Research Article

Adaptive Reduced Dimension Fuzzy Decoupling Control Method with Its Application to a Deployable Antenna Panel

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This paper addresses the multiple-input multiple-output (MIMO) control problem with an active deformation adjustment mechanism on a 5-metre deployable antenna panel. An adaptive reduced-dimension fuzzy decoupling (ARDFD) control strategy based on fusion functions is proposed to eliminate the effects uncertainties due to coupled subsystems by designing and incorporating a fuzzy decoupling disturbance observer. Moreover, the MIMO system can be decomposed into a number of single-input single-output (SISO) systems using the unit diagonal matrix decoupling theory. Based on Lyapunov stability theory, it was shown that the error of fuzzy interference observer and the error between the fuzzy rule post parameter matrix and optimal post parameter matrix converge to a small region. Results of the simulation also show that the ARDFD control strategy can significantly eliminate adverse effects of the coupled subsystems.

1. Introduction

To achieve long-term continuous meteorological observations, 5-metre deployable antennas can be arranged in geosynchronous orbit [1, 2]. The reflector can significantly affect meteorological observation performance due to deformation of the antenna caused by external factors. To offset the impact of the environmental load on the antenna panel deformation, an active control strategy must be adopted, which can control the surface deformation down to microscale precision.

In this paper, a high precision active control method is studied to overcome the problem of antenna surface deformation on space reflecting antennas. An active adjustment system for a 5-metre deployable antenna is proposed, as shown in Figure 1. Because the reflecting panel is supported by the more active actuators, the supporting actuators interact with one another during the adjustment process. Therefore, it is necessary to develop decoupling control methods for complex multivariable systems, in particular, to realize high precision active adjustments of the reflecting panel of 5-meter deployable antennas.

In real-world multiple-input multiple-output (MIMO) systems, a variety of factors may cause many parameters to change at runtime. Traditional fuzzy theory can be directly applied to MIMO systems; however, due the large number of relationships, the “rule explosion” problem occurs. That is, the number of fuzzy rules increases exponentially with the number of fuzzy input variables [3]. In the last decade, a great deal of progress has been made in relation to the control of MIMO systems, and numerous control strategies have been introduced [4–7]. Decoupling controllers are based on nonlinear dynamic inverse and decentralized methods, but cannot guarantee robustness when certain system characteristics are unknown [8]. The automatic adjustment of a proportional–integral–derivative (PID) decoupling controller using a neural network was previously proposed; however, its precision for MIMO systems with strong coupling cannot be guaranteed [9]. Intelligent control methods for MIMO systems have been put forward combining fuzzy control theory...
with artificial intelligence and adaptive control [10–12]. However, such methods are only applicable to cases wherein the output is a state quantity. An adaptive fuzzy controller was previously applied to a multivariable system with uncertainties and demonstrated some practical value [13]. Interferences were approximated using a specifically designed disturbance observer, and a single precise controller was successfully implemented on a system of multiple variables. Efficient observers for the decentralized control of uncertain MIMO systems have also been studied [14, 15]. Tian et al. proposed a novel inverted fuzzy decoupling scheme [16]; moreover, the Smith predictor (SP) based on a fuzzy decoupling scheme was proposed for a MIMO chemical process with multiple time delays [17]. The latter two schemes were achieved using fuzzy logic, thereby avoiding the reliance on model-based analyses. Further to this, Hamdy et al. proposed another fuzzy decoupling-based PI controller [18]. Fuzzy decoupling schemes are simple to design and easy to implement and demonstrate good decoupling capabilities.

For the active adjustment system of the 5-meter deployable antenna, the relationship between the input and output is unclear and previously adopted control strategies cannot effectively deal with the coupling of variables and influence of uncertainty factors.

This paper presents an adaptive reduced-dimension fuzzy decoupling (ARDFD) control strategy based on a number of fuzzy observers. The efficiency of the method was demonstrated via simulations. Moreover, compared to the aforementioned controllers designed for MIMO systems, the ARDFD controller has the following advantages:

(i) MIMO systems can be decomposed into a number of single-input single-output (SISO) systems using the unit diagonal matrix decoupling method.

(ii) The dimension of multidimensional systems can be reduced using fusion function theory. Furthermore, the controller can be constructed by introducing a fuzzy decoupling disturbance observer.

(iii) The value of disturbances can be approximately obtained based on the Lyapunov stability and using the fuzzy observer. Further, disturbances are considered by the addition of compensation signals to the ARDFD controller, which eliminates the influence of uncertainties due to coupled subsystems.

2. Problem Statement

Twenty-five actuators are arranged on the 5-meter space deployable antenna reflector. The distribution of the actuators is shown in Figure 2, and each actuator is labelled with a number between 1–25.

For active adjustment system of the 5-meter deployable antenna, the positions of the zero-displacement datum of the 25 actuators were chosen to directly represent the system state.

