Research Article

The Influence of Orbital Element Error on Satellite Coverage Calculation

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This paper studies the influence of orbital element error on coverage calculation of a satellite. In order to present the influence, an analysis method based on the position uncertainty of the satellite shown by an error ellipsoid is proposed. In this error ellipsoid, positions surrounding the center of the error ellipsoid mean different positioning possibilities which present three-dimensional normal distribution. The possible subaerial points of the satellite are obtained by sampling enough points on the surface of the error ellipsoid and projecting them on Earth. Then, analysis cases are implemented based on these projected subaerial points. Finally, a comparison report of coverage calculation between considering and not considering the error of orbital elements is given in the case results.

1. Introduction

The calculation of satellite and constellation coverage of ground targets is a basic problem of Earth observation systems. Various kinds of satellites and constellations are available and selectable according to different requirements of mission design. As the basic purposes of constellation design, continuous coverage and maximum regional coverage are often considered. Hence, many proposed methods were devoted to how to optimize these two goals. Cote [1] offered tips on how to maximize satellite coverage while cruising in the high-latitude West Coast waters of British Columbia. Yung and Chang [2] proposed a method for maximizing satellite coverage at predetermined local times for a set of predetermined geographic locations. Draim [3] described a new four-satellite elliptic orbit constellation giving continuous line-of-sight coverage of every point on Earth's surface.

During the implementation of a mission, the visible time windows of important targets are also usually considered. For example, Lian et al. [4] proposed two static and dynamic models of observation toward Earth by agile satellite coverage

to decompose area targets into small pieces and compute visible time window subarea targets. Wang et al. [5] analyzed four parameters to determine the coverage area of satellite. It also introduced and compared two current test methods of satellite area, which presents the composition of the required test equipment and test processes, and proved the effectiveness of the tests through engineering practices. Besides the numerical analysis methods, heuristic search algorithms such as particle swarm optimization [6] were also applied to the satellite coverage calculation. Huang and Dai [7] proposed an optimal design of constellation of multiple regional coverage based on NSGA-II with the aid of multiobjective optimization.

The error influence of orbital elements on the satellite positioning was mainly considered in the field GIS and radar location [8–10]. Analysis methods proposed in these papers usually employed a model that can transform the possible position of a satellite to the points in an error ellipsoid. Nelson [11] introduced and discussed some basic properties of the error ellipsoid. For example, the basic parameters of the error ellipsoid were depicted by the axial direction and the axial ratio among each axis. In the study of satellite

positioning error, the error ellipsoid was used in the positioning accuracy analysis of global navigation satellite system [12]. Some papers [13–15] introduced that the axis ratio of error ellipsoid for GPS should be controlled within a certain range. Wang et al. [16] selected eight sites to study the different shapes of the error ellipsoid in the global range. Alfano [17] showed the effects of positional uncertainty on the Gaussian probability computation for orbit conjunction. A model of position uncertainty for the TIS-B System was proposed [18]. Shi et al. [19] introduced a distributed location model and algorithm with position uncertainty. Trifonov [20] introduced position uncertainty measures for a particle on the sphere.

However, for coverage calculation, the influence of the orbital element error has not been considered, which is the research purpose of this paper. Because only the error influence of initial orbital elements on coverage calculation is studied in this paper, hence, the influence of other variables, such as perturbation, are not considered here. Because of the nonlinear function between satellite position and orbital elements, assuming the error of orbital elements presents normal distribution, therefore, the uncertainty of the satellite position will also present normal distribution in the three-dimensional space. The relationship between the error of satellite position and orbital element error can be obtained according to the knowledge of error propagation and the function relationship between the satellite position and orbital elements, which is the basis of coverage uncertainty analysis.

The rest of this paper is organized as follows. In Section 2, the proposed method is introduced. In Section 3, preparations for cases are given. In Section 4, five analysis cases are implemented. Section 5 is devoted to the analysis of experimental results and summary of this paper. Section 6 gives some conclusions.

