{AL})
\begin{bmatrix}
0 & 0 & 1 \\
0 & 1 & 0 \\
-1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\hat{\mathbf{i}}_1 \\
\hat{\mathbf{j}}_1 \\
\hat{\mathbf{k}}_1
\end{bmatrix} = R_2(-L_L) R_3(\lambda_{AL}) R_{B}
\begin{bmatrix}
\hat{\mathbf{i}}_c \\
\hat{\mathbf{j}}_c \\
\hat{\mathbf{k}}_c
\end{bmatrix}.
\tag{15}
\]

The BA frame is also related to the LI frame through a sequence of three rotations using the attitude angles \((\phi, \theta, \psi)\). The complete sequence is 
\[
[\hat{\mathbf{x}}_B, \hat{\mathbf{y}}_B, \hat{\mathbf{z}}_B]^T = R_1(\phi) R_2(\theta) R_3(\psi) [\hat{\mathbf{i}}_0, \hat{\mathbf{j}}_0, \hat{\mathbf{k}}_0]^T.
\]

In order to avoid ambiguities in the definition of the vectors of interest, the angles introduced in this section have to be constrained. Table 3 shows the limitation for the angles used in this work.

Any vector can be obtained with respect to any reference frame using the rotations defined in this section. These rotations can be used to define the rocket kinematics and dynamics through the motion, and Figures 6 and 7 portray all the reference frames used in this work, together with the related rotations.
4.2. Forces. The main term of the gravitational force \( G \) is given in the RV frame simply by

\[
\frac{G}{m} = -\frac{\mu_E}{r^3} \begin{bmatrix}
\cos \gamma_R \\
-\sin \gamma_R \\
0
\end{bmatrix}^T \begin{bmatrix}
\hat{n}_R \\
\hat{v}_R \\
\hat{h}_R
\end{bmatrix},
\]

(16)

The components of the aerodynamic force can be referred to the WA frame and then projected in the RV frame with a simple rotation. The three components are the lift force, the drag force, and the side force.

\[
\frac{A}{m} = \frac{L + D + Q}{m} = \frac{L\hat{n}_W - D\hat{v}_W + Q\hat{h}_W}{m},
\]

(17)

where

\[
L = L\hat{n}_W = \frac{1}{2} \rho v_W^2 C_L(\alpha, \beta, Re, M)\hat{n}_W,
\]

\[
D = -D\hat{n}_W = \frac{1}{2} \rho v_W^2 C_D(\alpha, \beta, Re, M)\hat{n}_W,
\]

\[
Q = Q\hat{n}_W = \frac{1}{2} \rho v_W^2 C_Q(\alpha, \beta, Re, M)\hat{n}_W.
\]

(18)

The aerodynamic forces are taken into account up to 100 km of altitude where the atmospheric density becomes negligible.

The thrust can be projected along the RV frame with the in-plane \( \alpha_T \) and the out-of-plane \( \beta_T \):

\[
\frac{T}{m} = \begin{bmatrix}
\frac{T}{m} \cos \beta_T \sin \alpha_T \\
\frac{T}{m} \cos \beta_T \cos \alpha_T \\
\frac{T}{m} \sin \beta_T
\end{bmatrix}^T \begin{bmatrix}
\hat{n}_R \\
\hat{v}_R \\
\hat{h}_R
\end{bmatrix},
\]

(20)

and to avoid ambiguities, the following constraints must hold:

\[
-\pi < \theta_i \leq \pi, \quad -\pi < \lambda_G \leq \pi, \quad -\pi < \omega \leq \pi, \quad -\pi < \alpha \leq \pi, \quad -\pi < \omega < \pi, \quad -\pi < \beta \leq \pi
\]

(21)

The thrust vector \( T \) is considered aligned with the rocket longitudinal axis so two angles, \( \alpha_T \) and \( \beta_T \) suffices to describe its direction.

4.3. Equations of Motion. The equations of motion that govern the three-dimensional rocket dynamics can be conveniently written in terms of \( \{r, \lambda_G, La, \gamma_R, v_R, \xi_R\} \). These variables refer to the relative motion in the RV frame. They form the state vector \( x_R \) of the launch vehicle (in rotating coordinates).