According to [16], the mathematical model of the active adjustment mechanism can be described as follows:

\[ Z(k+1) = Z(k) + B \ast U(k), \]
\[ Y(k) = Z(k), \]  \hspace{2cm} (1)

where \( Z(k) \) denotes the displacement before the \( k \)th adjustment, \( U(k) \) is the external force of the \( k \)th adjustment, and \( B \) represents the transfer matrix of the external force and actual displacement.

Using existing control strategies to adjust the active adjustment system of the 5-meter deployable antenna, the following problems are encountered:

(i) Mapping relation is unclear between the input and output of each subsystem; therefore, strong coupling effects are caused.

(ii) The “rule explosion” problem occurs if traditional fuzzy methods are used for the active adjustment system of the 5-meter deployable antenna.

(iii) With existing control methods, it is difficult to determine the values of the coupling term and unknown disturbances.

To solve the above problems, an ARDFD control strategy based on the fusion function is proposed. An important aspect of the ARDFD control strategy is that the strongly coupled MIMO system can be deconstructed into 25 independent subsystems using the unit diagonal matrix method, thereby reducing the dimensions of the MIMO system and
solving the “rule explosion” problem. Moreover, by designing the fuzzy decoupling observer, estimated values of the disturbance terms can be obtained based on the input and output information. Then, estimated values can be incorporated into the decoupling controller by adding a compensation signal to eliminate the uncertainties due to the coupled subsystems.

3. Adaptive Reduced-Dimension Fuzzy Decoupling Controller Design

3.1. Adjustment System Decoupling. Based on (1), the discrete multivariable control model with uncertainties can be described as

\[ Z(k + 1) = Z(k) + (B + \Delta B) \ast U(k), \]
\[ Y(k) = Z(k), \]

(2)

where \( \Delta B \) represents the uncertainty of the system. In the active adjustment system, any changes in inputs will affect the outputs, because there is a strong coupling between the input and output. To optimize the control, (1) can be decoupled using the unit diagonal matrix decoupling method.

For any MIMO system with a full rank transfer matrix, a compensator can be designed to decouple it [17].

Assuming \( W(s) \) is the transfer function matrix of (1), then \( W(s) = C(SI − A)^{-1}B \), where \( C \) and \( A \) are unit diagonal matrices and \( W(s) \) is the full rank matrix. Thus, a compensator can be designed to decouple the MIMO system.

Based on the decoupling principle for a unit diagonal matrix, (3) can be written as

\[ W_p(s)W(s) = I, \]

(3)

where \( W_p(s) \) is the transfer function matrix of the feed-forward compensator.

According to (3), we can derive

\[ W_p(s) = W^{-1}(s) = B^{-1}(SI − A)^{-1}C^{-1}. \]

(4)

From (4), the output matrix of the feed-forward compensator is \( B^{-1}; \) thus, the output equation of the feed-forward compensator can be written as

\[ X(k) = B^{-1}Z(k). \]

(5)

Rearranging (5), we obtain

\[ Z(k) = BX(k). \]

(6)

Substituting (6) into (2), we obtain

\[ BX(k + 1) = BX(k) + (B + \Delta B) \ast U(k). \]

(7)

Then, multiplying both sides of (7) by \( B^{-1} \), the equation can be expressed as

\[ X(k + 1) = X(k) + U(k) + B^{-1}\Delta B \ast U(k), \]

(8)

where \( X(k) \) is the state vector of the decoupled system, \( X(k) = [x_1(k), x_2(k), \ldots, x_{25}(k)] \), and \( B^{-1}\Delta B \) is an unknown vector representing the unmodeled disturbance matrix of the MIMO system.

Here, assuming \( G = B^{-1}\Delta B \), (8) can be rewritten as

\[ X(k + 1) = X(k) + U(k) + GU(k), \]

(9)

where \( G = [G_{11}^T, G_{12}^T, \ldots, G_{25}^T]^T \).

From (9), it can be seen that the discrete multivariable decoupled system can be decomposed into 25 independent single variable state equations. The coupling effects caused by other subsystems and unknown terms are regarded as unknown quantities \( G \). Thus, the active adjustment coupling system of the 5-meter deployable antenna is decoupled.

Since the unknown disturbance \( G \) cannot be obtained, the disturbance observer can be constructed to estimate the unknown disturbance using the fuzzy strategy. Furthermore, since the active adjustment system of the 5-meter deployable antenna is a MIMO system in this case, directly adopting the traditional fuzzy control method leads to the “rule explosion” problem. Thus, the fuzzy rules grow exponentially with the number of fuzzy inputs, which is unfavorable for controlling the active adjustments of a coupled system. However, using the fusion function method, the input state variables can be merged to reduce the dimensions of the system.