2. Coverage Uncertainty Related to Orbital Elements

The uncertainty analysis method proposed in this paper includes four steps, which are summarized as follows:

- Step 1: obtain the position vector covariance matrix based on the given orbital element error matrix and transformation matrix.
- Step 2: calculate the eigenvalues and eigenvectors of the position vector covariance matrix. The eigenvalue is the axial length of the error ellipsoid, and the eigenvector is the axial direction of the error ellipsoid. Then, perform the rotation and translation operations on the error ellipsoid.
- Step 3: extract the edge points of the error ellipsoid projection on the surface of Earth.
- Step 4: obtain the coverage analysis results to ground targets based on the extracted edge points.

In the above four steps, the implementations of steps 1 and 2 are to obtain the necessary parameters about error ellipsoid. Steps 3 and 4 are utilized to project all the points of error

ellipsoid surface on Earth for the further coverage calculation. Process simulation presented in Figure 1 shows that each extracted point of the projection edge is regarded as the worst position of satellite positioning, and all the analysis work are implemented based on these points. The method employed to extract the edge points is a convex hull algorithm shown in Table 1. The analysis method mentioned in step 4 is introduced in Table 2 with more details. Sections 2.1 and 2.2 describe the specific process of obtaining an error ellipsoid.

2.1. Obtain the Covariance Matrix of the Satellite Position. Considering the nonlinear relationship between the satellite position and orbital elements, the covariance matrix and propagation matrix between orbital elements and position uncertainty can only be obtained by solving partial derivative, which can be shown by

$$Dz z_{mm} = k D x x_{nn} k^T, \quad (1)$$

where $Dz z_{mm}$ is an $m * m$ square matrix. In this paper, $Dz z_{mm}$ represents the covariance matrix of satellite position in directions X , Y , and Z . $D x x_{nn}$ is the error matrix of orbital elements.

When z is a nonlinear function of x , such as

$$\begin{aligned} z_1 &= f_1(x_1, x_2, \dots, x_n), \\ z_2 &= f_2(x_1, x_2, \dots, x_n), \\ &\vdots \\ z_m &= f_m(x_1, x_2, \dots, x_n), \end{aligned} \quad (2)$$

the partial derivative of z by partial derivative of x needs to be obtained by the following method:

$$\begin{aligned} dz_1 &= \frac{\partial f_1}{\partial x_1} dx_1 + \frac{\partial f_1}{\partial x_2} dx_2 + \dots + \frac{\partial f_1}{\partial x_n} dx_n, \\ dz_2 &= \frac{\partial f_2}{\partial x_1} dx_1 + \frac{\partial f_2}{\partial x_2} dx_2 + \dots + \frac{\partial f_2}{\partial x_n} dx_n, \\ &\vdots \\ dz_m &= \frac{\partial f_m}{\partial x_1} dx_1 + \frac{\partial f_m}{\partial x_2} dx_2 + \dots + \frac{\partial f_m}{\partial x_n} dx_n. \end{aligned} \quad (3)$$

Then, (5) can be obtained based on (3) and represented by

$$\begin{aligned} z_{m1} &= \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_m \end{pmatrix}, \\ dz_{m1} &= \begin{pmatrix} dz_1 \\ dz_2 \\ \vdots \\ dz_m \end{pmatrix}, \end{aligned} \quad (4)$$

$$k_{mn} = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} \frac{\partial f_1}{\partial x_2} & \dots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} \frac{\partial f_2}{\partial x_2} & \dots & \frac{\partial f_2}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial f_t}{\partial x_1} \frac{\partial f_t}{\partial x_2} & \dots & \frac{\partial f_t}{\partial x_n} \end{pmatrix}. \quad (5)$$

The nonlinear function between the satellite position vector and orbital elements is shown in (6), (7), and (8), where a , e , E , i , Ω , and ω are orbital elements and P and Q are functions of Ω , ω , and i . Vector r is decomposed into three vectors in the directions x , y , and z , and in each direction, the partial derivatives of orbital elements can be obtained. For example, the partial derivative of z by that of a , e , E , i , Ω , and ω can be presented in (9). Similarly, gaining the partial derivatives of x and y by that of a , e , E , i , Ω , and ω , respectively, can finally obtain the error propagation matrix.