\[
\dot{r} = v_R \sin \gamma_R,
\]

\[
\dot{\lambda}_G = \frac{v_R \cos \gamma_R \cos \xi_R}{r \cos La},
\]

\[
\dot{\xi}_R = \frac{v_R \cos \gamma_R \cos \xi_R}{r} + \frac{T/m \cos \beta_T \sin \alpha_T + A_n}{v_R} + \frac{\omega_k^2}{v_R} \cos La \cos \gamma_R + \sin La \sin \gamma_R \sin \xi_R,
\]

\[
\dot{\xi}_R = \frac{v_R \tan La \cos \gamma_R \cos \xi_R + T/m \sin \beta_T}{v_R \cos \gamma_R} + \frac{A_n}{v_R \cos \gamma_R} \cos La \tan \gamma_R \sin \xi_R + \frac{\omega_k}{v_R \cos \gamma_R} \sin La \cos La \cos \xi_R - 2 \omega_k \sin La.
\]

(22)

The initial values for (22) are reported in Table 4. Equation (22) is valid for both the propelled arc and the ballistic flight of the ogive. The values for \( La \) and \( \lambda_G \) in Table 4 are referred to the Malindi launch base, while the launch is in the east direction (\( \xi_R \)). The control laws for \( (\alpha_T, \beta_T) \), are determined using three distinct guidance scheme, namely, the constant attitude guidance, the near radial guidance, and the sun-pointing guidance, which are described in the following section. The state \( x_R \) is continuous across the first stage separation, which occurs at time \( t_B \). After that, the payload continues its motion in a ballistic trajectory.

5. Guidance Algorithms

5.1. Constant Attitude Guidance. With this guidance, the rocket orientation does not change during the flight and
was actually implemented in the Black Brant X (cf. [17]). This is ideal to achieve nominal (perfect) microgravity conditions ($\Delta g = 0$), due to rotation rates equal to 0. The attitude angles $\phi, \theta, \psi$ remain constant; therefore, in this type of guidance, the following expressions can be assumed as follows:

$$\phi = \phi_0, \theta = \theta_0, \psi = \psi_0.$$  \hfill (23)

![Figure 6: Reference frames and related rotation matrices.](image)

![Figure 7: Main reference frames and related angles.](image)

<table>
<thead>
<tr>
<th>Table 4: Initial condition for (22).</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$ (km)</td>
</tr>
<tr>
<td>$v_R$ (km/s)</td>
</tr>
<tr>
<td>$\gamma_R$ (deg)</td>
</tr>
<tr>
<td>$\Lambda$ (deg)</td>
</tr>
<tr>
<td>$\zeta_R$ (deg)</td>
</tr>
<tr>
<td>$\lambda_{\text{li}}$ (deg)</td>
</tr>
</tbody>
</table>
The remaining angles \( \{\alpha_T, \beta_T, \alpha, \beta, \sigma\} \) can be found using analytical formulas at any times using the rotation matrices portrayed in Figure 6.

5.2. Near Radial Guidance. This guidance is called near radial because it is defined in the AO reference frame and the thrust direction is defined by

\[
\alpha_T = \gamma_{R0} + \frac{t}{t_0} (\frac{\pi}{2} - \gamma_{R0}) - \gamma_R,
\]

where \( \gamma_{R0} = \theta_0 \) is the pitch angle at the initial time and is equal to the initial flight path angle. With \( \beta_T = 0 \), the angle that the rocket forms with the local horizontal axis is given by \( (\alpha_T + \gamma_R) \). At the initial time, this angle corresponds to the initial flight path angle, while at the burnout time, it corresponds to \( (\alpha_T + \gamma_R) = \pi/2 \). The roll angle is can be chosen arbitrarily and is set to \( 0 \) deg. For the remaining angles \( \{\theta, \psi, \alpha, \beta, \sigma\} \), they are obtained through analytical expressions through the rotation matrices in Figure 6. The thrust direction tends to be aligned with the radial direction. Thus, in principle, this guidance leads to the mean maximal altitude for the rocket at hand. Moreover, with this guidance, the usual values of \( \Delta g \) are in the order of \( 10^{-4} \) m/sec².

5.3. Sun-Pointing Guidance. The sun-pointing guidance is based on maintaining constant attitude of the rocket with respect to the sun position. The attitude angles \( \{\phi, \theta, \psi\} \) are computed in an analytical fashion using the rotations in Figure 6. The first step to obtain the attitude of the sounding rocket is to locate the sun position with respect to the ECI frame:

\[
\hat{r}_S = \begin{bmatrix} \cos \lambda_S \cos \xi_S & \cos \lambda_S \sin \xi_S & \sin \lambda_S \end{bmatrix} [\vec{e}_1 \quad \vec{e}_2 \quad \vec{e}_3]^T,
\]

where \( \lambda_S \) and \( \xi_S \) are the sun declination and the right ascension, respectively. Then, projection of this unit vector into the ALI frame yields as follows:

\[
\begin{bmatrix} \hat{r}_S \end{bmatrix} = \begin{bmatrix} \hat{r}_0 \end{bmatrix},
\]

where

\[
\begin{bmatrix} \hat{x}_L \end{bmatrix} = \begin{bmatrix} \hat{x}_S \end{bmatrix} \begin{bmatrix} \cos \xi \cos \zeta & -\cos \lambda \sin \xi & \sin \lambda \sin \xi \\ \cos \lambda \cos \xi & \cos \lambda \sin \xi & \sin \lambda \sin \xi \\ -\sin \lambda \sin \xi & \sin \lambda \sin \xi & \cos \lambda \cos \xi \end{bmatrix} \begin{bmatrix} \hat{x}_S \end{bmatrix},
\]

\[
\hat{r}_S = \begin{bmatrix} \hat{r}_0 \end{bmatrix},
\]

\[
\begin{bmatrix} \hat{r}_0 \end{bmatrix} = \begin{bmatrix} \hat{r}_0 \end{bmatrix} R_S(-\lambda_S)R_L(L_L) \begin{bmatrix} \hat{E}_0 \end{bmatrix} \begin{bmatrix} \hat{N}_0 \end{bmatrix},
\]

\[
\begin{bmatrix} X_\odot \ Y_\odot \ Z_\odot \end{bmatrix} \begin{bmatrix} \hat{E}_0 \end{bmatrix} = \begin{bmatrix} \hat{N}_0 \end{bmatrix}.
\]

So the thrust direction is obtained by aligning the body axis of the launcher with the sun direction in the ALI frame, as shown in Figure 8. To avoid that the rocket will fly with a \( \gamma_r \) near to zero while trying to catch the sun slightly above the local horizon, the thrust direction is constrained to have a displacement from \( \hat{r}_S \) not exceeding the maximal value \( \eta = 10 \) deg (cf. Figure 8). So the thrust direction is defined imposing \( \xi_T = \xi_S \) and \( \lambda_T = \max(\lambda_S, \pi/2 - \eta) \). The angles \( \alpha_T \) and \( \beta_T \) are then obtained through rotation of \( \hat{T} \) from the ALI frame to the RV frame using the rotations reported in Figure 6 and considering that the thrust is aligned with the longitudinal axis of the rocket \( \hat{s}_B \). The pitch and yaw angles are then obtained with the following expressions:

\[
\sin \theta = \cos \lambda_T \cos \xi_T \rightarrow \theta,
\]

\[
\cos \psi = \sin \lambda_T, \quad \sin \psi = \cos \lambda_T \sin \xi_T \rightarrow \psi.
\]

As roll angle is arbitrary, it can be selected after choosing the angle \( \xi_S \) related to the sun position in the body frame. The roll angle \( \phi \) is obtained by comparing \( \hat{r}_S \) in the ALI frame with its projection in the RV frame.
From Figure 8(c), it is straightforward to obtain the following angles:

\[
\begin{align*}
\sin L_S &= \sin \lambda_T \rightarrow L_S, \\
\tan L_S &= \sin \xi_S, \\
\cos \lambda_T &= \cos \xi_S \rightarrow \xi_S.
\end{align*}
\]

They univocally define the vector \( \hat{r}_\oplus = [X_{B0} \ Y_{B0} \ Z_{B0} \ \hat{x}_B \ \hat{y}_B \ \hat{z}_B]^T \) in the body frame. Comparing the coordinates of \( \hat{r}_\oplus \) in the ALI frame to the ones in the BA frame leads the following expression:

\[
[X_{B0} \ Y_{B0} \ Z_{B0} \ R_B^T \ R_S^T (\psi) R_S^T (\theta) R_1^T (\phi)] = [X_{B0} \ Y_{B0} \ Z_{B0}].
\] (29)

Equation (29) leads to the roll angle \( \phi \).
Then, the aerodynamic angles $\alpha, \beta, \sigma$ can be obtained through the rotation matrices in Figure 6, which are not reported for the sake of brevity. In short, the sun-pointing guidance requires specifying the following parameters:

(i) $\theta_s$ which is the sun position in the ECI frame
(ii) $\eta$ which constraints the possible thrust direction
(iii) $\theta_{G0}$, defined by the date and hour of the launch
(iv) $\iota_s$, needed to obtain the roll angle $\phi$

As the attitude is also in this case constant, the sun-pointing guidance is a particular case of the constant attitude guidance. Hence, it maintains its main advantages like the ideally perfect microgravity condition ($\Delta g = 0$) and can provide the perfect conditions for thermal experiments in the space.