3.2. Fusion Function Design. Controlling this type of MIMO system using multidimensional fuzzy control strategies is difficult; therefore, a multilevel control method is adopted. Fusion function theory can be used to convert the complex fuzzy control strategy into a more simplified nested control strategy [18, 19], as follows:

\[ X_i(k) \xrightarrow{f_1} \tilde{X}_i(k) \xrightarrow{f_2} Y. \]

Equation (10) shows that the multivariable system is preliminary processed by \( f_1() \). Then, based on the output, \( f_1() \) and \( f_2() \) control the active adjustment of the coupled system. If the dimension of the output vector of \( f_1() \) is less than the dimension of \( X_i(k) \), the problem has been simplified.

In this paper, the reduction in dimensions can be achieved by transforming the 25 state variables \( x_i(k) \) into 5 synthetic errors \( E_i \), as follows:

\[ E_1 = [k_1 k_2 k_3 k_4 k_5][x_1(k) x_2(k) x_3(k) x_4(k) x_5(k)]^T, \]
\[ E_2 = [k_6 k_7 k_8 k_9 k_{10}][x_6(k) x_7(k) x_8(k) x_9(k) x_{10}(k)]^T, \]
\[ E_3 = [k_{11} k_{12} k_{13} k_{14} k_{15}][x_{11}(k) x_{12}(k) x_{13}(k) x_{14}(k) x_{15}(k)]^T, \]
\[ E_4 = [k_{16} k_{17} k_{18} k_{19} k_{20}][x_{16}(k) x_{17}(k) x_{18}(k) x_{19}(k) x_{20}(k)]^T, \]
\[ E_5 = [k_{21} k_{22} k_{23} k_{24} k_{25}][x_{21}(k) x_{22}(k) x_{23}(k) x_{24}(k) x_{25}(k)]^T, \]

(11)

where \( k_1, k_2, \ldots, k_{25} \) represent the comprehensive coefficients.
A number of assumptions can be made:

\[ K_1 = [k_1 k_2 k_3 k_4 k_5], \]
\[ K_2 = [k_6 k_7 k_8 k_9 k_{10}], \]
\[ K_3 = [k_{11} k_{12} k_{13} k_{14} k_{15}], \]
\[ K_4 = [k_{16} k_{17} k_{18} k_{19} k_{20}], \]
\[ K_5 = [k_{21} k_{22} k_{23} k_{24} k_{25}], \]

\[ Y_1 = [x_1(k) x_2(k) x_3(k) x_4(k) x_5(k)]^T, \]
\[ Y_2 = [x_6(k) x_7(k) x_8(k) x_9(k) x_{10}(k)]^T, \]
\[ Y_3 = [x_{11}(k) x_{12}(k) x_{13}(k) x_{14}(k) x_{15}(k)]^T, \]
\[ Y_4 = [x_{16}(k) x_{17}(k) x_{18}(k) x_{19}(k) x_{20}(k)]^T, \]
\[ Y_5 = [x_{21}(k) x_{22}(k) x_{23}(k) x_{24}(k) x_{25}(k)]^T. \]

The output of the fusion function can then be expressed as

\[ f_1(x) = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \end{bmatrix} = \begin{bmatrix} K_1 & K_2 & K_3 & K_4 & K_5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \end{bmatrix} \tag{13} \]

By designing the feedback gain matrix using the linear quadratic regulator (LQR) method, the optimization index can be defined as

\[ J = \frac{1}{2} \int_0^\infty (x^T(k)Qx(k) + u^T(k)Ru(k)) \, dk, \tag{14} \]

where \( Q \) is a positive semidefinite matrix and \( R \) represents a symmetrical and positive definite matrix.

The Riccati equation can be solved as follows:

\[ -PA - AP + PBR^T B^T P - Q = 0, \tag{15} \]

where \( P \) is a positive definite matrix of constants.

Then, the feedback matrix can be obtained as

\[ K^T = R^{-1}B^TP[k_1 k_2 k_3 k_4 k_5 \cdots k_{25}] = \frac{K^T}{||K||}, \tag{16} \]

It can be seen that the linear combination reduces the dimensions of the MIMO system, which greatly reduces the number of fuzzy rules.

3.3. ARDFD Controller Design. In this section, the 1st decoupled subsystem is taken as an example to introduce the ARDFD controller. According to (9), the state equation of the 1st decoupled subsystem can be written as

\[ x_1(k + 1) = x_1(k) + u_1(k) + G_1U(k). \tag{17} \]

Here, we define the following set:

\[ G_1U(k) = d_1(k). \tag{18} \]

Substituting (18) into (17), the decoupled subsystem, which takes into account disturbances caused by other subsystems, can be further expressed as

\[ x_1(k + 1) = x_1(k) + u_1(k) + d_1(k), \quad d_1(k) = \sum_{j=1}^{25} G_{1j}u_j(k), \tag{19} \]

where \( d_1(k) \) are the disturbances due to the coupled subsystems and contains the uncertainty terms. In this paper, the \( d_1(k) \) values of the 5-meter deployable antenna are unknown. Equation (19) represents an SISO system with disturbances and a one-to-one relationship between the input and output of the 1st decoupled subsystem exists.