$$r = a(\cos E - e) \cdot P + a\sqrt{1-e^2} \sin E \cdot Q, \quad (6)$$

$$P = \begin{pmatrix} \cos \Omega \cos \omega - \sin \Omega \sin \omega \cos i \\ \sin \Omega \cos \omega + \cos \Omega \sin \omega \cos i \\ \sin \omega \sin i \end{pmatrix}, \quad (7)$$

$$Q = \begin{pmatrix} -\cos \Omega \sin \omega - \sin \Omega \cos \omega \cos i \\ -\sin \Omega \sin \omega + \cos \Omega \cos \omega \cos i \\ \cos \omega \sin i \end{pmatrix}, \quad (8)$$

$$\begin{aligned} \frac{\partial z}{\partial a} &= \sin E \cos \Omega \sqrt{1-e^2} - \sin \Omega \sin i (e - \cos E), \\ \frac{\partial z}{\partial e} &= a \sin \Omega \sin i - \frac{ae \sin E \cos \Omega \sin i}{\sqrt{1-e^2}}, \\ \frac{\partial z}{\partial E} &= a \cos E \cos \Omega \sin i \sqrt{1-e^2} - a \sin E \sin \Omega \sin i, \\ \frac{\partial z}{\partial i} &= a \sin E \cos \Omega \sqrt{1-e^2} - a \cos i \sin \Omega (e - \cos E), \\ \frac{\partial z}{\partial \Omega} &= -a \cos \Omega \sin i (e - \cos E) - a \sin E \sin \Omega \sin i \sqrt{1-e^2}, \\ \frac{\partial z}{\partial \omega} &= 0. \end{aligned} \quad (9)$$

2.2. Rotation and Translation of the Error Ellipsoid. Assume that the major axis direction of error ellipsoid is

$$r = (x_m, y_m, z_m). \quad (10)$$

So the rotation angle of major axis around axis z should be

$$\theta_z = \arctan \left(\frac{y_m}{x_m} \right). \quad (11)$$

Then, the rotation angle of major axis around axis y is

$$\theta_y = \arcsin \left(\frac{|z_m|}{r_m} \right). \quad (12)$$

Subsequently, all points need to be rotated around axis x . Based on the previous rotations, the third rotation angle is equal to the angle between axis y and middle axis of error ellipsoid.

Finally, if the standard position of satellite is (x, y, z) , the point (x_0, y_0, z_0) needs to be translated to a new position $(x_0 + x, y_0 + y, z_0 + z)$.

2.3. Feature of the Error Ellipsoid. It is assumed that the measurement errors of six orbital elements present normal distribution. So the position error of satellite also presents three-dimensional normal distribution [5, 16]. The distribution density of possible satellite position in three-dimensional space is

$$\frac{\exp(-0.5(r^T D_{zz}^{-1} r))}{(2\pi)^{1.5} \sqrt{\Delta}}, \quad (13)$$

where r is the position increment.

The denominator of (13) is a constant, while the numerator is changeable. So all points with the same distribution density can be presented by

$$r^T D_{zz}^{-1} r = k^2. \quad (14)$$

Equation (14) is a presentation of ellipsoid cluster, and k is a magnification factor. If r is symbolized by

$$r = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}, \quad (15)$$

(14) can also be shown by

$$\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}^T \begin{bmatrix} \sigma_{xx}^2 & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy}^2 & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz}^2 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} = k^2. \quad (16)$$

Because it is not convenient to show the ellipsoid in coordinate system $OXYZ$, the principal axis coordinate system $OUVW$ is employed to present it. For a real symmetric matrix D_{zz} , an orthogonal matrix T can be utilized to make

$$\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} = T \begin{bmatrix} U \\ V \\ W \end{bmatrix}, \quad (17)$$

FIGURE 1: Simulation of uncertainty analysis process.

TABLE 1: Convex hull algorithm.