6. Numerical Results

Figures 9–11 portray the results of the computation, which are, respectively, for the constant attitude guidance.
Table 5: Optimal configurations for each payload mass, $\Delta T_{mg} = 10$ min.

<table>
<thead>
<tr>
<th>$m_U$ (kg)</th>
<th>50</th>
<th>100</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l$ (m)</td>
<td>7.29</td>
<td>8.25</td>
<td>9.69</td>
</tr>
<tr>
<td>$d$ (m)</td>
<td>0.36</td>
<td>0.41</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Table 6: Optimal configurations for each payload mass, $\Delta T_{mg} = 15$ min.

<table>
<thead>
<tr>
<th>$m_U$ (kg)</th>
<th>50</th>
<th>100</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l$ (m)</td>
<td>10.98</td>
<td>12.42</td>
<td>14.53</td>
</tr>
<tr>
<td>$d$ (m)</td>
<td>0.55</td>
<td>0.62</td>
<td>0.73</td>
</tr>
</tbody>
</table>

(Figure 9), the near radial guidance (Figure 10), and the sun-pointing guidance (Figure 11). From Figure 9 to Figures 11(b), 11(d), and 11(f), if the desired $\Delta T_{mg}$ is not chosen, it is possible to obtain a maximum $\Delta T_{mg}$ for each length. This result is particularly visible for the shorter lengths while is less evident for the bigger configurations.
Panels (a), (c), and (e) of Figures 9–11 show every \((l, d)\) combination which guarantee 600 and 900 sec of \(\Delta T_{mg}\). The optimal configuration corresponds to the point in which each curve is tangent to an equilateral isovolume hyperbola in the \((l, d^2)\) graph. As the tangence point always occurs in the lowest right part of the graph, two constraints were
inserted, for the sake of avoiding too slender or bulky configurations. In general, the constraint for the \( l/d \) is

\[
5 \leq \frac{l}{d} \leq 20.
\]  

(30)

It should be noticed that the optimal configurations for all the three guidance are nearly the same, and this means that the preliminary design of the sounding rocket can be done regardless of the guidance type. Tables 5 and 6 show the optimal rocket configuration for 600 and 900 sec of \( \Delta T \) and for all the payload masses. The configuration with \( l = 7.29 \) m and \( d = 0.36 \) m in Table 5 is particularly similar to the Black Brant VC, demonstrating that with similar \( m_U \) and \( \Delta T_{mg} \), the algorithm can be effective to optimize the size of the rocket.

Figures 12–14 portray the complete state vector elements and other variables for the optimal configuration with \( m_U = 50 \) kg and \( \Delta T_{mg} = 10 \) min, using the near radial guidance.

For the rocket structural integrity, the constraint \( Q\alpha_{tot} < 150000 \mathrm{Pa} \times \deg \) must be satisfied, where

\[
Q = \frac{1}{2} \rho v_r^2
\]  

(31)

and

\[
\alpha_{tot} = \arccos(\cos \alpha \cos \beta).
\]  

(32)

From Figure 13(f), it is apparent that the constraint is satisfied, and the same occurs for all the cases.

7. Conclusions

Designing a sounding rocket is a difficult task, and this work proposes a method to simplify the early stage modeling process and performance evaluation. The new methodology at hand has four principal phases: (i) mass and geometry sizing based on reasonable assumptions, (ii) scaled propulsion modeling, (iii) accurate aerodynamic modeling, and (iv) the performance evaluation using different guidance schemes. With regard to the guidance algorithms, three schemes were presented which are the constant attitude, near radial, and sun-pointing guidances. While the constant attitude guidance is a well-known scheme, the other two are novel guidances. In particular, the benefits of the sun-pointing guidance are presented, which are the nearly perfect micro-gravity condition and the possibility to orient the rocket with a defined attitude with respect to the sun. The results clearly show that the algorithm is capable of providing a fast and reasonable sizing in accordance with the mission requirements. The results obtained can be used as a reference point for a montecarlo campaign to analyze the impact points in the presence of local winds or as a first guess solution for more sophisticated optimization algorithms.

Nomenclature

- \( g \): Earth gravitational acceleration, \( \text{m/sec}^2 \)
- \( \mu_E \): Earth gravitational parameter, \( \text{km}^2/\text{sec}^3 \)
- \( d \): Rocket diameter, m
- \( l_{ogive} \): Ogive length, m
- \( C_{m_0} \): Dimensionless total initial mass
Conflicts of Interest

The authors declare that they have no conflicts of interest.

References


Submit your manuscripts at www.hindawi.com