Based on fuzzy theory, the fuzzy decoupling disturbance observer can be constructed to obtain the approximate disturbance values of the 1st decoupled subsystem. Moreover, an adaptive law to adjust the consequent parameter based on fuzzy rules can be simultaneously obtained.

Since \( d_1(k) \) is a nonlinear function of arbitrary accuracy, the real matrix \( \theta^*_1(k) \) exists [20, 21] and can be defined as

\[ d_1(k) = \theta^*_1(k)\xi(k) + \varepsilon, \tag{20} \]

where \( \theta^*_1(k) \) is the consequent parameter matrix of the fuzzy rules, \( \varepsilon \) represents a nonnegative constant, \( 1 > \varepsilon > 0 \), and \( \xi(k) \) is the basis function of the fuzzy rules.

Subsequently, the fuzzy disturbance observer can be constructed, and the estimated values of the disturbances can be obtained.

Suppose the 1st fuzzy rule is given as follows:

if \( x_1(k) \) is \( A_1^1 \) and \( x_2(k) \) is \( A_2^1 \) and \( \cdots \) and \( x_5(k) \) is \( A_5^1 \)
then \( \hat{d}_1(k) = d_1^1(k) \).

(21)

Then, the output of the fuzzy system can be expressed as

\[ \hat{d}_1(k) = \frac{\sum_{i=1}^{r} \hat{d}_1^i(k) \prod_{l=1}^{5} \mu_{A_l^i}(x_l(k))}{\sum_{i=1}^{r} \prod_{l=1}^{5} \mu_{A_l^i}(x_l(k))}, \tag{22} \]

where \( r \) is the number of fuzzy rules and \( \mu_{A_l^i} \) represents the membership functions of the input variables.
Now, suppose the basic function $\xi_i(k)$ of the $i$th fuzzy rule is expressed as

$$\xi_i(k) = \frac{\prod_{j=1}^{\gamma} \mu_{j_i}(x_i(k))}{\sum_{i=1}^{\gamma} \prod_{j=1}^{\gamma} \mu_{j_i}(x_i(k))}. \quad (23)$$

Then, the basic function vector of the fuzzy rules can be defined as

$$\xi(k) = [\xi_1(k), \xi_2(k), \ldots, \xi_\gamma(k)]^T. \quad (24)$$

Suppose the fuzzy posterior parameter matrix is

$$\theta_i(k) = [a_{i1}^T(k), a_{i2}^T(k), \ldots, a_{i\gamma}^T(k)]^T. \quad (25)$$

Based on (24) and (25), the output of the fuzzy system can be defined as

$$\hat{d}_i(k) = \theta_i^T(k)\xi(k). \quad (26)$$

Then, the fuzzy disturbance observer for the 1st decoupled subsystem can be defined as

$$\sigma_1(k + 1) = \sigma_1(k) + u_1(k) - \gamma_1 \sigma_1(k) + \gamma_1 x_1(k) + \hat{d}_1(k),$$

$$\hat{d}_1(k) = \theta_1^T(k)\xi(k), \quad (27)$$

where $\hat{d}_i(k)$ is the estimated value of the disturbance, $\gamma_i$ represents a parameter of the fuzzy interference term, and $\sigma_1(k)$ is the state of the fuzzy disturbance observer.

According to $\sigma_1(k)$ and $x_1(k)$, the observation error can be defined as

$$e_1(k) = x_1(k) - \sigma_1(k). \quad (28)$$

The error between the fuzzy rule posterior parameter matrix and the optimal fuzzy posterior parameter matrix can be expressed as

$$\hat{\theta}_1(k) = \theta_1^*(k) - \theta_1(k). \quad (29)$$

Furthermore, the control law based on the fuzzy disturbance observer can be designed as

$$u_1(k) = -hx_1(k) - \hat{d}_1(k), \quad (30)$$

where $h$ is the feedback control law parameter.