Input: Planar points set p ; Output: convex hull $\#(p)$ of points set p ; clockwise queue of $\#(p)$.
Step 1: Find point p_0 with the minimum ordinate in p . If more than one point meets this requirement, select the point with minimum horizontal ordinate.
Step 2: Order all the points according to the polar angle relative to p_0 . If more than one point locates on the same direction, retain the farthest one.
Step 3: Start from the minimum polar angle, and check whether the points in this direction can be contained in $\#(p)$. If not, put this point to $\#(upper)$ and update it.
Step 4: Repeat step 3 to all the points have been checked.

TABLE 2: Method of analyzing error influence.

Step 1: Find two points p_{l1} and p_{l2} from $\#(p)$ with the longest distance. Find two other points p_{s1} and p_{s2} which are perpendicular to the line $p_{l1}p_{l2}$.
Step 2: If The target is a point.
Step 3: Obtain the calculation result based on the points gained in step 1.
Step 4: If The target cannot be covered when subastral points are p_{s1} and p_{s2} .
Step 5: Draw the conclusion: This target cannot be covered completely.
Step 6: Else If The target cannot be covered when subastral points are p_{s1} and p_{s2} .
Step 7: Draw the conclusion: This target may be covered possibly.
Step 8: Else Draw the conclusion: This target can be covered completely.
Step 9: ELSE Obtain the calculation result based on the points gained in step 1.
Step 10: If The target cannot be covered when subastral points are p_{s1} and p_{s2} .
Step 11: Draw the conclusion: This target cannot be covered completely.
Step 12: Else If The target cannot be covered when subastral points are p_{l1} and p_{l2} .
Step 13: Obtain the least percentage that the area target can be covered.
Step 14: Else Draw the conclusion: This target can be covered completely.

and

$$T^T D z z T = \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{bmatrix}. \quad (18)$$

According to (18), (15) can be shown by

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix}^T \begin{bmatrix} \frac{1}{\lambda_1} & & \\ & \frac{1}{\lambda_2} & \\ & & \frac{1}{\lambda_3} \end{bmatrix} \begin{bmatrix} U \\ V \\ W \end{bmatrix} = k^2. \quad (19)$$

Equation (19) can be expanded to

$$\frac{u^2}{\lambda_1} + \frac{V^2}{\lambda_2} + \frac{W^2}{\lambda_3} = k^2. \quad (20)$$

Assume that each axis of the error ellipsoid is symbolized by a , b , and c ; hence,

$$\begin{aligned} a^2 &= k^2 \lambda_1, \\ b^2 &= k^2 \lambda_2, \\ c^2 &= k^2 \lambda_3. \end{aligned} \quad (21)$$

As mentioned before, the position of the satellite is an uncertainty position distribution within error ellipsoid interior. This distribution probability can be presented by

$$p = \iiint \frac{\exp(-0.5(U^2/\sigma_1^2 + V^2/\sigma_2^2 + W^2/\sigma_3^2))}{(2\pi)^{1.5} \sigma_1 \sigma_2 \sigma_3} dU dV dW, \quad (22)$$

giving a variable substitution, such as

$$\begin{aligned} u' &= \frac{u}{\sqrt{2}\sigma_1}, \\ v' &= \frac{v}{\sqrt{2}\sigma_2}, \\ w' &= \frac{w}{\sqrt{2}\sigma_3}. \end{aligned} \quad (23)$$

The probability in the ellipsoid is converted to the probability in the ball as follows:

$$p = \frac{1}{\pi^{1.5}} \int_0^{2\pi} d\theta \int_0^\pi d\varphi \int_0^{k/\sqrt{2}} \exp(-r^2) \sin \varphi dr = \frac{4}{\sqrt{\pi}} \int_0^{k/\sqrt{2}} e^{-r^2} r^2 dr. \quad (24)$$

In addition, considering that

$$e^{-r^2} = 1 - r^2 + \frac{r^4}{2!} - \frac{r^6}{3!} + \cdots + (-1)^{-1} \frac{r^{2n}}{n!} + \cdots, \quad (25)$$

$$r \in (-\infty, +\infty),$$

TABLE 3: Data of orbital elements.

Semimajor axis (km)	Eccentricity	True anomaly (°)	Inclination of orbit (°)	Argument of perigee (°)	Right ascension of ascending node (°)
10,424	0.3	0	45	45	20

TABLE 4: Data of axis length.