Substituting (30) into (19), the 1st decoupled subsystem can be rewritten as

$$x_1(k + 1) = x_1(k) - hx_1(k) - \hat{d}_1(k) + d_1(k)$$

$$= (1 - h)x_1(k) - \hat{d}_1(k) + d_1(k). \quad (31)$$

Then, substituting (20) and (26) into (30), the equation can be rewritten as

$$x_1(k + 1) = x_1(k) - hx_1(k) - \hat{d}_1(k) + d_1(k)$$

$$= (1 - h)x_1(k) - \theta_1^T(k)\xi(k) + \theta_1^T(k)\xi(k) \quad (32)$$

$$= (1 - h)x_1(k) + \hat{\theta}_1^T(k)\xi(k).$$

According to (17), (18), and (27), the observation error equation of an unknown disturbance can be represented as

$$e_1(k + 1) = e_1(k) - \gamma_1 e_1(k) + \hat{\theta}_1^T(k)\xi(k). \quad (33)$$

Then, based on (19), (27), and (30) for the 1st decoupled subsystem, the adaptive law of the fuzzy post parameter matrix can be written as

$$\theta_1(k + 1) - \theta_1(k) = \rho_1 x_1(k)\hat{\xi}_1(k) + \rho_1 x_1(k)\xi_1(k), \quad (34)$$

where $\rho_1$ is a positive constant, $\rho_1$ is the adaptive parameter, and $\rho_1 > 0$.

Now, the closed-loop model of the 1st decoupled subsystem can be constructed, as shown in Figure 3.

Therefore, the fuzzy system can be used to obtain approximate values of the disturbances based on the input and output information, which can be added to the decoupling controller as a compensation signal to eliminate the uncertainties due to the coupled subsystems.

Similarly, control laws for the other 24 decoupled subsystems can be defined and the fuzzy disturbance estimator can easily be obtained for each.

4. Stability Analysis

For the mathematical model expressed in (1), the fuzzy disturbance observer of (27) and the control law based on the fuzzy disturbance observer presented as (29), when $k = 1, 2, \ldots, n$, the error of the fuzzy disturbance observer and the error between the fuzzy rule posterior parameter matrix and optimal fuzzy posterior parameter matrix converge uniformly with discrete time $k$. 

![Figure 3: The closed-loop model of the 1st decoupled subsystem.](image-url)
The Lyapunov candidate function is defined as
\[ y(k) = \frac{1}{2} \rho_1 |x_i(k)|^2 + \frac{1}{2} e_i(k)^2 + \frac{1}{2\rho_1} \tilde{\theta}_i^T(k)\tilde{\theta}_i(k), \tag{35} \]
where \( \rho_1 > 0, \rho_1 > 0, y(k) > 0, \) and \( y(k) \) is a positive definite matrix.

According to (35),
\[ \Delta y(k) = y(k+1) - y(k). \tag{36} \]

Substituting (35) into (36), we obtain
\[ \Delta y(k) = \frac{1}{2} \rho_1 |x_i(k+1)|^2 - \frac{1}{2} \rho_1 |x_i(k)|^2 \]
\[ + \frac{1}{2} |e_i(k+1)|^2 + \frac{1}{2} \tilde{\theta}_i^T(k+1)\tilde{\theta}_i(k+1) \]
\[ - \frac{1}{2} \rho_1 |x_i(k)|^2 - \frac{1}{2} |e_i(k)|^2 - \frac{1}{2\rho_1} \tilde{\theta}_i^T(k)\tilde{\theta}_i(k). \tag{37} \]

Based on (32), the following can be derived:
\[ \frac{1}{2} \rho_1 |x_i(k+1)|^2 - \frac{1}{2} \rho_1 |x_i(k)|^2 \]
\[ = p_1 x_i(k)\tilde{\theta}_i^T(k)\tilde{\theta}_i(k) \]
\[ + \frac{1}{2} \rho_1 \left\{ (h^2 - 2h)|x_i(k)|^2 - 2hx_i(k)\tilde{\theta}_i^T(k)\xi_i(k) \right\} \]
\[ + \left[ \tilde{\theta}_i^T(k)\xi_i(k) \right]^2 \]
\[ = p_1 x_i(k)\tilde{\theta}_i^T(k)\tilde{\theta}_i(k) + \frac{1}{2} \rho_1 \left[ x_i(k) \right] \]
\[ \cdot \left[ \begin{array}{c} h^2 - 2h \\ -h \end{array} \right] \]
\[ \cdot \left[ \begin{array}{c} x_i(k) \\ -h \end{array} \right]. \tag{38} \]

According to (33), the following can be derived:
\[ \frac{1}{2} |e_i(k+1)|^2 - \frac{1}{2} |e_i(k)|^2 \]
\[ = e_i(k)\tilde{\theta}_i^T(k)\tilde{\theta}_i(k) \]
\[ + \frac{1}{2} \left\{ (\gamma^2 - 2\gamma)|x_i(k)|^2 - 2\gamma x_i(k)\tilde{\theta}_i^T(k)\xi_i(k) + \left[ \tilde{\theta}_i^T(k)\xi_i(k) \right]^2 \right\} \]
\[ = e_i(k)\tilde{\theta}_i^T(k)\tilde{\theta}_i(k) + \frac{1}{2} \left[ \begin{array}{c} e_i(k) \\ \tilde{\theta}_i^T(k)\tilde{\theta}_i(k) \end{array} \right] \]
\[ \cdot \left[ \begin{array}{c} \gamma^2 - \gamma \\ -\gamma \end{array} \right] \]
\[ \cdot \left[ \begin{array}{c} e_i(k) \\ \tilde{\theta}_i^T(k)\tilde{\theta}_i(k) \end{array} \right]. \tag{39} \]

Furthermore, based on (33), we can derive the following:
\[ \frac{1}{2\rho_1} \tilde{\theta}_i^T(k+1)\tilde{\theta}_i(k+1) - \frac{1}{2\rho_1} \tilde{\theta}_i^T(k)\tilde{\theta}_i(k) \]
\[ = \frac{1}{2\rho_1} \left[ 2\tilde{\theta}_i(k) - \rho_1 \Delta \right] \cdot \left[ -\rho_1 \Delta \right] \tag{40} \]
\[ = -\tilde{\theta}_i^T(k) + \frac{\rho_1}{2} \Delta^2 \Delta, \]

where
\[ \Delta = x_i(k)p_1\xi_i(k) + e_i(k)\xi_i(k). \tag{41} \]

Substituting (38), (39), and (40) into (37), the equation can be rewritten as
\[ \Delta y(k) = \frac{1}{2} \rho_1 \left[ \begin{array}{c} x_i(k) \\ \tilde{\theta}_i^T(k)\tilde{\theta}_i(k) \end{array} \right] \]
\[ \cdot \left[ \begin{array}{c} h^2 - 2h \\ -h \end{array} \right] \]
\[ \cdot \left[ \begin{array}{c} x_i(k) \\ -h \end{array} \right]. \tag{42} \]

In the case of \( \rho_1 > 0, \rho_1 > 0, y_i > 0, \) we have
\[ \frac{1}{2} \rho_1 \left[ \begin{array}{c} h^2 - 2h \\ -h \end{array} \right] = -h\rho_1 < 0, \tag{43} \]
\[ \frac{1}{2} \rho_1 \left[ \begin{array}{c} \gamma^2 - \gamma \\ -\gamma \end{array} \right] = -\gamma_i < 0. \]

Then, \( \Delta y(k) < 0. \)

According to the Lyapunov stability theorem of discrete systems, \( x_i(k), e_i(k), \) and \( \tilde{\theta}_i(k) \) uniformly converge with discrete time \( k. \) In a similar way, the stability analysis of the other 24 decoupled subsystems can be performed.

5. Simulation Verification

To test the validity of the ARDFD controller, a simulation analysis was performed. During the simulation, control matrix \( B \) was obtained using finite element software ANSYS (version 14.5).
\[ B = \left[ \begin{array}{c} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{array} \right]. \tag{44} \]
where

\[
B_1 = \begin{bmatrix}
90.775 & 0.907 & 4.138 & -1.646 & 0.616 \\
0.907 & 90.740 & 0.613 & 0.029 & 4.123 \\
4.138 & 0.613 & 6.313 & 0.005 & 0.280 \\
-1.643 & 0.032 & 0.008 & 3.647 & -0.002 \\
\end{bmatrix}
\]

\[
B_2 = \begin{bmatrix}
5.088 & 174.390 & 10.081 & 9.752 & 96.525 \\
4.138 & 0.613 & 6.313 & 0.005 & 0.280 \\
-1.643 & 0.032 & 0.008 & 3.647 & -0.002 \\
\end{bmatrix}
\]

\[
B_3 = \begin{bmatrix}
0.773 & 0.184 & 0.003 & -0.002 & -0.031 \\
0.187 & 115.940 & 0.002 & -1.924 & 0.047 \\
0.003 & -0.015 & 0.772 & 0.185 & 0.030 \\
0.002 & -1.923 & 0.189 & 115.930 & 0.049 \\
0.032 & 0.046 & 0.032 & 0.048 & 29.912 \\
\end{bmatrix}
\]

\[
B_4 = \begin{bmatrix}
25.795 & -0.004 & 0.059 & 0.017 & 0.174 \\
-0.009 & 25.683 & -0.012 & 0.058 & 0.175 \\
0.054 & 0.013 & 21.254 & -0.389 & -0.022 \\
0.017 & 0.064 & -0.384 & 21.260 & 0.028 \\
0.169 & 0.176 & 0.023 & 0.024 & 1.114 \\
\end{bmatrix}
\]

\[
B_5 = \begin{bmatrix}
11.516 & 17.269 & -0.115 & -0.985 & 0.341 \\
17.268 & 54.711 & -0.518 & -2.334 & 0.867 \\
-0.112 & -0.514 & 76.493 & -0.500 & 0.160 \\
-0.980 & -2.310 & -0.500 & 440.780 & -8.235 \\
0.340 & 0.870 & 0.160 & -8.239 & 286.420 \\
\end{bmatrix}
\]

The initial value \( Z(k) \) of the discrete state vector of the 5-meter deployable antenna adjustment system is

\[
Z(k) = [-1.141, 12.668, 12.656, -3.987, 4.032, 8.982, 0.676, -3.183, -12.143, -11.927, 0.297, 0.045, 0.465, -9.303, -14.889, 2.119, 2.003, 0.194, 11.079, 0.631, 1.194, 1.094, -9.187, 3.839, 2.257]^T.
\]

Units are in \( \mu \)m.