Major axis (km)	Middle axis (km)	Minor axis (km)
20.9535	14.6836	5.4130

TABLE 5: Data of rotation angle.

Rotation angle of the major axis around z-axis (°)	Rotation angle of the major axis around y-axis (°)	Rotation angle of the middle axis around x-axis (°)
55.262168	-29.999022	125.621076

(24) can be expanded to

$$p = \frac{4}{\sqrt{2\pi}} \left(\frac{k^3}{6} - \frac{k^5}{20} + \frac{k^7}{112} - \frac{k^9}{864} + \dots \right), \quad (26)$$

where the probability of the point distribution in the ellipsoid varies with the change of K as follows:

- (i) When $K = 1$, the probability of the point distribution in the error ellipsoid is 19.9%.

- (ii) When $K = 2$, the probability of the point distribution in the error ellipsoid is 73.9%.

- (iii) When $K = 3$, the probability of the point distribution in the error ellipsoid is 95%.

3. Preparation for Coverage Case

3.1. Calculation of the Error Ellipsoid. According to the measurement accuracy in actual engineering project, the error matrix of orbital elements is set as follows:

$$D_{xx} = \begin{pmatrix} 3^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.002^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & (0.05 \cdot H)^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & (0.05 \cdot H)^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & (0.05 \cdot H)^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & (0.05 \cdot H)^2 \end{pmatrix}, \quad (27)$$

where H is a conversion factor between angle and radian.

The orbital elements of a satellite are shown in Table 3. In addition, as a requirement of parameter transformation, the conversion relationship between eccentric anomaly E and true anomaly f is needed:

$$E = 2 \cdot a \tan \left(\frac{\tan(f/2)}{((1+e)/(1-e))^{0.5}} \right). \quad (28)$$

The covariance matrix of the satellite position D_{zz} is obtained as follows:

$$D_{zz} = \begin{pmatrix} 264.2010 & 99.6929 & 30.6575 \\ 99.6929 & 251.3009 & 168.8499 \\ 30.6575 & 168.8499 & 168.4551 \end{pmatrix}. \quad (29)$$

The feature vector v of D_{zz} is

$$v = \begin{pmatrix} 0.4935 & 0.8516 & 0.1766 \\ 0.7117 & -0.2786 & -0.6449 \\ 0.5000 & -0.4439 & 0.7436 \end{pmatrix}. \quad (30)$$

The data of axis length and rotation angle are shown in Tables 4 and 5, respectively. The standard position coordinate of the satellite position is (3600.6, 5193.1, 3648.4). The final simulation of the error ellipsoid with different views is shown in Figure 2.

3.2. Edge Extraction of Projection on Earth. The projection of error ellipsoid on Earth is a curved surface, so in order to obtain the edge points, a mapping on plane is more convenient. 421 edge points are obtained in the case by using convex hull algorithm, with different views of simulation results shown in Figure 3.

FIGURE 2: Space distribution of error ellipsoid.

FIGURE 3: Result of edge extraction.

FIGURE 4: Satellite orbit and point targets.

TABLE 6: Coordinate of the six point targets.

Group	Coordinate	Latitude and longitude
1	(3700.600, 5193.100, 3648.400)	54.526°E, 37.855°N
2	(3700.600, 5293.100, 3648.400)	55.041°E, 37.848°N
3	(3700.600, 5293.100, 3748.400)	55.041°E, 37.260°N
4	(3900.600, 5293.100, 3748.400)	53.612°E, 36.112°N
5	(3900.600, 5493.100, 3748.400)	54.621°E, 36.117°N
6	(3900.600, 5493.100, 3948.400)	54.621°E, 35.068°N

TABLE 7: Results of case 1.

Group	Geocentric angle without considering error (°)	Geocentric angle with considering error (°)
1	1.356686	1.580021
2	1.595898	1.713935
3	1.940826	2.012707
4	4.564908	4.720740
5	6.070766	6.134848
6	7.341740	7.391859

FIGURE 5: Simulation comparison of case results.

FIGURE 6: Changes of axial length.