First, the decomposed state vector \( X(k) \) was given based on (5) and (6).

Then, the semipositive definite matrix was designed according to the actual requirements as follows:

\[
Q = \text{diag} [80, 200, 100, 80, 150, 50, 200, 100, 500, 200, 100, 300, 200, 200, 150, 500, 200, 100, 150, 100, 500, 200, 100, 300].
\]

The symmetric matrix is \( R = 1 \). Further, the state feedback matrix \( K \) was obtained as

\[
K = [10, 5.58, 11.05, 10, 13.29, 8.14, 15.18, 11.05, 3018.35, 15.18, 15.18, 13.29, 23.38, 15.18, 15.18, 13.29, 11.05, 23.38, 15.18, 18.34, 13.28, 11.04, 23.38, 15.18, 18.35].
\]

The comprehensive coefficient was then obtained based on (16) as follows:

\[
K_1 = [k_1, k_2, k_3, k_4, k_5] = [0.132, 0.074, 0.146, 0.132, 0.175],
\]

\[
K_2 = [k_6, k_7, k_8, k_9, k_{10}] = [0.107, 0.201, 0.146, 0.242, 0.2001],
\]

\[
K_3 = [k_{11}, k_{12}, k_{13}, k_{14}, k_{15}] = [0.2001, 0.175, 0.308, 0.2001, 0.2001],
\]

\[
K_4 = [k_{16}, k_{17}, k_{18}, k_{19}, k_{20}] = [0.175, 0.146, 0.308, 0.2001, 0.242],
\]

\[
K_5 = [k_{21}, k_{22}, k_{23}, k_{24}, k_{25}] = [0.175, 0.146, 0.308, 0.2001, 0.242].
\]

Based on fusion theory, the dimensions of \( X(k) \) were reduced to

\[
X(k) = \begin{bmatrix}
Y_1 \\
Y_2 \\
Y_3 \\
Y_4 \\
Y_5
\end{bmatrix},
\]

where

\[
Y_1 = [-0.1911, 0.1311, 4.0682, -2.4169, 0.2832]^T,
\]

\[
Y_2 = [0.0399, 0.0366, -0.0467, -0.0584, 0.0189]^T,
\]

\[
Y_3 = [0.1054, 0.1334, 0.7029, -0.044, -0.5334]^T,
\]

\[
Y_4 = [-0.0338, 0.162, -0.3691, 0.4334, 0.9486]^T,
\]

\[
Y_5 = [0.0604, -0.0098, 0.0272, -0.0805, -0.0541]^T.
\]
The uncertainty matrix $\Delta B$ was randomly generated in the range of $\pm 5\%$ based on the 2-norm of control matrix $B$, defined as

$$
\Delta B = \begin{bmatrix}
\Delta B_1 \\
\Delta B_2 \\
\Delta B_3 \\
\Delta B_4 \\
\Delta B_5 
\end{bmatrix},
$$

(52)

where

$$
\Delta B_1 = \begin{bmatrix}
0.057 & 0.046 & -0.040 & -0.027 & -0.061 \\
0.073 & 0.044 & 0.032 & -0.055 & 0.053 \\
-0.067 & -0.019 & 0.028 & -0.045 & -0.034 \\
0.074 & 0.028 & -0.061 & 0.021 & 0.005 \\
0.024 & -0.059 & -0.069 & -0.005 & -0.060 
\end{bmatrix},
$$

$$
\Delta B_2 = \begin{bmatrix}
0.074 & -0.029 & -0.037 & 0.068 & -0.043 \\
-0.057 & 0.072 & 0.044 & 0.009 & 0.019 \\
-0.043 & -0.024 & -0.056 & 0.022 & 0.038 \\
0.074 & 0.028 & -0.061 & 0.021 & 0.005 \\
-0.064 & -0.070 & 0.034 & 0.016 & -0.050 
\end{bmatrix},
$$

$$
\Delta B_3 = \begin{bmatrix}
-0.083 & 0.020 & 0.029 & -0.071 & -0.084 \\
0.069 & 0.021 & 0.003 & -0.023 & 0.011 \\
0.074 & 0.065 & 0.085 & -0.054 & 0.069 \\
0.053 & 0.055 & 0.027 & -0.002 & 0.030 \\
-0.072 & 0.014 & 0.054 & -0.029 & -0.056 
\end{bmatrix},
$$

$$
\Delta B_4 = \begin{bmatrix}
0.073 & 0.084 & -0.044 & 0.019 & -0.007 \\
0.068 & 0.006 & -0.050 & -0.055 & 0.029 \\
-0.043 & -0.071 & 0.062 & -0.046 & -0.027 \\
0.017 & 0.020 & -0.028 & 0.075 & 0.029 \\
-0.017 & 0.046 & 0.052 & -0.001 & 0.043 \\
-0.009 & -0.046 & 0.081 & -0.056 & 0.015 \\
-0.067 & -0.019 & 0.028 & -0.045 & -0.034 \\
0.074 & 0.034 & 0.031 & -0.063 & 0.030 \\
0.023 & -0.025 & -0.011 & -0.080 & -0.075 
\end{bmatrix},
$$

(53)

The deformation was adjusted by the LQR controller, and the adjustment curve is presented in Figure 4. After adding the uncertainty matrix to the actuator, the displacement control was adjusted by LQR controller according to the curve shown in Figure 4. Uncertainty led to excessive overshooting of the system, which caused the curve converge.

Based on the same amount of interference, the 1st actuator was adjusted by the ARDFD controller, as shown in Figure 5. To assess the ability of the ARDFD controller to deal with the uncertainty of an observer, we have the following constraints on the parameters. The parameters of the fuzzy disturbances $\gamma_i \in [0.3, 1.2]$, $(i = 1, 2, 3, \ldots, 25)$, the adaptive algorithm parameters of the fuzzy observers $\rho_i = 0.01$, $p_i = 1$, $(i = 1, 2, 3, \ldots, 25)$, the parameters of the feedback control laws $h_i \in [0.6, 1.0]$, $(i = 1, 2, 3, \ldots, 25)$. Specific parameters can be adjusted according to actual simulation. The parameters of the fuzzy disturbances for the 1st to 25th decoupled subsystems were analyzed.

$$
\gamma = [\gamma_1 \gamma_2 \gamma_3 \cdots \gamma_{25}] = [0.4 \ 0.6 \ 0.3 \ 0.6 \ 1.2 \ 0.4 \ 0.6 \ 0.3 \ 0.6 \ 1.2 \ 0.4 \ 0.6 \ 0.3 \ 0.6 \ 1.2 \ 0.4 \ 0.6 \ 0.3 \ 0.6 \ 1.2]^{T}. 
$$

(54)
Adaptive algorithm parameters of the fuzzy observers were derived:

\[
\rho = [\rho_1 \rho_2 \rho_3 \cdots \rho_{25}]
= [0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01],
\]

\[
P = [p_1 p_2 p_3 \cdots p_{25}]
= [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1],
\]

Finally, parameters of the feedback control laws were obtained:

\[
h = [h_1 \ h_2 \ h_3 \cdots h_{25}]
= [0.8 \ 0.7 \ 0.7 \ 1.0 \ 0.6 \ 0.8 \ 0.7 \ 0.7 \ 1.0 \ 0.6 \ 0.8 \ 0.7 \ 0.7 \ 1.0 \ 0.6 \ 0.8 \ 0.7 \ 0.7 \ 1.0 \ 0.6]^{T}.
\]

As seen in Figure 5, when the uncertainty matrix \(\Delta B\) is added to the 1st actuator, the curves do not show large fluctuations with the ARDFD controller and the system adjustments are very small. At the 10th step, the system reaches stability. Because the ARDFD controller uses a fuzzy system to realize the disturbance observer and estimate the uncertainty values of each coupled subsystem, the system can be controlled with high precision.

The root mean square (RMS) error curve for the 25 actuators is presented in Figure 6. The RMS value is 0.124 mm before the adjustment. After adjustment using the ARDFD controller, the displacements of all adjustment actuators are below \(10^{-3}\) mm after the 45th adjustment. Accuracy is shown to be in the micron scale and demonstrates a tendency to narrow.

### 6. Conclusions

In this paper, the ARDFD controller for the active deformation adjustment mechanism of a 5-meter deployable antenna panel was studied. First, based on the unit diagonal matrix decoupling method, the MIMO system is decomposed into a number of SISO systems. Then, the ARDFD controller is constructed by introducing the fuzzy decoupling disturbance observer and using fusion function theory. Finally, the results of the simulation demonstrate that the proposed method for active deformation adjustment of a 5-meter deployable antenna panel is effective.

Furthermore, once the rule structure of the fuzzy system is defined during the design stage, it cannot be modified. Future research will consider a self-organized fuzzy system to improve the approximation capabilities of fuzzy systems.

### Data Availability

The authors confirm that the data used to support the findings of this study are included within the article. All the data supporting the results were shown in the paper and can be applicable from the corresponding author.

### Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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