FIGURE 7: Changes of axial ratio.

TABLE 8: Orbital element of each satellite.

Semi major axis (km)	Eccentricity	True anomaly (°)	Inclination of orbit (°)	Argument of perigee (°)	Right ascension of ascending node (°)
10,424	0.3	0	45	45	20
10,424	0.3	0	45	45	40
10,424	0.3	0	45	45	60

FIGURE 8: Positions of points A and B.

4. Numerical Case

Five cases with different configurations are designed and implemented in this paper. Firstly, the uncertainty influence of single satellite positioning on multiple point target coverage is given in case 1. Then, in order to deeply understand the shape change of “error ellipsoid” when the satellite moves to different positions, the changes of axial length and axial ratio are given in case 2. Lastly, the uncertainty influence of a constellation positioning on multiple point targets, constellation positioning on a single area target, and constellation positioning on two area targets are given in cases 3 to 5, respectively.

The reference system used in this paper is ECI. When the satellite is in the initial position perigee, the latitude and longitude of the satellite are 55.2°E and 30°N, respectively. The orbit and ground point targets are shown in Figure 4 with 3D and 2D views.

4.1. Influence of Single Satellite Position Uncertainty on Multiple Point Target Coverage. The purpose of case 1 is to explain the necessity of considering orbital element error in coverage calculation. Six point targets are set in this case, whose coordinates are presented in Table 6, obtaining the different minimum coverage requirements of these point targets in terms of geocentric angles considering and not considering positioning uncertainty, respectively. Case results listed in Table 7 show that the geocentric angle needs to be larger to cover all the aimed targets when considering error of orbital elements. The results shown in Figure 5 obviously present the relative percentage of needed geocentric angle when considering uncertainty.

TABLE 9: Minimum coverage radius of satellites.

Satellite	Point target	Minimum covering radius (km)
1	A	469.590510
2	A	1484.017928
2	B	460.448878
3	B	1459.206737

4.2. Shape Change of the “Error Ellipsoid.” The purpose of case 2 is to research on the shape change of the error ellipsoid. Analyze the axial length variation while true anomaly changes from 0° to 360°. Focus on the shape change by studying the variations of axial length and axial ratio, which are shown in Figures 6 and 7, respectively. Interesting conclusions can be drawn from the case results. The most significant one is that the 3D shape of error ellipsoid is a time-dependent function, and the error ellipsoid is rather slender when true anomaly changes around 110°. While anomaly reaches around 180°, the shape is stouter.

4.3. Uncertainty Influence of a Constellation Positioning on Multiple Point Targets. The implementation of case 3 is to study the coverage uncertainty of constellation containing 3 satellites whose orbital elements are shown in Table 8. Two point targets A (3159.6162, 5472.44908, and 3648.4) and B (1097.44509, 6222.5133, and 3648.4) shown in Figure 8 are set in this case. The latitude and longitude of these two points are (60°E, 30°N) and (80°E, 30°N), respectively. Case results in Table 9 show that when considering positioning uncertainty, the minimum required covering radius is larger than that of not considering uncertainty. If it is needed to cover both

TABLE 10: Coverage results of a circular area when the sensor angle is 7° .

Coverage times	Target number	Coverage times	Target number	Coverage times	Target number	Coverage times	Target number
31	2525	37	404	43	0	49	0
32	2780	38	193	44	0	50	0
33	1918	39	38	45	0	51	0
34	1398	40	0	46	0	52	0
35	1082	41	0	47	0	53	0
36	818	42	0	48	0	54	0

TABLE 11: Coverage results of a circular area when the sensor angle is 7.4° .

Coverage times	Target number	Coverage times	Target number	Coverage times	Target number	Coverage times	Target number
31	1810	37	2145	43	1392	49	821
32	5354	38	4878	44	2160	50	724
33	5371	39	3507	45	1268	51	516
34	3882	40	2590	46	1316	52	344
35	3745	41	2006	47	1331	53	215
36	4106	42	2102	48	1061	54	38,589

TABLE 12: Coverage results of a circular area when the sensor angle is 7.7° .

Coverage times	Target number	Coverage times	Target number	Coverage times	Target number	Coverage times	Target number
31	0	37	902	43	675	49	501
32	0	38	836	44	1185	50	469
33	0	39	1756	45	612	51	257
34	0	40	1441	46	1160	52	168
35	0	41	1179	47	775	53	99
36	198	42	1152	48	440	54	82,792

TABLE 13: Coverage results of a circular area when the sensor angle is 8° .

Coverage times	Target number	Coverage times	Target number	Coverage times	Target number	Coverage times	Target number
31	0	37	0	43	0	49	0
32	0	38	0	44	0	50	0
33	0	39	0	45	0	51	0
34	0	40	0	46	0	52	0
35	0	41	0	47	0	53	0
36	0	42	0	48	0	54	96,597

points A and B at the same time, the minimum radius must be 1484.017928 km.

4.4. Uncertainty Influence of Constellation Positioning on a Single Area Target. We have mentioned that 421 edge points are obtained in the case by using convex hull algorithm. In order to reduce the calculation expense, 54 edge points are chosen evenly to implement the coverage case containing 96,576 ground target points. Only when each of the 96,576 sampling points can be covered by the satellite, whose sub-satellite point is any one of the 54 edge points, the area is deemed covered completely. Assuming that the target is a circular

area, and the radius of this area is 100 km, compute the minimum sensor angle meeting the coverage requirements of this area when the angle of the satellite sensor changes from 7° . Case results are shown in Tables 10, 11, 12, and 13. In order to present this change significantly, relevant simulation results of these data are shown in Figures 9–12, respectively. As well, the percentage change of area coverage is also shown in Figure 13.

4.5. Uncertainty Influence of Constellation Positioning on Two Area Targets. Assuming that A and B are circular areas with circle radius = 100 km. The minimum coverage radius

FIGURE 9: Coverage results of a circular area when the sensor angle is 7°.

FIGURE 10: Coverage results of a circular area when the sensor angle is 7.4°.

FIGURE 11: Coverage results of a circular area when the sensor angle is 7.7°.

FIGURE 12: Coverage results of a circular area when the sensor angle is 8°.

FIGURE 13: Percentage change of area coverage.

should be calculated with respect to the requirement of the complete coverage from satellite 2. From case results shown in Table 14, we know that the minimum coverage radius of satellite 2 for coverage requirement is 1684.017928 km.

5. Results and Discussion

Above cases have shown the influence of orbital element errors on coverage calculation. It is obvious that the minimum angle requirement is larger when considering error of orbital elements. From case 1, it can be seen that the influence of orbital element error on coverage is more significant when the required geocentric angle is smaller. As the required geocentric angle increases, the impact of orbital element error on coverage will be smaller and smaller. From case 2, an interesting phenomenon can be seen that when true anomaly is about 180°, and it is a turning point for the changes of axial length and axial ratio. Cases 3–5 also show the analysis results of different coverage problems based on the Monte Carlo simulation for area targets. These cases show that

TABLE 14: Minimum coverage radius for each target.

Satellite	Point target	Minimum covering radius (km)
1	A	669.590510
2	A	1684.017928
2	B	660.448878
3	B	1659.206737

under the case settings in this paper, the extra requirements of sensor angle grows by about 14.3%. The analysis implemented in this paper is based on the fixed positions of satellite or constellation. Hence, in future research, this work can be extended to analyze the real-time coverage error related to positioning uncertainty.

6. Conclusions

In this paper, we present a method of accurately calculating the constellation coverage when considering position uncertainty. The position uncertainty of satellite is shown by an “error ellipsoid,” whose spatial shape is also analyzed in the paper. For the illustration of proposed method, coverage calculations for a single satellite and constellation are both considered to carry out the coverage cases. The cases show that when considering the position uncertainty related to influence of orbital error, in order to surely cover the target points, the covering radius needs to be larger than not considering position uncertainty. The work presented in this paper also has important reference value for the reliability analysis of constellation design, which can effectively deal with the influence of position uncertainty.

Conflicts of Interest

The authors declare that there is no conflict of interests regarding the publication of this paper.

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